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Hyper-Accurate Gate Set Tomography

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Talk outline

- Motivation for tomography
- Problems with quantum state and quantum process tomography.
- Gate set tomography (GST) framework
  - Linear gate set tomography
  - Least-squares gate set tomography
- How to characterize the “accuracy” of GST
  - Simulated characterization
  - Experimental characterization

Towards true QIP

- Want gate set $\mathcal{G}$

e.g. $\mathcal{G} = \{I, X_{\frac{\pi}{2}}, Y_{\frac{\pi}{2}}\}$

- Have $\mathcal{G}_{exp}$

- Can know $\mathcal{G}_{est}$

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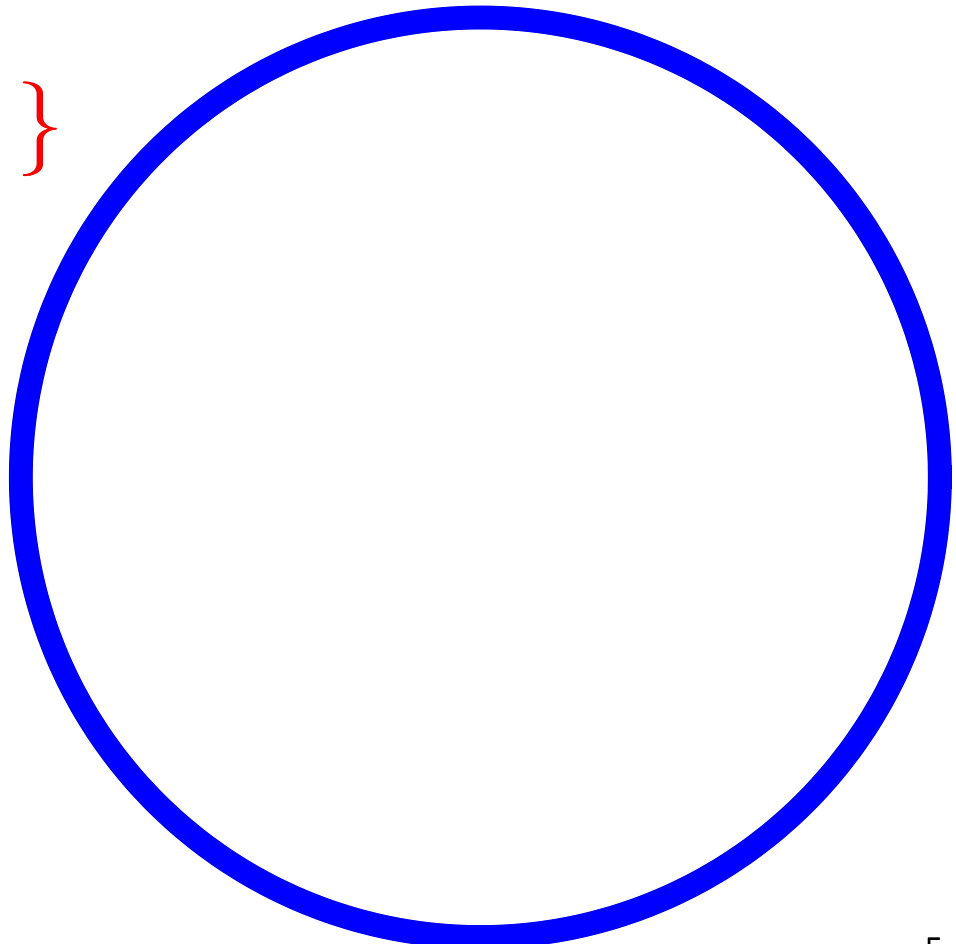

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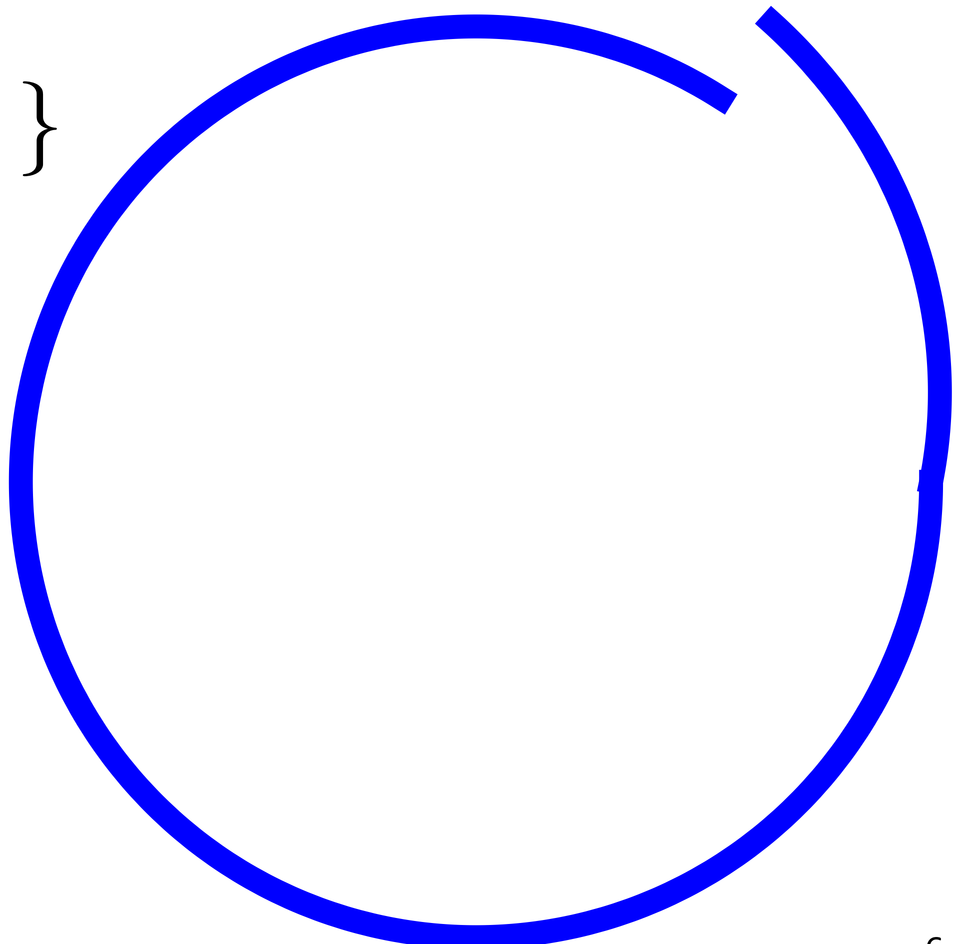

