



Exploring model-form uncertainties in Large Eddy Simulation (LES)

*Exceptional
service
in the
national
interest*

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1541, Comp. Thermal and Fluids Mechanics
September 6th, 2016

With contributions from:
Professor G. Iaccarino and Dr L. Jofre, Stanford University



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Outline

Abnormal/Thermal Environment; Turbulence Basics

Methodology for Validation (Step 1:n)

Step 1: Definition of the Validation Study

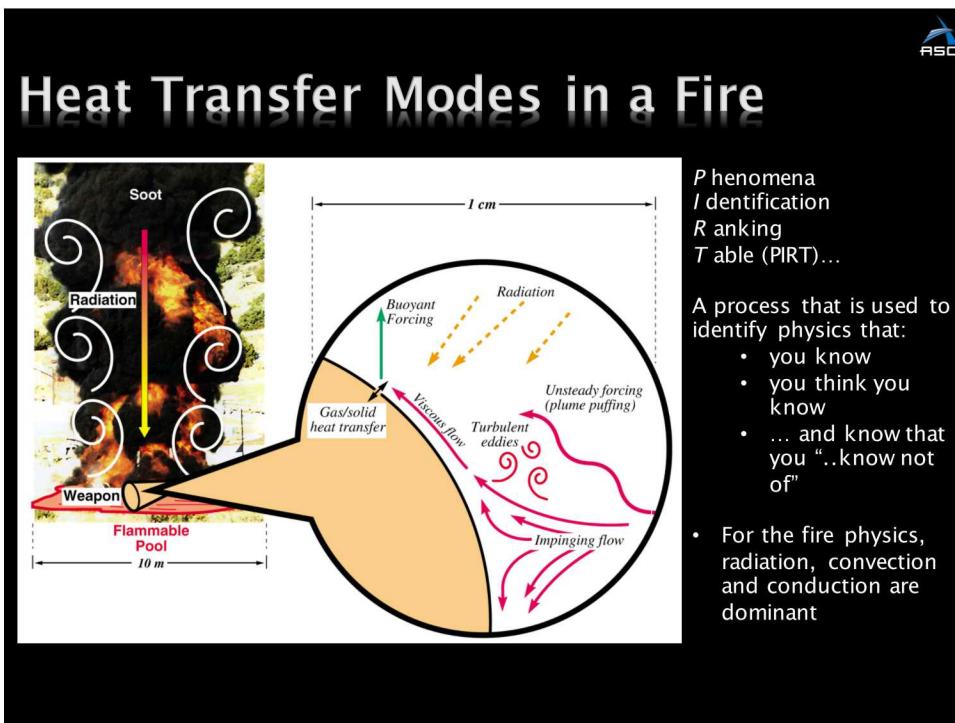
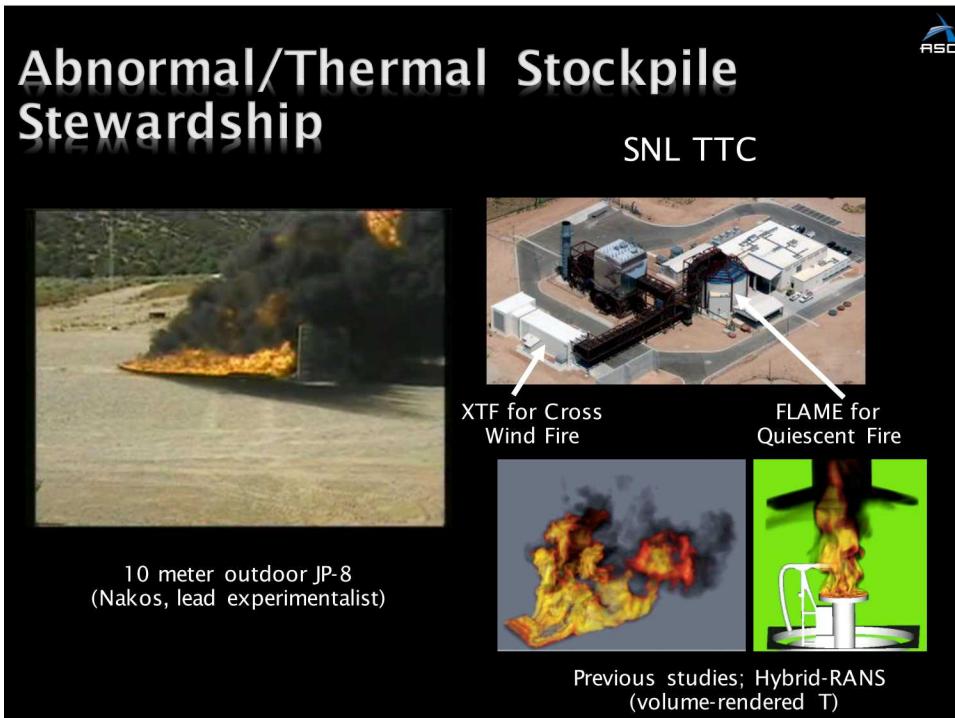
Step 2: Code Verification

Step 3: Solution Verification

Step 4: Advanced LES Model-form Sensitivity

What about *Physics-form* Error?

Future Efforts





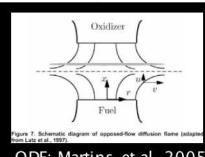
Evolution of a Mindset... *Quiescence*

"I PURPOSE, in return for the honor you do us by coming to see what are our proceedings here, to bring before you, in the course of these lectures, the Chemical History of a Candle"

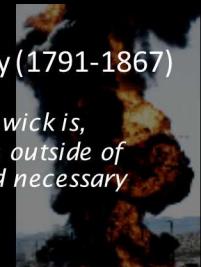
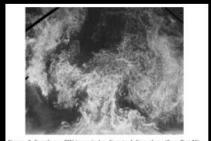
The Chemical History of a Candle, Michael Faraday (1791-1867)



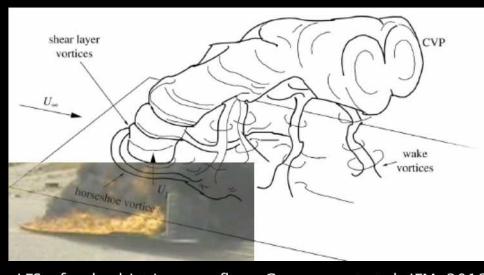
"In the middle of the flame, where the wick is, there is this combustible vapor; on the outside of the flame is the air which we shall find necessary for the burning of the candle"



ODF; Martins et al., 2005



Evolution of a Mindset... *Cross Flow*



Conclusion: The inclusion of a cross-flow wind profile couples vorticity of the pool and streamwise momentum which drives the formation of column vortices, increases the importance of mixing and, therefore, convective loads on the object become more important



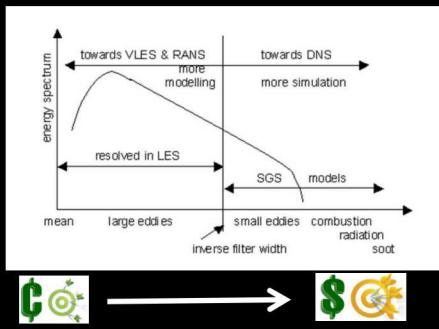
Fire whirls from a 3-meter diameter pool in the Fire Laboratory for Accreditation of Modeling by Experiment, or FLAME, facility at Sandia National Laboratories. (Photo by Richard Simpson; A. Hanlin, lead experimentalist)



Change in mindset: Invest in Validation cases highlight the importance of convection physics

Resolved Scale is User Defined

Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and Reynolds-Averaged Navier-Stokes (RANS) approaches define the computationally required time- and length scales



<https://sites.google.com/site/smokeisnojoke/lesrans2.jpg>



Direct Numerical Simulation:

- Simulation captures all relevant length- and time-scales
- No turbulence modeling
- Very expensive; total cost $\sim Re^3$

Large Eddy Simulation

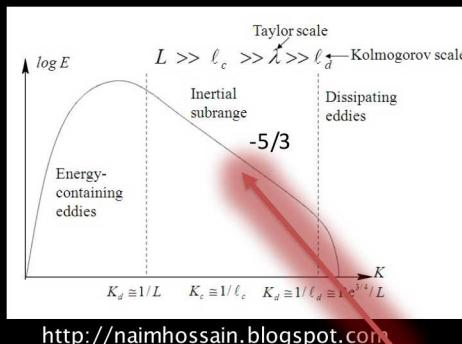
- Resolve the large-scale motion that contains most of the flow's energy
- Model small scale based on scale- similarity
- Model = $f(D)$

Reynolds-Averaged Navier-Stokes

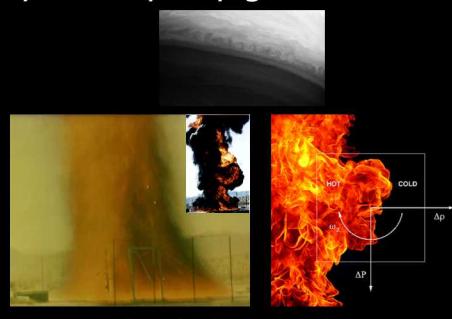
- Model the turbulence spectrum
- Empirical in nature

Towards LES Subgrid Models (PEM¹)

The standard turbulence decay (at least at the affordable LES scale) for momentum driven flows (shown below) does not apply to buoyancy generated turbulent flows



<http://naimhossain.blogspot.com>



Rayleigh/Taylor and Kelvin/Helmholtz instabilities

Are we ever really this resolved in engineering LES?

