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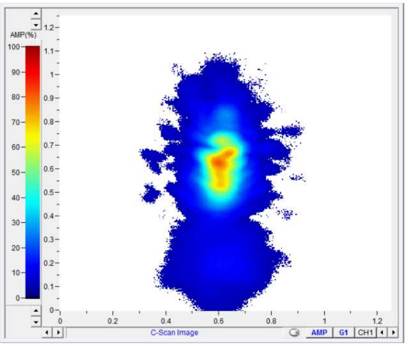

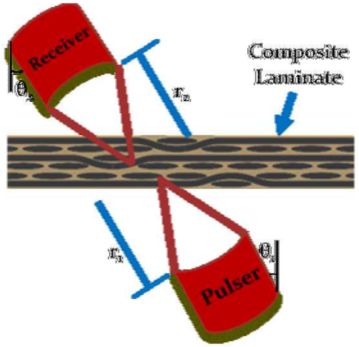

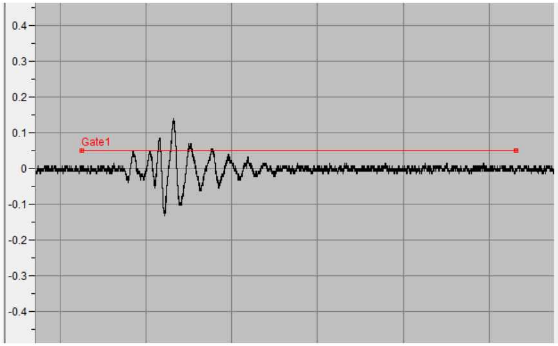

Combining Inspection Data, Modeling and Optimization Techniques to Identify the Elastic Constants of a Woven Fiber Reinforced Laminated Composite

Sarah L. Stair, David G. Moore, and Byron P. Newberry

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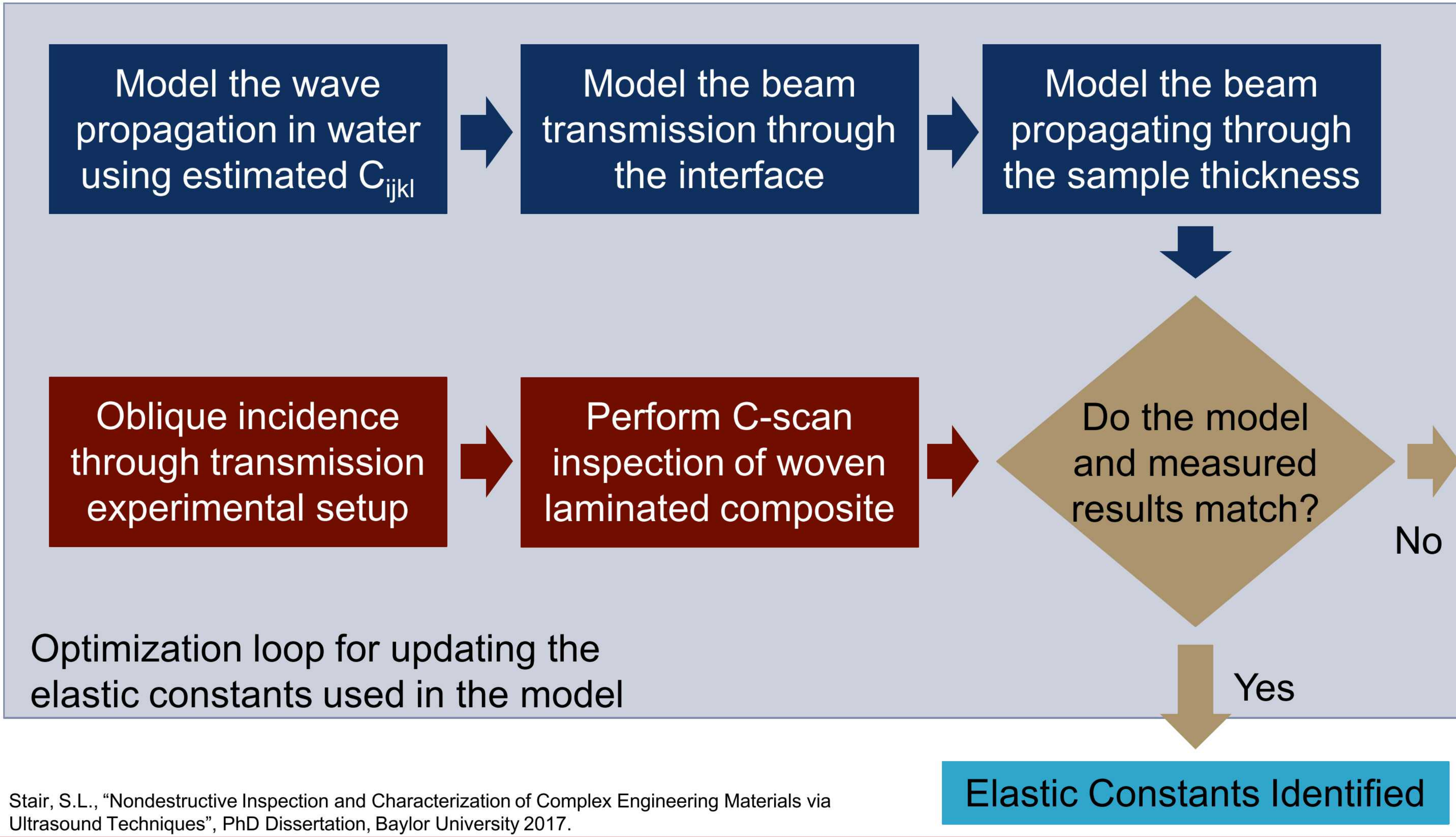
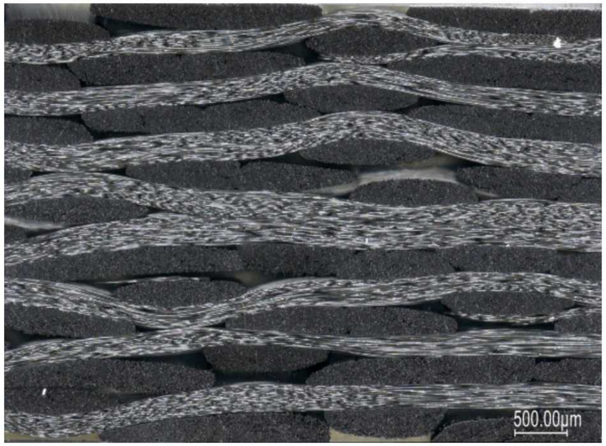
Project Objective



Stair, S.L., "Nondestructive Inspection and Characterization of Complex Engineering Materials via Ultrasound Techniques", PhD Dissertation, Baylor University 2017.

