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Subsurface Applications for Peridynamics

6th European Congress on Computational Mechanics

MS62 - Nonlocal Models and Methods for Material Failure and Damage Simulation

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SAND2018--????C



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Outline

- Peridynamics Review**
- Seismic Inversion and Wellbore Stimulation Review
- Inverse Problems in Heterogeneous Fractured Media with Peridynamics
- Borehole Stimulation with Peridynamics
- Conclusions

What is Peridynamics?

□ Peridynamics is a nonlocal extension of classical solid mechanics

□ Peridynamic equation of motion (**integral, nonlocal**)

$$\rho \ddot{u}(x, t) = \int_{H_x} f(u(x') - u(x), x' - x) dV' + b(x, t)$$

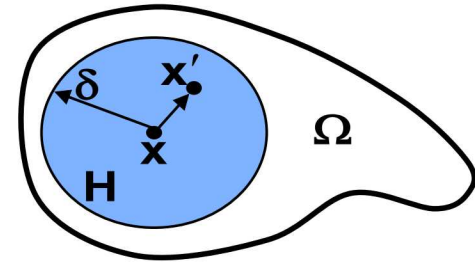
- Replace PDEs with integral equations
- **Utilize same equation everywhere; nothing “special” about cracks**
- No assumption of differentiable fields (admits fracture)
- No obstacle to integrating nonsmooth functions
- $f(\cdot, \cdot)$ is “force” function; contains constitutive model
- $f = 0$ for points x, x' more than δ apart (like cutoff radius in MD!)
- Peridynamics is “continuum form of molecular dynamics”

□ Impact

- Nonlocality
- Larger solution space (fracture)
- Account for material behavior at small & large length scales (**multiscale material model**)

□ Ancestors

- Kröner, Eringen, Edelen, Kunin, Rogula, etc.



Point x interacts directly with all points x' within H

“It can be said that all physical phenomena are nonlocal. Locality is a fiction invented by idealists.”

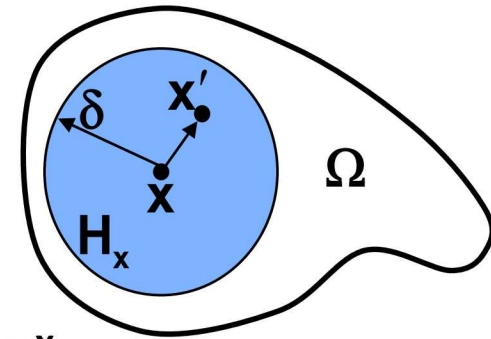


A. Cemal Eringen

Peridynamics: The Basics

□ Horizon and family

- Point \mathbf{x} interacts directly with all points with distance δ (**horizon**)
- Material within distance δ of \mathbf{x} is denoted H_x (**family of x**)



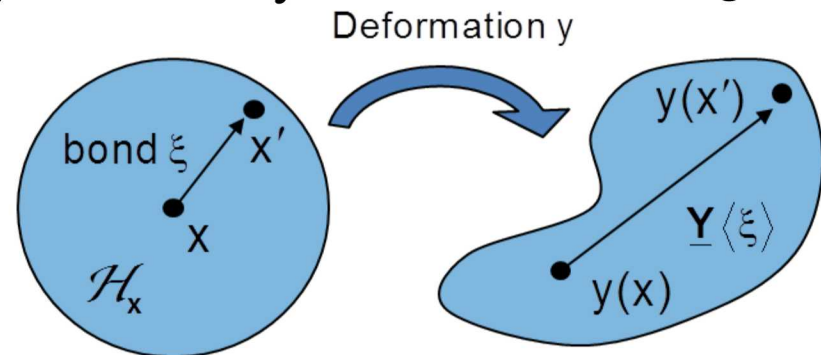
□ Bonds and bond forces

- Vector between \mathbf{x} and any point in its family is called a **bond**: $\xi = \mathbf{x}' - \mathbf{x}$
- Each bond has **pairwise force density vector** applied at both points: $\mathbf{f}(\mathbf{x}', \mathbf{x}, \mathbf{t})$
- This vector is determined jointly by collective deformation of H_x and collective deformation of $H_{x'}$
- Bond forces are antisymmetric: $\mathbf{f}(\mathbf{x}', \mathbf{x}, \mathbf{t}) = -\mathbf{f}(\mathbf{x}, \mathbf{x}', \mathbf{t})$
- Bond degrade and fail, admitting damage, failure, and fracture.

□ Deformation state

- Deformation state operator $\underline{\mathbf{Y}}$ maps each bond ξ into its deformed image

$$\underline{\mathbf{Y}}\langle \xi \rangle = \mathbf{y}(\mathbf{x}') - \mathbf{y}(\mathbf{x})$$



Undeformed family of x

Deformed family of x

Peridynamics: The Basics

□ Bonds and states

- $\mathbf{f}(\mathbf{x}', \mathbf{x})$ has contributions from material models at both \mathbf{x} and \mathbf{x}'

$$\mathbf{f}(\mathbf{x}', \mathbf{x}) = \underline{\mathbf{T}}[\mathbf{x}, \mathbf{t}] \langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}', \mathbf{t}] \langle \mathbf{x} - \mathbf{x}' \rangle$$

- $\underline{\mathbf{T}}[\mathbf{x}]$ is the **force state** – it maps bonds onto bond force densities
- $\underline{\mathbf{T}}[\mathbf{x}]$ is determined by the constitutive model $\underline{\mathbf{T}} = \hat{\mathbf{T}}(\underline{\mathbf{Y}})$, where $\hat{\mathbf{T}}$ maps deformation state to force state

□ Peridynamics vs. standard equations

- Peridynamic operators and relationships are nonlocal analogues of standard theory

Relation	Peridynamic theory	Standard theory
Kinematics	$\underline{\mathbf{Y}} \langle \mathbf{x}' - \mathbf{x} \rangle = \mathbf{y}(\mathbf{x}') - \mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \frac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho \ddot{\mathbf{u}}(\mathbf{x}) = \int_{H_x} (\underline{\mathbf{T}}[\mathbf{x}] \langle \mathbf{x}' - \mathbf{x} \rangle - \underline{\mathbf{T}}[\mathbf{x}'] \langle \mathbf{x} - \mathbf{x}' \rangle) dV_{x'} + \mathbf{b}(\mathbf{x})$	$\rho \dot{\mathbf{y}}(\mathbf{x}, t) = \nabla \cdot \boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\underline{\mathbf{T}} = \hat{\mathbf{T}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\int_{H_x} \underline{\mathbf{Y}} \langle \mathbf{x}' - \mathbf{x} \rangle \times \underline{\mathbf{T}} \langle \mathbf{x}' - \mathbf{x} \rangle dV_{x'} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = \mathbf{W}_{\underline{\mathbf{Y}}} \text{ (Frechet derivative)}$	$\boldsymbol{\sigma} = \mathbf{W}_{\mathbf{F}} \text{ (tensor gradient)}$
First law of thermodynamics	$\dot{\boldsymbol{\varepsilon}} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} + \mathbf{h} + \mathbf{r}$	$\dot{\boldsymbol{\varepsilon}} = \boldsymbol{\sigma} \cdot \dot{\mathbf{F}} + \mathbf{h} + \mathbf{r}$

Peridynamic Material Models

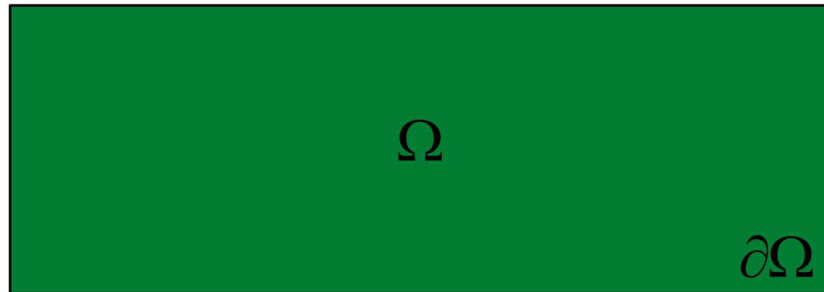
- **Linear Peridynamic Solid (LPS)***
- Nonlocal analog to linear isotropic elastic solid
- k is bulk modulus, μ is shear modulus

$$\rho \ddot{\mathbf{u}}(\mathbf{x}, \mathbf{t}) = \int_{\mathcal{H}} \left(\mathbf{T}[\mathbf{x}, \mathbf{t}] \langle \mathbf{x}' - \mathbf{x} \rangle - \mathbf{T}[\mathbf{x}', \mathbf{t}] \langle \mathbf{x} - \mathbf{x}' \rangle \right) dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}, \mathbf{t})$$

