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Applying Matlab LOCO to the NSLSII Storage Ring Commissioning

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1. Introduction

Linear Optics from Closed Orbits (LOCO) has been a powerful beam-based diagnostics and optics control method for storage rings and synchrotrons worldwide ever since it was established at NSLS by J. Safranek [1]. This method measures the orbit response matrix and optionally the dispersion function of the machine. The data are then fitted to a lattice model by adjusting parameters such as quadrupole and skew quadrupole strengths in the model, BPM gains and rolls, corrector gains and rolls of the measurement system. Any abnormality of the machine that affects the machine optics can then be identified. The resulting lattice model is equivalent to the real machine lattice as seen by the BPMs. Since there are usually two or more BPMs per betatron period in modern circular accelerators, the model is often a very accurate representation of the real machine. According to the fitting result, one can correct the machine lattice to the design lattice by changing the quadrupole and skew quadrupole strengths. LOCO is so important that it is routinely performed at many electron storage rings to guarantee machine performance, especially after the Matlab-based LOCO code [2] became available. The Matlab version includes a user-friendly interface, with many useful fitting and analysis options.

2. Basic LOCO Algorithm

If the linear optics in a storage ring is known, then the closed orbit response matrix can be easily calculated. The LOCO algorithm reverses this process. Given the measured orbit response matrix, the code determines the actual linear optics of the ring, including the errors in the optics. The quadrupole strengths can then be adjusted to correct the errors.

The closed orbit response matrix is defined as

$$\begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{bmatrix} \vec{\theta}_x \\ \vec{\theta}_y \end{bmatrix}, \quad (1)$$

where (*x, y*) is the horizontal and vertical shift in closed orbit at all the BPMs for a change in strength of the steering magnets of (θ_x, θ_y). The size of the orbit response matrix is the number of BPMs times the number of steering magnets, so the measured orbit response matrix typically contains thousands or tens of thousands of data points. The measurement accuracy of the closed orbit is usually in the order of a micron, so the orbit response matrix contains a huge number of data points reflecting the optics of the ring.

The way to find the optics model that best fits the data is to minimize the chi-squared difference between the measured data and the model,

$$\chi^2 = \sum_{i,j} \frac{(M_{ij}^{\text{meas}} - M_{ij}^{\text{model}})^2}{\sigma_i^2} \equiv \sum_{k=i,j} E_k^2 \qquad E_k = \frac{M_{ij}^{\text{meas}} - M_{ij}^{\text{model}}}{\sigma_i}, \quad (2)$$

where σ_i is the measured noise level on the i^{th} BPM, and E_k is the error vector. Minimizing the length of the vector E is equivalent to minimizing chi-squared.

Finding the best fit optics model simply requires minimizing a single function, χ^2. The algorithm used in LOCO is Gauss-Newton minimization. Model parameters are adjusted to generate a ΔE such that

$$E_k^{new} = E_k + \frac{\partial E_k}{\partial K_l} \Delta K_l, \quad (3)$$

where K_l are the parameters varied to fit the response matrix, and each column of the matrix $\partial E_k/\partial K_l$ corresponds to the change in the orbit response matrix error vector with some fit parameter K_l.

Depending on the fit parameter, the LOCO code uses an analytical calculation of $\partial E_k / \partial K_l$ or uses AT [3] to numerically calculate the derivative. The model fitting is now a linear algebra problem,

$$-E_k = \frac{\partial E_k}{\partial K_l} \Delta K_l, \quad (4)$$

where the matrix $\partial E_k / \partial K_l$ is inverted to find the ΔK_l that best cancels the difference between the measured and model response matrices. There are far more data points in the response matrix than there are fit parameters, so the equation is over-constrained. LOCO uses singular value decomposition (SVD) [4] to invert the response matrix.

The change in the response matrix with quadrupole gradients is not entirely linear, so the fitting must be iterated a few times to converge to the best solution. If the initial model is far from the actual ring optics, the problem may be sufficiently nonlinear that the iterations do not converge. In the past, such problems were addressed (with varying results) by reducing the number of singular values used in SVD and/or reducing the number of fit parameters in initial iterations. More recently, Levenberg-Marquadt minimization has been added as a fitting option in LOCO when the problem is too nonlinear for Gauss-Newton minimization [5].

3. LOCO Fit Parameters

The standard parameters varied when fitting the orbit response matrix are:
quadrupole gradients, BPM gains, steering magnet calibrations, skew quadrupole gradients, BPM rolls and deformation parameters, steering magnet rolls.

The parameters in blue are primarily associated with the uncoupled quadrants of the response matrix, *M*, in equation 1 (i.e. the diagonal quadrants giving the horizontal shift at BPMs from changing horizontal steering magnets and the vertical shift when changing vertical steerers). The parameters in red are associated with the coupled, off-diagonal, quadrants of *M*. The red parameters only need to be included when the coupled parts of the response matrix are included in the fit.

Depending on the desired results or error sources, other parameters that are sometimes fit include: normal and skew gradients in sextupoles or insertion devices, steering magnet or BPM longitudinal positions, and steering magnet energy shifts.

4. LOCO with Constraints and Improved Fitting Technique

Two ways to improve the LOCO technique have been implemented – the constrained fitting method and the Levenberg-Marquadt algorithm [5-8].