Background

- Benefits of fiber reinforced composite laminates
  - High strength to weight ratio
  - Properties are tailorable depending on ply stacking sequence and choice of fiber fabric
  - Recent increase in use for reducing fuel consumption and improving impact resistance in automobiles
- Challenges posed by composite laminates
  - Non-homogeneous material
  - Viscoelastic – properties change with time
  - Ultrasound waves attenuate and scatter within the laminate
  - Features, such as fiber waviness and overlapping tows, are difficult to identify

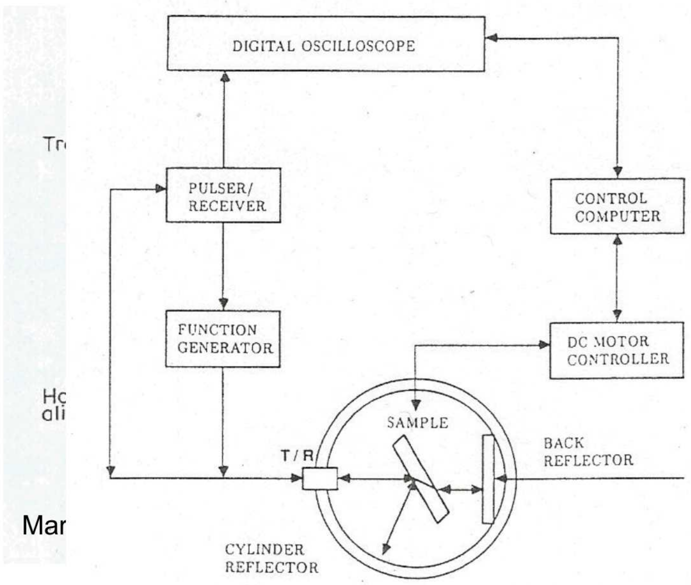


H. Brinson and C. Brinson. *Polymer Engineering Science and Viscoelasticity: An Introduction*, *1st Edition.* Springer US, 2008.

Stair, S., Moore, D. and Jack, D. Longitudinal and Shear Velocity Measurements in a Woven Fiber Reinforced Laminated Composite. *American Society of Non-Destructive Testing 24th Research Symposium*, Anaheim, California, March 2015.

Previous Work (1 of 2)

- Markham (1970)
  - Determined transversely isotropic elastic constants
  - Through transmission, immersion ultrasound
  - Determination made via A-scans
  - Sample rotated relative to probes
- Rokhlin and Wang (1989, 1992)
  - Double through transmission ultrasound
  - Part rotated relative to the transducer
  - Unidirectional laminate
  - Identified that Markham used the group velocity instead of the phase velocity



Rokhlin and Wang (1989)

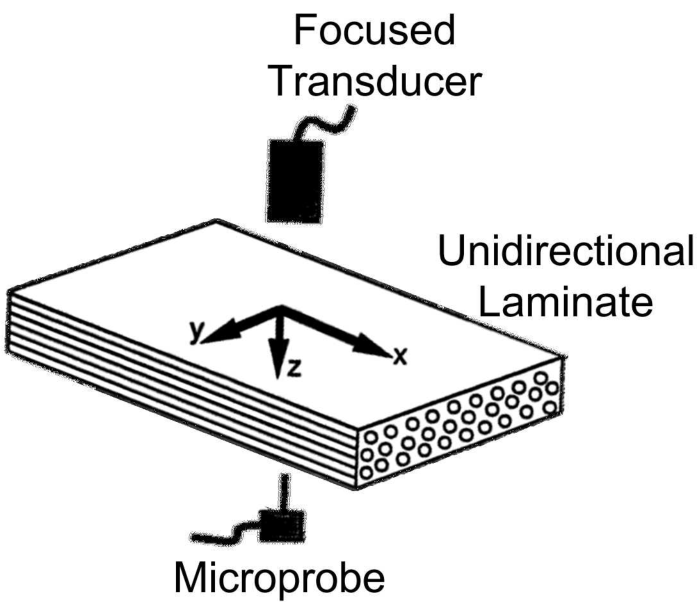
Markham, M.F., "Measurement of the Elastic Constants of Fibre Composites by Ultrasonics," Division of Molecular Science, National Physical Laboratory, Teddington, Middlesex, England, *Composites*, March, 1970.

Rokhlin, S.I. and W. Wang, "Ultrasonic Evaluation of In-Plane and Out-of-Plane Elastic Properties of Composite Materials," *Review of Progress in Quantitative Nondestructive Evaluation,* **8B,** D.O. Thompson and D.E. Chimenti (Eds.), Plenum, New York, 1989.

Rokhlin, S.I. and W. Wang, ""Double Through-Transmission Bulk Wave Method for Ultrasonic Phase Velocity Measurements and Determination of Elastic Constants of Composite Materials," The Ohio State University, Columbus, Ohio, *American Journal Acoustics Society*, **91**, June 6, 1992.

Previous Work (2 of 2)

- Newberry (1988, 1989)
  - Through transmission ultrasound
  - Transversely isotropic laminate
  - Developed mathematical model using paraxial approximation of ultrasound wave
  - Compared model with results



Newberry (1988)

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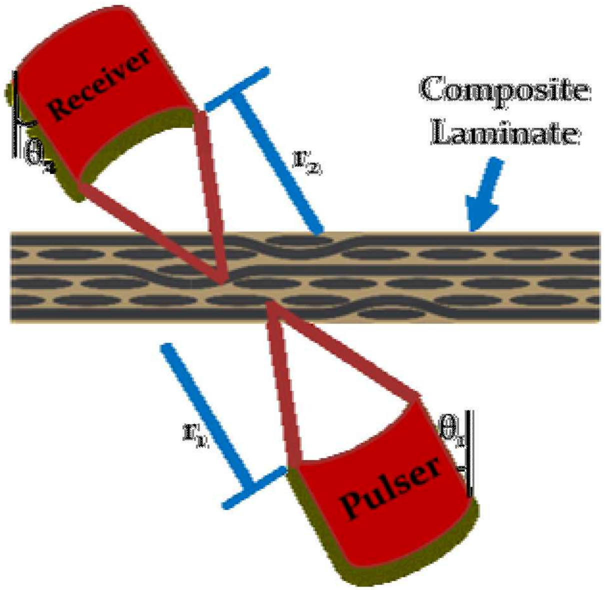
Oblique Incidence Through Transmission Inspection

- Sample Description
  - 8 HS carbon fiber fabric
  - Fabric preimpregnated with UF3352 TCR™ resin
  - Laminate stacking sequences
    - 4 ply $[0, 90]_{2,s}$
    - 8 ply $[0, 90]_{4,s}$
    - 12 ply $[0, 90]_{6,s}$
- Inspection
  - Oblique incidence through transmission
  - 5 MHz spherically focused probes
  - C-scan data stored for post-processing



