

**Sandia  
National  
Laboratories**

# Deep Learning for Turbulence Modeling

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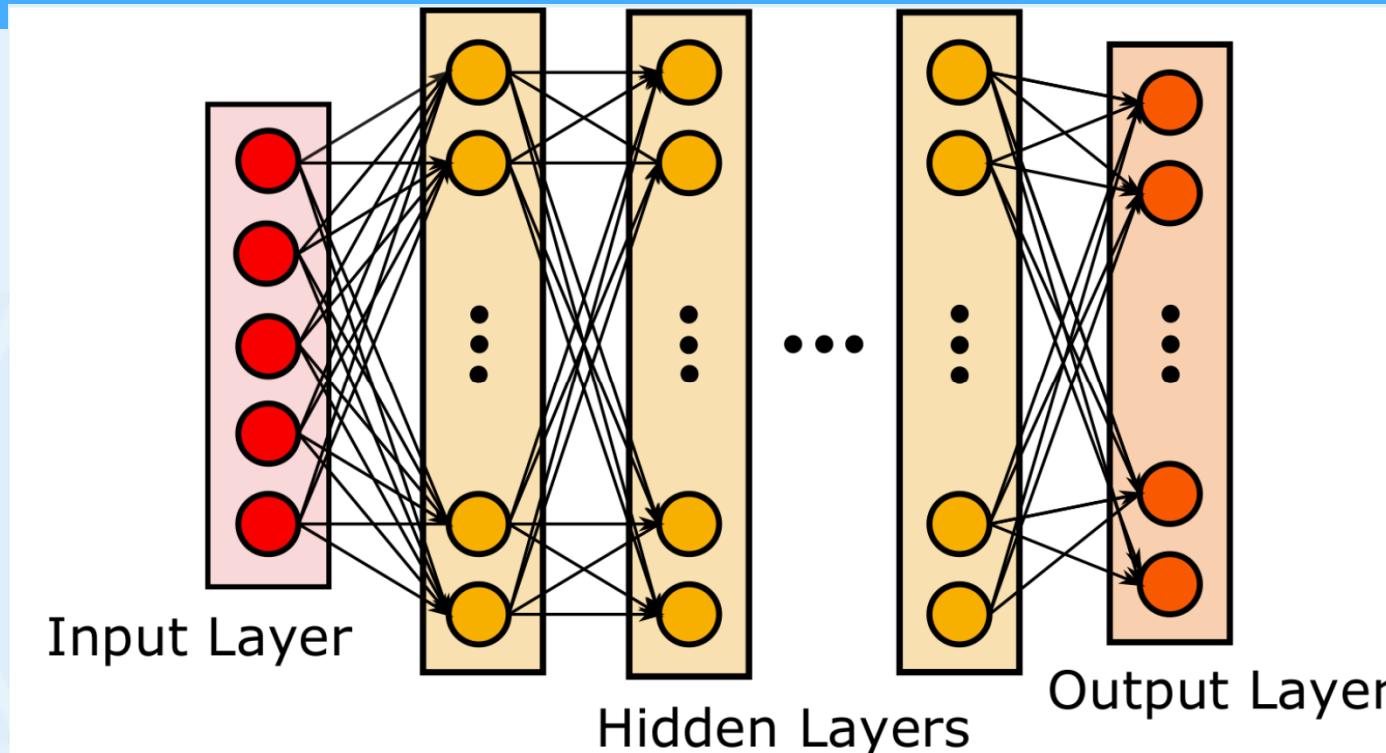
# What is Machine Learning?