- **LOCO Fitting with Constraints**

The constrained fitting is introduced to cure the degeneracy problem caused by the coupling between fitting parameters (mainly the neighboring quadrupoles). This is a common problem that occurs to many machines in different severity. The constraints are implemented by putting penalties for the step sizes between the solutions of successive iterations. It has been shown to be an efficient way to remove the less restrictive patterns from the solution and it results in fitted lattices with small changes from the starting point. This enables precise control over the machine optics, even for machines where the degeneracy problem has made the original form of LOCO not useful.

- **Levenberg-Marquadt Fitting Algorithm**

The Levenberg-Marquadt algorithm is a robust solver for general nonlinear least square problems. It is useful for LOCO in cases when the initial guessed solution is not close enough to the minimum. In such cases the Gauss-Newton solver may fail because the solution it finds could lie outside of the region where linearization of the model is valid. The Levenberg-Marquadt algorithm is based on the trust-region strategy – the solution it finds is confined in a region where the linear model is valid. The size of the trust region is controlled implicitly or explicitly and is adjusted after every iteration.

5. LOCO Applications

There are a variety of results that can be achieved with LOCO:

1. Finding actual gradient errors.
2. Finding changes in gradients to correct betas.
3. Finding changes in gradients to correct betas and dispersion.
4. Finding changes in local gradients to correct ID focusing.
5. Finding changes in skew gradients to correct coupling and vertical dispersion η_y.
6. Finding transverse impedance.

These are some of the most common LOCO applications. LOCO can also be used for various other applications, such as beam-based alignment of sextupoles (fitting normal and skew gradients in sextupoles), local chromaticity calibration (fitting response matrices measured at varying RF frequencies), and local transverse impedance measurements.

6. LOCO Applications to the NSLSII Storage Ring Commissioning

- **Application List**
  1. Calibrate/control optics using orbit response matrix and dispersion(x)
  2. Determine quadrupole gradients (x)
  3. Correct coupling and vertical dispersion (x)
  4. Calibrate BPM gains, steering magnets (to do)
  5. Measure local chromaticity and transverse impedance (to do)

1). Degeneracy Problem and Solution: A general nonlinear least-square problem is to minimize the merit function $f(\mathbf{p}) = \chi^2 = \sum[y_i - y(x_i;\mathbf{p})]^2$where *p* is a vector of the fitting parameters, (x_i, y_i) are the measured data, and *y*(*x*;p) is a nonlinear model function. The error vector is a column vector *r* whose components are $r_i = y_i - y(x_i;\mathbf{p})$. Jacobian matrix *J* is defined as, $J_{ij} = \frac{\partial r_i}{\partial p_j}$. Each column of *J* is the derivative of the error vector with respect to a fitting parameter. In a fitting, two parameters can be deeply coupled, therefore, their contributions to the merit function are indistinguishable. The coupling is characterized by$\rho_{12} = \frac{\mathbf{J}_1^T\mathbf{J}_2}{\|\mathbf{J}_1\|\|\mathbf{J}_2\|}$, $\|\cdot\|$ stands for the 2-norm of its argument. In an extreme case, it is impossible to determine the two fitting parameters. Their corresponding columns of *J* differ by only a scaling (*ρ*=1). More generally, phase advance between two quads (Δϕ) determines their coupling strength. Strong coupling occurs when $\Delta\phi \approx n\pi$, here n=0, 1, 2, etc., no matter the two quadrupoles are physically next to each other or separated.

The correlation of QH1 quad (#1) in the lattice with all the quads, including itself, is plotted in Fig. 1. Quad number 1-30, 31-60, 61-90, 91-150, 151-210, 211-240, 241-270, and 271-300 are for QH1, QH2, QH3, QM1, QM2, QL1, QL2, and QL3 quadrupoles respectively.

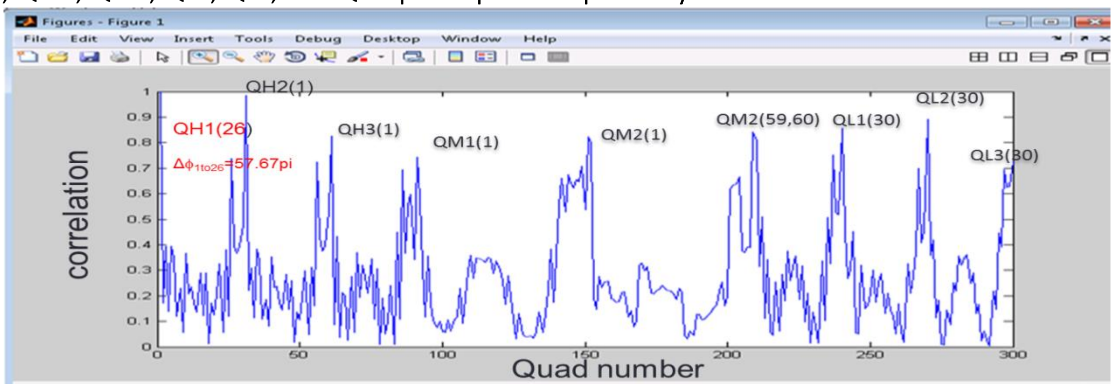

Fig. 1 The correlation of QH1 (#1) quad in the lattice with all the quads including itself.