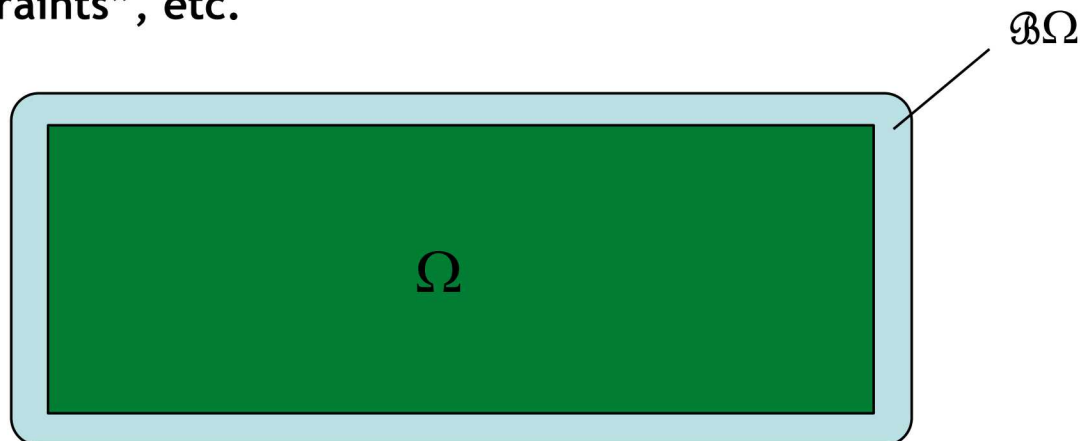
$$\mathbf{T}[\mathbf{x}, \mathbf{t}] \langle \mathbf{x}' - \mathbf{x} \rangle = \left(\frac{3k\theta}{m} \underline{\underline{\omega}} \mathbf{x} + \frac{15\mu}{m} \underline{\underline{\omega}} \mathbf{e}^d \right) \frac{\mathbf{x}' - \mathbf{x}}{\|\mathbf{x}' - \mathbf{x}\|}$$

Nonlocal Boundary Conditions

- For local models (for example, PDE-based models), we apply boundary conditions on the boundary of the domain (hence the name).



- A Peridynamic “boundary” becomes a volumetric region, sometimes called a “nonlocal boundary”, “collar”, etc.
- Boundary conditions for these models are called “nonlocal boundary conditions”, “volume constraints”, etc.



Fragmenting Cylinder

Fragmenting Cylinder

- Motivated by tube fragmentation experiments of Winter (1979), Vogler (2003)*

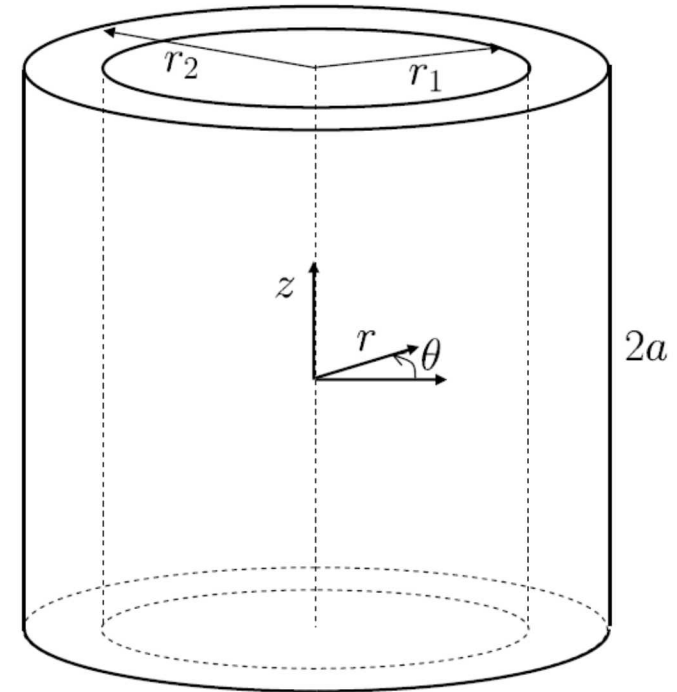
Material properties

- Inner radius $r_1 = 0.020$ m
- Outer radius $r_2 = 0.025$ m
- Length $2a = 0.1$ m
- Density $\rho = 7800$ kg/m³
- Bulk modulus $k = 130$ GPa
- Shear modulus $\mu = 78$ GPa
- Yield stress $Y = 500$ GPa
- Ultimate stress $\sigma = 700$ GPa
- Elongation at failure $\varepsilon = 0.02$

Initial Velocity

- $v(r) = V_{r0} - V_{r1}(a/z)^2$
- $v(z) = V_{z0}(a/z)$
- $v(\theta) = 0$

where $V_{r0} = 200$ m/s, $V_{r1} = 50$ m/s, $V_{z0} = 100$ m/s

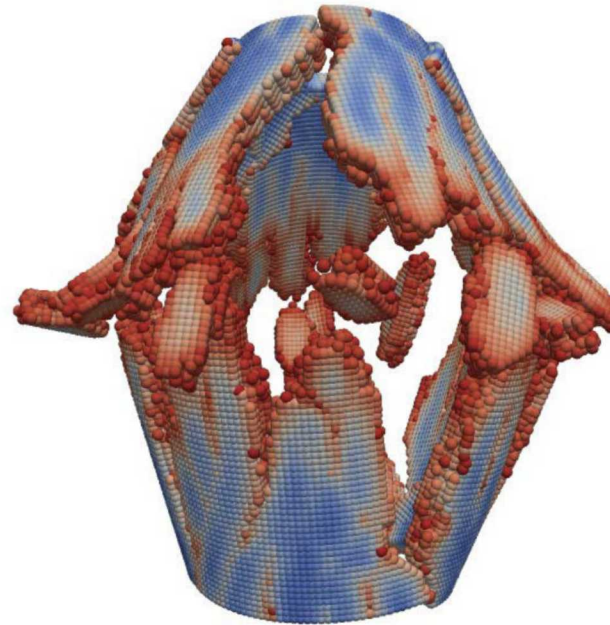


Fragmenting Cylinder

Simulation performed
with Peridigm



Before



**After
(brittle failure)**

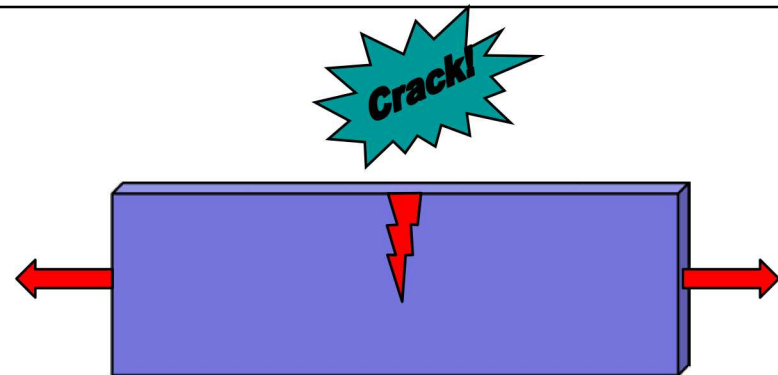


Fracture in Glass Plate

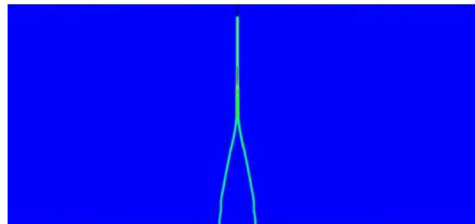
- ❑ With Florin Bobaru (Nebraska), Youn-Doh Ha, & Stewart Silling (Sandia)
- ❑ Soda-lime glass plate (microscope slide)
 - ❑ Dimensions: 3" x 1" x 0.05"
 - ❑ Density: 2.44 g/cm³
 - ❑ Elastic Modulus: 79.0 Gpa
- ❑ Dawn (LLNL): IBM BG/P (500 TF; 147,456 cores)
 - ❑ Mesh spacing: 35 microns
 - ❑ Approx. 82 million particles
 - ❑ Time: 50 microseconds (20k timesteps)
 - ❑ 6 hours on 65k cores