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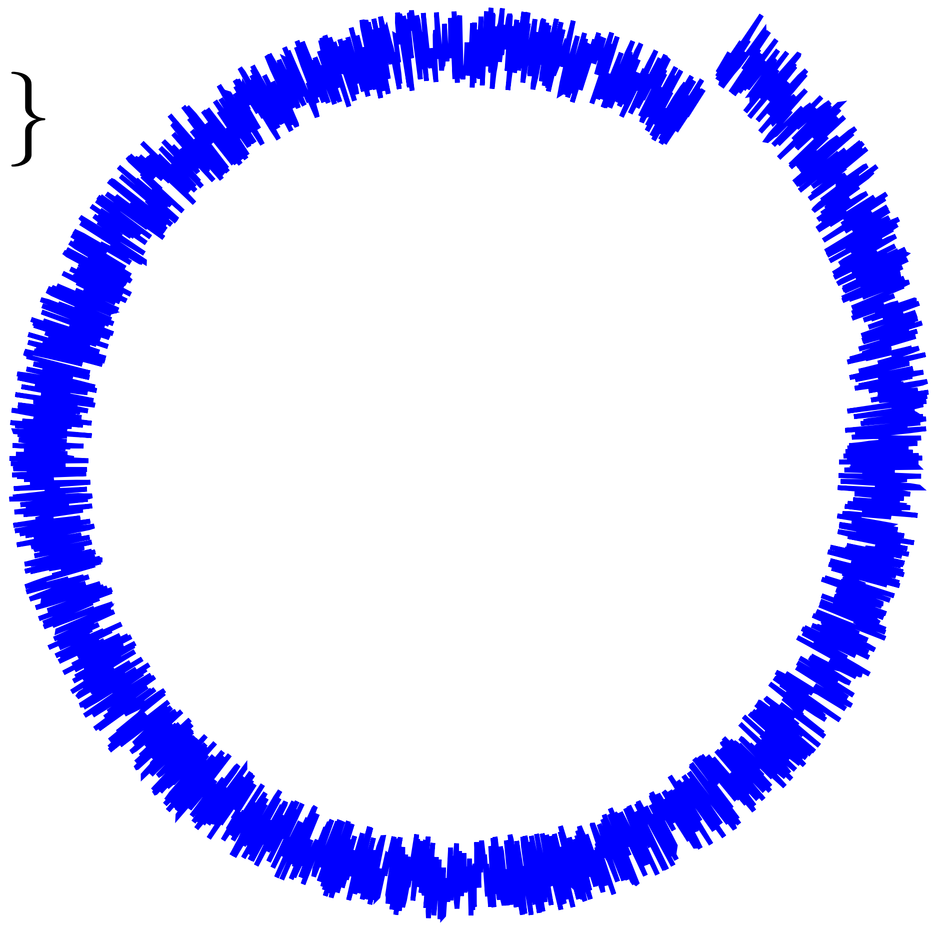
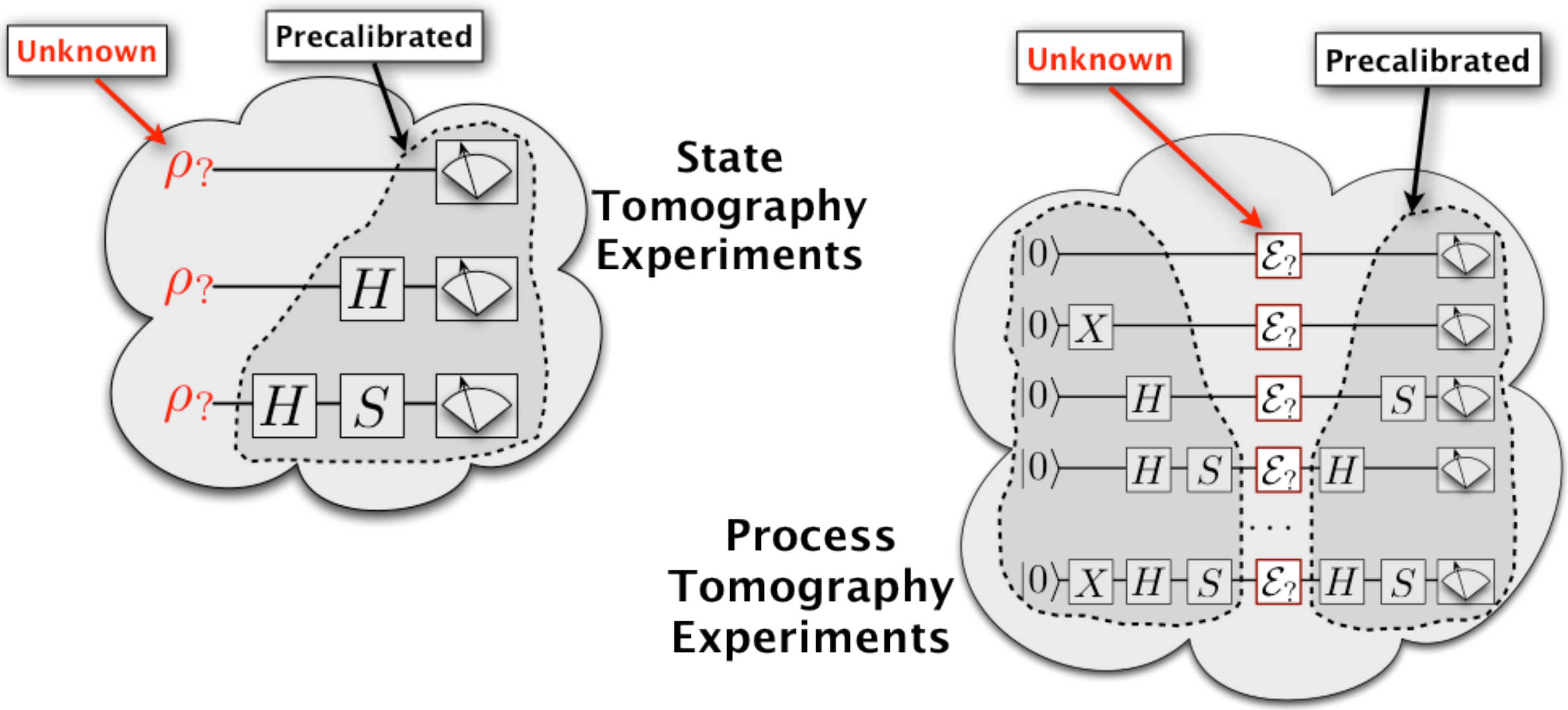


$$G_{ij}^{est} = G_{ij}^{exp} + \varepsilon_{ij}$$

Goal of tomography:

Make ε_{ij} as small as possible as cheaply as possible.

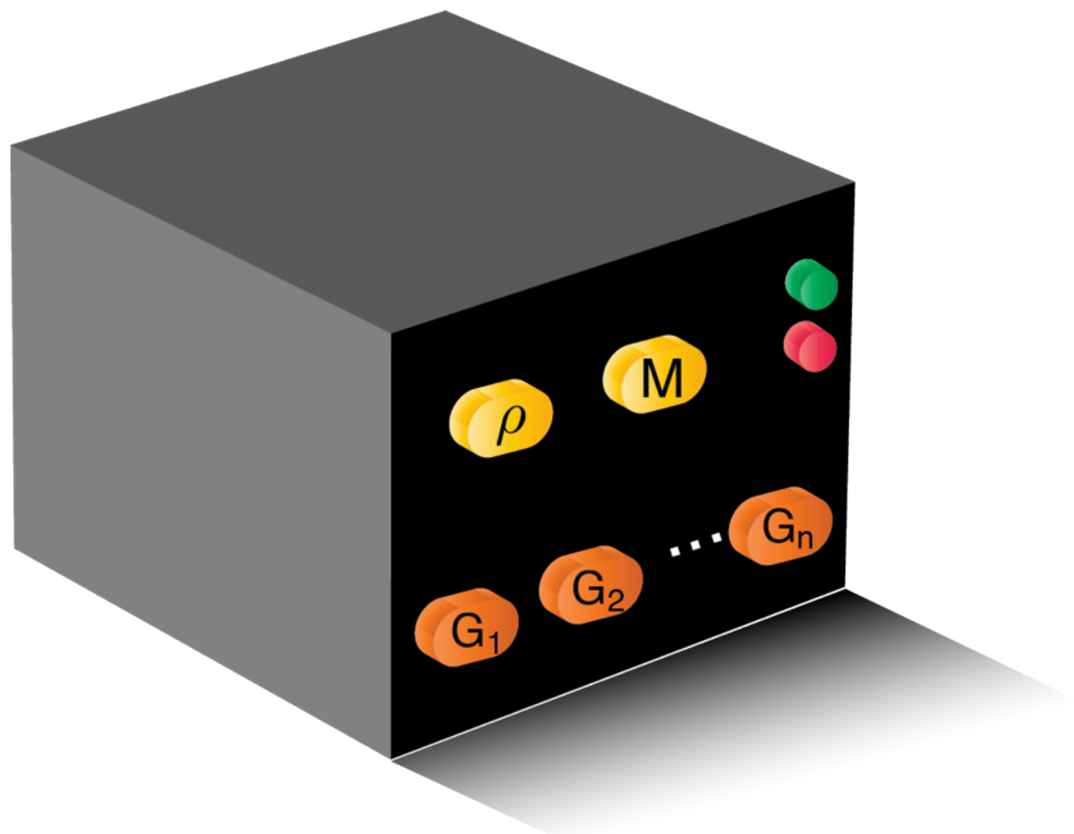
The problem with tomography



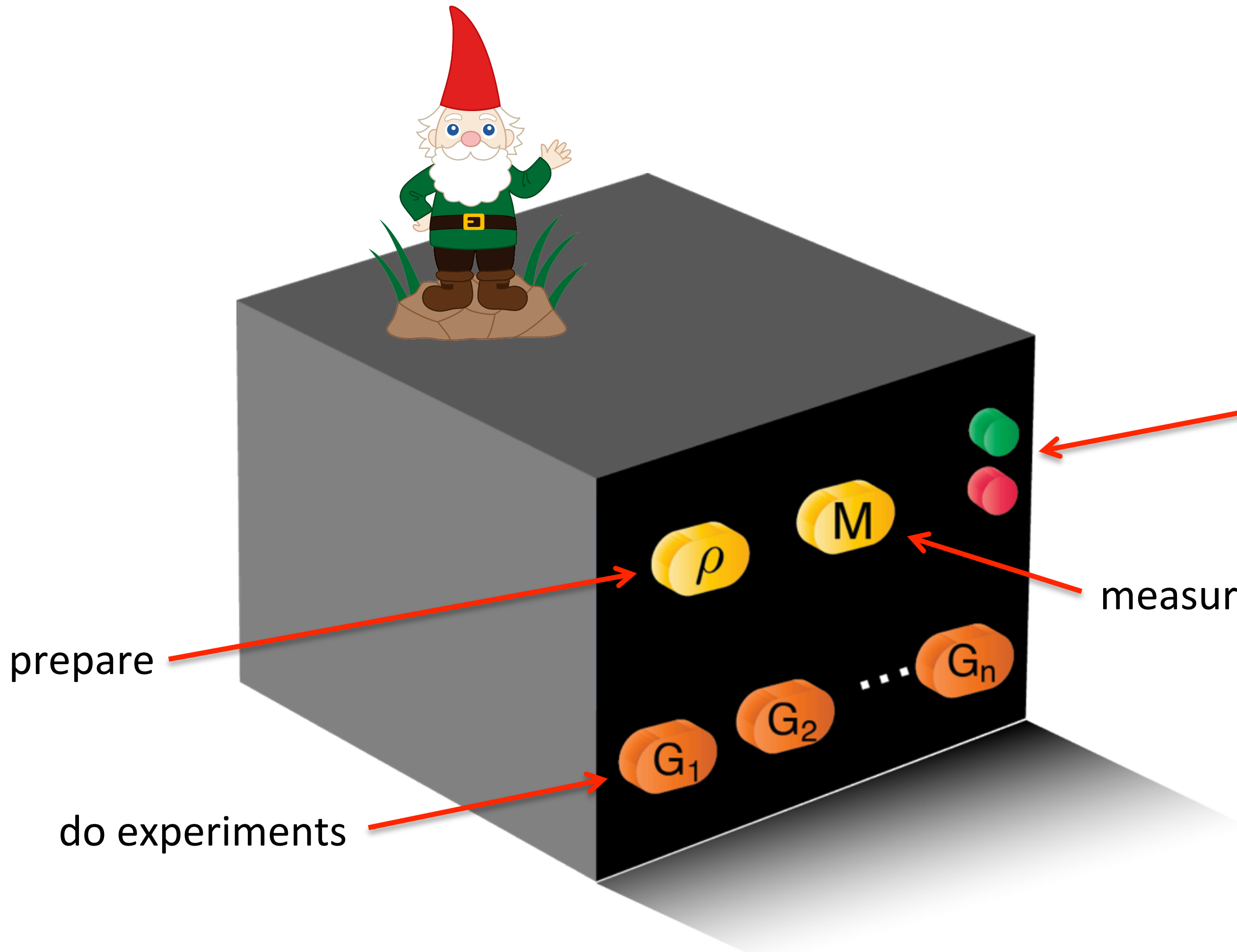
Critical problem: relies on precalibrated reference frames that don’t really exist in hardware!