- Data-driven algorithms to discern patterns and make predictions on big, high-dimensional data
- Linear regression, support vector machines, neural networks

The collage illustrates various applications of machine learning:

- Top Left:** A screenshot of a recommendation system titled "Inspired by your Wish List". It shows book covers for "THE INNER LIVES of MARKETS" by Ray Fisman and Tim Sullivan, "Who Gets What—and Why" by Alvin E. Roth, "PHISHING FOR PHOOLS" by George A. Akerlof and Robert J. Shiller, "THE SEVENTH SENSE" by Joshua Cooper Ramo, "OUT STEALING HORSES" by Karl Ove Knausgård, and "THE TEARDROP RIOT" by Theda Skocpol and Vanessa Williamson.
- Bottom Left:** A screenshot of a news aggregator. It shows a list of most emailed and most viewed articles. The most emailed articles are: 1. Julian Assange Repeat to U.S., 2. WNBA Players in Turmoil Rise in Terror, 3. Alec Baldwin to host 'Saturday Night Live' on Feb. 11, and 4. Mark Zuckerberg, in Suit, Testifies in Oculus Intellectual Property Trial.
- Middle Left:** An iPhone displaying the "amazonPrime" app. The screen shows an "Original audio series" with a thumbnail of a person speaking.
- Bottom Right:** A screenshot of the Pandora music streaming service. The "Now Playing" station is "We All Love Ennio Morricone" by Yo-Yo Ma. The station list on the left includes: Shuffle, Thumprint Radio, K-Pop Radio, Naturally 7 Radio, Moby Radio, Israel 'IZ' Kamakawiwi..., Robert Johnson Radio, I Heard It Through Th..., Bliss N Eso Radio, Francis Cabrel Radio, Hip hop, Electropop, and Classical (which is selected).

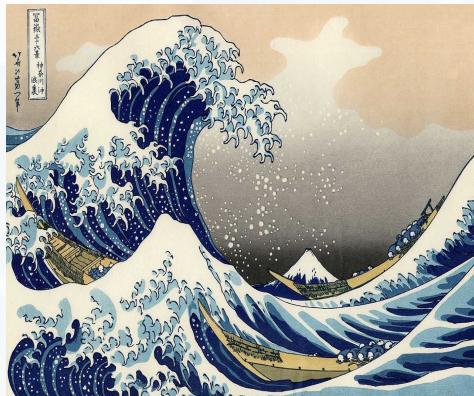
# Neural Networks and Deep Learning



$$y = f(w^T x)$$

# Turbulence Simulations

- Many physical processes are inherently multi-scale and require constitutive models
- Growing interest in applying machine learning to constitutive modeling
- Leverage the massive data sets from high fidelity simulations and high-res experiments
- We present a method for using deep neural networks to learn a turbulence model



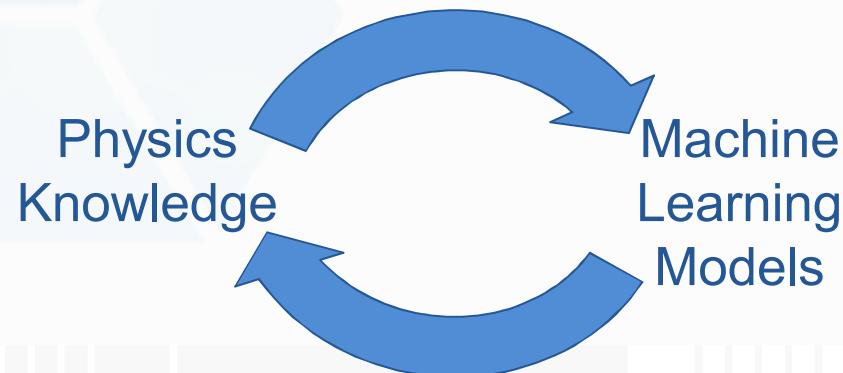
Hokusai (c 1830)



<http://www.windturbinesyndrome.com/2011/wind-turbine-turbulence-what-are-the-micro-climate-effects/>