1. Development of subgrid LES models that include the effect of buoyancy; PL, Domino (P&EM)



Filtered LES low-Mach Equation Set

$$\begin{aligned}
 \int \frac{\partial \bar{\rho}}{\partial t} dV + \int \bar{\rho} \tilde{u}_j n_j dS &= 0 & \text{Given filter: } \overline{\phi(\mathbf{x}, t)} \equiv \int_{-\infty}^{+\infty} \phi(\mathbf{x}', t) G(\mathbf{x}' - \mathbf{x}) d\mathbf{x}' \\
 \int \frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} dV + \int \bar{\rho} \tilde{u}_i \tilde{u}_j n_j dS &= \int \bar{\sigma}_{ij} n_j dS - \int \tau_{ij}^{sgs} n_j dS + \int (\bar{\rho} - \rho_\circ) g_i dV \\
 \int \frac{\partial \bar{\rho} \tilde{h}}{\partial t} dV + \int \bar{\rho} \tilde{h} \tilde{u}_j n_j dS &= - \int \bar{q}_j n_j dS - \int \tau_{h,j}^{sgs} n_j dS - \int \frac{\partial \bar{q}_i^r}{\partial x_i} dV \\
 &\quad + \int \left(\frac{\partial \bar{P}}{\partial t} + \tilde{u}_j \frac{\partial \bar{P}}{\partial x_j} \right) dV + \int \tau_{ij} \frac{\partial u_i}{\partial x_j} dV
 \end{aligned}$$

Subgrid Stress (SGS): $\tau_{ij}^{sgs} \equiv \bar{\rho}(\tilde{u}_i \tilde{u}_j - \bar{u}_i \bar{u}_j)$ Closure: $\tau_{ij}^{sgs} - \frac{1}{3} \delta_{ij} \tau_{kk}^{sgs} = -2\mu^t S_{ij}^*$
 $\tau_{h,j}^{sgs} \equiv \bar{\rho}(\tilde{h} \tilde{u}_j - \bar{h} \bar{u}_j)$
 Smagorinsky: $\mu_t = \rho (C_s \Delta)^2 |\tilde{S}|$
 $\int \frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} dV + \int \bar{\rho} \tilde{u}_i \tilde{u}_j n_j dS + \int \left(\bar{P} + \frac{2}{3} \bar{\rho} k \right) n_i dS =$
 $\int 2(\mu + \mu_t) \left(\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{kk} \delta_{ij} \right) n_j dS + \int (\bar{\rho} - \rho_\circ) g_i dV$
 Ksgs: $\mu_t = C_{\mu_\epsilon} \Delta k^{sgs \frac{1}{2}}$
 WALE: $\mu_t = \rho (C_w \Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(S_{ij} S_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}}$
 $\mu_t = \rho (C_s \Delta)^2 |\tilde{S}|$



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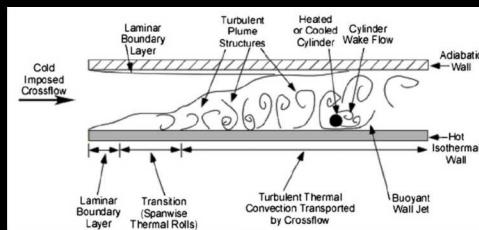
Step 4: Advanced LES Model-form Sensitivity

What about *Physics-form* Error?

Future Efforts

Definition of Validation Case

Model Configuration: SNL-based Sean Kearney Experiment, "Experimental investigation of a cylinder in turbulent convection with an imposed shear flow", AIAA, 2005



Kearney experimental configuration, 2005

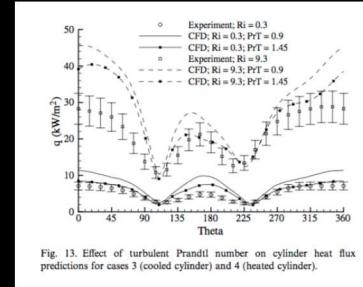


Fig. 13. Effect of turbulent Prandtl number on cylinder heat flux predictions for cases 3 (cooled cylinder) and 4 (heated cylinder).

Laskowski et al., 2007

RANS Conclusion: The presence of the heated bottom wall significantly challenged ability to predict the QoI; q''

RANS-based simulation (v2-f, k-e) study conducted by Laskowski et al., AIAA 2007

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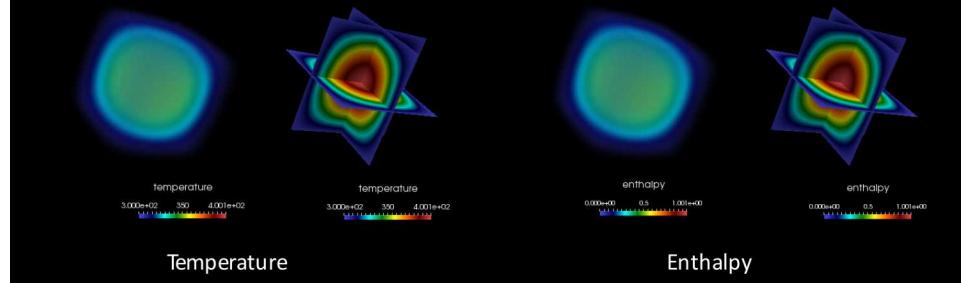
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Variable Density MMS, non-Isothermal

Density is a function of static enthalpy transport via standard ideal gas $\rho = f(P, M, R, T)$

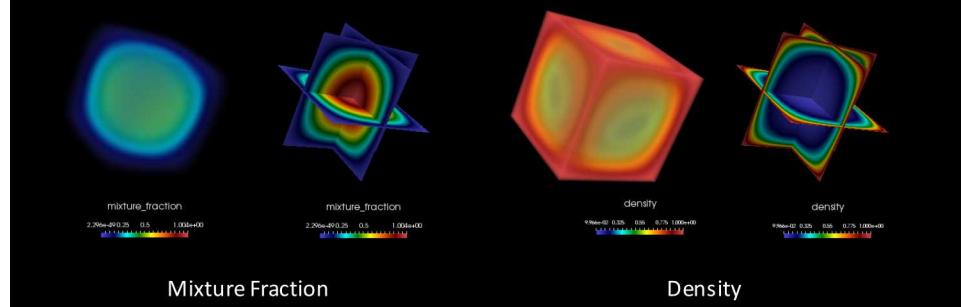
Temperature range ~maps to Kearney (400-500 K); g_i is arbitrary and misaligned



Variable Density MMS, non-Uniform

Density is a function of species transport via standard mixture fraction weighting $\rho = f(Z)$

Density ratio maps to a helium plume, e.g., $\sim 10x$; g_i is arbitrary and misaligned



CVFEM Discretization

The core discretization used in the NW low-Mach code base has been the Control Volume Finite Element Method, CVFEM

An elemental basis is defined from which interpolation and gradients within the element are determined

The test function is defined to be piece-wise constant

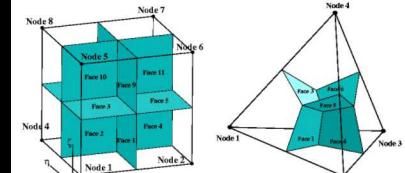
This method can best be described as a Petrov-Galerkin method

The canonical 27-point stencil is recovered

$$\int w \frac{\partial \bar{\rho} \tilde{u}_j \tilde{\phi}}{\partial x_j} d\Omega = - \int \bar{\rho} \tilde{u}_j \tilde{\phi} \frac{\partial w}{\partial x_j} d\Omega + \int w \bar{\rho} \tilde{u}_j \phi n_j d\Gamma$$

$$w = w_i; \frac{\partial w_i}{\partial x_j} = -\delta(x - x_{\text{scs}})$$

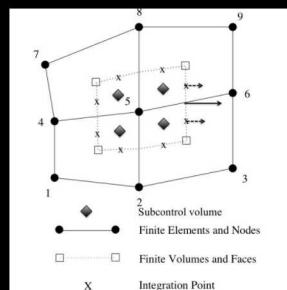
$$\int w \frac{\partial \bar{\rho} \tilde{u}_j \tilde{\phi}}{\partial x_j} d\Omega = \sum_{ip} (\bar{\rho} \tilde{u}_j)_{ip} \tilde{\phi}_{ip} n_j dS = \sum_{ip} m_{ip} \tilde{\phi}_{ip}$$



Hexahedral Dual Mesh (L)
Tetrahedral Dual Mesh (R)