Goal: Calibration-free tomography.

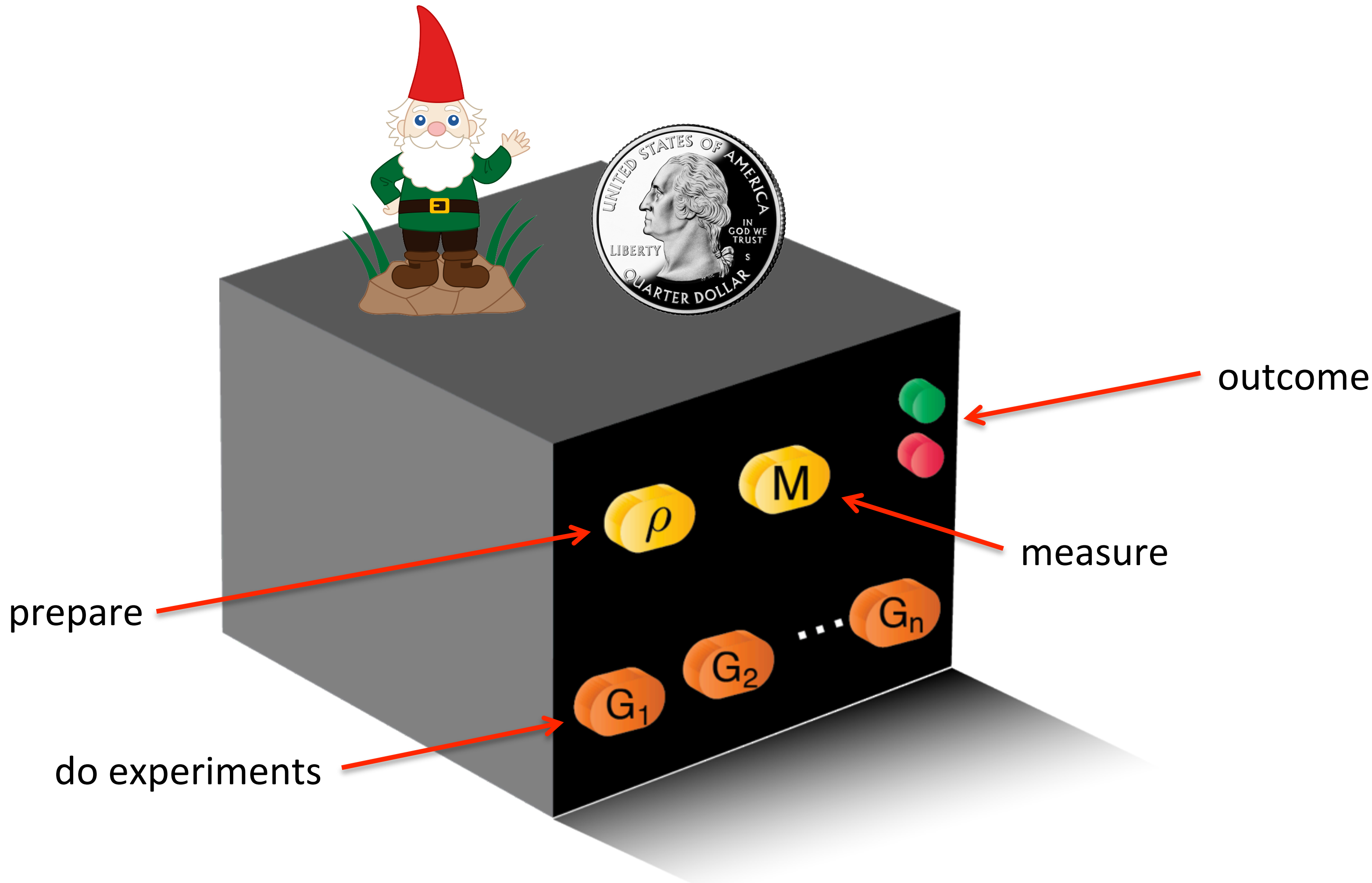
"Black box picture" of quantum information processor



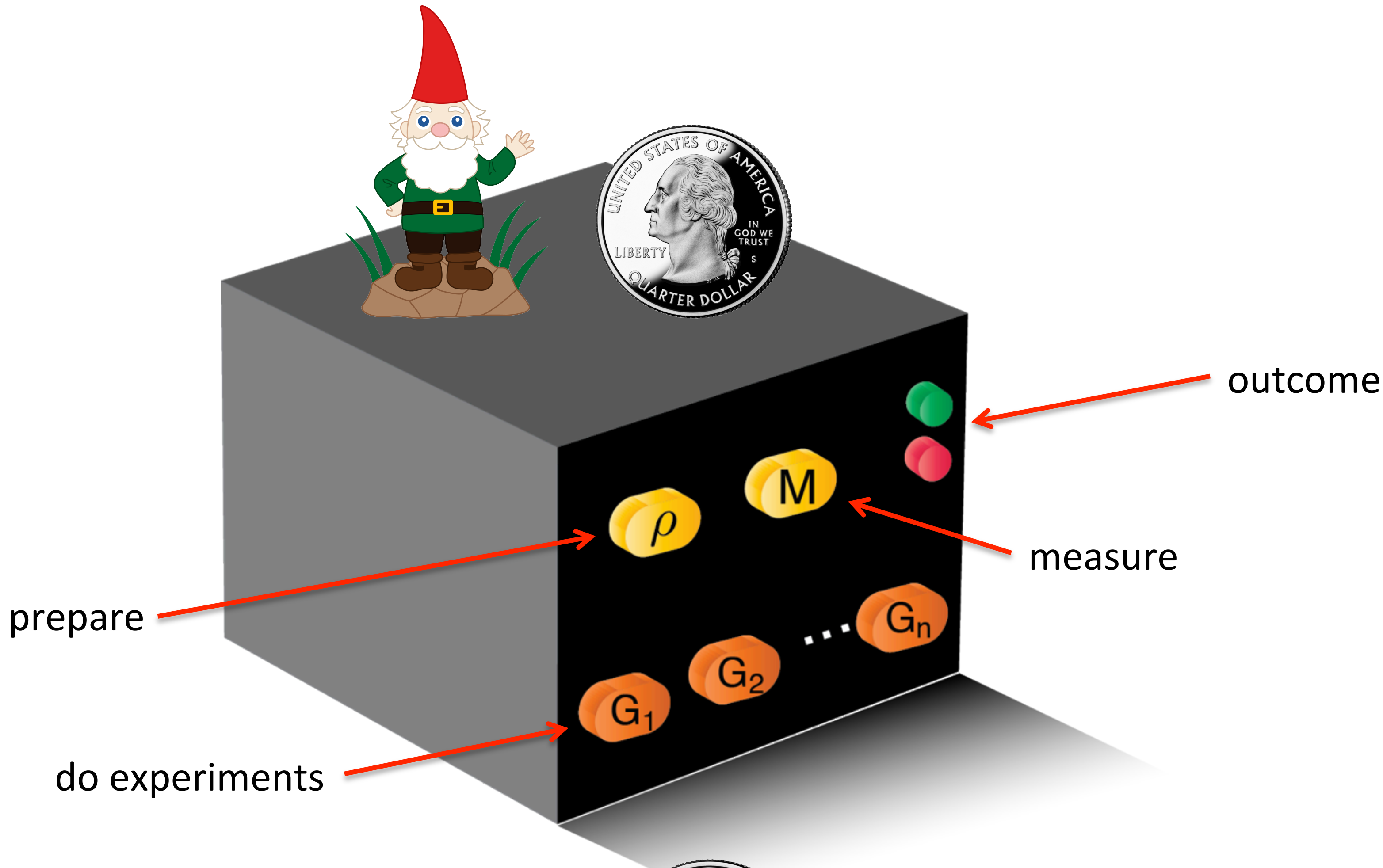
“Black box picture” of quantum information processor