Setup

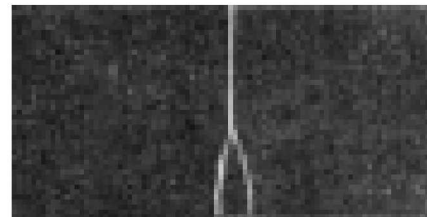
- ❑ Glass microscope slide
- ❑ Dimensions: 3" x 1" x 0.05"
- ❑ Notch at top, pull on ends



Results



Peridynamics



Physical Experiment*

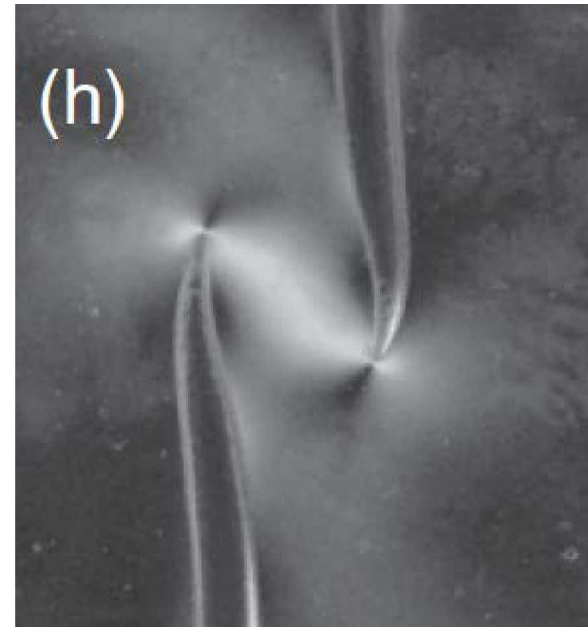
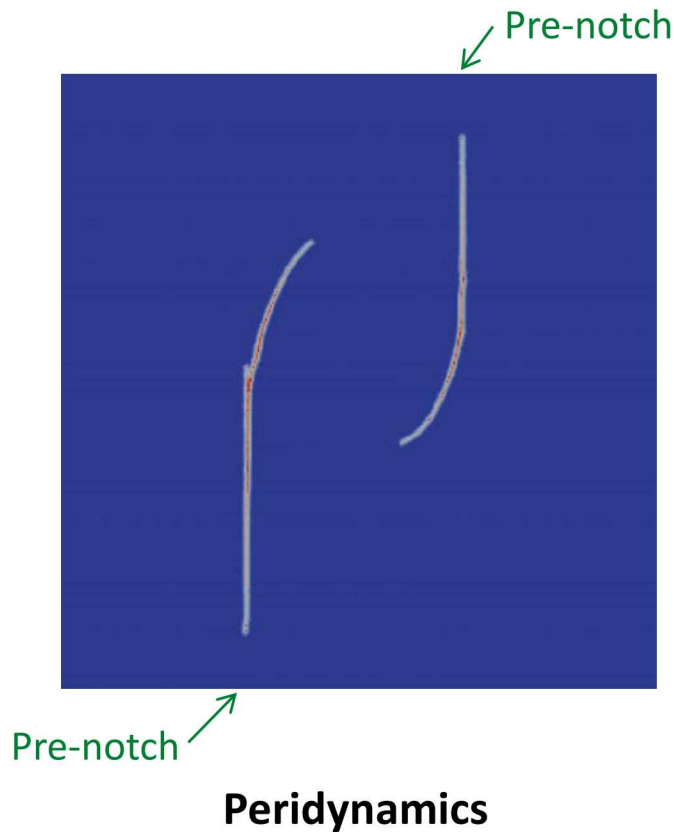


Strain Energy
Density

Two Interacting Cracks

- ❑ Offset notches thin rectangular elastic plate
- ❑ Uniaxial strain applied from sides
- ❑ Approaching cracks produce “en passant” crack pattern

Simulation performed with PDLAMMPS



Codes

❑ PDLAMMPS (Peridynamics-in-LAMMPS) (Open source, C++)

- ❑ Developers: Parks, Seleson, Plimpton, Silling, Lehoucq
- ❑ Particular discretization of PD has computational structure of molecular dynamics (MD)
- ❑ LAMMPS: Sandia's open-source massively parallel MD code (lammps.sandia.gov)
- ❑ More info & user guide: www.sandia.gov/~mlparks

❑ Peridigm (Open Source, C++)

- ❑ peridigm.sandia.gov; github.com/peridigm/peridigm
- ❑ Developers: Parks, Littlewood, Mitchell, Silling
- ❑ Intended as Sandia's primary open-source PD code
- ❑ Built upon Sandia's Trilinos Project (trilinos.sandia.gov)
- ❑ Massively parallel
- ❑ Explicit, implicit time integration
- ❑ State-based linear elastic, elastic-plasticity, viscoelastic models
- ❑ DAKOTA interface for UQ/optimization/calibration, etc. (dakota.sandia.gov)



Outline

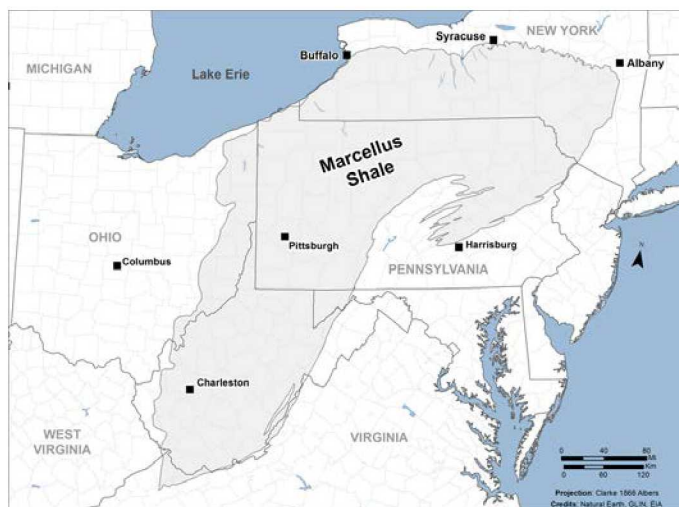
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Hydrocarbon Exploration

- ❑ Search for oil and gas (hydrocarbon) deposits in the subsurface
- ❑ Expensive, high-risk operation generally undertaken by large companies or national governments.
- ❑ Deep water wells can cost \$100M+.
- ❑ Marcellus shale may hold 50 trillion cubic feet (TCF) and 500 TCF of natural gas.