The correlation peaks appear at the neighboring quadrupoles. LOCO is able to accurately fit the combined integrated gradient of the two magnets but it would not be able to distinguish the individual contributions when ($\Delta\phi \approx n\pi$). No single "best solution". Any solution differs from the global minimum by less than a certain value determined by the noise level is a valid and equivalent solution. Therefore, LOCO is not easy to carry out in NSLSII Storage Ring.

In some cases, LOCO fitting converges to an unrealistic solution with large changes to the quadrupole strengths Δ*K*. The quadrupole gradient changes can be so large that the resulting lattice model fails to find a closed orbit and subsequent iterations become impossible. It happens in the Storage Ring (SR) commissioning at the shift on April 30th 2014. Scaled Levenberg-Marquardt method was chosen in the LOCO fitting. The solution has Δ*K* larger than realistic and along with a spurious zigzag pattern between adjacent quadrupoles. Chi-square was reduced by a factor of 400 at the 3rd iteration, the ratio of quadrupole strengths *K*(4) and the design values *k*(1) is plotted in Fig.2.

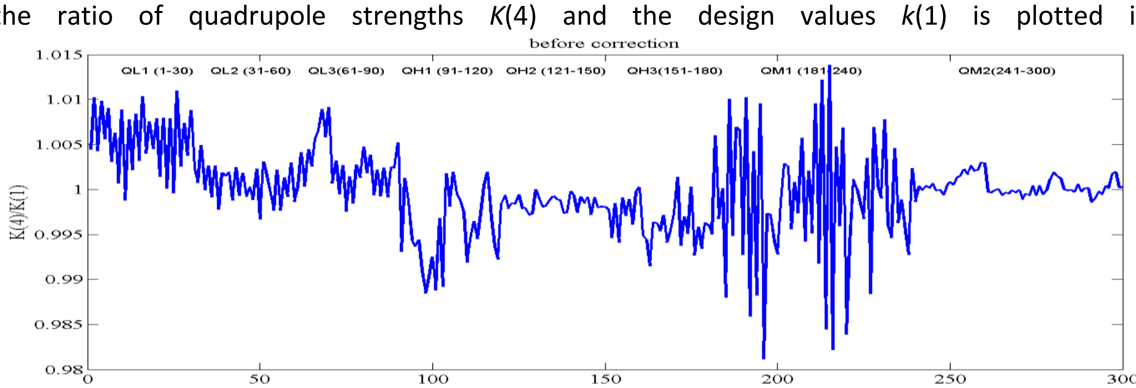

Fig.2 The ratio of quadrupole strengths *K*(4) at the 3rd iteration and the design values *k*(1).

The fitting scheme is less restrictive over certain patterns of changes to these quadrupoles with which the correlated quadrupoles fight each other and the net effect is very inefficient χ2 reduction, i.e., small χ2 reduction with large changes of Δ*K*. Such a solution is often not very useful in optics correction because after the solution is dialed in, the quadrupoles do not respond as predicted by the lattice model due to magnet hysteresis.

We will show that adding constraints to the fitting parameters is an effective way to combat this problem of LOCO [6-8]. Optimizing the weights of less constrained fit parameters and applying them for the SR commissioning *via* the following numerical study: In the error lattice, the amplitude of random quad strength errors +/- 0.3% is applied to all the quadrupoles. Quad gradient fit errors, as shown in Fig.3, imply more weights should be given to QL1, QL3, QH1, and QM1 families. We chose ten times larger weight, 0.1, for them.

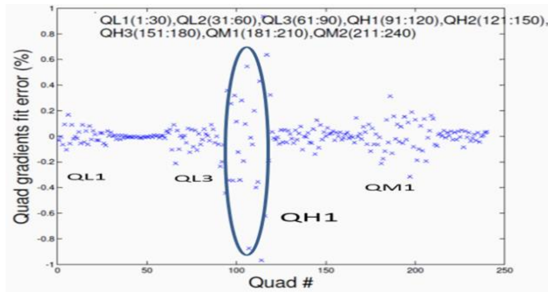

Fig.3 Quad gradient fit error, which is defined as the difference between the quadruple gradient found by LOCO and the gradient in the initial error lattice divided by the gradient in the ideal lattice.

Apply constrains to the LOCO fitting: At the 3rd iteration, chi-square is reduced by a similar amount; however, the ratio of the quadrupole strengths *K*(4) and the design values *k*(1) is reduced by a factor three in rms, as shown in Fig.4.

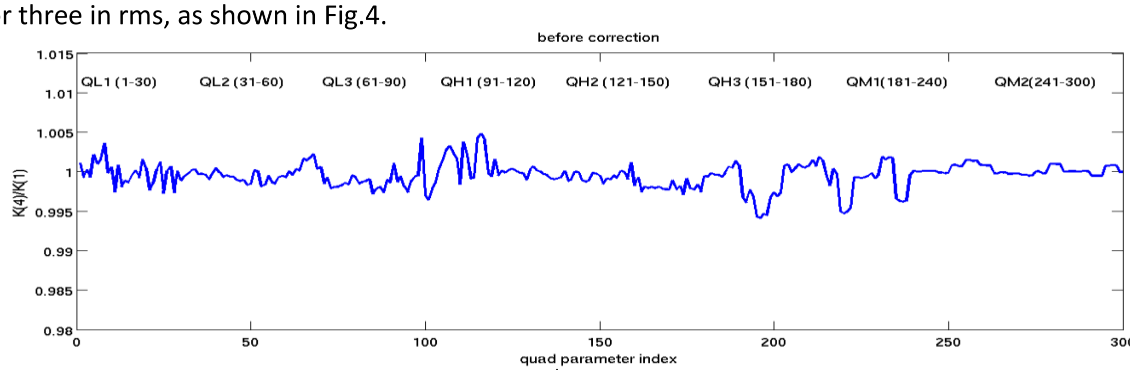

Fig.4 The ratio of the quadrupole strengths *K*(4) at the 3rd iteration of LOCO fitting with constraints and the design values *k*(1).