“Black box picture” of quantum information processor

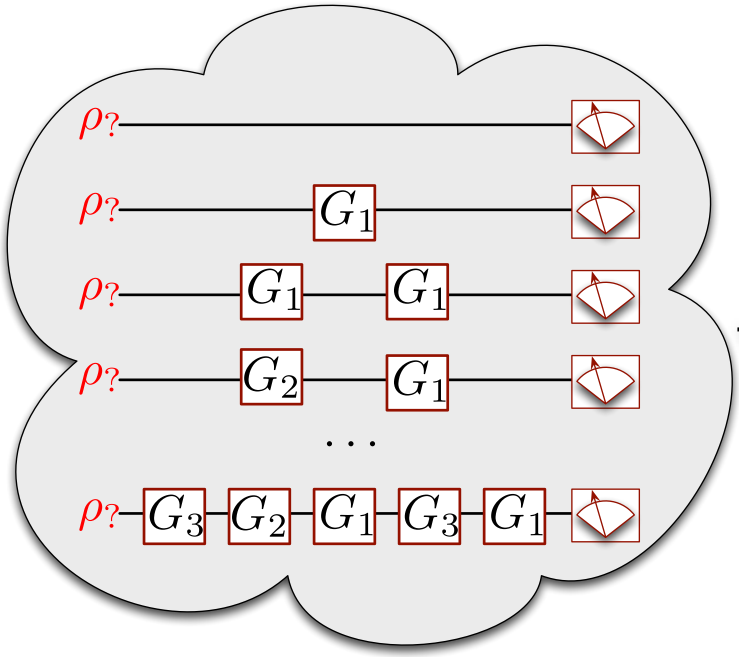


"Black box picture" of quantum information processor



Markovian model: $\dfrac{d}{dt}$  $= 0$

Gate Set Tomography Framework



1. Perform collection of experiments.

$$f_1 \approx \langle E|G_1|\rho\rangle$$

$$\vdots$$

$$f_m \approx \langle E|G_{i_1}\cdots G_{i_l}|\rho\rangle$$

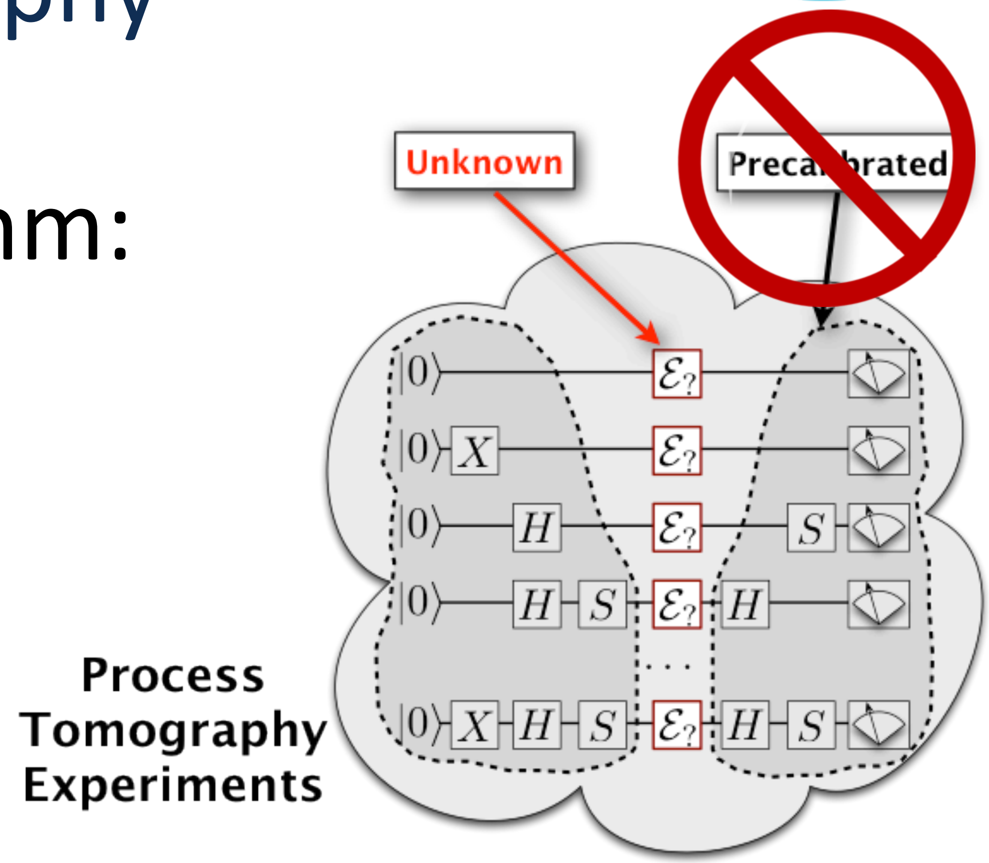
2. Compute:

$$\mathcal{G}_{est} = F(f_1 \cdots f_m)$$

Many choices for gate strings, estimator *F*.

Gate Set Tomography

- Simplest algorithm: Linear Inversion (LGST)
- “Process tomography without calibration”.



Gate Set Tomography

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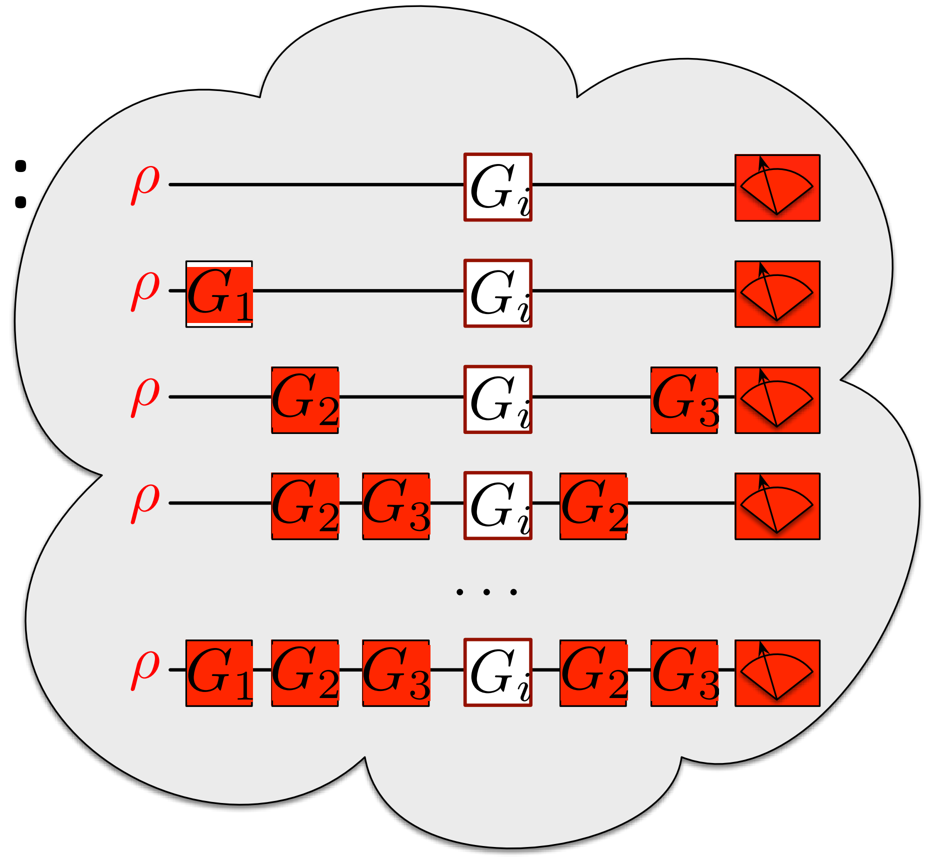
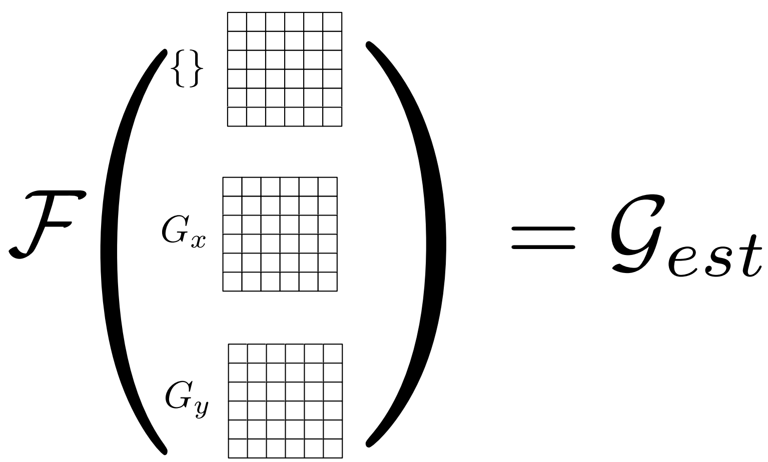
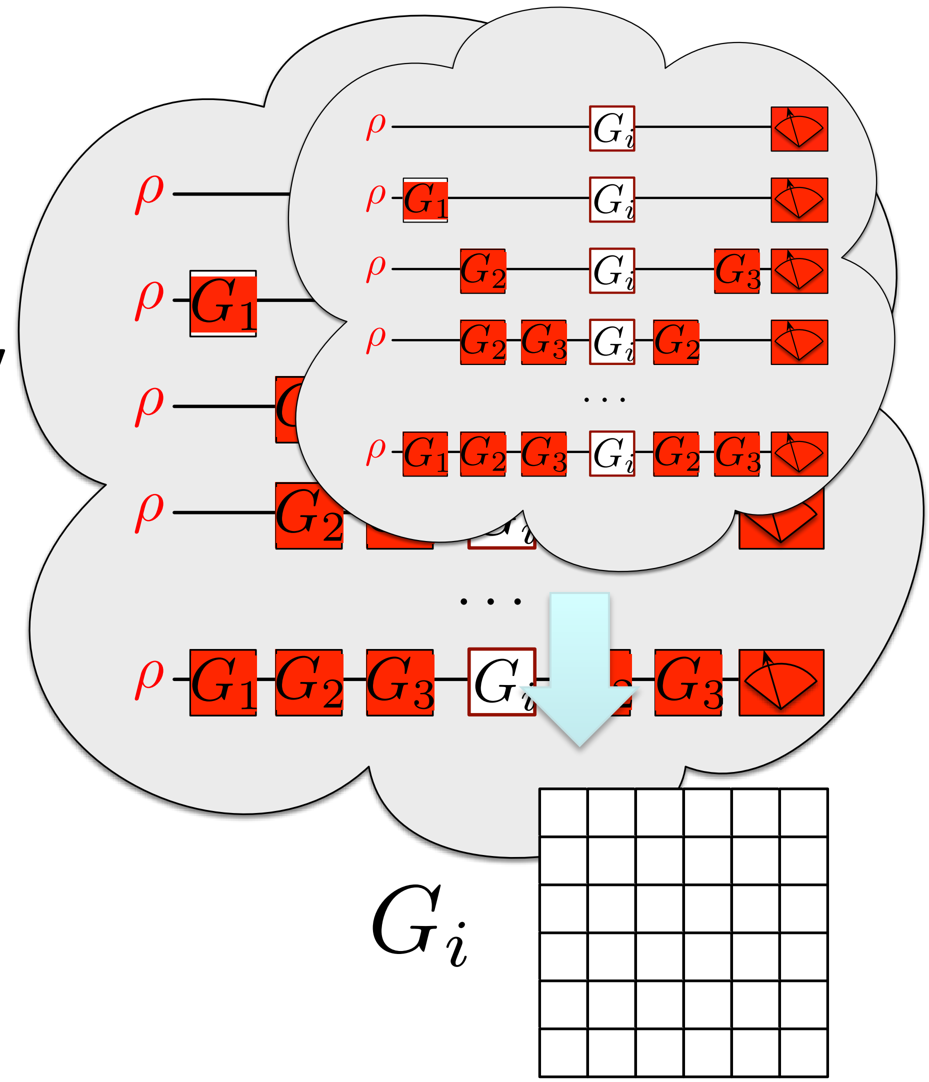