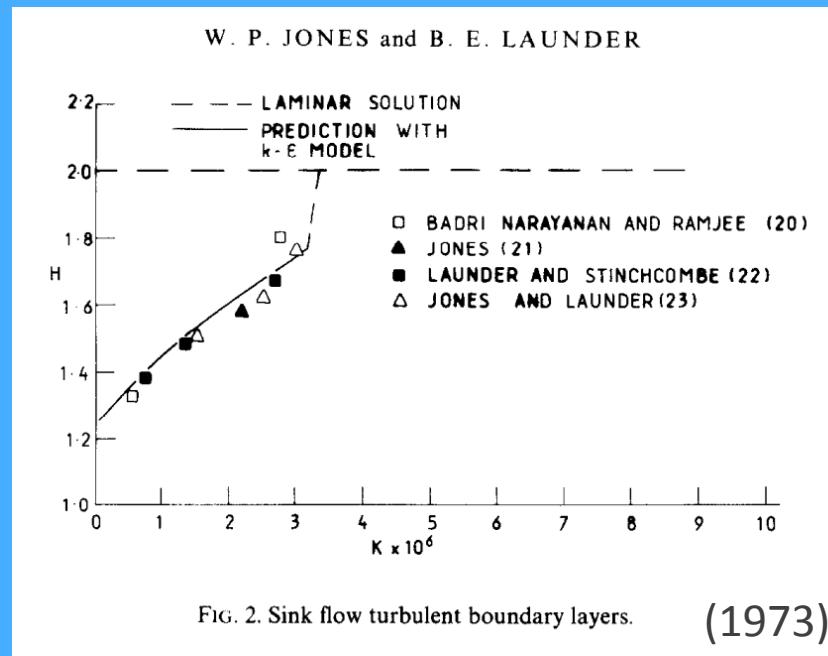
<https://brilliant.org/wiki/rocket-physics/>



# Deep Learning for Turbulence Modeling



- In Reynolds Averaged Navier Stokes (RANS), use simplifying assumptions to get computational efficiency
  - Need model for unknown term: the Reynolds stress anisotropy tensor  $\mathbf{A}$
- Default model: Linear Eddy Viscosity Model
  - Based on theory + sparse experimental data
- Our approach: Deep neural network
- Inputs: Mean strain rate tensor  $\mathbf{S}$ , mean rotation rate tensor  $\mathbf{R}$
- Outputs: Reynolds stress anisotropy  $\mathbf{A}$



# Deep Learning for Turbulence Modeling



- Inputs: Tensors  $\mathbf{S}$ ,  $\mathbf{R}$
- Output: Tensor  $\mathbf{A}$
- Would like to enforce Galilean invariance
  - Invariance to inertial coordinate frame transformations

$$\mathbf{A}(\mathbf{Q}\mathbf{S}\mathbf{Q}^T, \mathbf{Q}\mathbf{R}\mathbf{Q}^T) = \mathbf{Q}\mathbf{A}(\mathbf{S}, \mathbf{R})\mathbf{Q}^T$$

- Borrow some ideas from group theory, representation theory
- All Galilean invariant tensors that are a function of  $\mathbf{S}$  and  $\mathbf{R}$  lie on a tensor basis: the *integrity basis* of  $\mathbf{S}$  and  $\mathbf{R}$  for the orthogonal group