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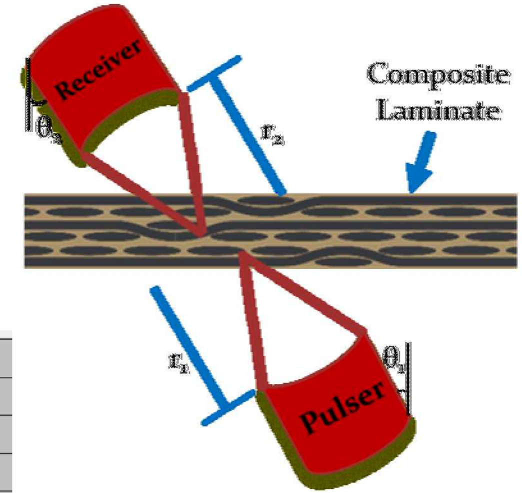
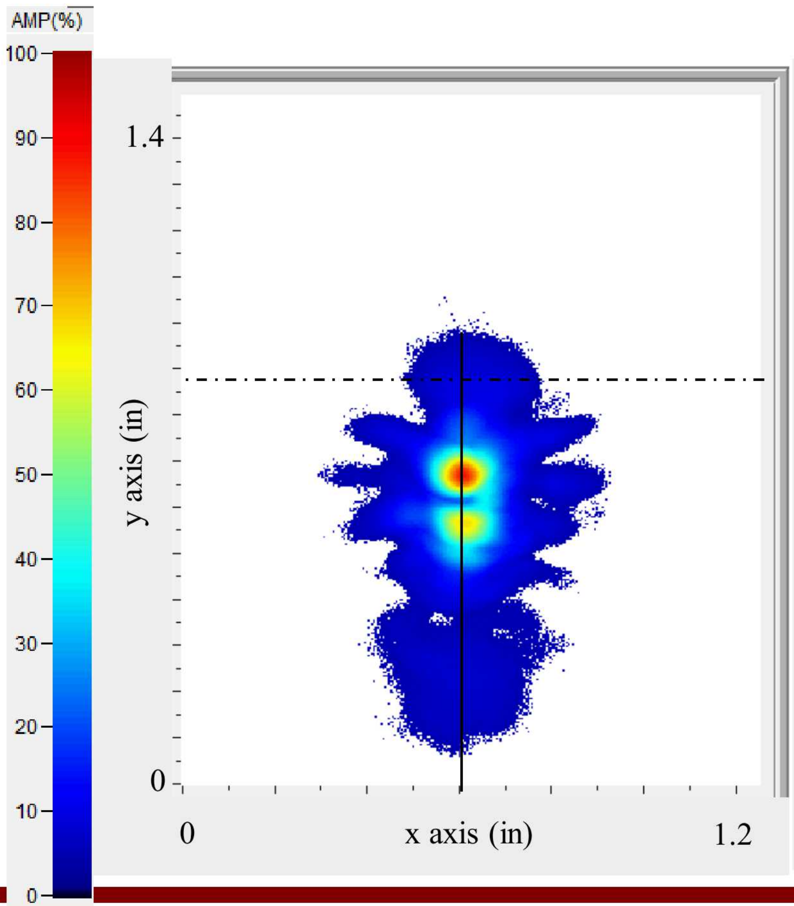
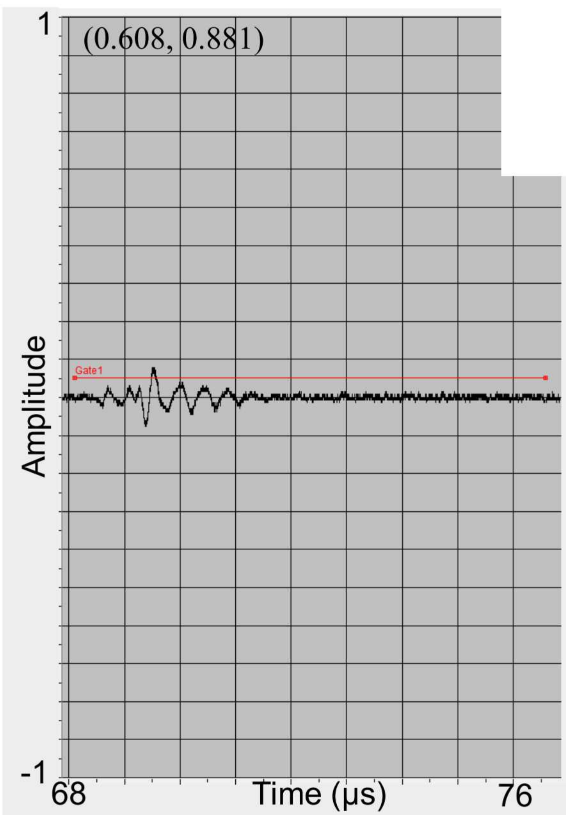
Through Transmission Scan Results (1 of 3)