Linear gate set tomography

- Use unknown gates as uncalibrated “fiducials”.
- Run “process tomography” on each gate, *and* on empty gate string.





- Linear algebra → gate set.
- arXiv:1310.4492

How does LGST perform?

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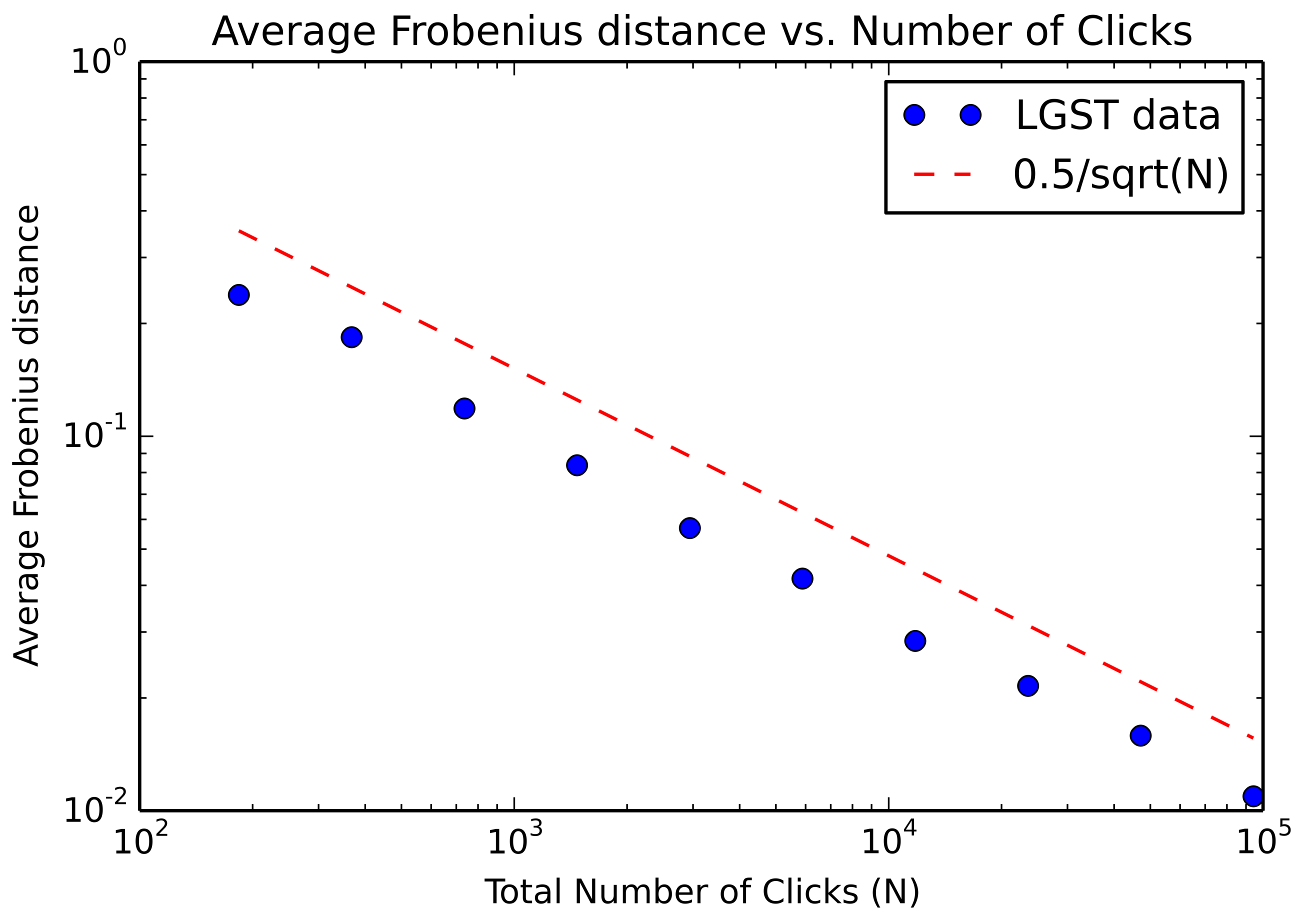
"RMS Frobenius distance":

$$G_{ij}^{est} = G_{ij}^{exp} + \varepsilon_{ij}$$

$$\text{Error} = \langle \varepsilon_{ij} \rangle_{RMS}$$

$$\text{Error} = \sqrt{\frac{1}{k(\dim G)^2} \sum_{i=1}^{k} \left(||G_{est}^{(k)} - G_{exp}^{(k)}||_F \right)^2}$$

LGST on simulated data



Works nicely; limited by $\frac{1}{\sqrt{N}}$

LGST review

1. N increases: $\varepsilon \rightarrow 0$
2. No self-calibration problem
3. Experimentally demonstrated
4. ε decreases slowly. (1/Sqrt(N))