Deepwater Offshore Drilling Rig



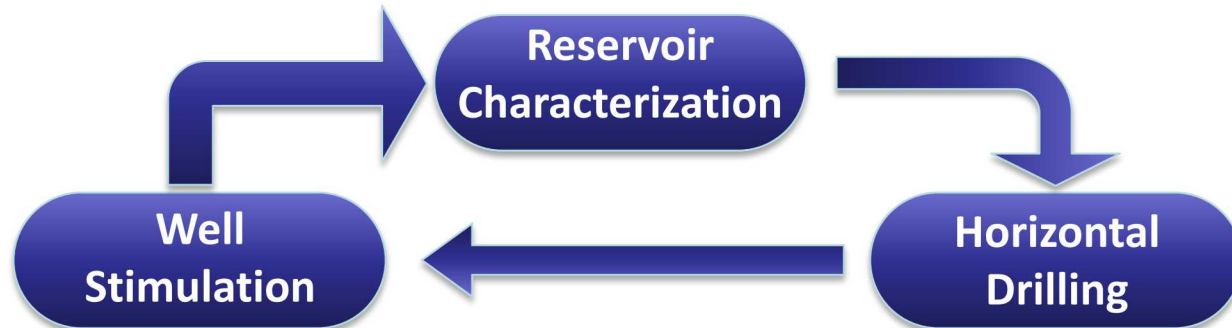
Marcellus Shale



Natural Gas Derrick, Pennsylvania 15

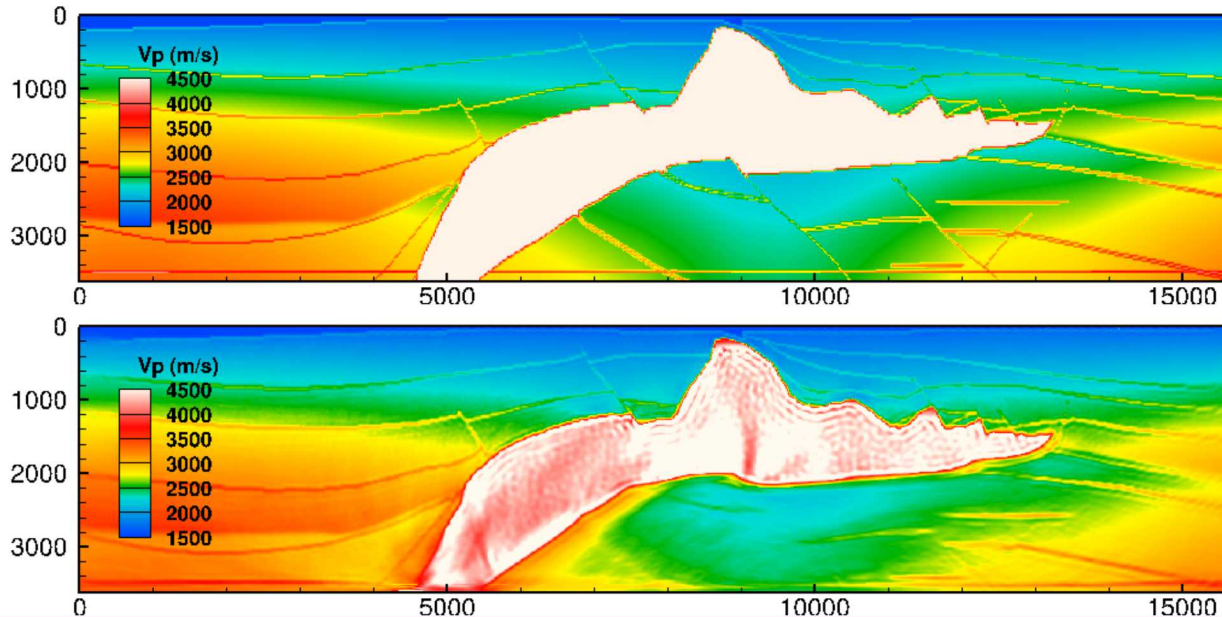
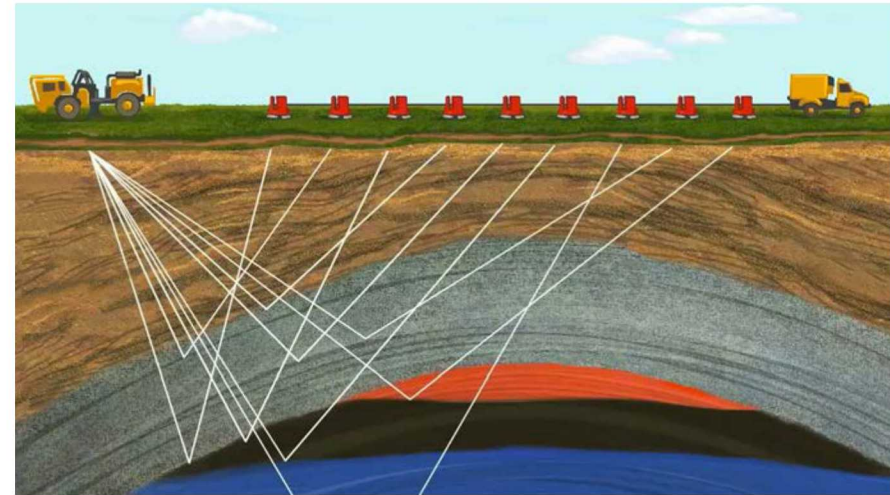
Hydrocarbon Exploration

- ❑ Computational modeling can play numerous roles. I'll discuss two aspects today, where nonlocal (peridynamics) models have been applied to both.
- ❑ **Reservoir Characterization via Seismic Inversion**
 - ❑ Transform seismic reflection data into quantitative rock properties that are descriptive of the reservoir.
- ❑ **Well Stimulation**
 - ❑ Well intervention technique performed on an oil or gas well to increase production by improving the flow of hydrocarbons from the drainage area into the well bore.
 - ❑ This is especially important for so-called unconventional resources (shale gas, tight gas, tight oil plays) with low reservoir permeability



Reservoir Characterization via Seismic Inversion

- ❑ Accurate subsurface models to reduce risk of drilling poorly producing wells and maximize production of reservoirs with fewest number of wells.
- ❑ Seismic inversion transforms seismic reflection data into quantitative rock-property description of reservoir.

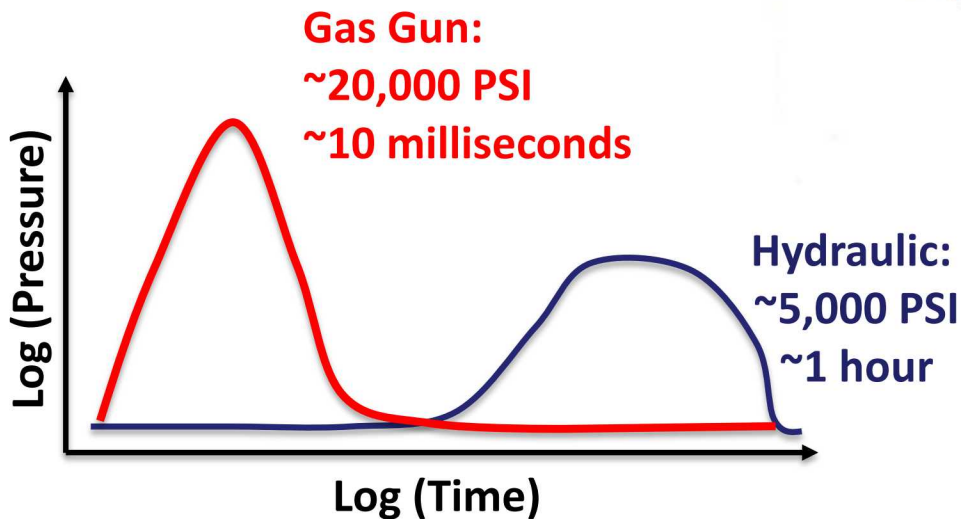
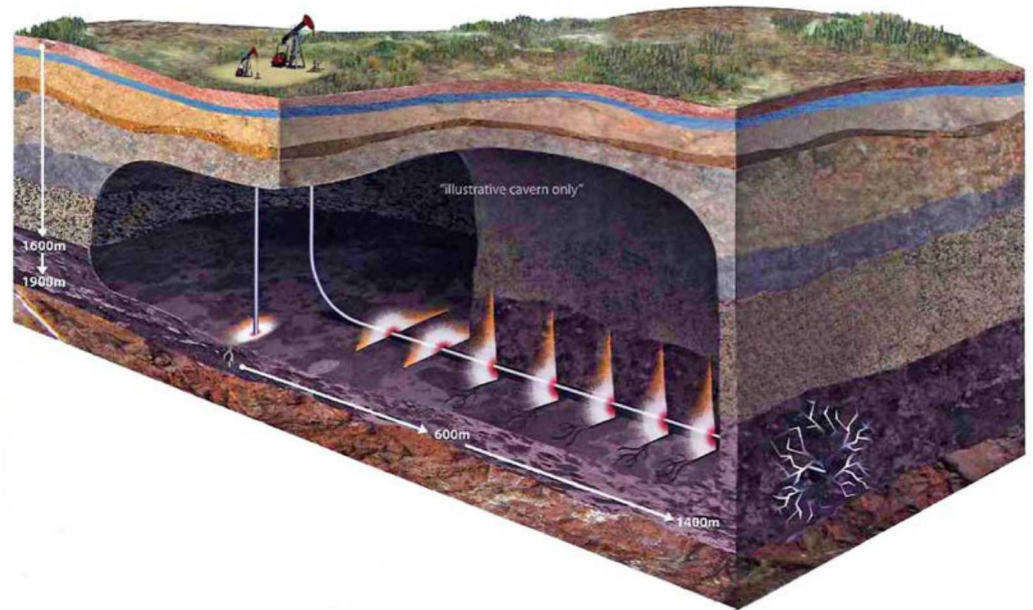


Top: The true compressional wave speed for the SEG Salt Model*.

Bottom: Inverted results. Salt structure well defined. Steep faults under the salt not captured due to the lack of illumination from the source & receiver locations.

