BNL-211214-2019-TECH
NSLSII-ASD-TN-282

# Beam-induced power in HEX SCW

A. Blednykh,

September 2018

Photon Sciences
**Brookhaven National Laboratory**

**U.S. Department of Energy**
USDOE Office of Science (SC), Basic Energy Sciences (BES) (SC-22)

# DISCLAIMER

| NSLS II TECHNICAL NOTE<br>BROOKHAVEN NATIONAL LABORATORY | NUMBER<br>**NSLSII-ASD-TN282** |
|---|---|
| AUTHOR<br>**V. Smaluk, A. Blednykh, B. Podobedov, T. Tanabe, C. Hetzel** | DATE<br>**09/05/2018** |
| TITLE<br>**Beam-induced power in HEX SCW** | |


### *Abstract*

*A superconducting wiggler (SCW) is planned to be installed in NSLS-II for HEX beamline. Interaction of the electron beam with the resistive-wall impedance of the low-gap SCW liner and with the geometric impedance of the transition sections results in excitation of electromagnetic fields, dissipation of which contributes heat load in the cold volume. The beam-induced power caused by the beam-impedance interaction is estimated using analytical formulae and computer simulations.*


## Coherent energy loss

The beam-induced power in a structure with non-zero impedance is calculated by the formula:

$$P = k_{||} \frac{I^2}{N_b f_0}\,, \qquad (1)$$

where $I$ is the average beam current, $N_b$ is the number of bunches in the train, $f_0$ is the revolution frequency. Here $k_{||}$ is the longitudinal loss factor

$$k_{||} = \frac{1}{\pi} \int_0^\infty Z_{||}(\omega) \tilde{\lambda}^2(\omega) d\omega\,, \qquad (2)$$

where $Z_{||}(\omega)$ is the longitudinal impedance and $\tilde{\lambda}(\omega)$ is the bunch spectrum (Fourier transform of the longitudinal bunch profile $\lambda(t)$. For a Gaussian bunch, $\tilde{\lambda}^2(\omega) = e^{-\omega^2 \sigma_t^2}$, where $\sigma_t = \frac{\sigma_z}{c}$ and $\sigma_z$ is the r.m.s. bunch length.

The impedance includes geometric and resistive-wall components, the first one is obtained by numerical solution of Maxwell equations, and the second one is calculated using analytical formulae.

## Geometric impedance of the transitions

The geometric impedance is calculated by Gdfidl code [1] using a simplified model of the SCW vacuum chamber. The chamber includes a flat part with the length of 1.2 m and with the vertical aperture of 10 mm representing the liner and two tapered transitions with the vertical aperture changing linearly from $2b = 10$ mm up to $2d = 25$ mm. Fig. 1 shows the model used for the simulation.

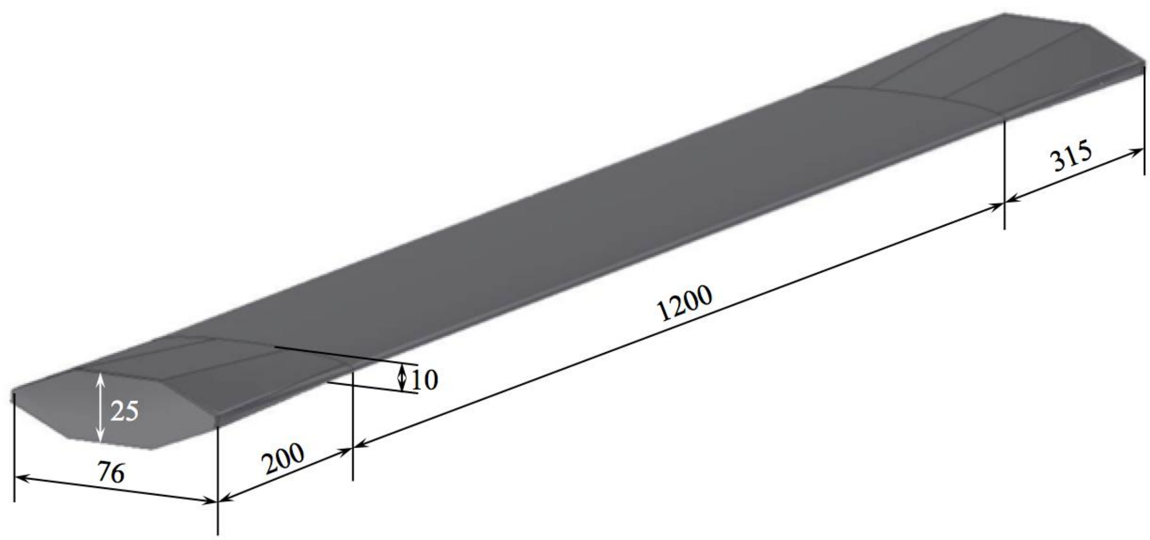


Fig. 1. SCW model used for wakefield simulations.