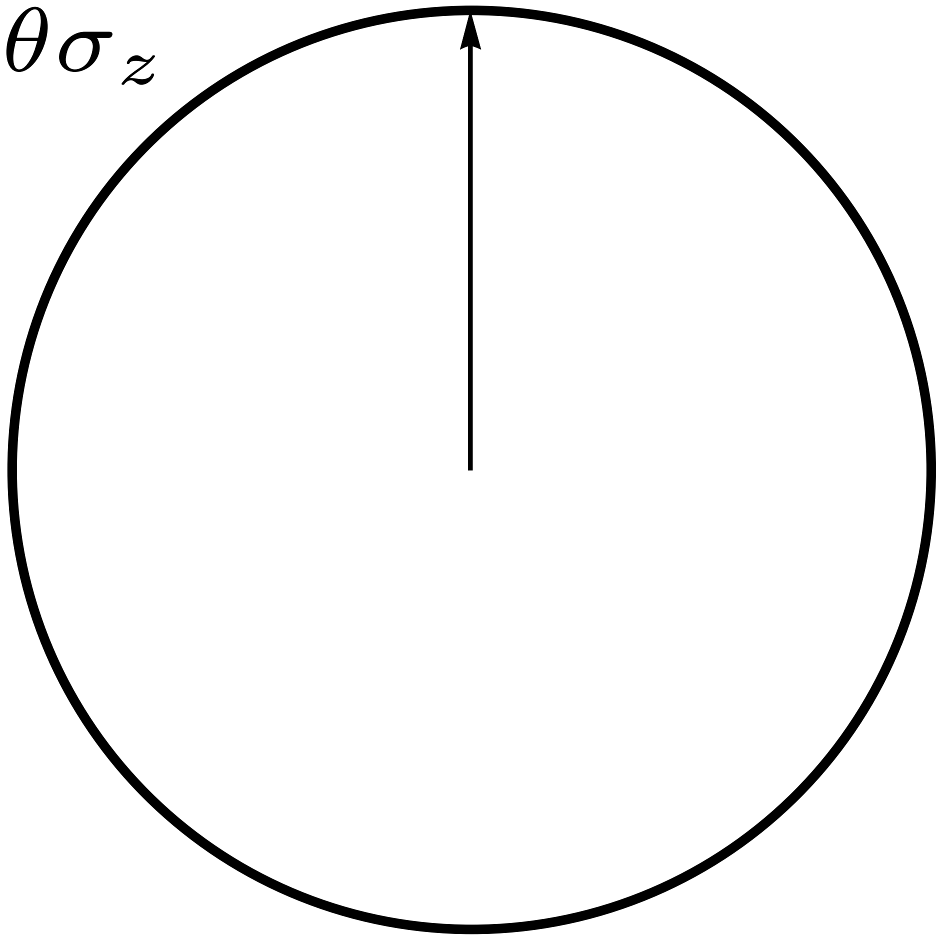
Can we do better?

Can we beat $\frac{1}{\sqrt{N}}$?

Want to be sensitive to small errors.

$$G = e^{-i\theta\sigma_z}$$

$$\theta << 1$$

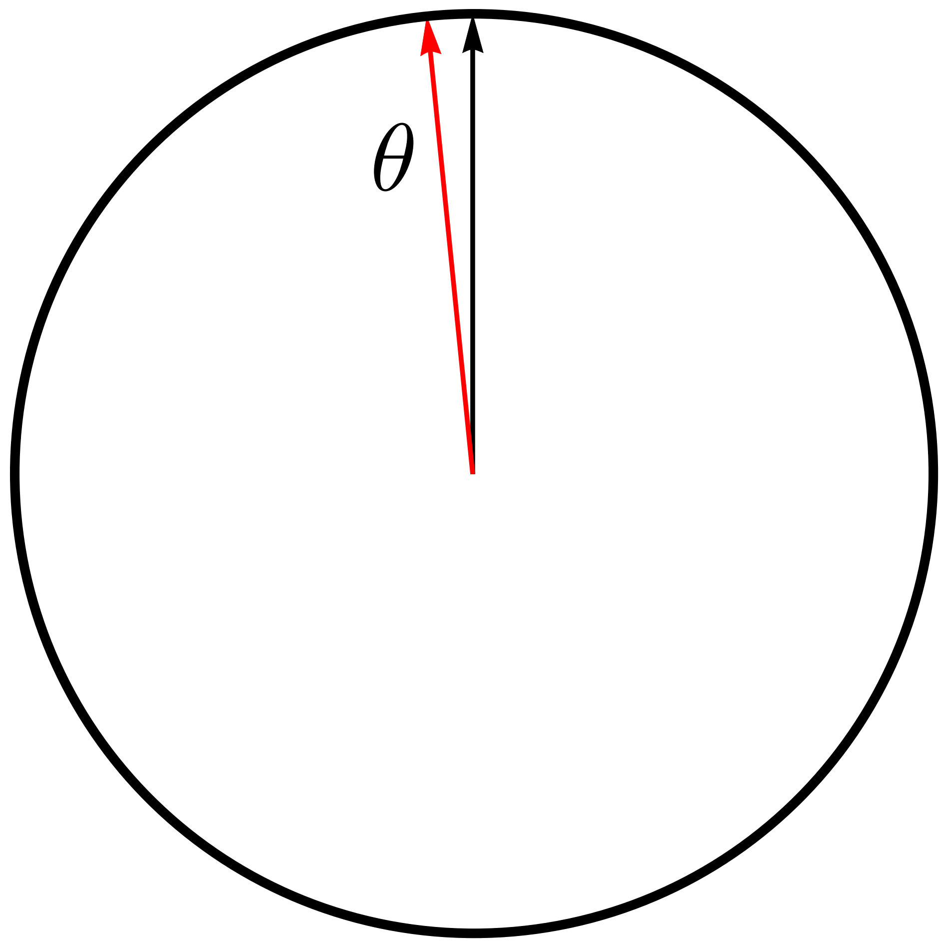


Can we beat $\frac{1}{\sqrt{N}}$?

Push G_1 :

$$\langle \sigma_z \rangle \approx 1 - \frac{\theta^2}{2}$$

Need O(θ^{-2}) measurements to distinguish from *I.*

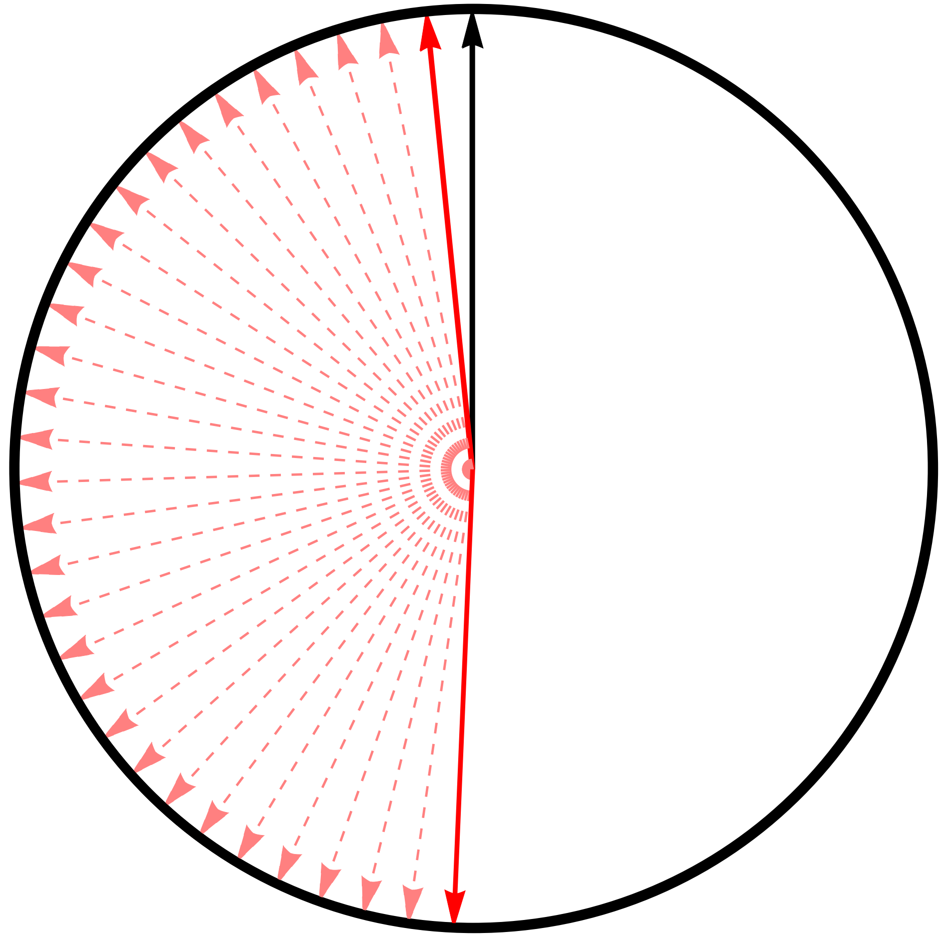


Can we beat $\frac{1}{\sqrt{N}}$?

Push G_1 L times:

$$\langle \sigma_z \rangle = \cos L\theta$$

$$\langle \sigma_z \rangle \ll 1 - \frac{\theta^2}{2}$$



Can amplify coherent errors!