Besides, we have checked the degeneracy of all other fit parameters. There are no correlations between different BPM calibrations and different corrector calibrations. There exist some correlations between quad gradient and BPM calibration, quad gradient and corrector calibration, and corrector and BPM calibrations. They are often small, except the one between corrector and BPM calibrations. However, including the dispersion in the fit can provide an absolute calibration of the BPMs and correctors. Without dispersion there is always degeneracy between the scaling of BPMs and corrector magnets. This will create a small singular value per plane which needs to be removed. If dispersion is included and the RF frequency is not fit (which is often the case since the accuracy of RF changes is usually very good) then the horizontal degeneracy should disappear. If there is substantial vertical dispersion then the vertical degeneracy will also disappear. To force this, the coupling is sometimes increased just to obtain a good vertical BPM calibration. [5].

2). LOCO Improvements

Problems:

- It takes 80min for a complete orbit response matrix (ORM) measurement (360x360). The size of a Jacobian matrix *J* is (360x360) by n_{fit}. n_{fit} is the number of fit parameters, ~(4x360+300+30).

- SVD(*J*) is slow, several minutes.

Solutions:

- Implement SVD(J^T•*J*) successfully in the routine 'loco.m' before the commissioning. SVD(J^T•*J*) takes seconds. Here, the size of (J^T•*J*) is n_{fit}xn_{fit}.
- Reduce the number of correctors used in the ORM measurement from 180 per plane to 60 while keeping all the BPMs for maximizing the information in the shift of April, 17th, 2014, no significant degradation appears in the LOCO fitting result since there are still two correctors per betatron period. The time for a LOCO measurement is ~27min, a factor of three less.

It is realistic to apply LOCO for the beta beating and coupling correction since it takes only 30min for an iteration.

3). Beta Beating Correction

Fix twiss using normal quads. Iterate several times!

Apply the LOCO fitting result in Fig. 4 to correct the linear optics. Tunes measured using the turn-by-turn (TBT) data are 33.21 in x and 16.24 in y, and LOCO fitted tunes are 33.207 and 16.243. They agree reasonably well. Beta beat was reduced down to 2.5% (rms) in x, and 4.1% in y. Beta x (top) and beta y (bottom) are shown in Fig.5. Here, red and blue curves are the LOCO fitted result and the design model respectively. Injecting to the SR using this lattice, the injection efficiency improves to ~95 to 96%. Beta beat x (top) and beta beat y (bottom) are shown in Fig.6.

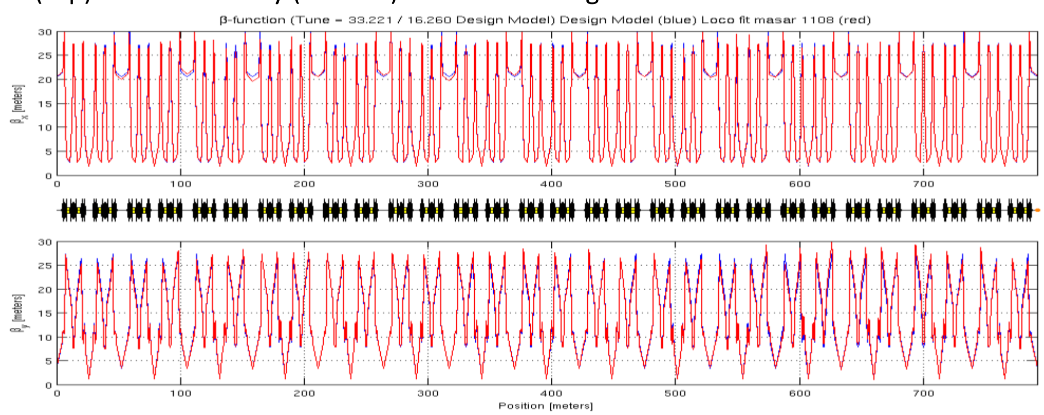

Fig.5 Beta x (top) and beta y (bottom). Here, red and blue curves are the LOCO fitted result and the design model respectively.