- 8 ply laminate
- $\theta_1 = 5^{o}$ and $\theta_2 = 14^{o}$

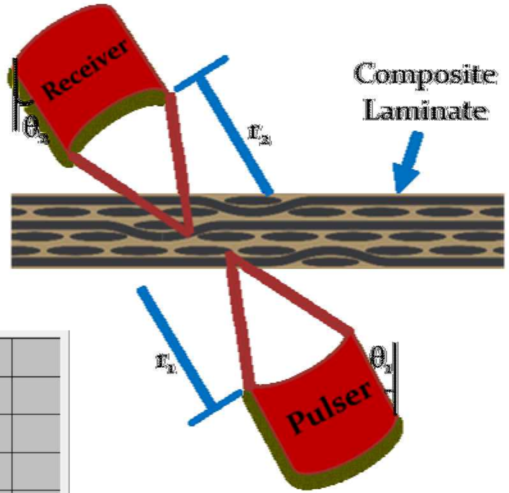


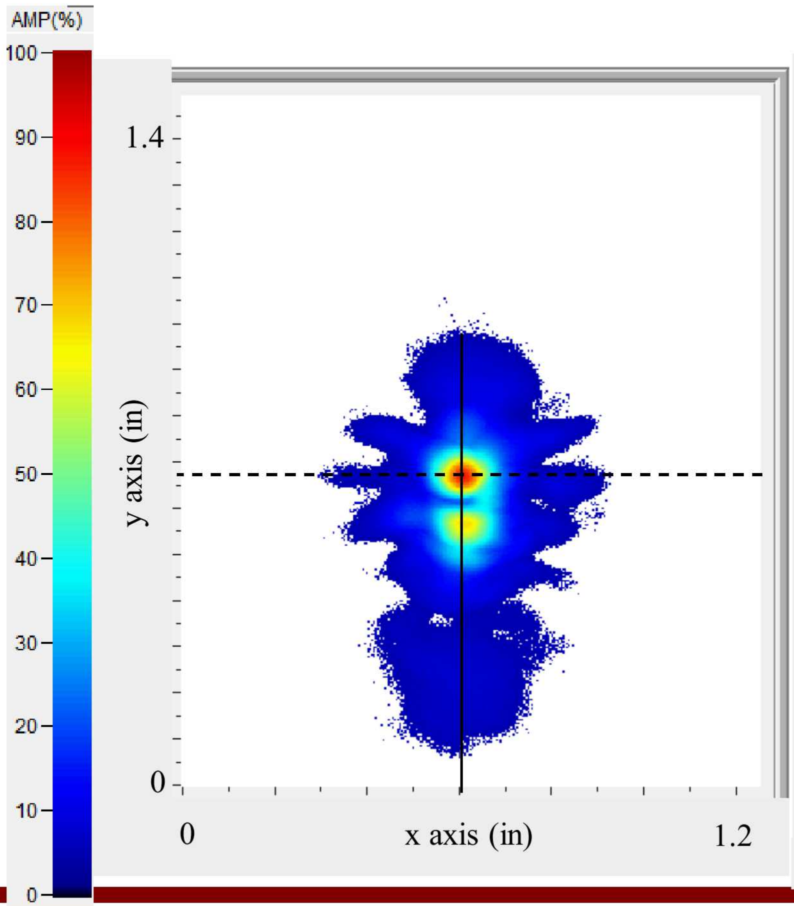


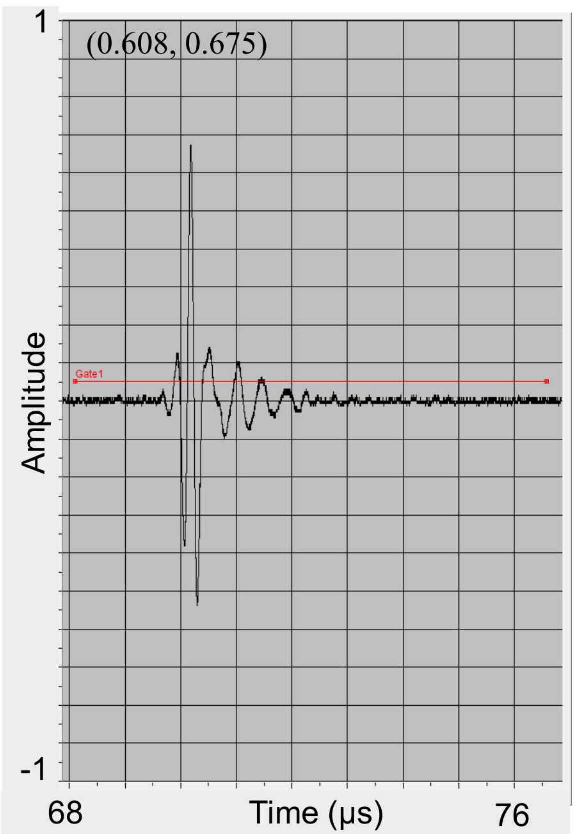


Through Transmission Scan Results (2 of 3)

