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Emittance Increase Estimate ByThree-Pole Wigglers In NSLS-II Storage Ring

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This technical note presents a derivation of a simple formula to estimate the total horizontal emittance increase by the installation of 3-pole wigglers (3PWs) in the NSLS-II storage ring as a function of the number of 3PWs and the vertical magnetic field strength of each 3PW. The final formula was originally derived by S. Krinsky.

The formula for the relative emittance increase due to the presence of wigglers is as follows (Eq. (10.72) of Ref. [1]):

$$\frac{\epsilon_w}{\epsilon_0} = \frac{1 + \langle \mathcal{H}/\rho^3 \rangle_w / \langle \mathcal{H}/\rho^3 \rangle_o}{1 + \langle 1/\rho^2 \rangle_w / \langle 1/\rho^2 \rangle_o} = \frac{1 + \left(\frac{1}{C} \oint \frac{\mathcal{H}}{\rho_w^3} ds \right) / \left(\frac{1}{C} \oint \frac{\mathcal{H}}{\rho_o^3} ds \right)}{1 + \left(\frac{1}{C} \oint \frac{1}{\rho_w^2} ds \right) / \left(\frac{1}{C} \oint \frac{1}{\rho_o^2} ds \right)}, \tag{1}$$

where ϵ_0, ϵ_w, $\mathcal{H}$, ρ_0, ρ_w, and C are the initial natural emittance, emittance with the effect of wigglers, dispersion invariant in the horizontal plane, the bending radius in the main dipole magnets and the wigglers, and the circumference of the storage ring, respectively. The first integral to evaluate is

$$\oint \frac{1}{\rho_o^2} ds = \frac{L_{\mathrm{bends}}}{\rho_o^2} = \frac{L_{\mathrm{bends}}}{\rho_o} \cdot \frac{1}{\rho_o}. \tag{2}$$

where L_{bends} is the total length of all the bending magnets. By definition,

$$\frac{L_{\mathrm{bends}}}{\rho_o} = 2\pi. \tag{3}$$

For 3 GeV beam energy, the magnetic rigidity is

$$B\rho \approx 10[\mathrm{T} \cdot \mathrm{m}]. \tag{4}$$

Hence, for the NSLS-II bending magnets (0.4 T), we have

$$\frac{1}{\rho_o[\mathrm{m}]} \approx \frac{B_o[\mathrm{T}]}{10} = \frac{0.4}{10} = 0.04\ [\mathrm{m}^{-1}]. \tag{5}$$

Using the results (3) and (5), Eq. (2) evaluates to

$$\oint \frac{1}{\rho_o^2} ds = 2\pi \times 0.04 = 0.251[\mathrm{m}^{-1}] \tag{6}$$

The second integral to evaluate is

$$\begin{aligned} \oint \frac{1}{\rho_w^2} ds &= \oint \left(\frac{B_w}{B\rho} \right)^2 \approx \oint \left(\frac{B_w[\mathrm{T}]}{10} \right)^2 ds\ (\because (4)) \\ &= 0.01\, N \int_{1\ \mathrm{wiggler}} (B_w[\mathrm{T}])^2 ds, \end{aligned} \tag{7}$$

where N and B_w are the number of 3PWs installed in the ring and the vertical field of the 3PWs. The natural horizontal emittance (Eq. 3.1.4 (22) of Ref. [2]) can be computed as

$$\epsilon_{x0} = C_q \frac{\gamma^2}{J_x} \frac{I_{5x}}{I_2}, \tag{8}$$

where $C_q = 3.8319 \times 10^{-13}[\mathrm{m}]$, $\gamma = 5871.8$ for 3 GeV, and $J_x = 0.998$ from the particle tracking code Tracy [3] for the official Day-1 NSLS-II bare lattice, while the radiation integrals I_{5x} and I_2 are defined as

$$I_{5x} = \oint \frac{\mathcal{H}}{\rho_o^3} ds, \tag{9}$$

and

$$\begin{aligned} I_2 &= \oint \left(\frac{1}{\rho_{xo}^2} + \frac{1}{\rho_{yo}^2} \right) ds \; (\because \text{Eq.3.1.4 (2) of Ref.[2]}) \\ &= \oint \frac{1}{\rho_o^2} ds \; \left(\because \rho_{xo} = \rho_o \;\&\; \rho_{yo} = 0 \right) = 0.251 \; [\mathrm{m}^{-1}] \left(\because (6) \right). \end{aligned} \tag{10}$$