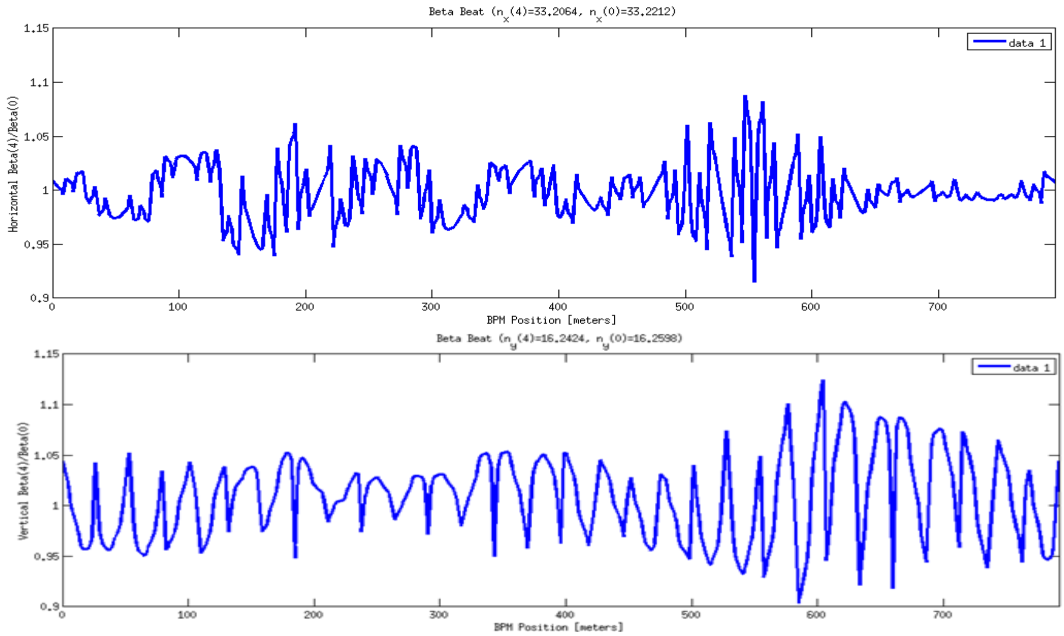

Fig.6 Beta beat x (top) and beta beat y (bottom). Beta beat is defined as the ratio of the beta at the 3rd iteration of the LOCO fitting and the design beta.

Apply the LOCO correction again for the purpose of bringing the tunes closer to the design values. The TBT measured tunes are 33.22 and 16.25. The LOCO fitted tunes are 33.217 and 16.252. They agree well. Beta beat is corrected down to 2.7% (rms) in x, and 3.1% in y. Beta x (top) and beta y (bottom), as shown in Fig 7, red and blue curves are the LOCO fitted beta and design model beta respectively.

Injecting using this lattice, the injection efficiency further improves to ~99%.

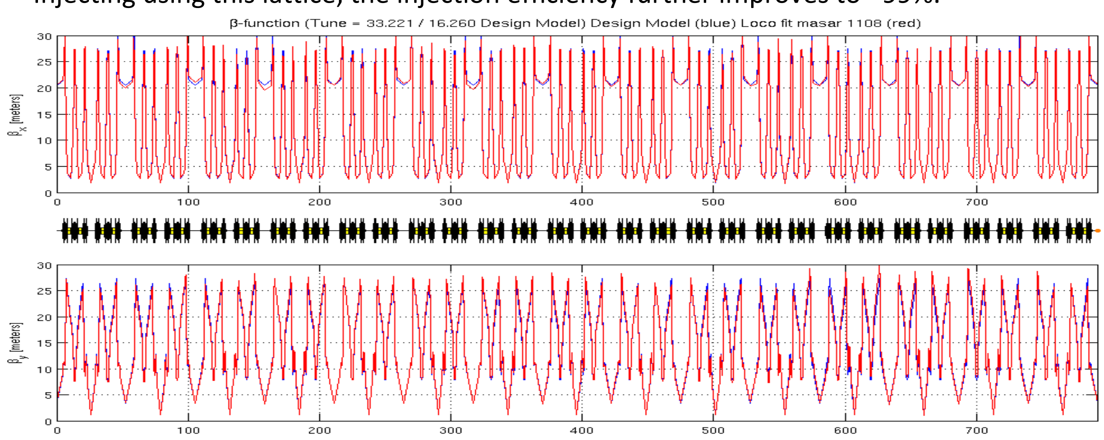

Fig.7 Beta x (top) and beta y (bottom). Red and blue curves are the LOCO fitted beta and ideal model beta respectively.

After the beta beating correction, the changes of all the quad strengths relative to their design values are plotted in Fig.8, each quadrupole family per plot.