- 8 ply laminate
- $\theta_1 = 5^o$ and $\theta_2 = 14^o$

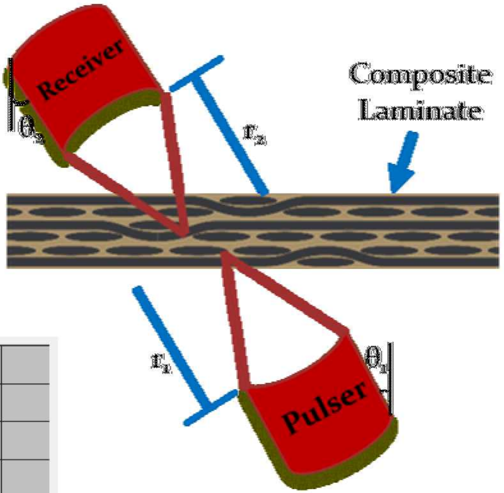


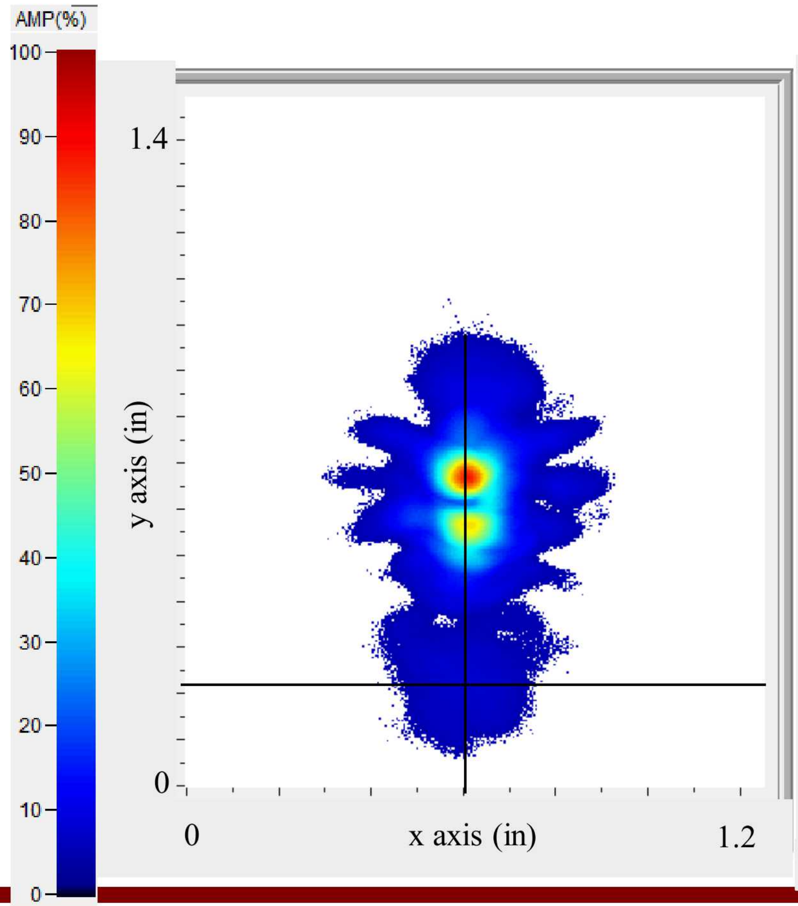


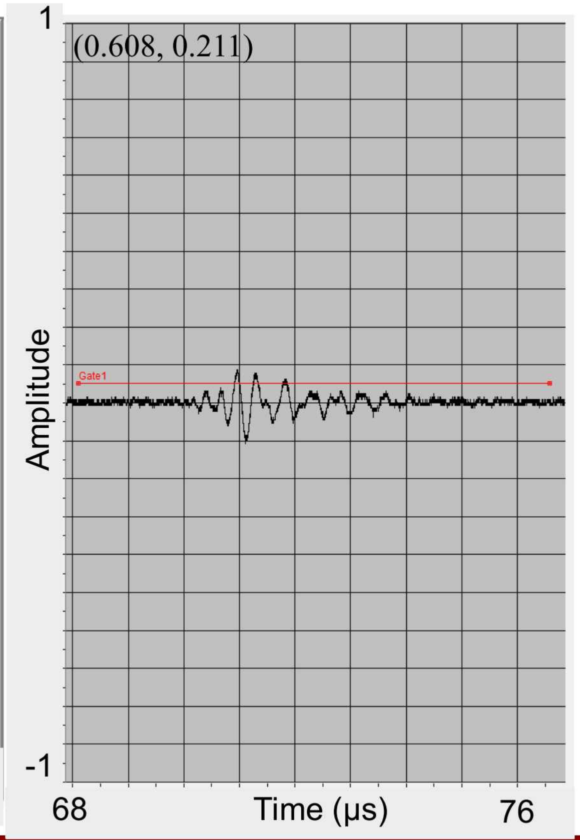


Through Transmission Scan Results (3 of 3)

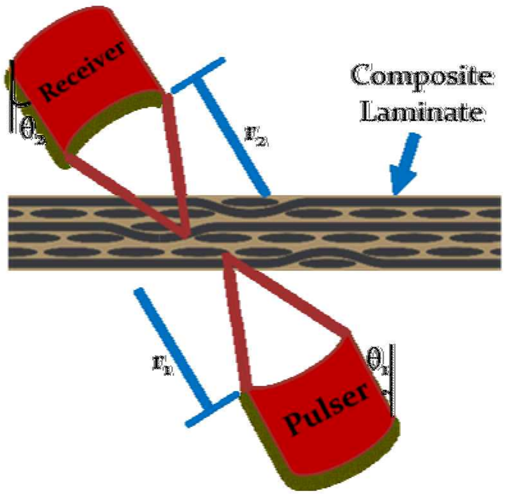
- 8 ply laminate
- $\theta_1 = 5^o$ and $\theta_2 = 14^o$







Mathematical Model Outline



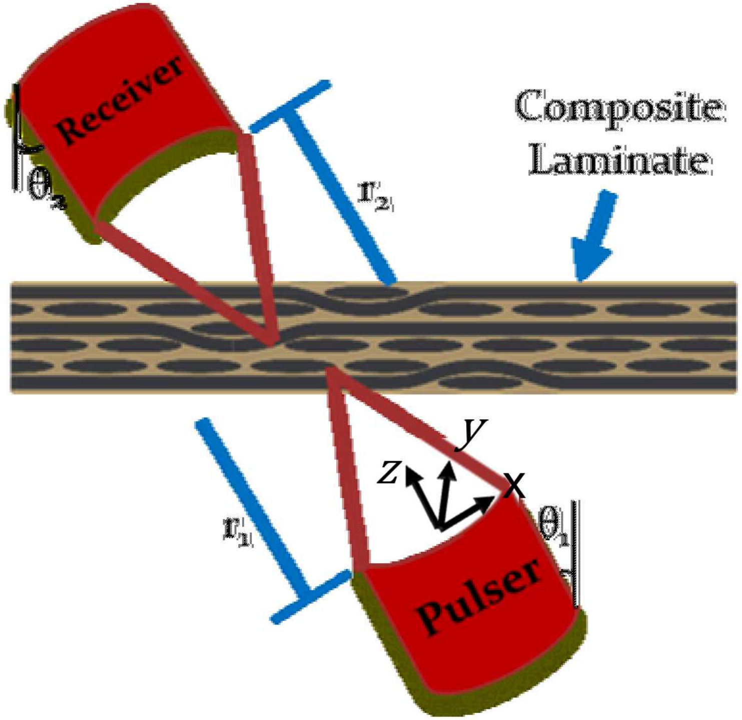
- Three important regions to consider in the model
  - Wave propagation in the water
  - Wave propagation across the water-to-composite interface
  - Wave propagation through laminate thickness
- For the receiver being focused at the back surface of the laminate, the assumption is made that the receiver can be treated as a point receiver. This would mean the wave propagation from the back surface of the sample and to the face of the receiving probe would not need to be calculated.
- If this assumption proves to be invalid, then future work would include accounting for the wave propagation in this area also.

Modeling the Wave in the Water

- Define the sound pressure wave exiting the ultrasound probe

$$p_o(x,y) = \begin{cases} p^* e^{\left(\frac{jk}{2}\right)\left(\frac{x^2}{f}+\frac{y^2}{f}\right)} & \text{if on the probe face} \\ 0 & \text{elsewhere} \end{cases}$$

$k =$ wavenumber
$f =$ focal length of probe
$x, y =$ spatial coordinates
$p_o =$ pressure at the probe face
$p^* =$ initial pressure



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Modeling the Wave Pressure Field

- Calculate the pressure field in the water where the pressure p is

$$p(x,y,z) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} G_{mn}\Phi_{mn}(x,y,z)$$

and

$$\Phi_{mn}(x,y,z) = \sqrt{\frac{w_x(0)w_y(0)}{w_x(z)w_y(z)}}\, e^{-jkz} e^{j[(2m+1)\psi_x(z)+(2n+1)\psi_y(z)]}$$

$$e^{-\left(\frac{jk}{2}\right)\left[\frac{x^2}{q_x(z)}+\frac{y^2}{q_y(z)}\right]} H_m\left[\frac{\sqrt{2}}{w_x(z)}x\right] H_n\left[\frac{\sqrt{2}}{w_y(z)}y\right]$$

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Defining Complex Coefficients

- The complex coefficients, G_{mn}, are defined as

$$G_{mn} = \left(\frac{1}{w_x(0)w_y(0)\pi 2^{m+n-1} m!\,n!}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_o(x,y)\Phi_{mn}(x,y,0)\,\mathrm{dydx}$$

where

$$\Phi_{mn}(x,y,0) = \mathrm{e}^{\left(\frac{-\mathrm{j}k}{2}\right)\left[\frac{x^2}{q_x(0)}+\frac{y^2}{q_y(0)}\right]}\mathrm{H}_m\left[\frac{\sqrt{2}}{w_x(0)}x\right]\mathrm{H}_n\left[\frac{\sqrt{2}}{w_y(0)}y\right]$$

and H_m and H_n are the m^{th} and n^{th} order Hermite polynomials evaluated at the points identified within the square brackets