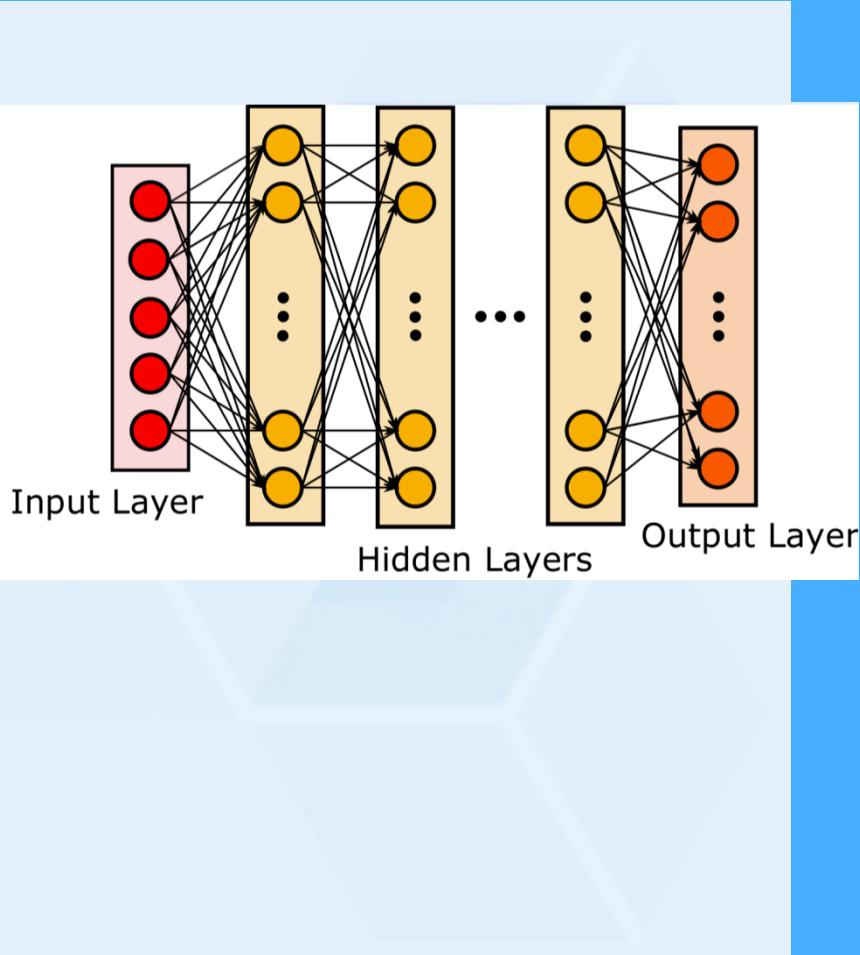
$$\mathbf{A} = \sum_{n=1}^{10} f^{(n)} \mathbf{B}^{(n)}$$