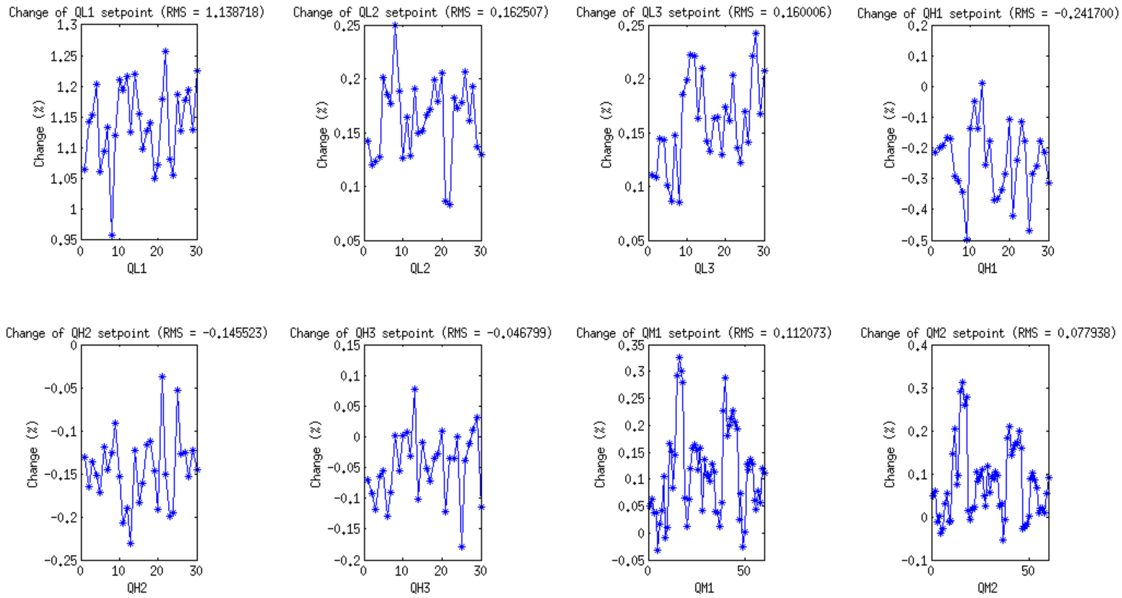

Fig.8 The change of quad strengths relative to their design values in %, each quad family per plot.

4). Coupling and vertical dispersion correction:

Sources of dispersion: There are two main terms that can create vertical dispersion:

$$\eta_y'' + K\eta_y = \frac{1}{\rho_y} - K_s \eta_x$$

– Dipole errors (steering magnets, misalignments, …) or intentional vertical bending magnets

– Skew quadrupole fields at the location of horizontal dispersion (due to quadrupole tilts, or vertical offsets in sextupoles)[9]

$$\kappa_{\eta_y} = \int ds\, K_s \eta_x \sqrt{\beta_y} e^{i\phi_{\eta_y}}$$

$$\frac{\phi_{\eta_y}}{2\pi} = \mu_y(s) - \frac{s}{C}(\nu_y - 5)$$

Vertical dispersion directly causes the increase of the vertical emittance by quantum excitation.

Dispersion correction scales like product of horizontal dispersion times the square root of vertical beta function time skew quadrupole strength. Dispersion from steering magnets scales like the bending angle.

Use accelerator toolbox (Andrei Terebilo) and Matlab LOCO for the simulations.

Response Matrix fitting -'deterministic', small number of iterations.

Optimize weight of dispersion in LOCO fit

- The relative contribution of vertical dispersion and coupling to the vertical emittance depends on the particular lattice (and the particular error distribution).
- Therefore the optimum weight for the dispersion in the LOCO fit has to be determined (experimentally or in simulations).
- The larger the weight factor, the better the vertical dispersion gets corrected, but eventually the coupling 'explodes'.
- Set weight to optimum somewhat below that point.

In the SR commission, we set comparably large weight ~60 (5 times higher than the LOCO default value) on the vertical dispersion in order to correct vertical dispersion effectively. As shown in Fig.9, the vertical dispersion was significantly reduced from the blue curve (before the correction) to the red curve (after the correction). Further increasing the weight doesn't reduce the vertical dispersion and made the x-y coupling worse.

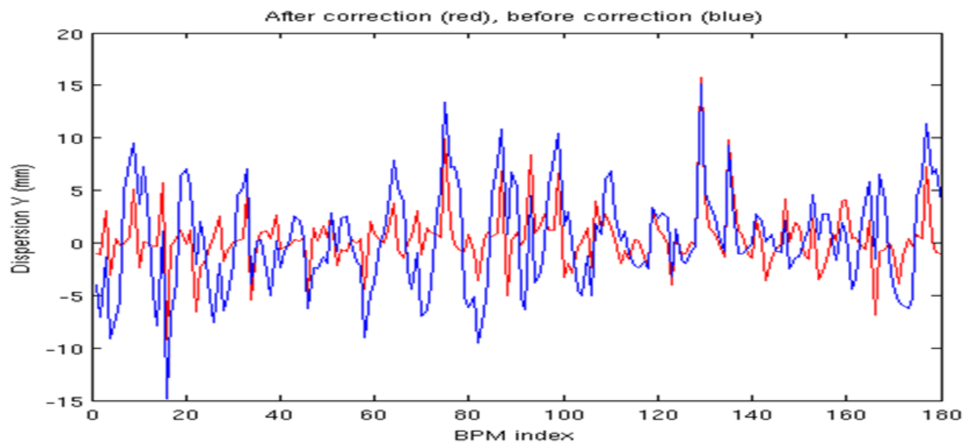

Fig.9. The vertical dispersion before the correction (blue) and after the correction (red).

The coupling calculated using the LOCO fitted machine model is ~0.3% after the beta beating and vertical dispersion correction. The equilibrium beam envelope is calculated using Ohmi's beam envelope formalism [10]. The coupling is defined as the ratio of the mean vertical and horizontal beam emittance.

Radiation-Diffusion matrix is calculated only for the elements with radiation, including dipoles, quadrupoles, and sextupoles, and set to zero for the rest elements. The cumulative radiation-diffusion matrix at the ring entrance is needed for the envelope matrix calculation. The equilibrium beam envelope, including 6-by-6 equilibrium envelope matrix, tilt angle of the XY ellipse, and rms size along the principal axis of a tilted ellipse, is obtained for each ring element. The tilt in degree (top left), emittance x in nm·rad (top right), emittance y (bottom right), and horizontal and vertical beam sizes in μm (bottom left) are shown in Fig.10.