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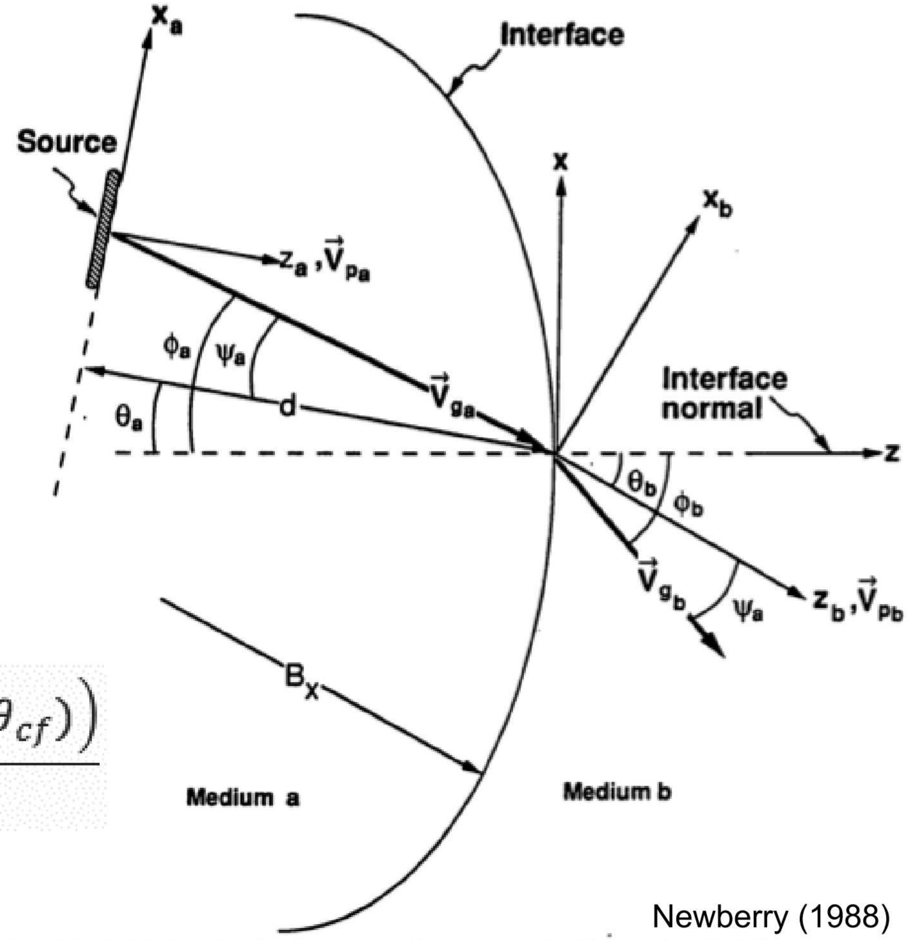
Wave Transmission Through the Water-to-Composite Interface

- The transmission of the ultrasound wave through the water-to-composite interface must be accounted for mathematically.



Newberry (1988)

$$\left(\frac{1}{q_x}\right)_T = \left(\frac{S_w \cos^2(\psi_{cf})\cos^2(\phi_w)}{S_{cf}\cos^2(\psi_w)\cos^2(\phi_{cf})}\right)\left(\frac{1}{q_x}\right)_I - \frac{\cos^2\psi_{cf}\left((S_w/S_{cf})\cos(\theta_w) - \cos(\theta_{cf})\right)}{B_x\cos^2(\phi_{cf})} \rightarrow 0$$

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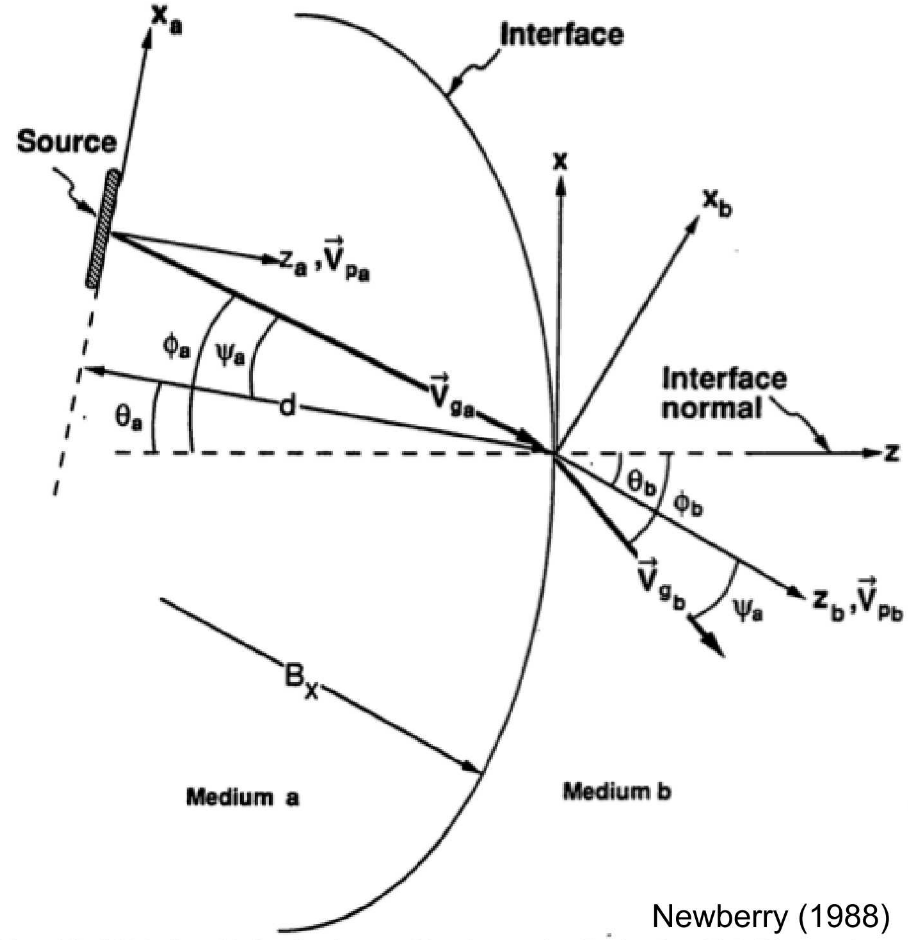
Wave Transmission Through the Water-to-Composite Interface

- The transmission of the ultrasound wave through the water-to-composite interface must be accounted for mathematically.

$$\left(\frac{1}{q_y}\right)_T = \left(\frac{S_w}{S_{cf}}\right)\left(\frac{1}{q_y}\right)_I - \frac{\left(\frac{S_w}{S_{cf}}\right)-1}{B_y}$$