Unknown coefficients      Known Tensor Basis

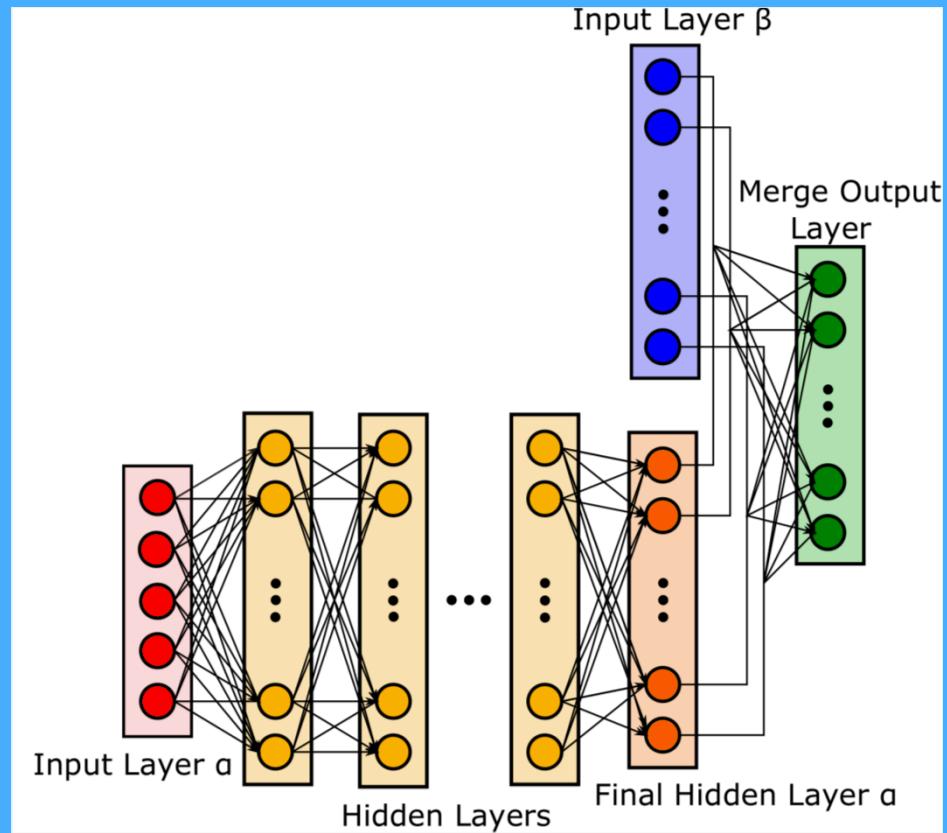
# Embedding Galilean Invariance



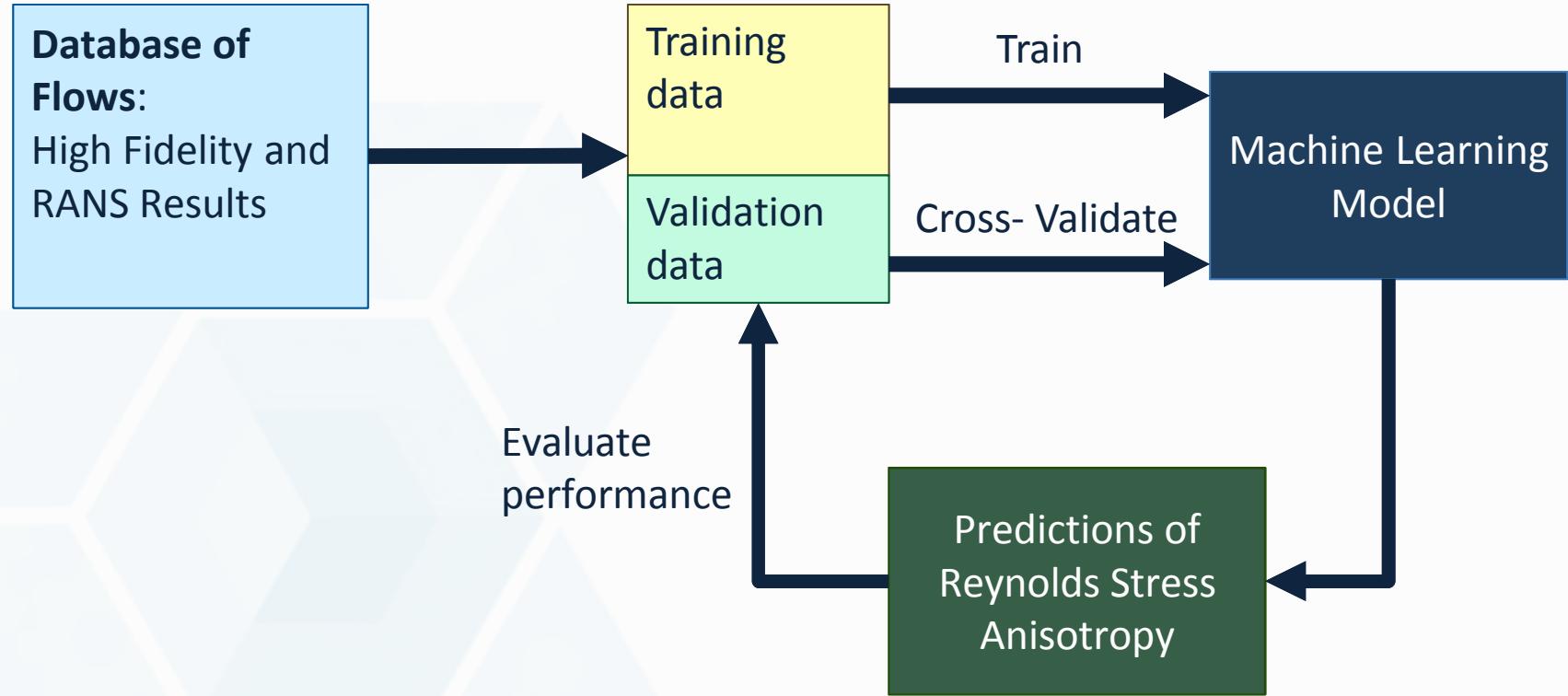
## Multi-Layer Perceptron (MLP)