Wellbore Stimulation

- ❑ Restore or enhance well production by improving the connection of the wellbore with the reservoir.
- ❑ This can be done in multiple ways; Two prominent methods are
 - ❑ Hydraulic fracturing (fracking)
 - ❑ Gas guns
- ❑ These have different pressure profiles and different effects on surrounding rock.



Stimulation of Horizontally Drilled Well

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Peridynamic Inverse Problem

□ Given the demonstrated ability of peridynamics to model fractures, we seek to demonstrate if it can be used in the context of inversion.

□ Specifically,

□ Can we invert for material properties $(k(x) \mu(x))$ with peridynamics?

□ Can we invert in the presence of fractures?

□ Formulate nonlinear constrained least-squares optimization problem (IDE-constrained optimization):

$$\min_{\mathbf{u}, k, \mu} \mathbf{g}(\mathbf{u}, k, \mu) \quad \text{subject to} \quad \begin{cases} \mathbf{c}(\mathbf{u}, k, \mu) = \mathbf{0} & \text{in } \Omega \\ \mathbf{u} = \mathbf{u}_0 & \text{in } \Omega^u \quad (\text{nonlocal Dirichlet bc}) \\ \mathbf{b} = \mathbf{q} & \text{in } \Omega^q \quad (\text{nonlocal Neumann bc}) \end{cases}$$

where

$$\mathbf{g}(\mathbf{u}, k, \mu) = \sum_{j=1}^{N_{\text{obs}}} \int_{\Omega} \|\mathbf{u} - \mathbf{u}^*\| \delta(\mathbf{x} - \mathbf{x}_j) d\Omega + \frac{\alpha_k}{2} \int_{\Omega} k^2 d\Omega + \frac{\alpha_{\mu}}{2} \int_{\Omega} \mu^2 d\Omega$$

$$\mathbf{c}(\mathbf{u}, k, \mu) = \int_{H_x} \mathbf{T}[\mathbf{x}] \langle \mathbf{x}' - \mathbf{x} \rangle - \mathbf{T}[\mathbf{x}'] \langle \mathbf{x} - \mathbf{x}' \rangle d\Omega$$

Optimality Condition

- Introduce Lagrange multiplier field (adjoint variable) to give

$$L(\mathbf{u}, \boldsymbol{\lambda}, \mathbf{d}) = g(\mathbf{u}, \mathbf{d}) + \int_{\Omega} (\mathbf{c}^T \boldsymbol{\lambda} - \mathbf{d}^T \boldsymbol{\lambda}) \, d\Omega$$

- where \mathbf{d} contains the material parameters k and μ .
- Linearizing and taking variations with respect to \mathbf{u} , \mathbf{d} , $\boldsymbol{\lambda}$ gives

$$\mathbf{c} = \mathbf{0} \quad \text{in } \Omega \text{ (state)}$$

$$\mathbf{J}_u^T \boldsymbol{\lambda} + \mathbf{g}_u = \mathbf{0} \quad \text{in } \Omega \text{ (adjoint)}$$

$$\mathbf{J}_d^T \boldsymbol{\lambda} + \mathbf{g}_d = \mathbf{0} \quad \text{in } \Omega \text{ (decision)}$$

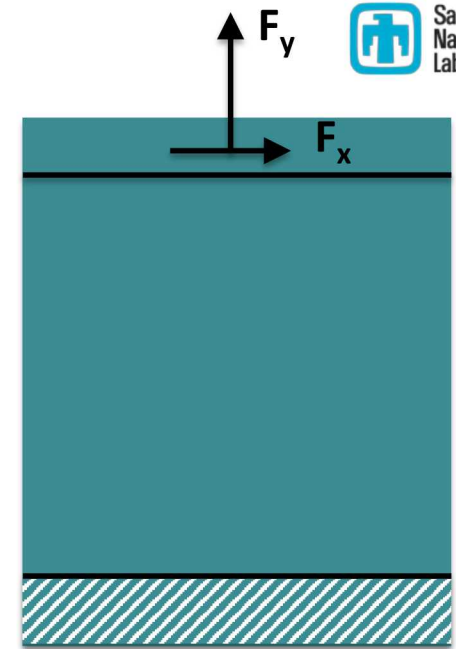
- After discretization:

$$\begin{aligned} \mathbf{u}, \boldsymbol{\lambda} \in \mathbb{R}^M & \quad \mathbf{J}_u = \partial \mathbf{c} / \partial \mathbf{u} \in \mathbb{R}^{M \times M} & \quad \mathbf{g}_u = \partial g / \partial \mathbf{u} \in \mathbb{R}^M \\ \mathbf{d} \in \mathbb{R}^P & \quad \mathbf{J}_d = \partial \mathbf{c} / \partial \mathbf{d} \in \mathbb{R}^{M \times P} & \quad \mathbf{g}_d = \partial g / \partial \mathbf{d} \in \mathbb{R}^P \end{aligned}$$

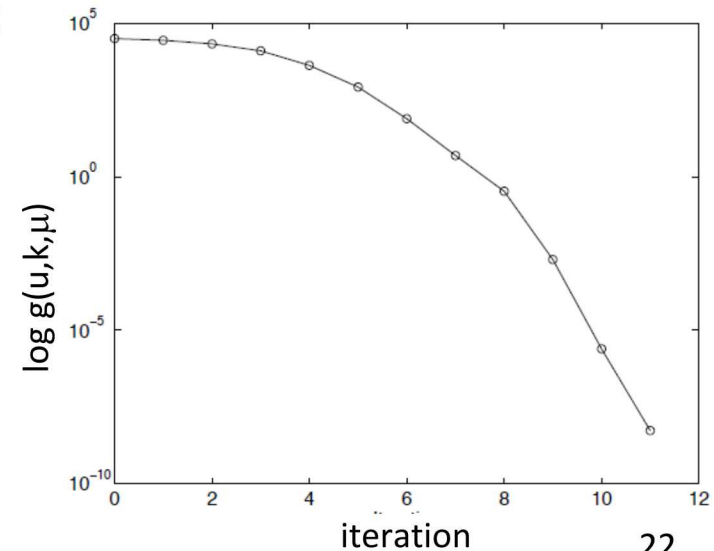
- Solve using trust-region algorithm with conjugate gradient solver.

Numerical Example #1

- Verification for constant material parameters (P=2)
- Consider a 2D, square domain $L \times L$ domain with bottom held fixed and force applied to top.
 - $L = 1000$,
 - $F_x = F_y = 1000$
 - $h = 100$, piecewise constants
- Observed displacement u^* manufactured by solving forward problem with $k = 666.67$ and $\mu = 400.0$.
- Solve for k, μ , given loading and least squares misfit of computed displacement field, u , to observed displacement field u^* . $k_{init} = \mu_{init} = 100$.
- No regularization used ($\alpha_k = \alpha_\mu = 0$).



$$u_x = u_y = 0$$



Parameter	Exact	Computed	Error (%)
k	666.67	666.6700	0.0000644
μ	400.00	399.9984	0.0003950

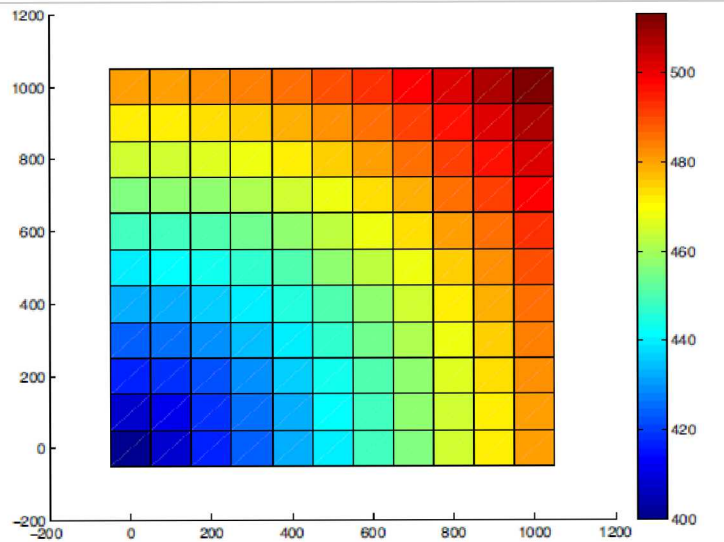
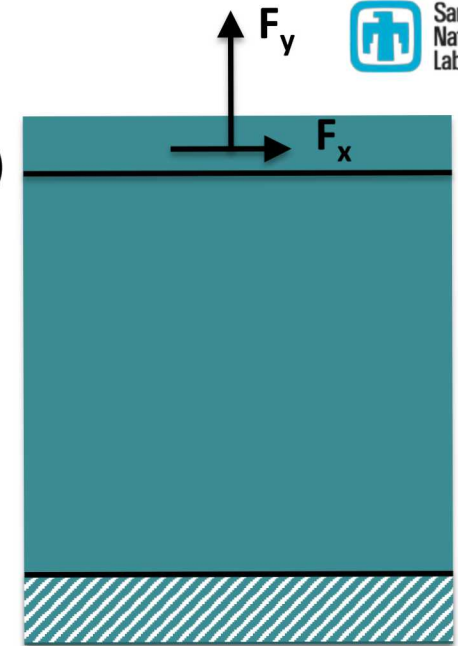
Numerical Example #2