Edge-based Vertex Centered Discretization

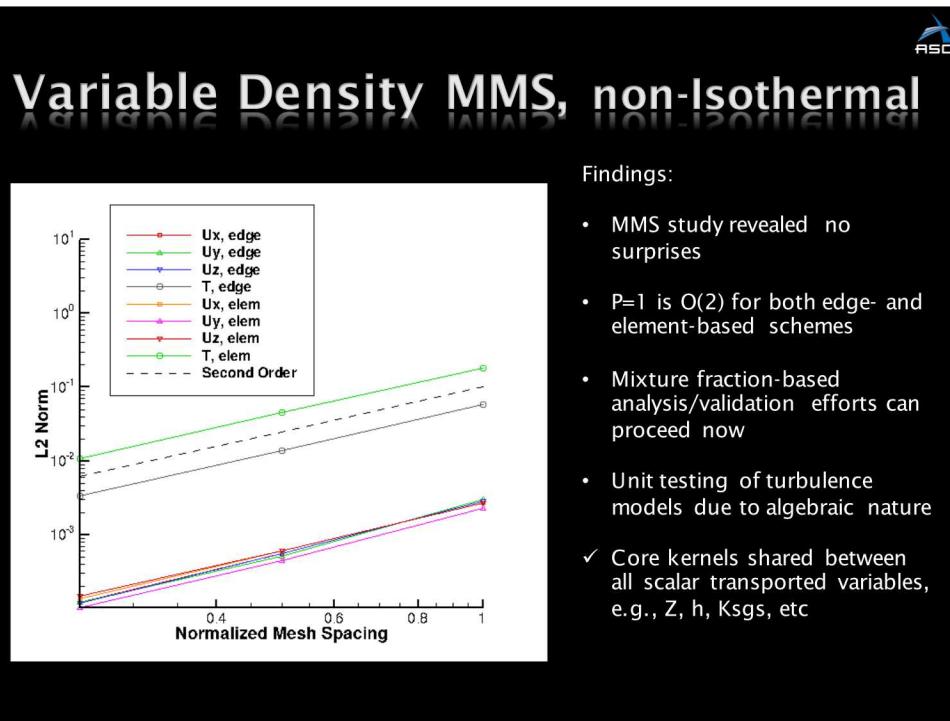
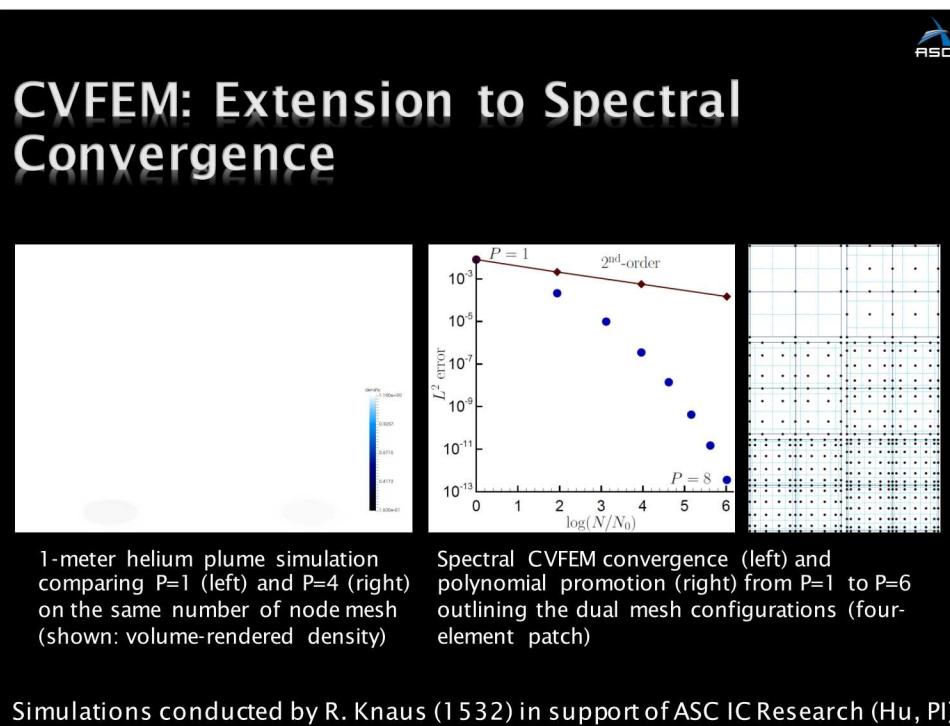
In this method, the dual mesh is defined to establish geometric values at the edge midpoint (area vector) and node (volume)

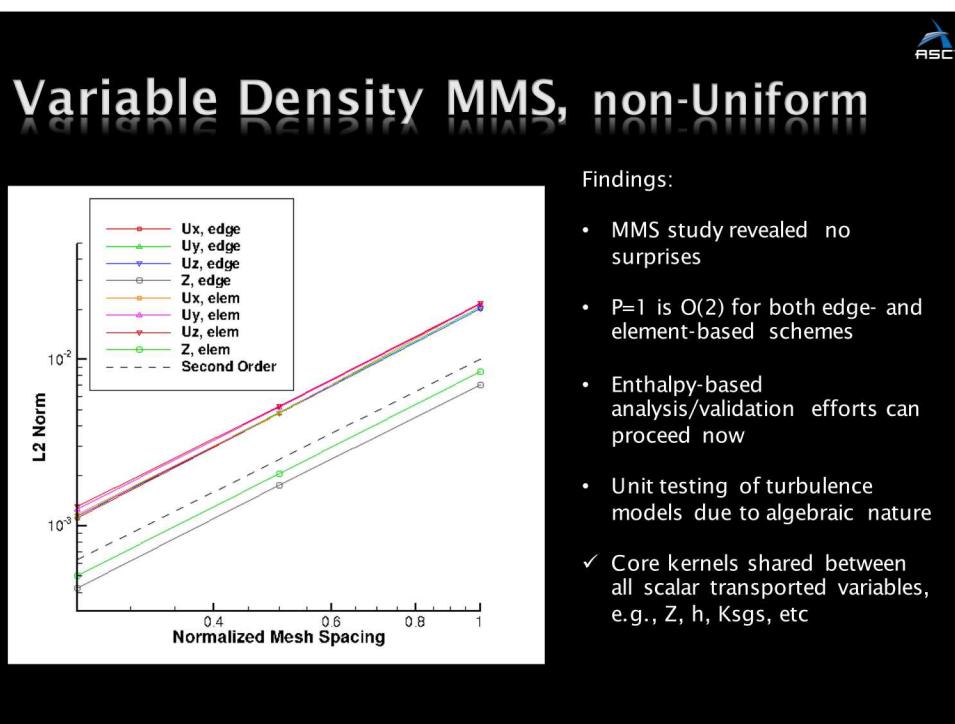


Quadrature points for edge-based scheme

Ramifications for the edge-based finite volume (EBFV) structure are as follows:

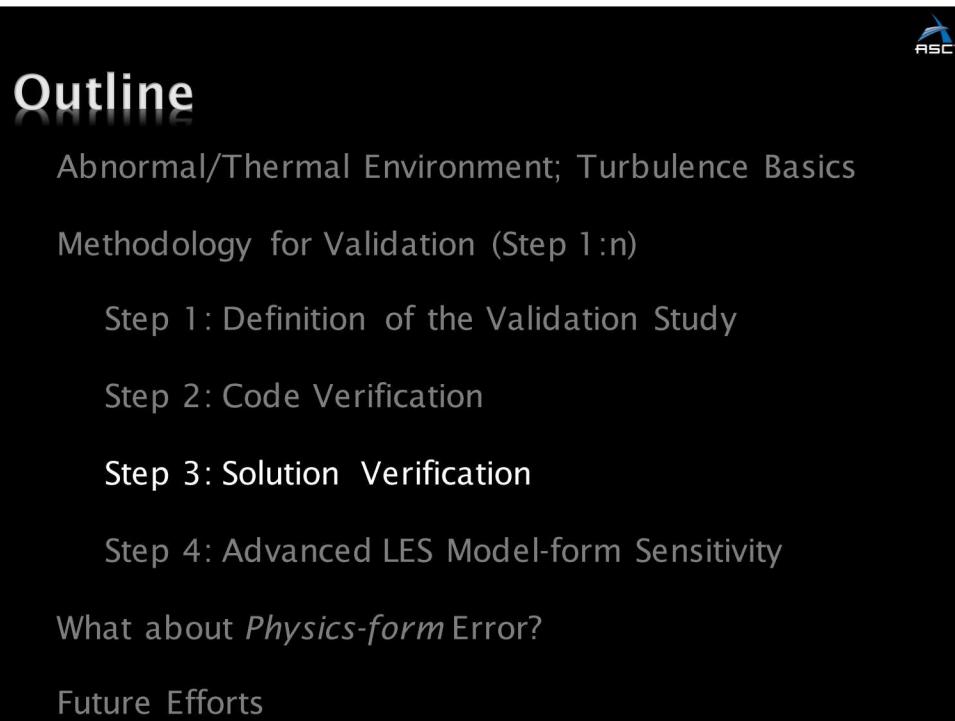
- Reduced stencil (27-point to 7-point for structured hex)
- Simple L/R data structure allows for simple interpolation and orthogonal gradient contributions
- Lack of elemental basis requires a diffusion operator in terms of orthogonal to the edge and non-orthogonal correction that requires projected nodal gradients
- 2-4x faster than CVFEM or GFEM
- Limited to second order in-space





Findings:

- MMS study revealed no surprises
- $P=1$ is $O(2)$ for both edge- and element-based schemes
- Enthalpy-based analysis/validation efforts can proceed now
- Unit testing of turbulence models due to algebraic nature
- ✓ Core kernels shared between all scalar transported variables, e.g., Z , h , $Ksgs$, etc



Testing Matrix

Three numerical methods:

Low order edge- element-based ($P=1$); element-based ($P=2$); with code verification

Three core LES models

Three mesh resolutions (R0, R1 and R2)

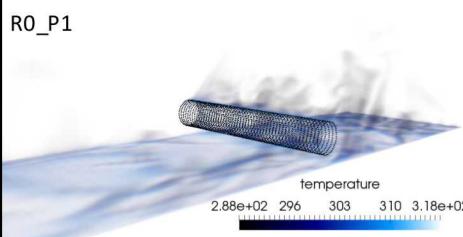
$O(10)$ simulations run to establish model-form error

$O(10)$ simulations run to establish physics-form error

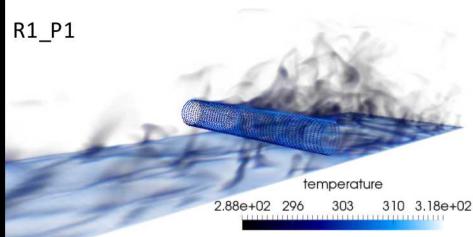
In the end, ~ 2.5 million CPU hours, however, not capability-based computing (2048 max core)

Thermal Plume Structure as a Function of R- and P- Refinement

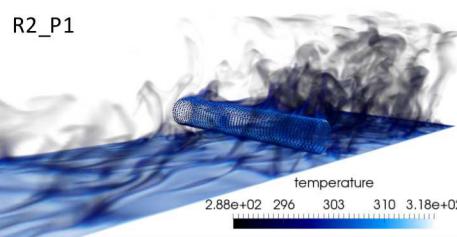
R0_P1



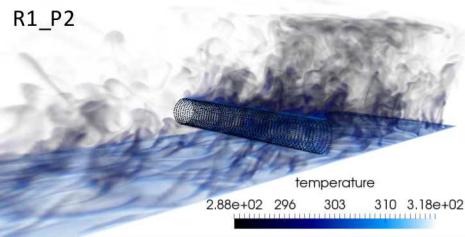
R1_P1



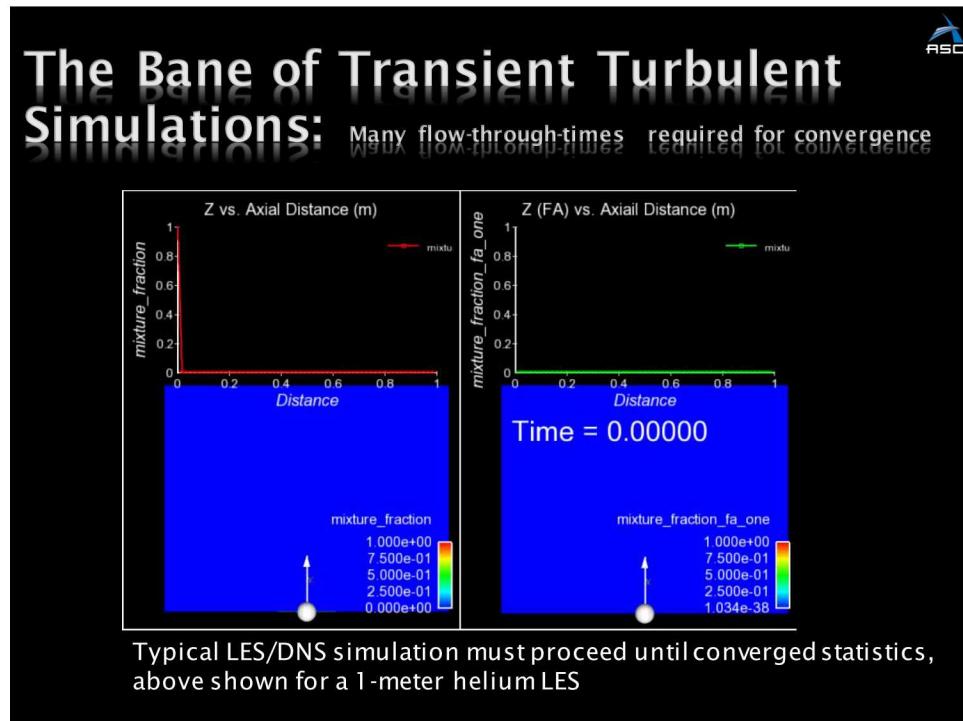
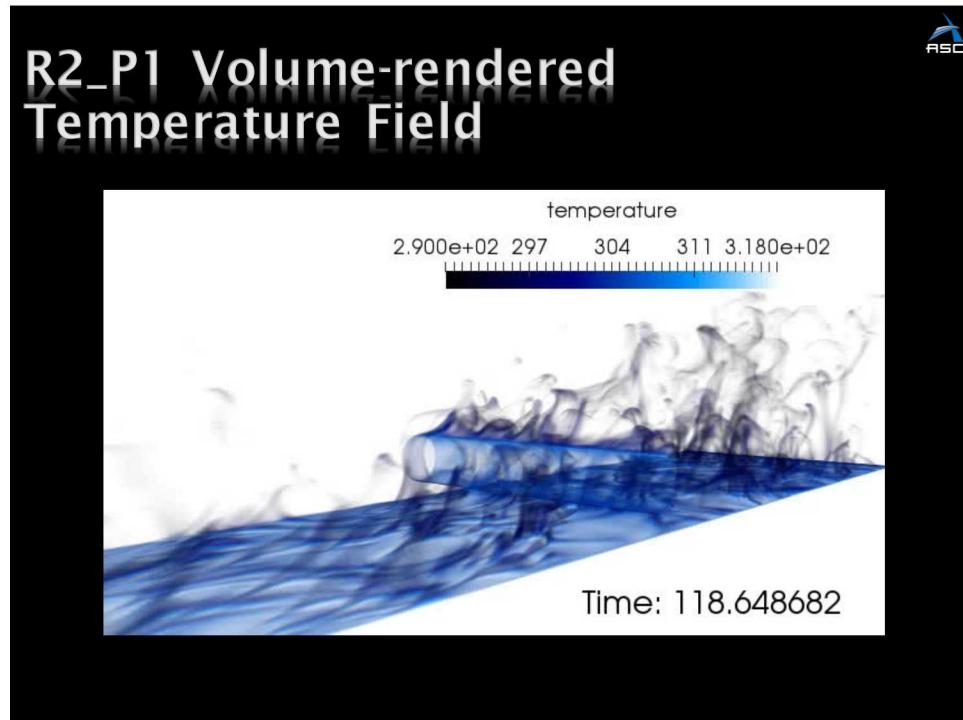
R2_P1

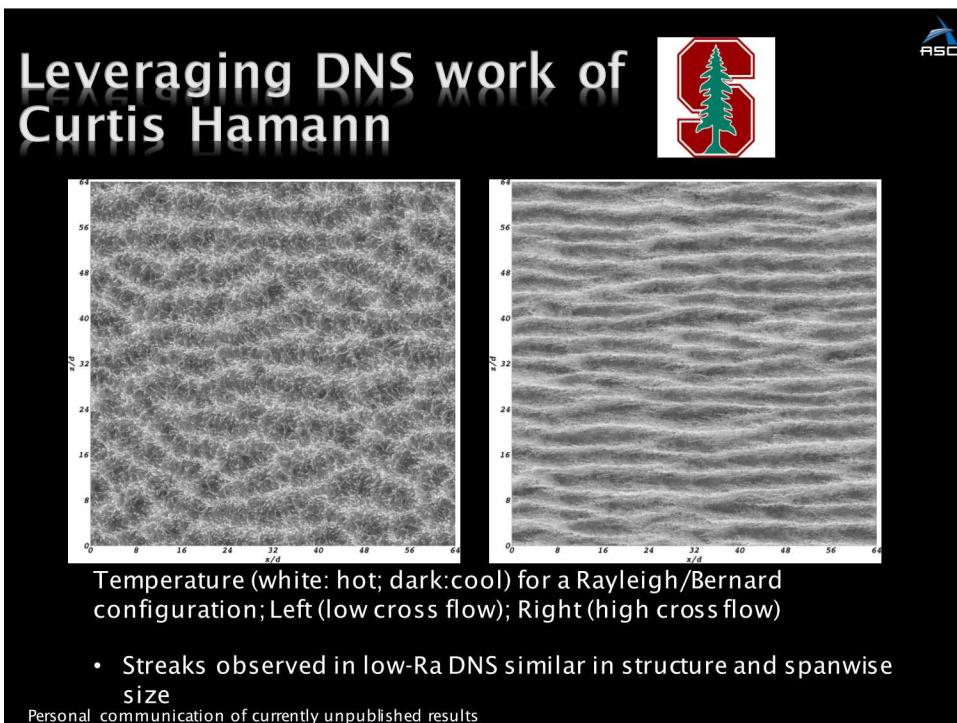
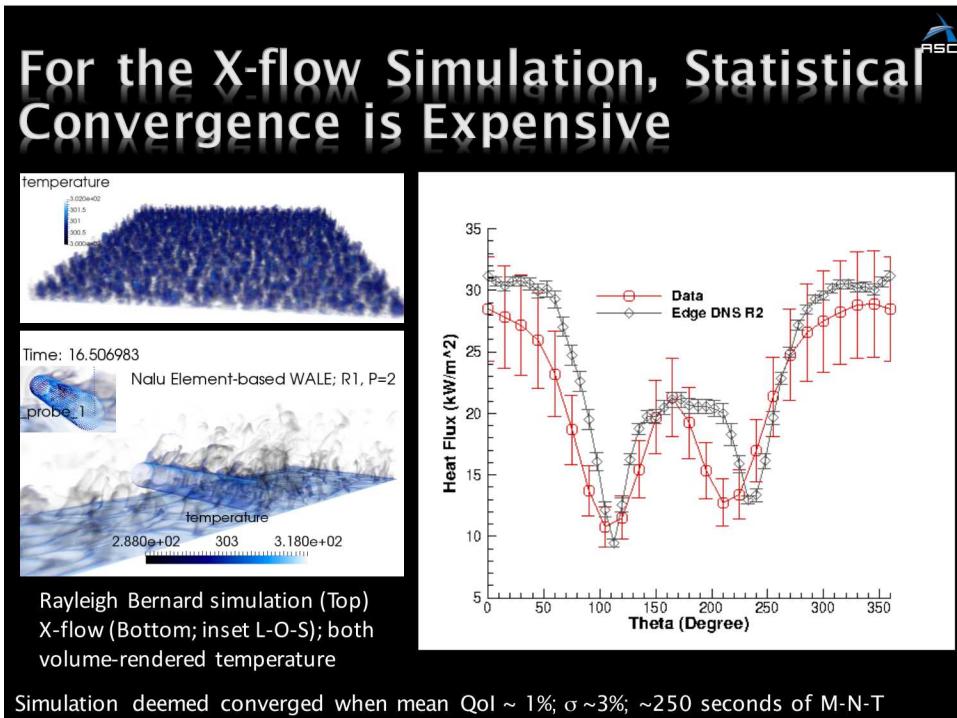