0



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Christoffel Analysis

- With the known sample density, ρ, and the estimated stiffness, C_{ijkl}, determine the phase velocity and polarization direction for each wave propagating in the sample

$$\left(C_{ijkl}\tau_i\tau_l - \rho v_p^2 \delta_{jk}\right)\widehat{d_k} = 0$$

- Repeat the analysis for multiple propagation directions and determine the slowness surface for the longitudinal and shear waves
- Near the direction of wave propagation, perform a regression on the slowness surface to obtain the constants A through E and S_o.

$$\left(\frac{k}{\omega}\right) \cong S_o + A\left(\frac{k_x}{\omega}\right) + B\left(\frac{k_y}{\omega}\right) + C\left(\frac{k_x}{\omega}\right)^2 + D\left(\frac{k_x}{\omega}\right)\left(\frac{k_y}{\omega}\right) + E\left(\frac{k_y}{\omega}\right)^2$$

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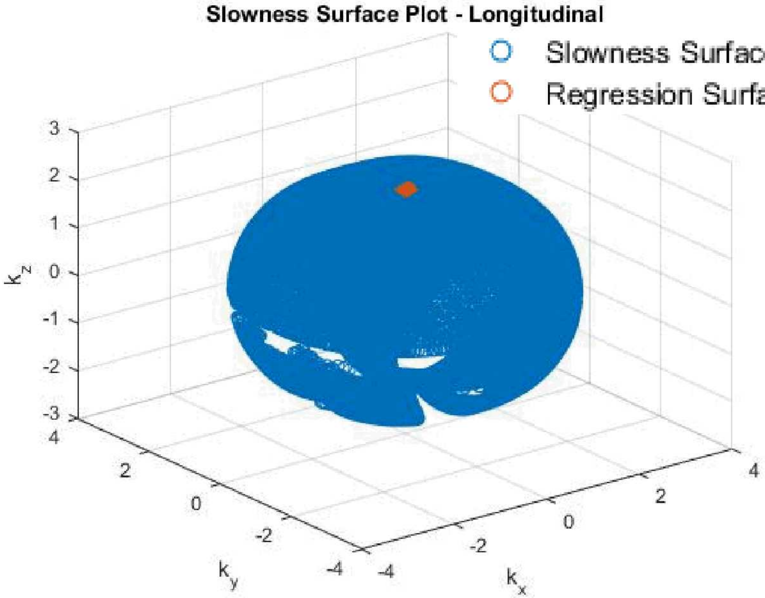

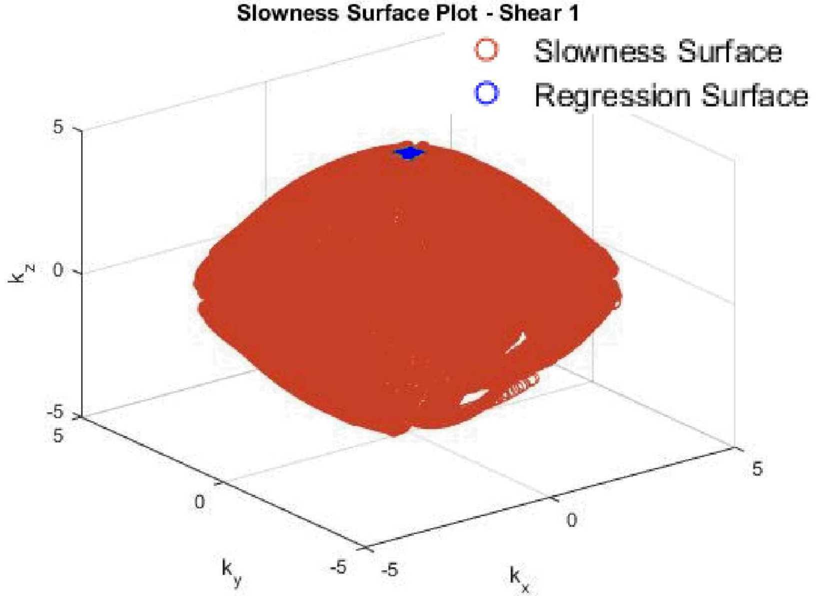

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Calculate Wave Displacement

- The wave displacement, $\underline{u}$, within the sample is given by

$$\underline{u}(x,y,z,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \mathrm{G}_{mn}\underline{u}_{mn}(x,y,z,t)$$

where

$$\underline{u}_{mn}(x,y,z,t) = \hat{d}^i(0,0)u_o \sqrt{\frac{w_{xo}w_{yo}}{w_x(z)w_y(z)}}\, \mathrm{e}^{(\mathrm{j}\omega_o(t-S_o z))}$$

$$\mathrm{e}^{\{\mathrm{j}[(2m+1)\psi_x(z)+(2n+1)\psi_y(z)]\}}$$

$$\mathrm{e}^{\left\{\frac{-\mathrm{j}\pi}{\lambda_z}\left[\frac{(x+Az)^2}{q_x(z)}+\frac{(y+Bz)^2}{q_y(z)}\right]\right\}}$$

$$\mathrm{H}_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]\mathrm{H}_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

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Identifying the Terms in the Wave Displacement Calculation

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$$\underline{u}_{mn}(x,y,z,t) = \hat{d}^i(0,0)u_o\sqrt{\frac{w_{xo}w_{yo}}{w_x(z)w_y(z)}}\,e^{(j\omega_o(t-S_oz))}$$

$$e^{\{j[(2m+1)\psi_x(z)+(2n+1)\psi_y(z)]\}}$$

$$e^{\left\{\frac{-j\pi}{\lambda_z}\left[\frac{(x+Az)^2}{q_x(z)}+\frac{(y+Bz)^2}{q_y(z)}\right]\right\}}$$

$$H_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]H_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

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$$e^{\{j[(2m+1)\psi_x(z)+(2n+1)\psi_y(z)]\}}$$

$$e^{\left\{\frac{-j\pi}{\lambda_z}\left[\frac{(x+Az)^2}{q_x(z)}+\frac{(y+Bz)^2}{q_y(z)}\right]\right\}}$$

$$H_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]H_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

The terms highlighted in red were obtained from the slowness surface regression.