The real part of the geometric impedance of two tapered transitions is shown in Fig. 2 together with the bunch power spectrum $\tilde{\lambda}^2(\omega)$ calculated for a Gaussian bunch with r.m.s. length $\sigma_z = 3$ mm.

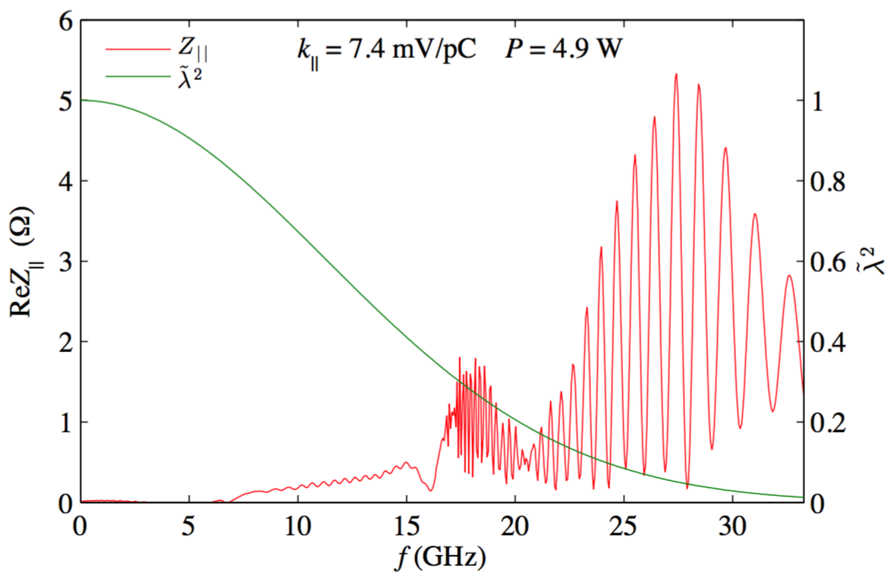


Fig. 2. Geometric impedance of SCW model vacuum chamber.

The loss factor calculated using the impedance shown in Fig. 2 and a Gaussian bunch with $\sigma_z = 3$ mm is 7.4 mV/pC. Assuming $I = 500$ mA and $N_b = 1000$, the beam-induced power is 4.9 W.

### Resistive-wall impedance of the transitions (normal skin effect)

To estimate the contribution of the tapered transitions, we assume they are normal-conducting copper plates. If the transition is modeled by two infinite plates, the longitudinal resistive-wall impedance per unit length $Z_{||}/L$ is:

$$Z_{||}/L = \frac{1-i}{2\pi b}\sqrt{\frac{Z_0 \mu_r \omega}{2c\sigma_c}}\,, \tag{3}$$

here $b$ is the vertical half-aperture. Following the impedance, the beam-induced power per unit length $P_{||}/L$ is inversely proportional to the aperture. For a tapered transition, the major power loss is concentrated at the narrow end of the transition, close to the cold part of the device, as shown in Fig. 3.

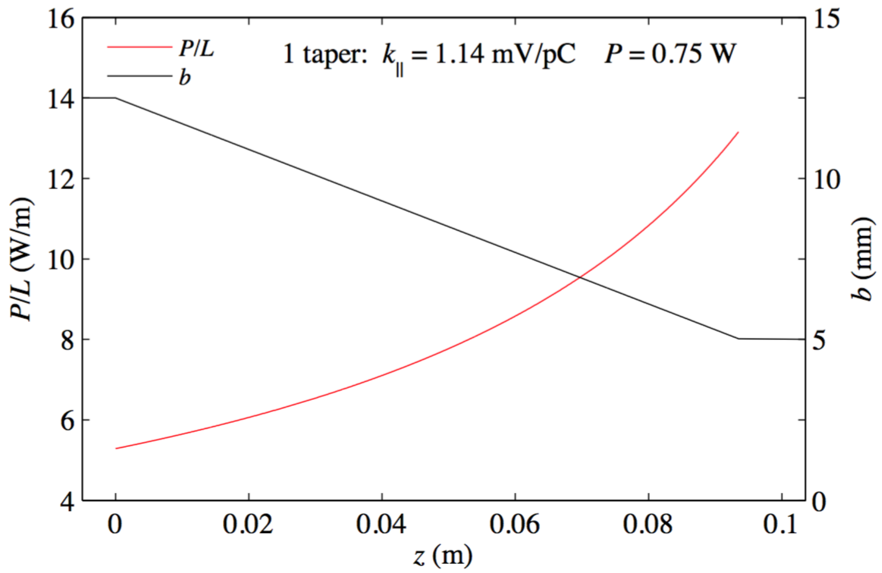


Fig. 3. Beam-induced power per unit length caused by resistive-wall impedance of one tapered transition.