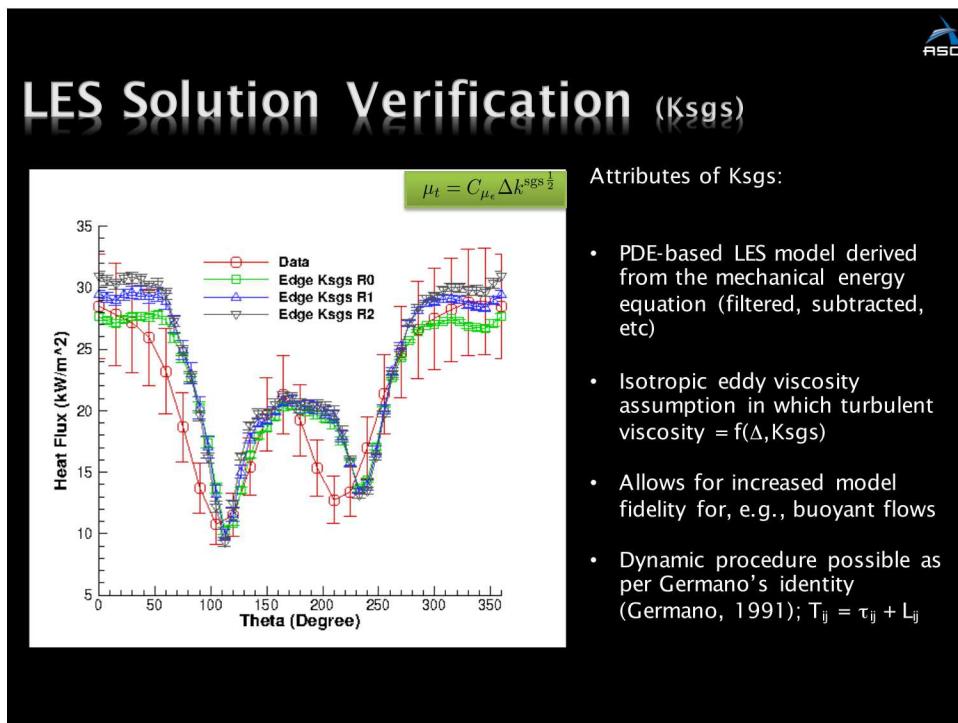
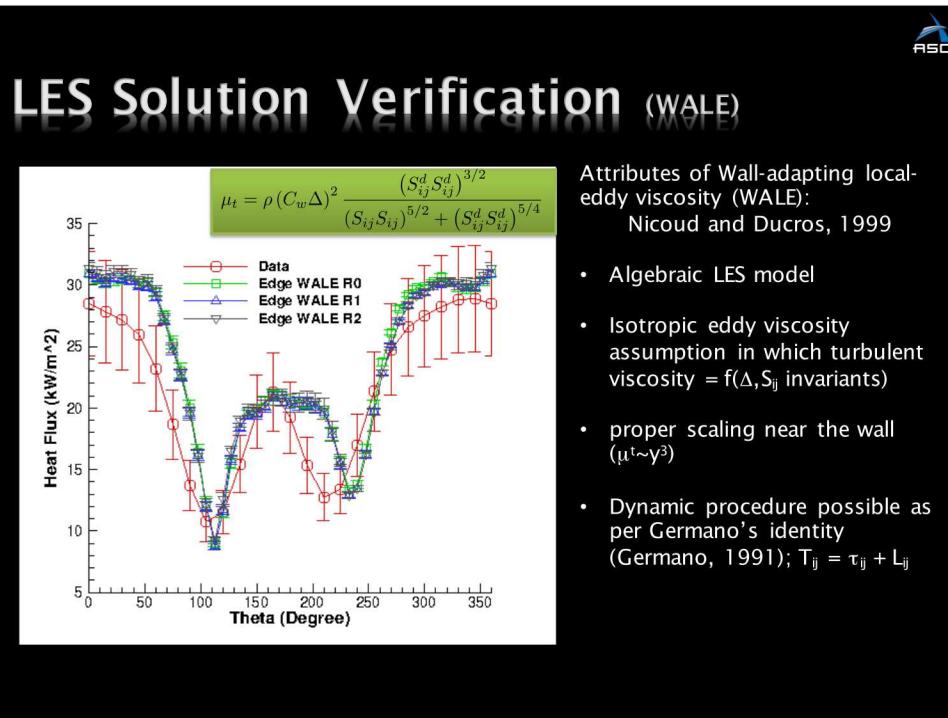
R1_P2

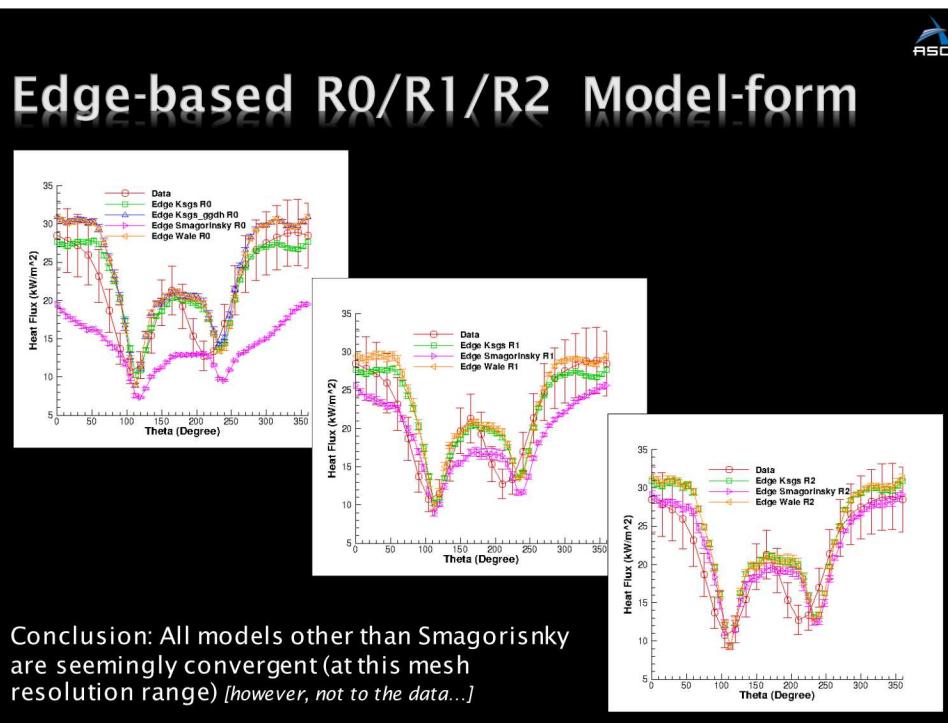
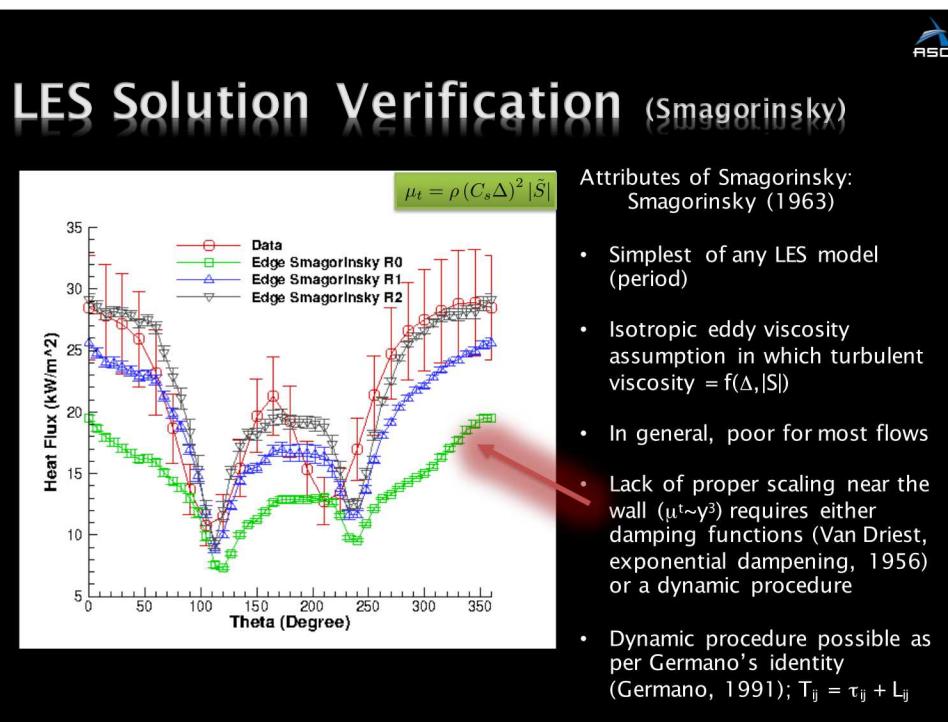