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$$e^{\{j[(2m+1)\psi_x(z)+(2n+1)\psi_y(z)]\}}$$

$$e^{\left\{\frac{-j\pi}{\lambda_z}\left[\frac{(x+Az)^2}{q_x(z)}+\frac{(y+Bz)^2}{q_y(z)}\right]\right\}}$$

$$H_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]H_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

The wavelength in the *z* direction. When the wavelength is combined with the Λ terms below, we can obtain the anisotropy parameters (Λ/λ_z) in the *x* and *y* directions as seen on the next slide.

$$\Lambda_x = \frac{2\pi}{\omega_o S_o}(1 - CS_o)$$

$$\Lambda_y = \frac{2\pi}{\omega_o S_o}(1 - ES_o)$$

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Identifying the Terms in the Wave Displacement Calculation

Sandia National Laboratories

$$\underline{u}_{mn}(x,y,z,t) = \hat{d}^i(0,0)u_o\sqrt{\frac{w_{xo}w_{yo}}{w_x(z)w_y(z)}}\,e^{(j\omega_o(t-S_oz))}$$

$$e^{\{j[(2m+1)\psi_x(z)+(2n+1)\psi_y(z)]\}}$$

$$e^{\left\{\frac{-j\pi}{\lambda_z}\left[\frac{(x+Az)^2}{q_x(z)}+\frac{(y+Bz)^2}{q_y(z)}\right]\right\}}$$

$$H_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]H_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

Complex parameter that is defined based on the initial width of the beam, initial radius of curvature for the beam, the anisotropy parameter (Λ/λ_z) and the position *z*.

$$q_x(z) = q_{xo} + \left(\frac{\Lambda_x}{\lambda_z}\right)z$$

$$q_y(z) = q_{yo} + \left(\frac{\Lambda_y}{\lambda_z}\right)z$$

B.P. Newberry. *Paraxial Approximations for Ultrasonic Beam Propagation in Liquid and Solid Media with Applications to Nondestructive Evaluation*, PhD Dissertation, Iowa State University, Engineering Mechanics, 1988.

Newberry, B.P. and R.B. Thompson, “A Paraxial Theory for the Propagation of Ultrasonic Beams in Anisotropic Solids,” *J. of the Acoustical Society of America*, **85**(6), June 1989.

Stair, S., Moore, D. and Jack, D. Determining the Elastic Coefficients of a Carbon Fiber Reinforced Laminated Composite using an Oblique Incidence Through Transmission Ultrasound Technique. *American Society of Non-Destructive Testing 24th Research Symposium*, Anaheim, California, March 2015.

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$$e^{\left\{\frac{-j\pi}{\lambda_z}\left[\frac{(x+Az)^2}{q_x(z)}+\frac{(y+Bz)^2}{q_y(z)}\right]\right\}}$$

$$\mathrm{H}_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]\mathrm{H}_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

The initial width of the ultrasound beam in the *x* and *y* directions

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$$H_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]H_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

The width of the ultrasound beam in the *x* and *y* directions as a function of position along the z direction

$$w_x(z) = \left[\frac{-\lambda_z}{\pi \mathrm{Im}\left(\frac{1}{q_x(z)}\right)}\right]^{1/2}$$

$$w_y(z) = \left[\frac{-\lambda_z}{\pi \mathrm{Im}\left(\frac{1}{q_y(z)}\right)}\right]^{1/2}$$

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$$\mathrm{H}_m\left[\frac{\sqrt{2}}{w_x(z)}(x+Az)\right]\mathrm{H}_n\left[\frac{\sqrt{2}}{w_y(z)}(y+Bz)\right]$$

The difference in phase angle between the initial q_o and the present q.

$$\Psi_x(z) = \frac{1}{2}[\angle q_{xo} - \angle q_x(z)]$$

$$\Psi_y(z) = \frac{1}{2}[\angle q_{yo} - \angle q_y(z)]$$

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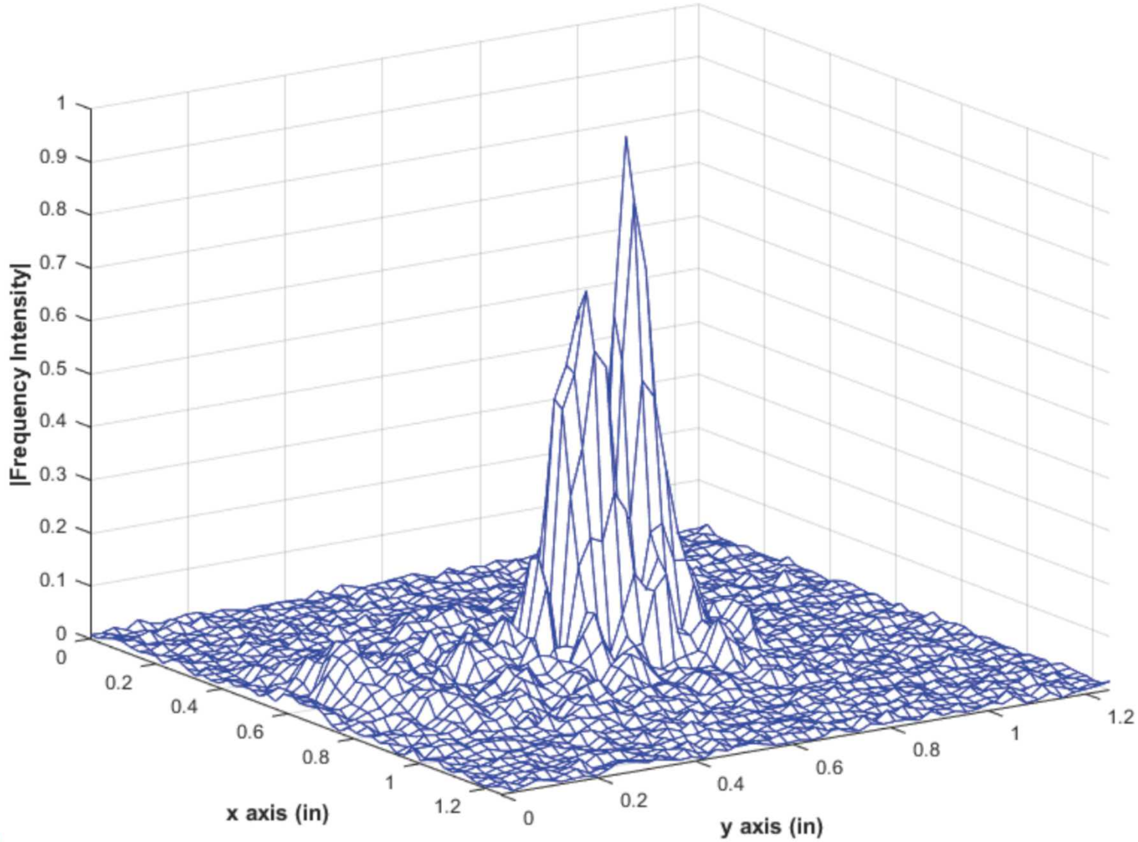
Frequency Intensity Plot Comparison

- Plot of the intensity of the 5 MHz component of the ultrasound wave collected in the C-scan measurement. This is the data that will be compared with the model results.



Future Work

- Refine the optimization method for comparing the measured and modeled wave displacement results
- Compare the elastic constants as determined from the optimization method to those obtained experimentally
- Determine applicability of this approach to bonded samples

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Questions?

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