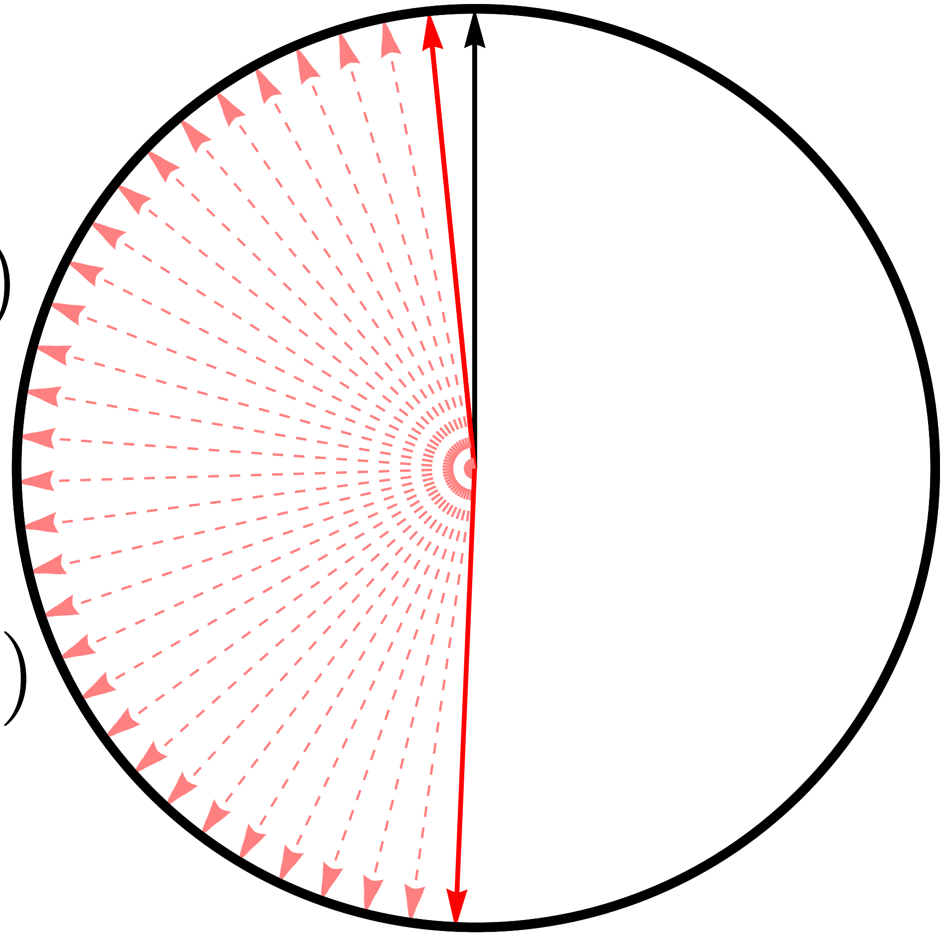
Can we beat $\frac{1}{\sqrt{N}}$?

Can measure with accuracy $\pm\theta$:

$$L = \mathcal{O}(1),\ N = \mathcal{O}(\theta^{-2})$$

$$OR$$

$$L = \mathcal{O}(\theta^{-1}),\ N = \mathcal{O}(1)$$



If we don't know θ, proceed iteratively.

$$(L = 1, 2, 4, \cdots, 512)$$

Can we beat $\frac{1}{\sqrt{N}}$?

Can each experiment just be a different gate repeated many times?

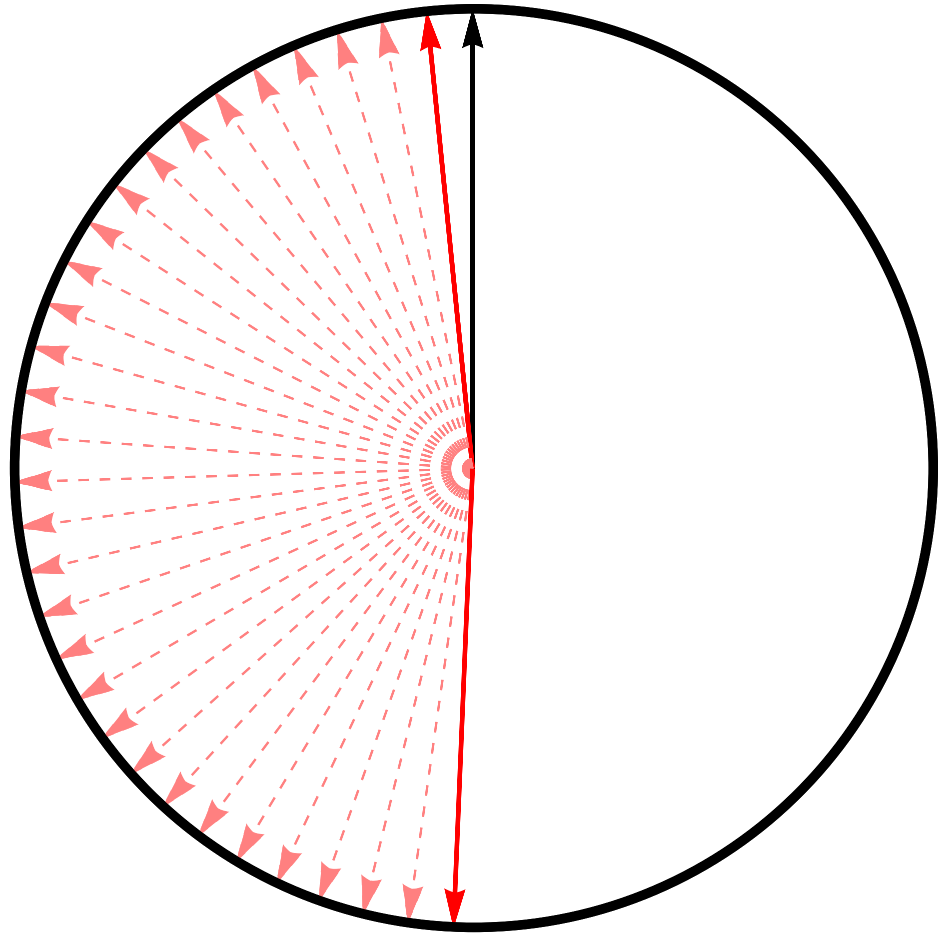
e.g. G_x^2, G_x^4,..., G_y^2, G_y^4...

Not sufficient. Need to amplify other errors as well.

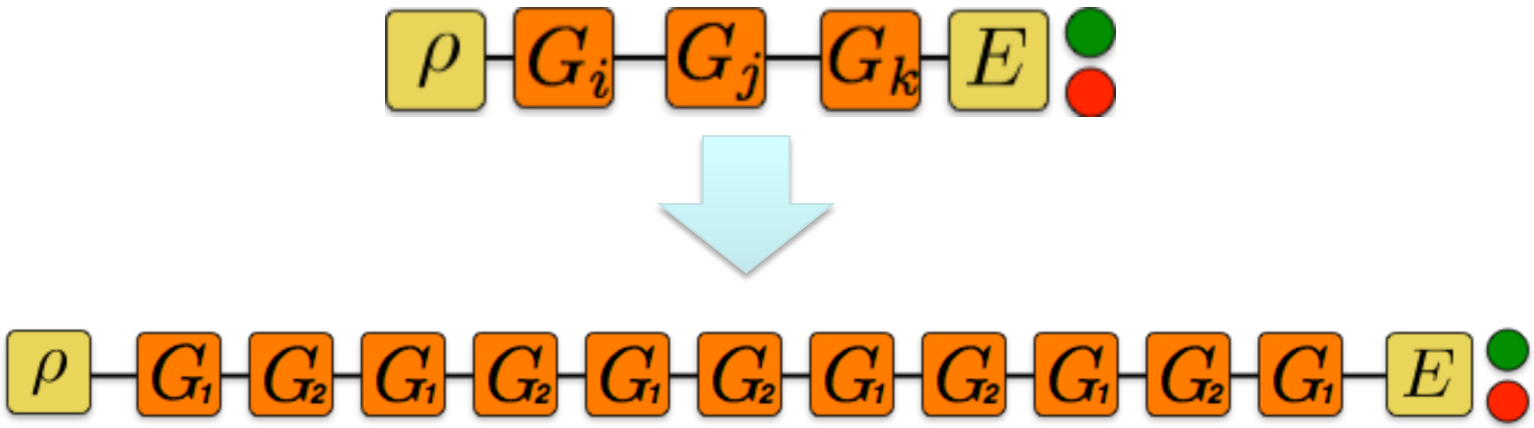
e.g. Tilt error

Also want sequences like

G_xG_y, $(G_xG_y)^2$, $(G_xG_y)^4$...



Can we beat $\frac{1}{\sqrt{N}}$?



Call these short sequences *germs.* Germs chosen to amplify errors. (E.g. tilt, over-rotation, dephasing.)

Do LGST on successively longer "powers".

We call this *extended linear gate set tomography (eLGST).*

Can we beat $\frac{1}{\sqrt{N}}$?

- eLGST can give us a rich data set, but how to best analyze it?

- Maximum likelihood?

1. Use LGST to estimate, e.g., G_x, G_y.
2. Use LGST to estimate sequences of length 2: e.g. G_x^2, G_y^2, G_xG_y.
3. Refine G_x, G_y estimates to be consistent with step 2.
4. Repeat steps 2, 3 with sequences of length 4, 8, etc.

Does estimate fit data?

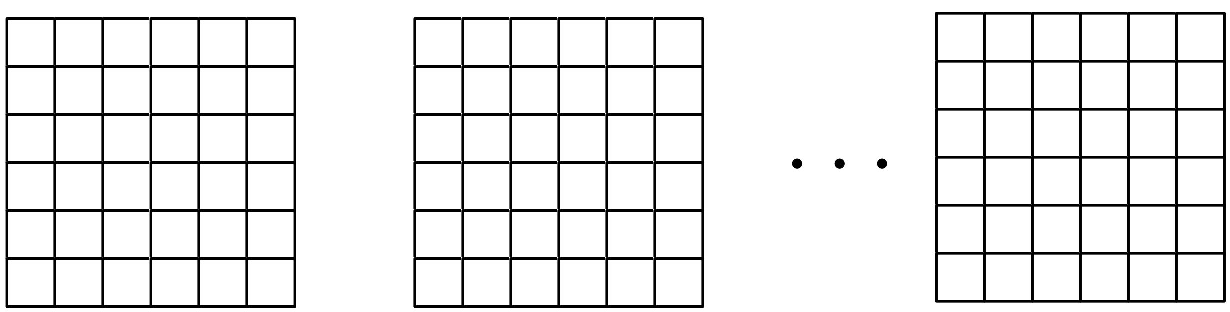
- Can use this eLGST procedure to estimate gates; works, but we can do better.
- How do we measure success? χ^2 statistic- "goodness of fit".

$$\text{Sequence } j =$$

$$\chi_j^2 = N_j \frac{(p_j - f_j)^2}{p_j(1-p_j)}$$

- Minimize total χ^2 at each step.