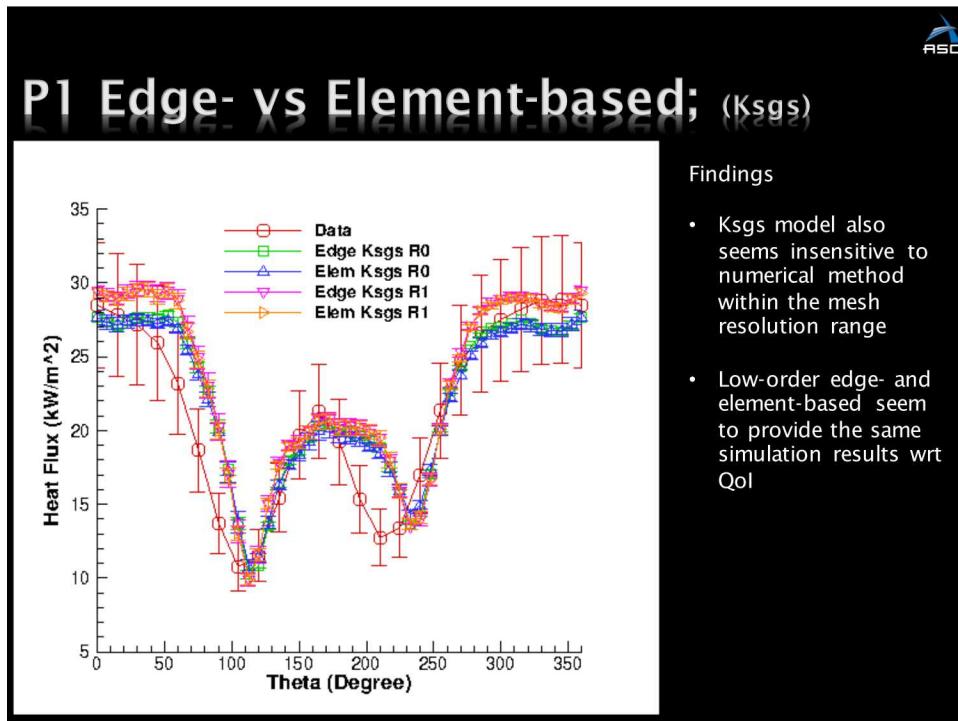
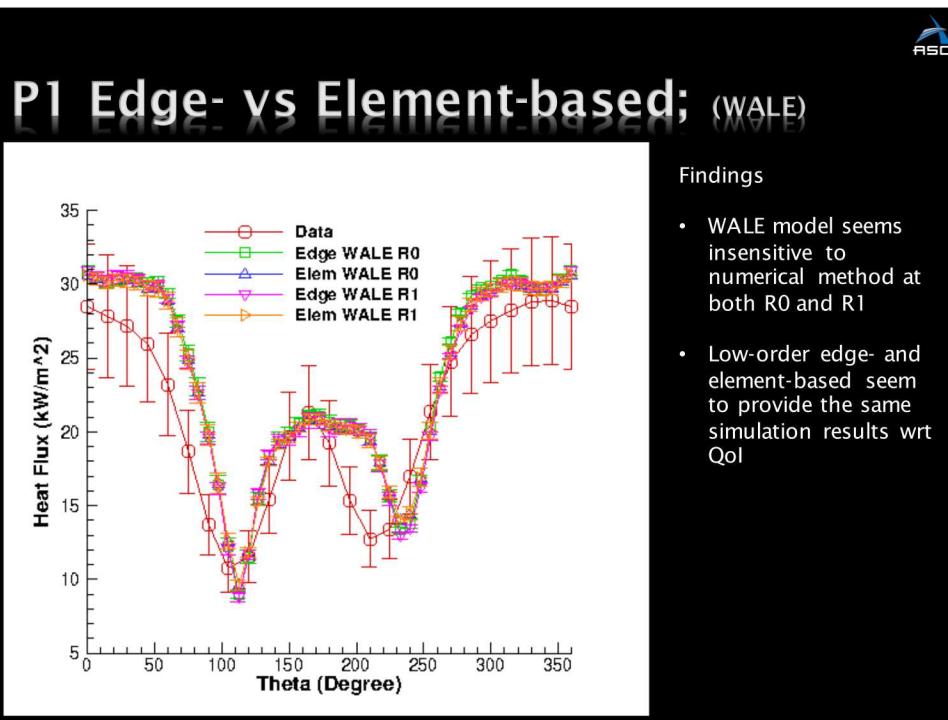
Note: Increased resolution yields finer plume structure; higher order ++

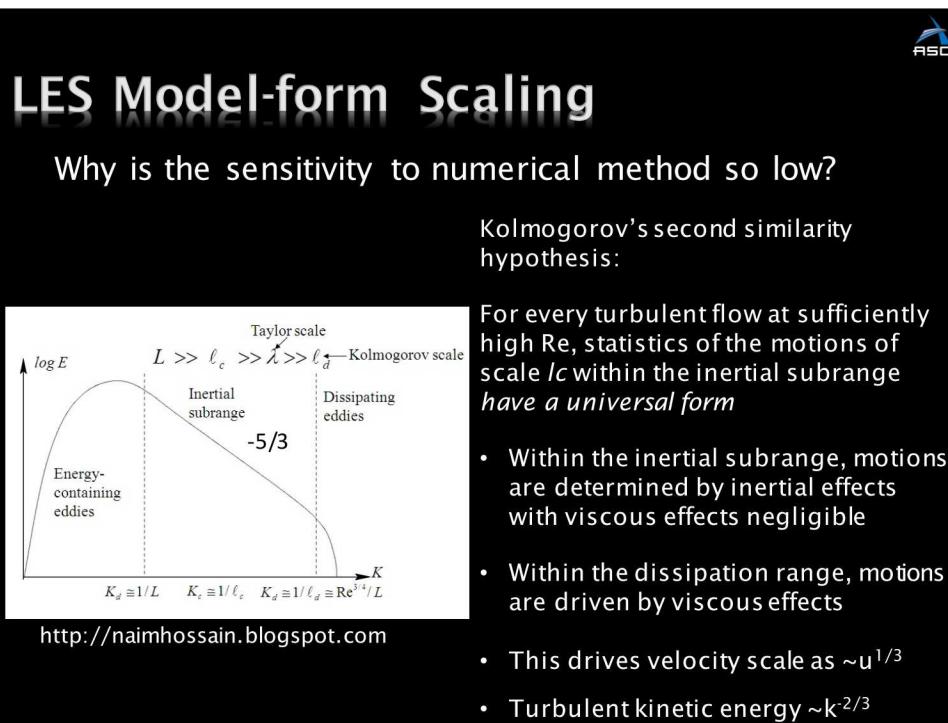
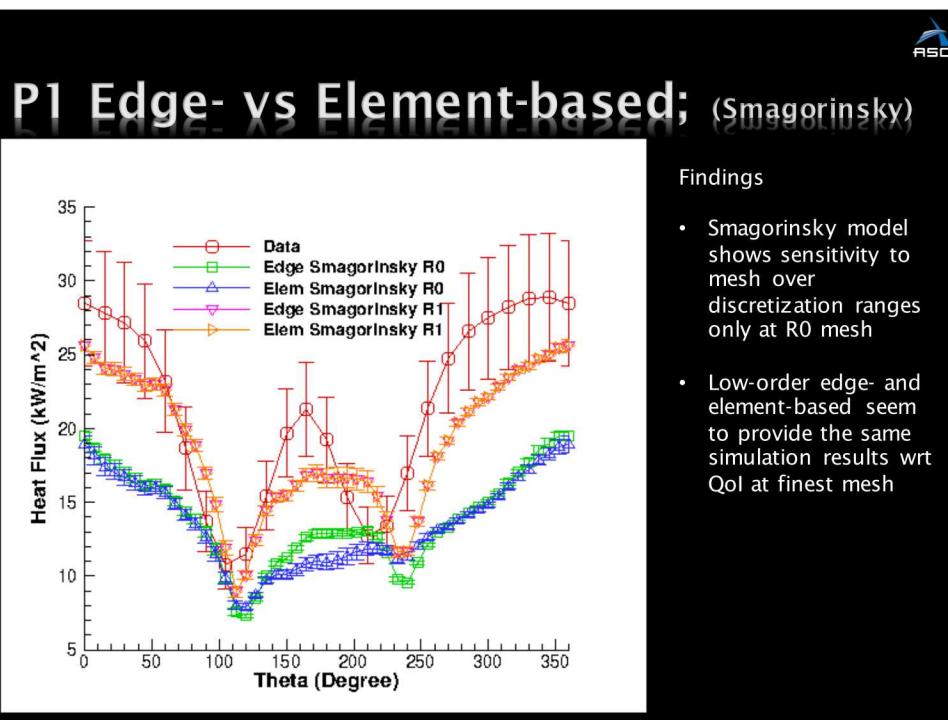