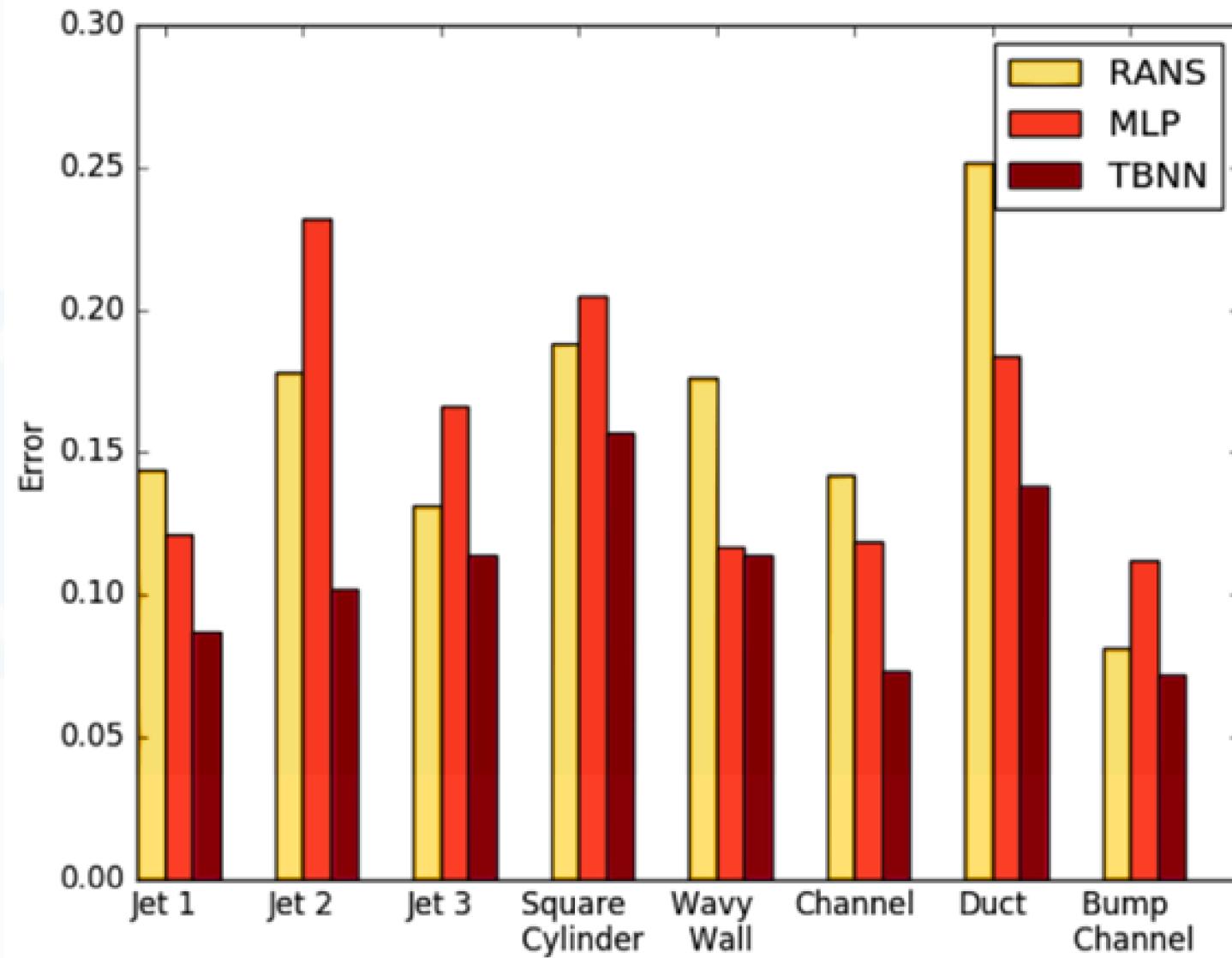
## Tensor Basis Neural Network (TBNN)