- Verification for heterogeneous material parameters (P=121)
- Observed displacement u^* manufactured by solving forward problem with

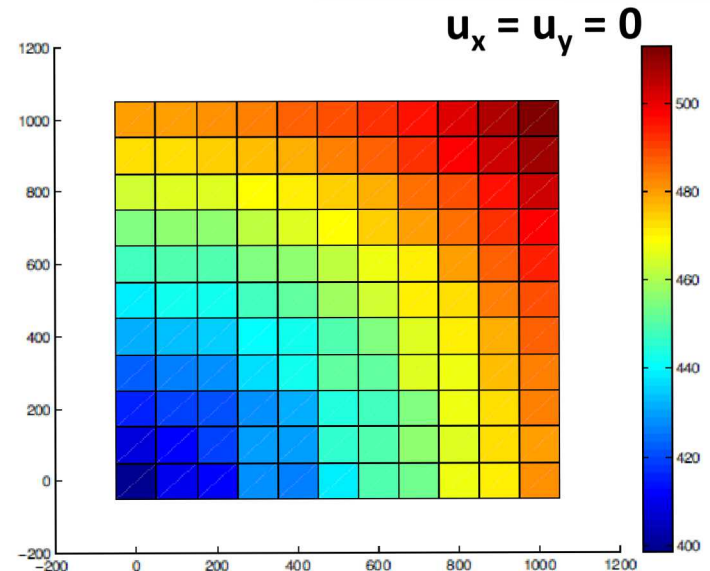
$$k(x) = -\gamma_k r k_0 + k_0$$

$$\mu(x) = -\gamma_\mu r \mu_0 + \mu_0$$

- $\gamma_k = 0.00025$
- $\gamma_\mu = 0.00020$
- $k_0 = 666.67, \mu_0 = 400.0$
- Invert for μ . $\mu_{\text{init}} = \mu_0$.



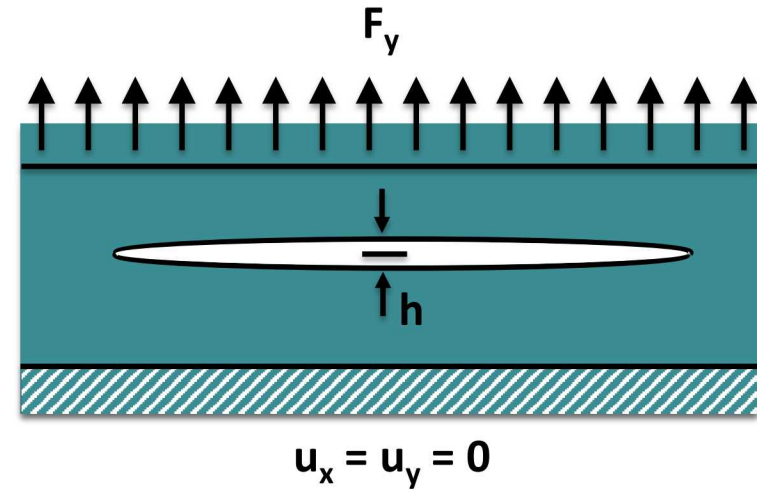
True $\mu(x)$



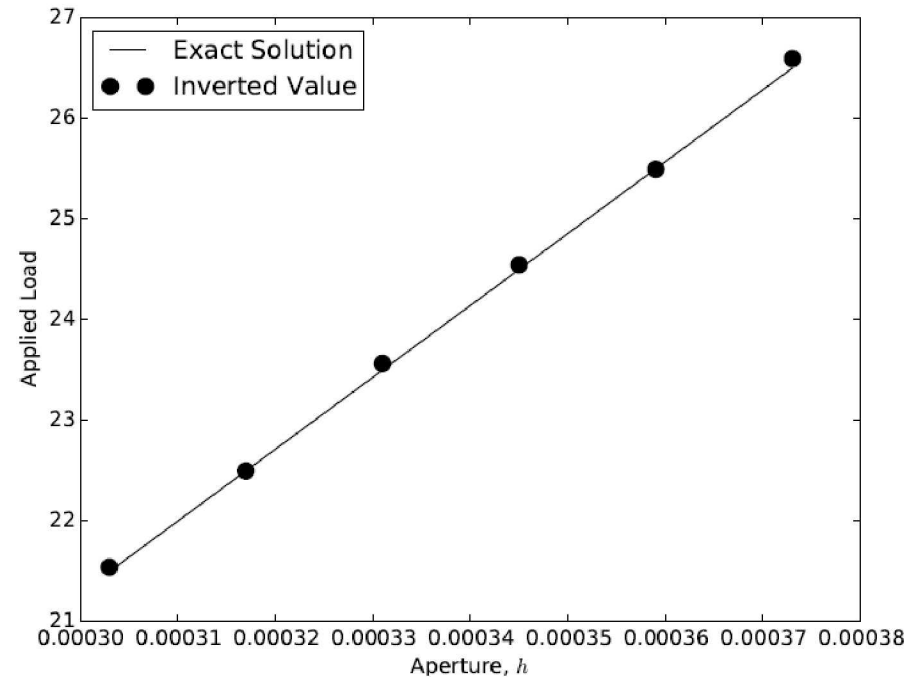
Inverted $\mu(x)$ (1.2% error)

Numerical Example #3

- Inversion for load state with fracture
- Consider 2D domain with bottom held fixed and force applied to top.
- Pre-crack applied in center
 - $\mu = 40$ GPa
 - $k = 66.6$ GPa
- Observed crack aperture h^* manufactured by solving forward problem with applied load F_y
- Invert for crack aperture h



$$\mathbf{g}(h) = \frac{1}{2} \sum_{j=1}^{N_{\text{obs}}} \int_{\Omega} (h - h^*)^2 \delta(\mathbf{x} - \mathbf{x}_j) d\Omega$$



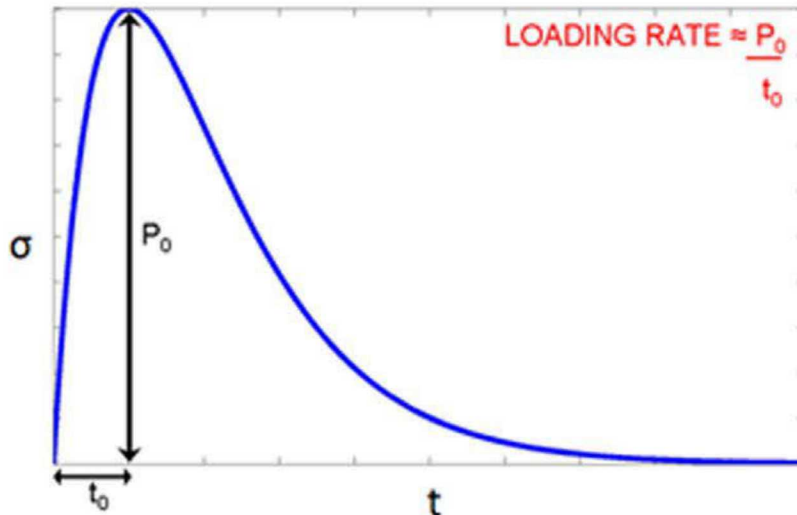
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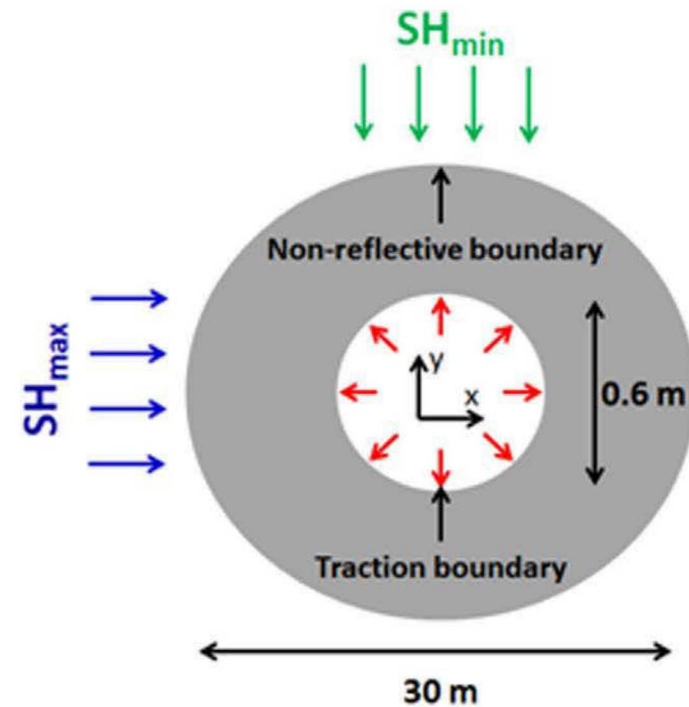
Well Stimulation