Assuming a Gaussian bunch with $\sigma_z = 3$ mm, $I = 500$ mA and $N_b = 1000$, the loss factor is 2.3 mV/pC and the integrated power is about 1.5 W for two transitions.

### Resistive-wall impedance of the liner (anomalous skin effect)

To calculate the resistive-wall impedance of the superconducting copper liner (anomalous skin effect), a set of formulae published in [2-4] are used.

For a round vacuum chamber, the longitudinal coupling impedance per unit length $Z_{||}/L$ is proportional to the surface impedance $Z_s$:

$$Z_{||}/L = \frac{Z_s}{2\pi b}\,, \tag{4}$$

where $b$ is the chamber radius. The real part of the surface impedance $Z_s$ is the surface resistance

$$R_s = \sqrt[3]{\frac{\sqrt{3} Z_0^2 \omega^2}{16\pi c^2} l/\sigma_c}\,(1 + 1.157\varsigma^{-0.2757})\,, \tag{5}$$

where $\varsigma = \frac{3}{2}\left(\frac{l}{\delta_s}\right)^2$, $l$ is the electron mean free path, $\delta_s = \sqrt{\frac{2c}{Z_0 \mu_r \sigma_c \omega}}$ is the skin depth, $Z_0 = 377\Omega$ is the free-space impedance, $\mu_r$ is the relative permeability, $\sigma_c$ is the conductivity. At cryogenic temperatures, the conductivity is proportional to the residual resistivity ratio defined as $RRR = \frac{\rho(273\text{ K})}{\rho(4.2\text{ K})}$. Commercially pure copper has the residual resistivity ratio of 50 to 500, whereas very high-purity copper, well-annealed, could have an RRR of around 2000 [5]. Electrical resistivity of annealed oxygen-free copper is shown in Fig. 4 as a function of temperature. Several curves corresponding to RRR from 30 to 3000 are presented.

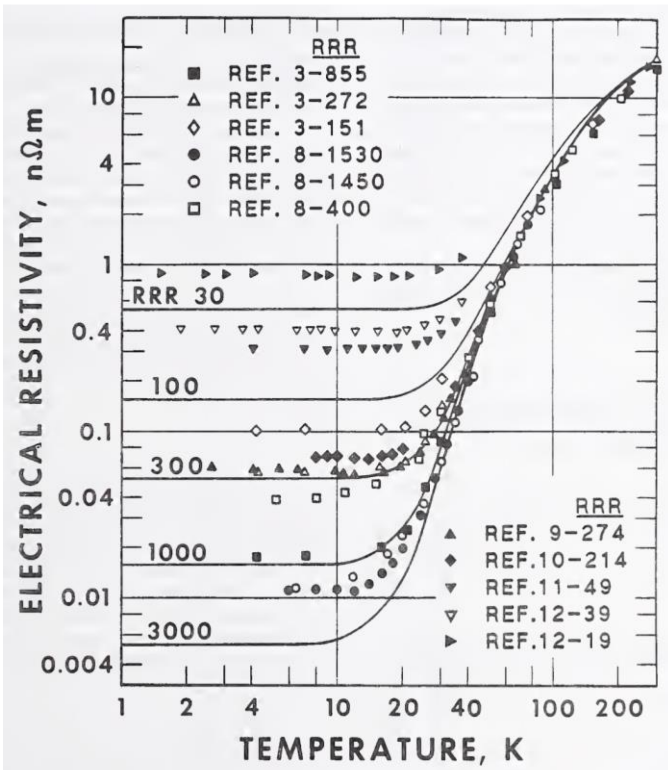


Fig. 4. Resistivity of annealed oxygen-free copper as a function of temperature.

The ratio $l/\sigma_c$ is independent of temperature, $l/\sigma_c = 6.6 \cdot 10^{-16}\Omega\text{m}^2$ for copper. Formula (4) is valid for $\varsigma \geq 3$. If $l \gg \delta_s$ (extreme anomalous skin effect), the second term in the brackets becomes negligible and the surface resistance is

$$R_{s\infty} = \sqrt[3]{\frac{\sqrt{3} Z_0^2 \omega^2}{16\pi c^2} l/\sigma_c} \,. \tag{6}$$

Fig. 5 shows the loss factor (2) and the beam-induced power (1) as a function of the residual resistivity ratio. The calculations are done for the HEX SCW with the length of 1.2 m and the gap of 5 mm using NSLS-II machine and beam parameters: $I = 500$ mA, $N_b = 1000$, $f_0 = 378.6$ kHz, $\sigma_z = 3$ mm. The dashed lines represent the loss factor and the power calculated in the limit of extreme anomalous skin effect.