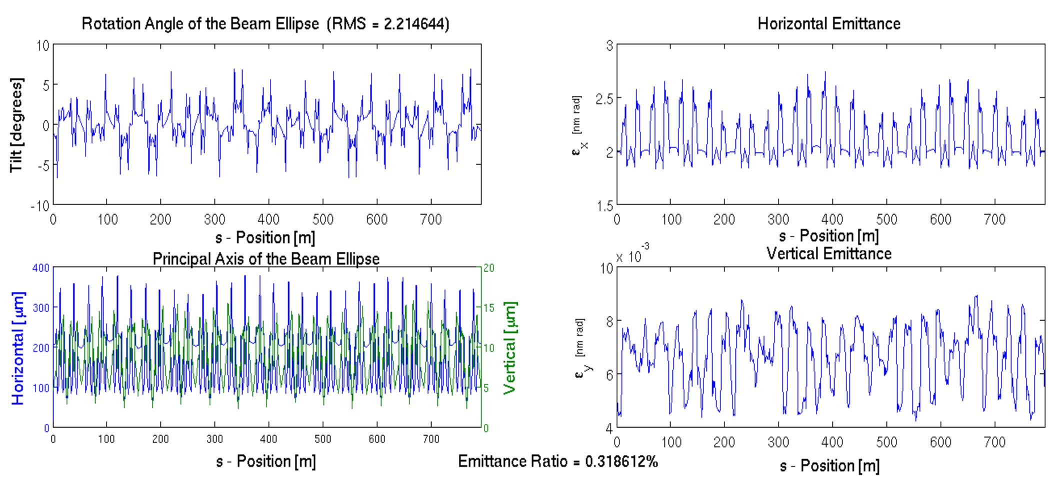

Fig.10 The tilt in degree (top left), emittance x in nm·rad (top right), emittance y (bottom right), and horizontal and vertical beam sizes in μm (bottom left).

7. TBT twiss measurement:

NSLSII SR is equipped with multiple TBT BPMs. The TBT beam position signal is a combination of various source signals. **x**(t) = **As**(t) + **n**(t). **A** is the mixing matrix. There are only a few meaningful source signals, such as betatron oscillation and synchrotron oscillation. Form a matrix of the BPM data

$$\mathbf{X} = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(T) \\ x_2(1) & x_2(2) & \cdots & x_2(T) \\ \vdots & \vdots) & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(T) \end{pmatrix}$$

, m BPMs and T turns [11,12].

It has been proven [13] that when the BPM reading contains only one betatron mode, i.e.

$$x_m(t) = \sqrt{2J(t)\beta_m}\cos(\phi(t)+\psi_m) \,.$$

Then there are only two non-trivial SVD eigen-modes because each BPM sees different phase.

$$\mathbf{x} = \mathbf{USV}^{\mathrm{T}} = s_+\mathbf{u}_+\mathbf{v}_+^{\mathrm{T}} + s_-\mathbf{u}_-\mathbf{v}_-^{\mathrm{T}}$$

$$u_{+,m} = \frac{1}{s_+}\sqrt{<J>\beta_m}\cos(\phi_0+\psi_m), \qquad v_+(t) = \sqrt{\frac{2J(t)}{T<J>}}\cos(\phi(t)-\phi_0),$$

$$u_{-,m} = \frac{1}{s_-}\sqrt{<J>\beta_m}\sin(\phi_0+\psi_m) \qquad v_-(t) = -\sqrt{\frac{2J(t)}{T<J>}}\sin(\phi(t)-\phi_0)$$

u: spatial vector **v**: temporal vector

Beta function and betatron phase advance can be calculated from the spatial vector.

$$\psi_m = \tan^{-1}\left(\frac{s_-u_{-,m}}{s_+u_{+,m}}\right) \qquad \beta_m = \frac{1}{<J>}[(s_+u_{+,m})^2 + (s_-u_{-,m})^2]$$

Independent Component Analysis (ICA) for TBT Data Analysis

What SVD essentially does is to de-correlate the sample data to find an orthogonal basis to reinterpret the sample data. Although there are numerous transformations that de-correlate the sample data, the result is unique because SVD makes sure the first component has the largest variance and the second component has the second largest variance and so on. However, SVD does not guarantee separation of the source signals, especially when the singular values (SVs) approach each other, SVD always produces modes with mixing. On the other hand, ICA does not show any dependence on the relative magnitudes of the SVs. ICA makes use of the fact that the power spectra of source signals are distinct and the autocorrelation covariance matrices are diagonal to find the source signals [11].

ICA method can de-couple the normal modes in the presence of linear coupling. It finds the paired betatron modes automatically, as shown in Fig.11, beta x mode (left) and beta y mode (right); therefore, the phase and beta at all the BPM positions are obtained.