- ❑ Fracture patterns dependent on
 - ❑ in-situ stress
 - ❑ pressure profile
- ❑ Investigate combination of both
- ❑ Study conducted with PMB model

$$P_b(t) = e^{-at} P_0 \frac{e^{-at} - e^{-\beta t}}{e^{-at_0} - e^{-\beta t_0}}$$



Property	Value
Density (ρ)	2700.0 kg/m ³
Bulk modulus (k)	37.0 GPa
Energy release rate (G0)	298 J/m ²
Rock diameter (D)	30 m
Borehole diameter (db)	0.6 m



Well Stimulation

Loading Rates

$$P_b(t) = e^{-at} P_0 \frac{e^{-at} - e^{-\beta t}}{e^{-at_0} - e^{-\beta t_0}}$$

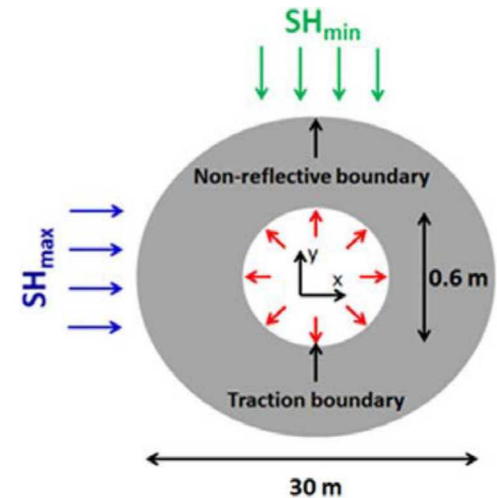
Case	P_0 (MPa)	t_0 (ms)	β/α
1	100.0	10.0	1.5
2	100.0	1.0	1.5
3	100.0	0.1	1.5
4	100.0	1.0	131.3
5	100.0	0.1	2120.0

Effect of loading rate

Effect of decay rate

In-situ Stress States

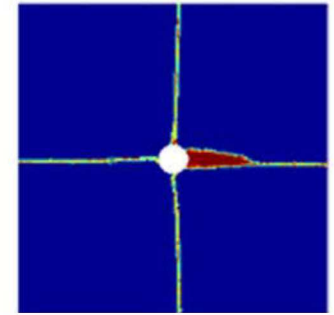
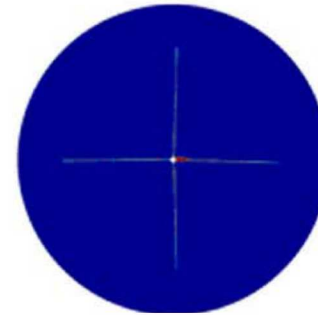
Case	SH_{max} (MPa)	SH_{min} (MPa)
A	0.0	0.0
B	10.0	10.0
C	12.5	7.5
D	15.0	5.0



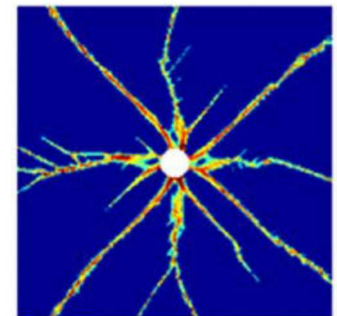
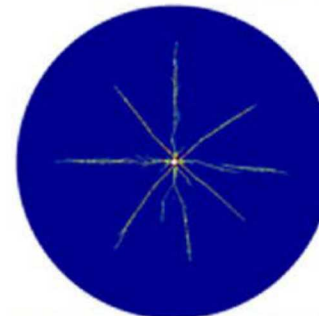
Influence of Loading Rate

- ❑ No in-situ stress
- ❑ Comparison of slow, fast, & fastest loading rates.
- ❑ Higher loading rate produces more radial fractures
- ❑ Slower loading rate produces fewer, longer radial fractures
- ❑ Close to borehole, higher loading rates produce ring of highly damaged material, consistent with region of crushed or pulverized material typically seen under explosive loading

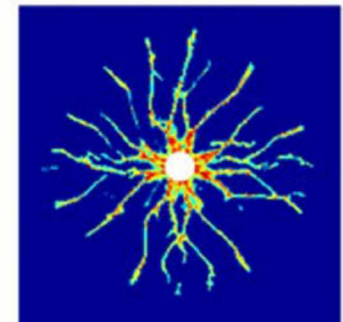
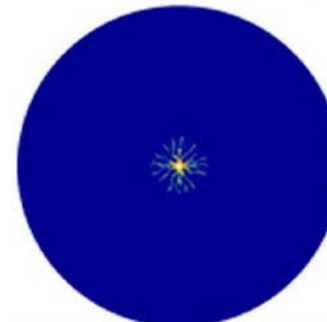
Case A-1 (slow)



Case A-2 (fast)



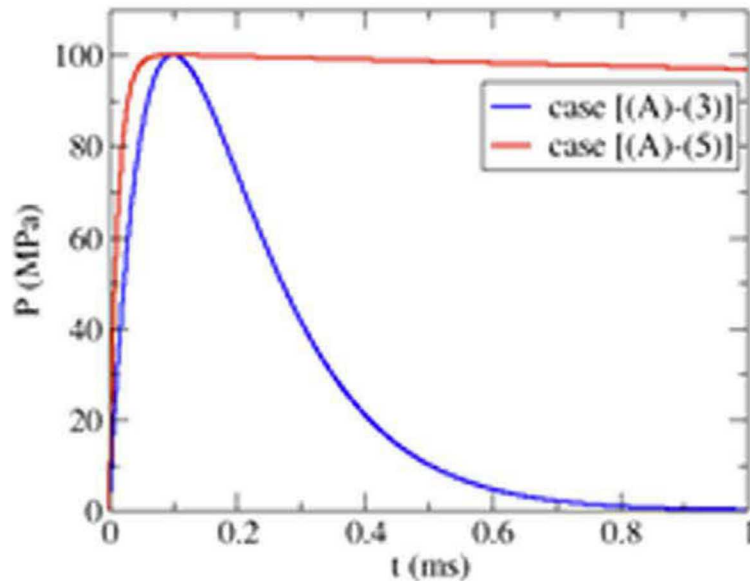
Case A-3 (fastest)



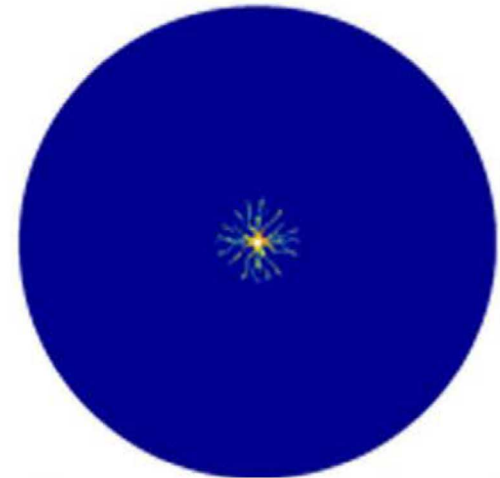
Influence of Decay Rate

- No in-situ stress
- Comparison of slow, fast, & fastest loading rates.