Therefore, Eq. (8) can be rearranged as follows to evaluate the third integral:

$$\begin{aligned} \oint \frac{\mathcal{H}}{\rho_o^3} ds = I_{5x} = \frac{I_2 J_x \epsilon_{xo}}{C_q \gamma^2} &= \frac{0.251 \cdot 0.998}{3.8319 \times 10^{-13} \cdot 5871.8^2} \epsilon_{xo} \\ &= 1.899 \times 10^4 \; \epsilon_{xo} [\mathrm{m \cdot rad}]. \end{aligned} \tag{11}$$

The fourth integral to evaluate is

$$\begin{aligned} \oint \frac{\mathcal{H}}{\rho_w^3} ds &= \mathcal{H} \oint \frac{1}{|\rho_w^3|} ds \; (\because \mathcal{H} \equiv \text{const @ 3PWs}) \\ &= \mathcal{H} N \int_{\text{1 wiggler}} \frac{|B_w[\mathrm{T}]|^3}{10^3} ds \;\; (\because (7)) \\ &= 7.76 \times 10^{-6} N \int |B_w[\mathrm{T}]|^3 ds, \end{aligned} \tag{12}$$

because $\mathcal{H} = 7.76 \times 10^{-3} [\mathrm{m}]$ at 3PWs according to Tracy. Finally, plugging in the results (6), (7), (11) and (12) into Eq. (1) yields

$$\begin{aligned} \frac{\epsilon_w}{\epsilon_0} &= \frac{1 + \left(\oint \frac{\mathcal{H}}{\rho_w^3} ds \right) / \left(\oint \frac{\mathcal{H}}{\rho_o^3} ds \right)}{1 + \left(\oint \frac{1}{\rho_w^2} ds \right) / \left(\oint \frac{1}{\rho_o^2} ds \right)} \approx \frac{1 + \frac{7.76 \times 10^{-6} N \int |B_w[\mathrm{T}]|^3 ds}{1.899 \times 10^4 \epsilon_o [\mathrm{m \cdot rad}]}}{1 + \frac{0.01 \, N \int (B_w[\mathrm{T}])^2 ds}{0.251}} \\ &= \frac{1 + \frac{4.09 \times 10^{-10}}{\epsilon_o [\mathrm{m \cdot rad}]} N \int |B_w[\mathrm{T}]|^3 ds}{1 + 0.0398 \cdot N \int (B_w[\mathrm{T}])^2 ds}. \end{aligned} \tag{13}$$

Multiplying ϵ_0 on both sides of Eq. (13) finally leads to

$$\boxed{\epsilon_w [\mathrm{nm \cdot rad}] \approx \frac{\epsilon_o [\mathrm{nm \cdot rad}] + 0.4 \cdot N \int |B_w[\mathrm{T}]|^3 ds}{1 + 0.04 \cdot N \int (B_w[\mathrm{T}])^2 ds}.} \tag{14}$$

References:

[1] H. Wiedemann, “Particle Accelerator Physics I,” Springer, 2nd ed. (2003).
[2] A. W. Chao and M. Tigner, “Handbook of Accelerator Physics and Engineering,” World Scientific, Singapore, 3rd printing (2006).
[3] J. Bengtsson, E. Forest, and H. Nishimura, “Tracy User Manual,” Internal SLS document, PSI, Villigen (1997).