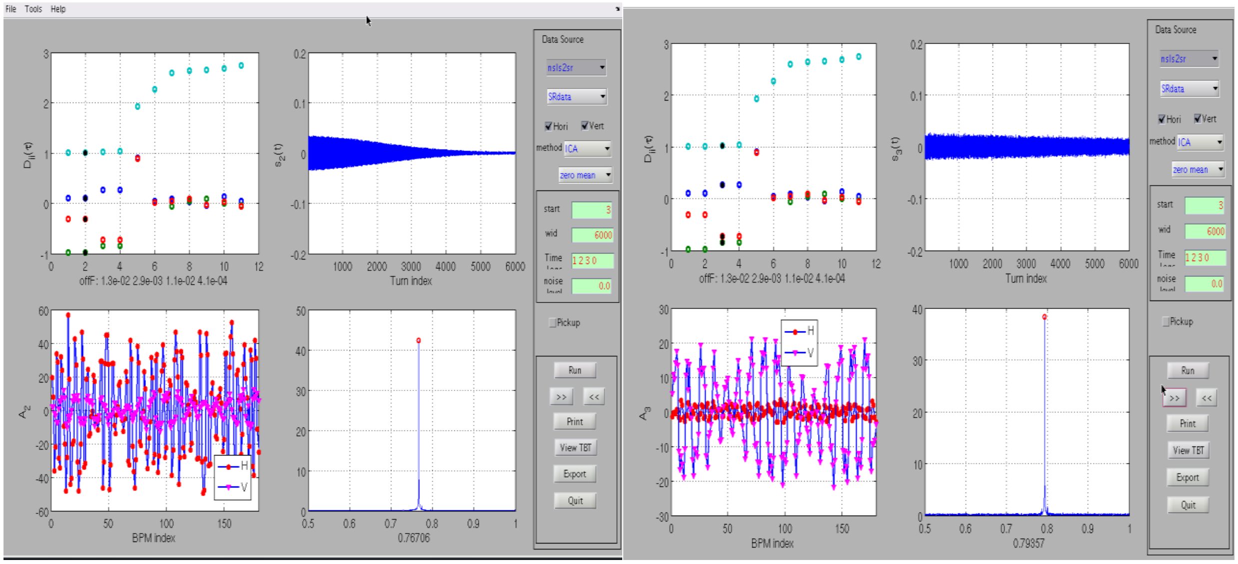

Fig.11 Beta x mode (left) and beta y mode (right). There are four figures for each of them. We label the top left figure as Figure 1 and other figures Figure 2, 3, 4, clockwise. Figure 1 shows the diagonal elements after joint approximate diagonalization for each mode. Different colors are used for the time-lags. The current mode is marked with '*'. Two modes are paired if their diagonal elements are the same. Figure 2 shows the temporal pattern, which is normalized so that ||s||=1. Figure 3 shows the FFT of the temporal pattern. A red circle is used to mark the peak frequency, which is shown in x-label, too. Figure 4 shows the spatial pattern. If both horizontal and vertical data are used, there will be two curves with different colors, red for H, magenta for V.

The TBT and ORM data were taken at the shift on April 18th 2014. Beta beats fitted by LOCO (blue) and measured by TBT data (red) are shown in Fig. 12, beta beat in x (left) and beta beat in y (right). They are in a reasonably good agreement. The LOCO fitted tunes are 0.2263/0.1997, and the TBT measured tunes are 0.2329/0.2064. The agreement on the tunes is not as good as the measurements, which were done after the beta beating correction and are described in 'Beta Beat Correction'.

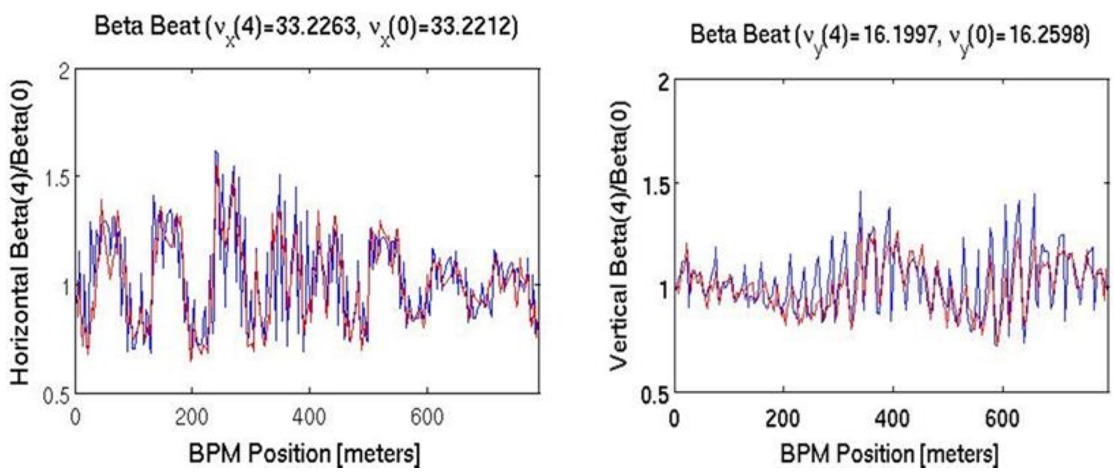

Fig.12 Beta beats fitted by LOCO (blue) and measured by TBT data (red). Beta beat x (left) and beta beat y (right).

For the two different approaches of the beta beating correction: LOCO and TBT method, the strategy is to apply the TBT method to correct beta beating down to a several-percentage level since it is faster; afterwards, apply LOCO to correct beta beating further down to a sub-percentage level. This two-step approach has been approved successfully in the SR commissioning.

Acknowledges:

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