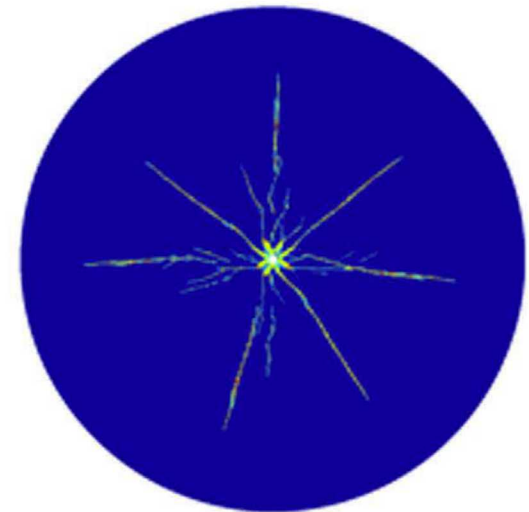
- Initiation nearly same for A-3, A-5.
- In A-5, pressure held nearly constant after initiation; fractures much longer before arrest.
- Number of fractures appears to depend on loading rate; length on decay rate.



Case A-3 (fastest loading; fast decay)



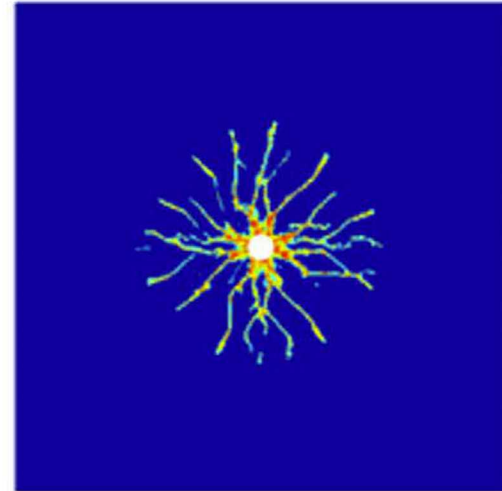
Case A-5 (fastest loading; long decay)



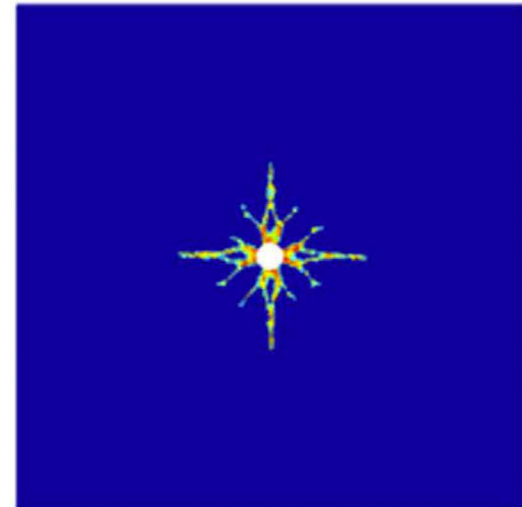
Influence of In-Situ Stress (1)

- ❑ Comparison of no in-situ stress vs. 10 MPa uniform in-situ stress for & fastest loading rate
- ❑ Maximum fracture length for B-3 about 50% that for A-3
- ❑ If the system is under compression, a larger amount of energy is required to generate a tensile state that initiates and propagates fractures.

Case A-3 (fastest loading; no stress)



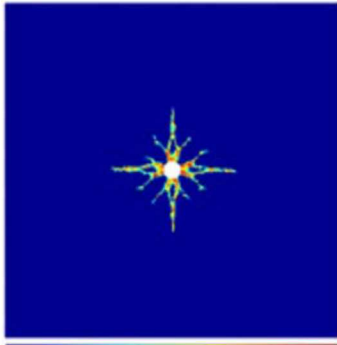
Case B-3 (fastest loading; uniform stress)



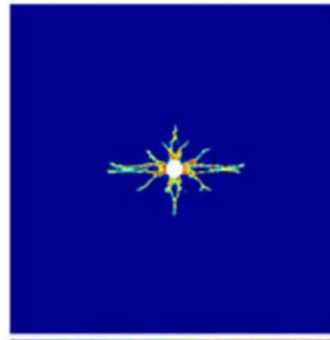
Influence of In-Situ Stress (2)

- ❑ Comparison of uniform in-situ stress vs. anisotropic in-situ stress
- ❑ For slower loading rates, stress anisotropy results in fractures oriented perpendicular to minimum confining stress direction, similar to what is expected in hydraulic fracturing

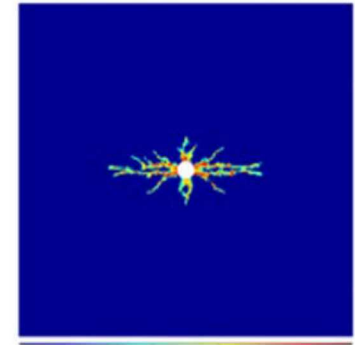
Case B-3
(fastest loading; no stress)



Case C-3
(fastest loading;
anisotropic stress)



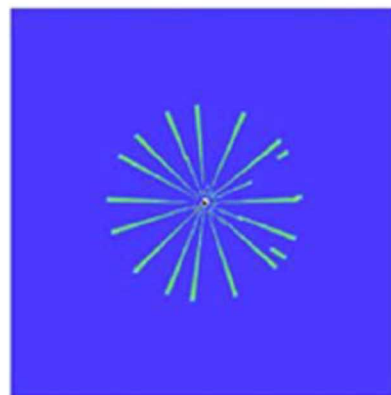
Case D-3
(fastest loading;
very anisotropic stress)



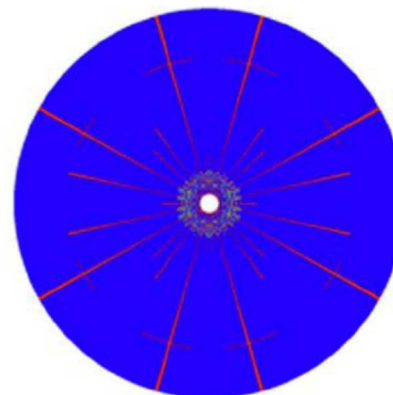
Comparison with other models

- Qualitative comparison to other models.
- LS-DYNA fracture patterns do not turn or branch. Current study show turning and branching fracture patterns, in qualitative agreement with experiment.

Johnson-Holmquest
Model (LS-DYNA)^a



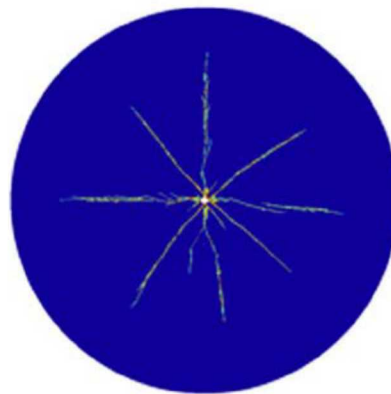
(a)



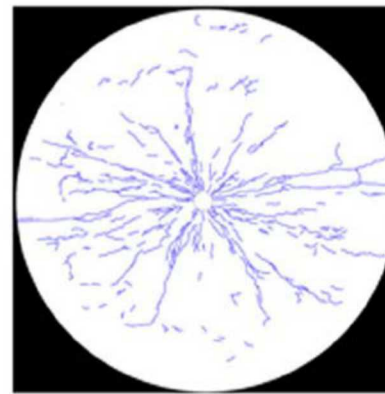
(b)

Johnson-Holmquest
Model (LS-DYNA)^a

Peridynamics
(current study)



(c)



(d)

Physical Experiment^b

Summary

- ❑ Demonstrated IDE-constrained optimization and borehole stimulation with peridynamics.

- ❑ Inversion with Peridynamics:
 - ❑ Daniel Turner, Bart van Bloemen Waanders and M.L.P, Inverse problems in heterogeneous and fractured media using peridynamics, *Journal of Mechanics of Materials and Structures*, 10(5), pp. 573-590, 2015.

- ❑ Borehole Stimulation with Peridynamics:
 - ❑ Rohan Panchadhara, Peter A. Gordon, and Michael L. Parks, Modeling propellant-based stimulation of a borehole with peridynamics, *International Journal of Rock Mechanics and Mining Sciences*, 93, pp. 330-343, 2017.

- ❑ Questions?