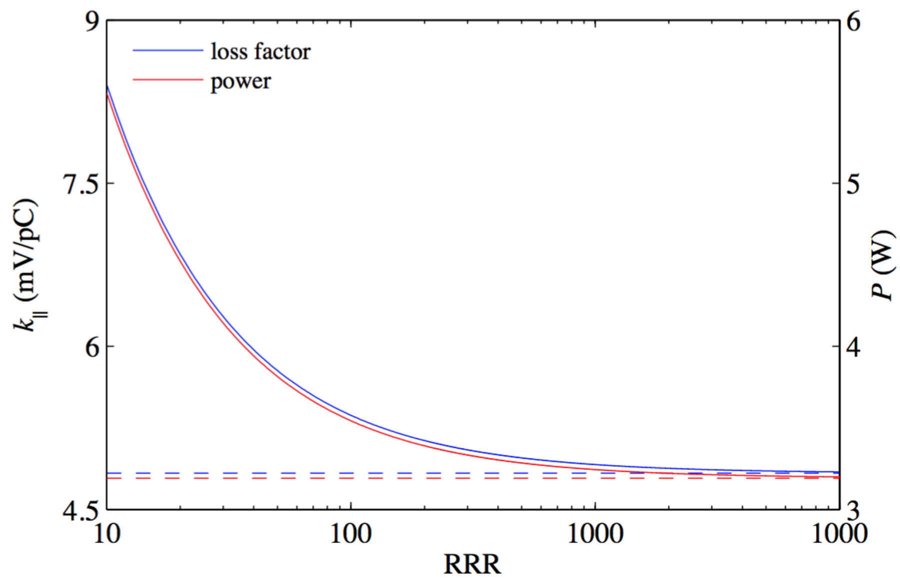


Fig. 5. Loss factor and beam-induced power caused by resistive-wall impedance of the cold liner.

#### Total beam-induced power

Combining the contributions from both geometric and resistive-wall impedance including the cold liner and warm transitions, we estimate the total beam-induced power in the HEX SCW. Fig. 6 shows the loss factor (2) and the beam-induced power (1) as a function of the r.m.s. bunch length. There three lines correspond to the liner resistive-wall impedance for RRR=30, RRR=100, and for the extreme anomalous skin effect.

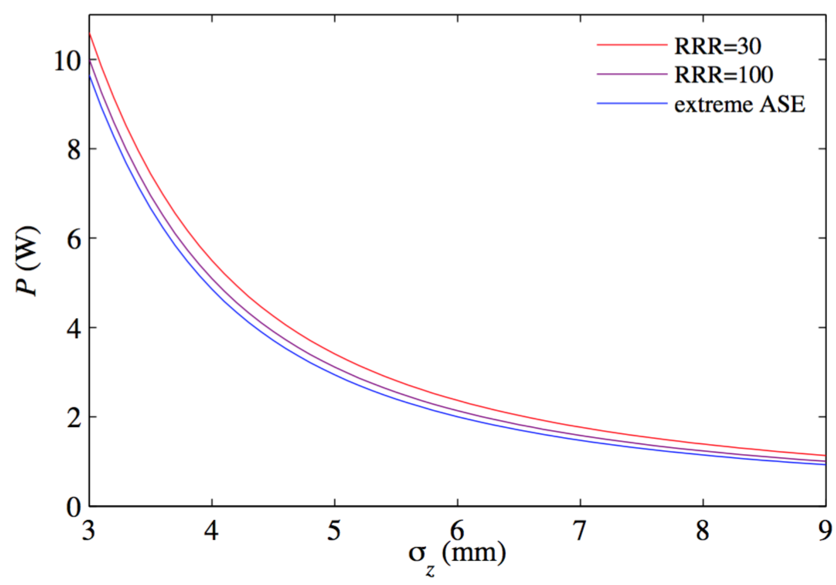


Fig. 6. Beam-induced power caused by geometric and resistive-wall impedance.

Note that the power induced by the beam interaction with geometric impedance is not necessarily to dissipate where the wakefields are generated, a part of it can propagate in the vacuum chamber and dissipate in other locations without contribution to the SCW heat load.

## References


1. The GdfidL Electromagnetic Field simulator, http://www.gdfidl.de/

2. G. E. H. Reuter and E. H. Sondheimer, "The theory of the anomalous skin effect in metals", Proc. R. Soc. A 195, 336 (1948).

3. S. Casalbuoni, A. Grau, M. Hagelstein, et al, "Beam heat load and pressure rise in a cold vacuum chamber", Phys. Rev. ST Accel. Beams 10, 093202 (2007).

4. B. Podobedov, "Resistive wall wakefields in the extreme anomalous skin effect regime", Phys. Rev. ST Accel. Beams 12, 044401 (2009).

5. N. J. Simon, E. S. Drexler, and R. P. Reed, "Properties of Copper and Copper Alloys at Cryogenic Temperatures", NIST Monograph 177 (1992) p.301