# Model Development



# Deep Learning for Turbulence Modeling

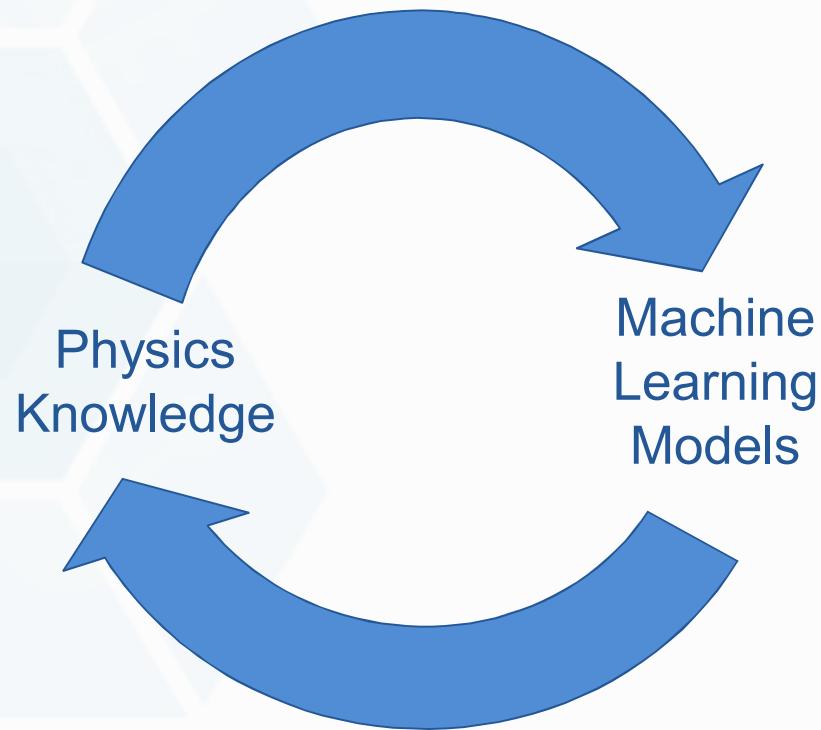


# Conclusions

- Developed network architecture to embed known tensor invariance property
- Demonstrated significant improvement over conventional eddy viscosity models, at orders of magnitude lower computational cost than DNS
- First application of deep learning to RANS turbulence modeling

# Machine Learning on Physics Systems

Directly embedding scientific domain knowledge into machine learning models can give improved performance, especially in data-limited scenarios



# References

- J. Ling, A. Kurzawski, and J. Templeton, “Reynolds Averaged Turbulence Modeling using Deep Neural Networks with Embedded Invariance,” *Journal of Fluid Mechanics*, (2016).
- J. Ling and J. Templeton, “Evaluation of machine learning algorithms for prediction of regions of high Reynolds averaged Navier Stokes uncertainty,” *Physics of Fluids*, (2015).
- J. Ling, A. Ruiz, G. Lacaze, and J. Oefelein, “Uncertainty analysis and data-driven model advances for a Jet-in-Crossflow,” *Journal of Turbomachinery*, (2016).
- J. Ling, R. Jones, and J. Templeton, “Machine Learning Strategies for Systems with Invariance Properties,” *J. Comp. Phys.*, (2016).

# Questions?