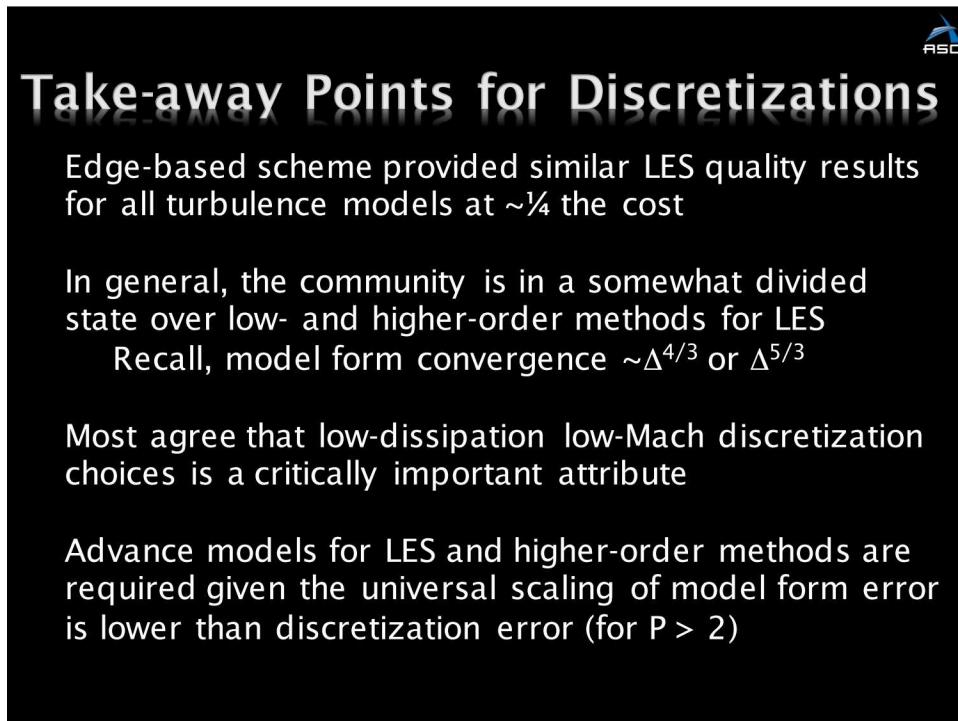
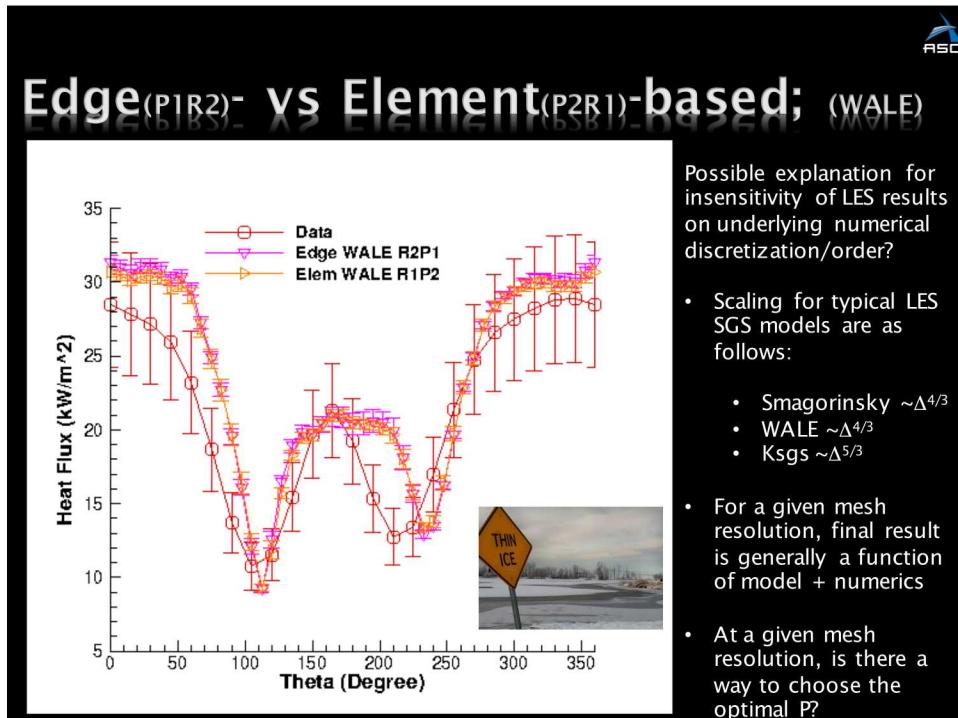












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Future Efforts



Model-form Error in LES?

Algorithmic and modeling constraints for the

^ application of LES to design

Sanjeeb Bose (Cascade, ICME-Stanford) & Frank Ham (Cascade, CTR)
L Shunn, G Bres, D Philips, M Emory, A Saghafian, D Kim, B Hejazi, P Quang [Cascade Tech.]



Relatively little is known about modeling errors
in the absence of validation data

In the limit when grid convergence to an asymptotic (dissipative) range is
infeasible and no DNS/expt. data is available, what can we say about the
simulation error?

Current lack a rigorous approach for estimating *modeling* errors in LES
... novel attempts have been made for RANS modeling (Emory et al 2013)

Should make efforts to use the same standard that's used in experiments

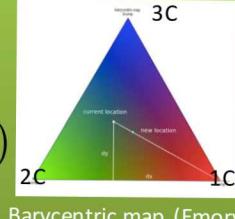
In what way can we drive model
form *estimation* for LES?

- Topic recently highlighted by Dr Sanjeeb Bose; July 20th, 2016 CTR summer program
- Developing state-of-the-art:
 - parametric studies of model constants, thermal/diffusion parameters, etc.
 - PCE (see Safta et al., IJNMF, 2016)
 - High-/low- fidelity (Eldred and Geraci, 2016)
- Is there a way to incorporate previous RANS-based sensitivity studies to the LES application space?

Eigen-value Decomposition of b_{ij}

“Estimating model-form uncertainty in Reynolds-Averaged Navier-Stokes Closures”, M. Emory, 2014

$$\begin{aligned}
 a_{ij} &= \overline{u'_i u'_j} - \frac{2}{3} k \delta_{ij} \quad b_{ij} = \frac{a_{ij}}{2k} \\
 &< u'_i u'_j > = 2k \left(\frac{1}{3} + b_{ij} \right) \\
 &< u'_i u'_j > = 2k \left(\frac{1}{3} + v_{in} \Gamma_{nl} v_{jl} \right) \\
 &\text{Six-dof perturbation + k} \\
 &= 2k^* \left(\frac{1}{3} + v_{in}^* \Gamma_{nl}^* v_{jl}^* \right) \\
 &\Gamma = \text{matrix of perturbed eigenvalues, } v \text{ eigenvectors}
 \end{aligned}$$

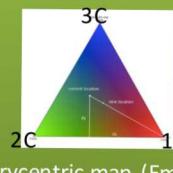


Barycentric map (Emory)

Extension to Scalar SGS Closure

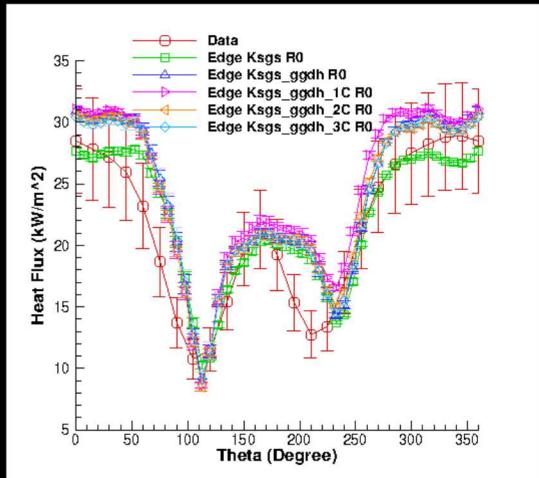
“A framework for epistemic UQ for turbulent scalar flux models for RANS”, C. Gorle, 2013

$$\begin{aligned}
 \tau_{h,j}^{sgs} &= -\frac{\mu_t}{\text{Pr}_t} \frac{\partial \tilde{h}}{\partial x_j} && \text{Gradient diffusion hypothesis} \\
 &&& \text{(fails for a simple shear flow)} \\
 \tau_{h,j}^{sgs} &= -\alpha_c \tau_c \tau_{jk}^{sgs} \frac{\partial h}{\partial x_k} && \text{Generalized} \\
 &&& \text{gradient diffusion} \\
 &&& \text{hypothesis;} \\
 &&& \text{Daly and Harlow,} \\
 &&& 1970 \\
 &< u'_i u'_j > = 2k \left(\frac{1}{3} + v_{in} \Gamma_{nl} v_{jl} \right) \\
 &\text{QL-decomposition} \\
 &\text{using fast-3x3} \\
 &\text{algorithm} \\
 &= 2k^* \left(\frac{1}{3} + v_{in}^* \Gamma_{nl}^* v_{jl}^* \right) \\
 &&& * = \text{perturb}
 \end{aligned}$$



Barycentric map (Emory)

Perturbation to 1C, 2C & 3C Poles (instantaneous τ_{ij} within scalar SGS only)



Findings:

- Minimal sensitivity in Scalar SGS closure; possibly due to plain-shear channel flow

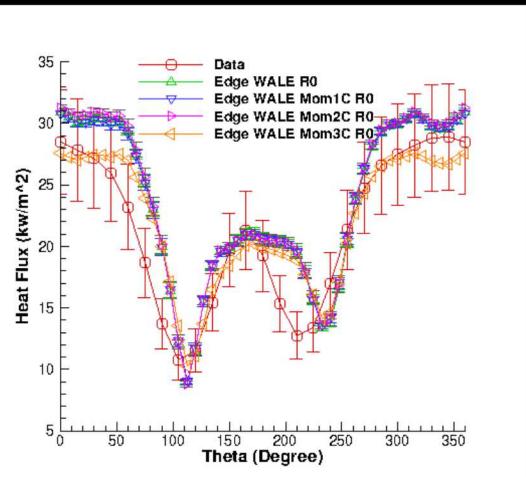
$$\text{GDH: } \bar{v'c} = -\frac{\nu_t}{\sigma_t} \frac{\partial C}{\partial y}$$

$$\text{GGDH: } \bar{v'c} = -\alpha_c \tau_c \bar{v'v'} \frac{\partial C}{\partial y}$$

$$\text{Q: } \bar{v'c} = -\alpha_c \tau_c \frac{1}{k} [(\bar{u'v'})^2 + (\bar{v'v'})^2] \frac{\partial C}{\partial y}$$

- Model form modification pushed closer to DNS, however, further from experimental data (later)
- Is this legal?

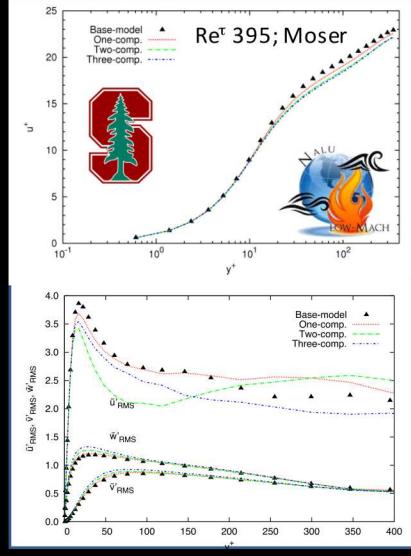
Perturbation to 1C, 2C & 3C Poles (instantaneous τ_{ij} within momentum SGS only)



Findings:

- Still seems that there is a minimal sensitivity in SGS closure
- However, three-component biasing drastically increased mixing
- Model form modification pushed closer to DNS, however, further from experimental data (later)
- Is this legal?
- Perhaps a more fundamental flow should be explored?

Questions? Reverting to *Plain Channel Flow* is “your best option”



Findings:

- Bounding of data seems to be conceptually different from RANS
- Still on-going questions as to the realizable of the SGS stress tensor (Vreman, JFM, 1994)
- Foundational work is planned both this year (PSAAP-2 interaction) and next FY (subject to V&V project renewal)
- Stanford partnership with G. Iaccarino and L. Jofre during the 2016 CTR summer program
- Interesting WMLES ramifications
- CU@ SIAM CSE2017, “UQ in Turbulent flow Simulations”

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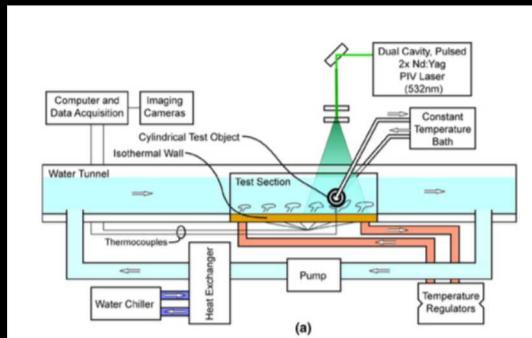
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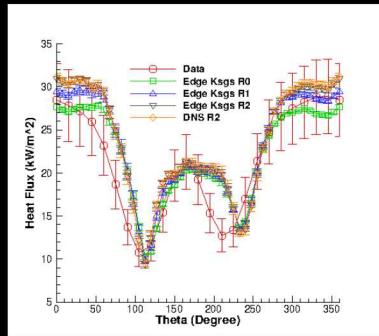


Physics-Form Error

Finding: The LES is consistent with DNS, however, is different from Kearney's experiment. Why?



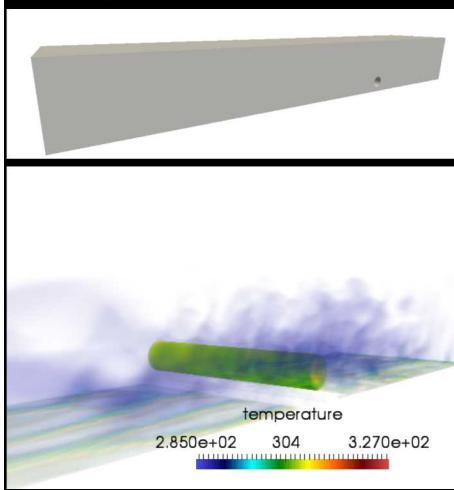
Kearney experimental configuration



Ksgs convergence with DNS

Physics-Form Error: \pm inner thermal physics

Classic Fluids/Thermal Conjugate Heat Transfer application



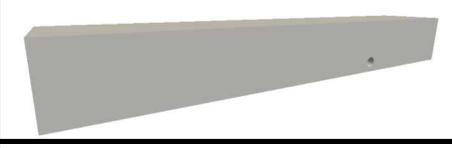
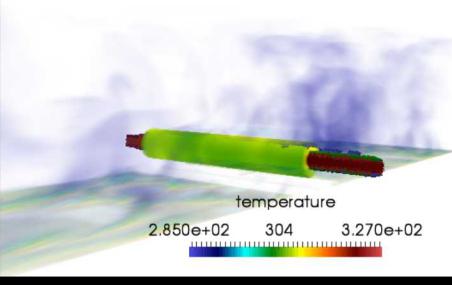
New Specifications

- Internal heat conduction physics added (uniform properties of SS)
- Inner heat flux applied (based on energy balance from experiment)
- Not without precedent; Kang et al, JCP 2009 (DNS with optimization loop)

Physics-Form Error: \pm inner thermal physics \pm inner fluids physics



More complex Fluids/Thermal/Fluids Conjugate Heat Transfer application

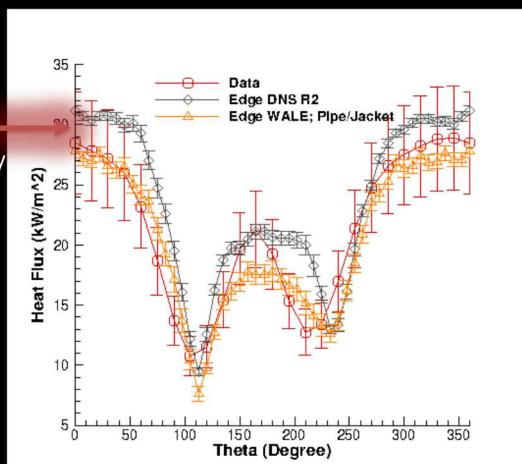
New Specifications

- Inner fluid (water) 1.12 GPM; 327 K; Re \sim 150K
- As reported in the experiment, inlet and outlet pipe temperature within \sim 1K
- Disparity in Re-numbers suggest a more sophisticated operator split time advancement; lock step for now (Picard loop)

Physics-Form Error: \pm inner thermal physics \pm inner fluids physics



Prediction improved with each added physics!



Inner q''
This approach slightly drops stagnation point heat flux

Physics-form prevails... In this study it is #1....

Conclusions



Systematic validation methodology outlined for a low-Mach LES DSW problem-of-interest; #1 Physics Form ε !

Remember the steps! Code verification before validation

Solution verification, although problematic for LES, should be performed (simple due diligence)

Model-form studies for LES are pedestrian (no offence)

Model-form uncertainties via eigenvalue perturbations is a novel, possibly promising area

Standard LES models within Solution verification converge rate lower than $P=2$ numerics....

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Domino, PI

David Womble and the ATCC-1 committee
~2+ million CPU hours consumed on Cielo;
dedicated Skybridge queue for CTR visiting scholar position

Robert Knaus



Stanford team

The SNL pioneers



<https://github.com/spdomin/Nalu>