G_i



$L = 1$ $\quad L = 2$ $\quad \cdots \quad$ $L = 512$

Algorithm summary

1. Start with experimental gate set, eg. {Gi, Gx, Gy}
2. From knowledge of target gate set, determine set of germs e.g. {Gx, Gy, Gi, GxGy, GxGyGi, GxGiGy, GxGiGi, GyGiGi, GxGxGiGy, GxGyGyGi, GxGxGyGxGyGy}
3. For varying maximum sequence length (L=1,2,…,512), perform "process tomography" experiments on each "extended germ"
4. Using least-squares, iteratively find gate set estimates that minimize χ^2.
5. Compare to target gate set.

How do we measure success?

1. Can we find "good estimate" with high probability?

2. Can we find accurate estimate cheaply? (Can we beat $N^{-0.5}$?)

3. Can we find estimate that fits the data well (small χ^2)?

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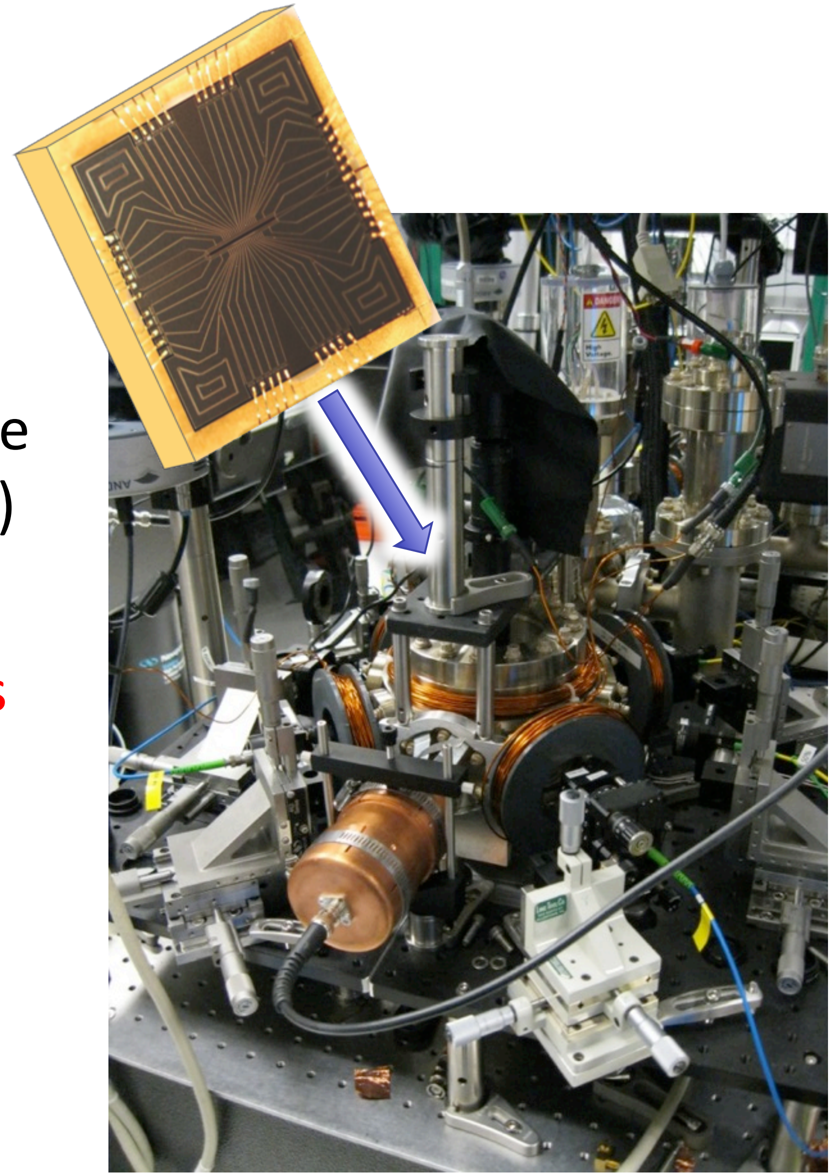
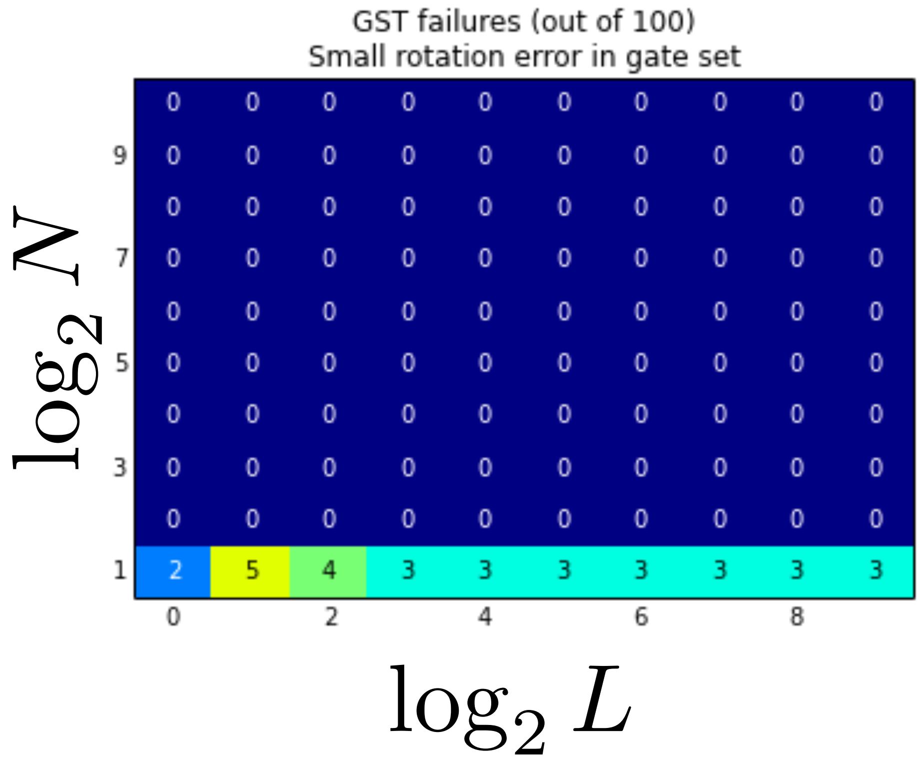
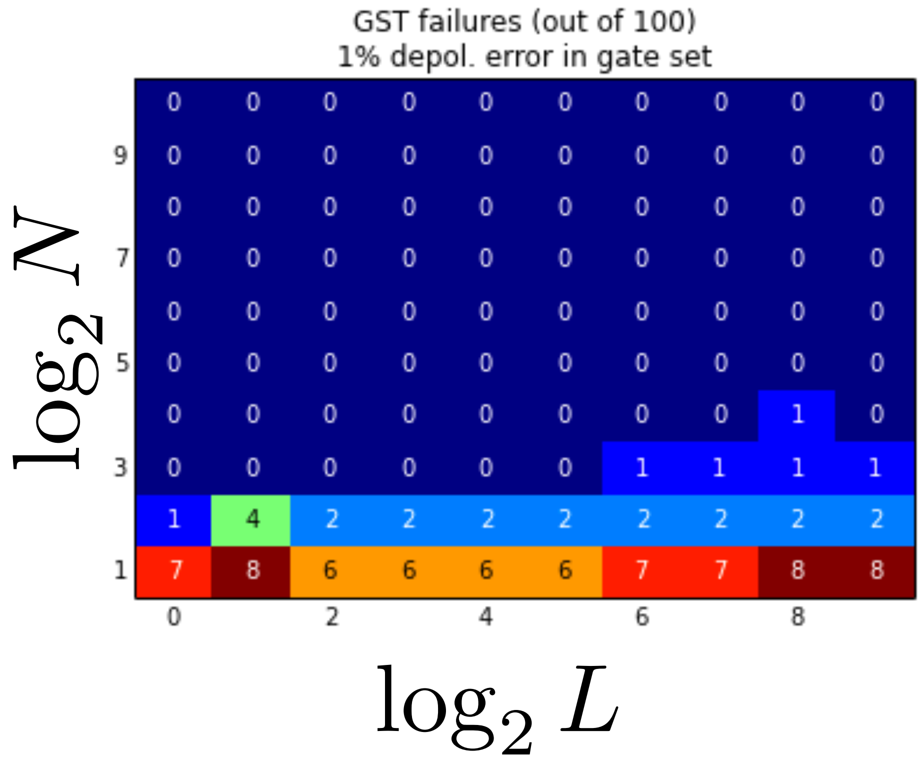


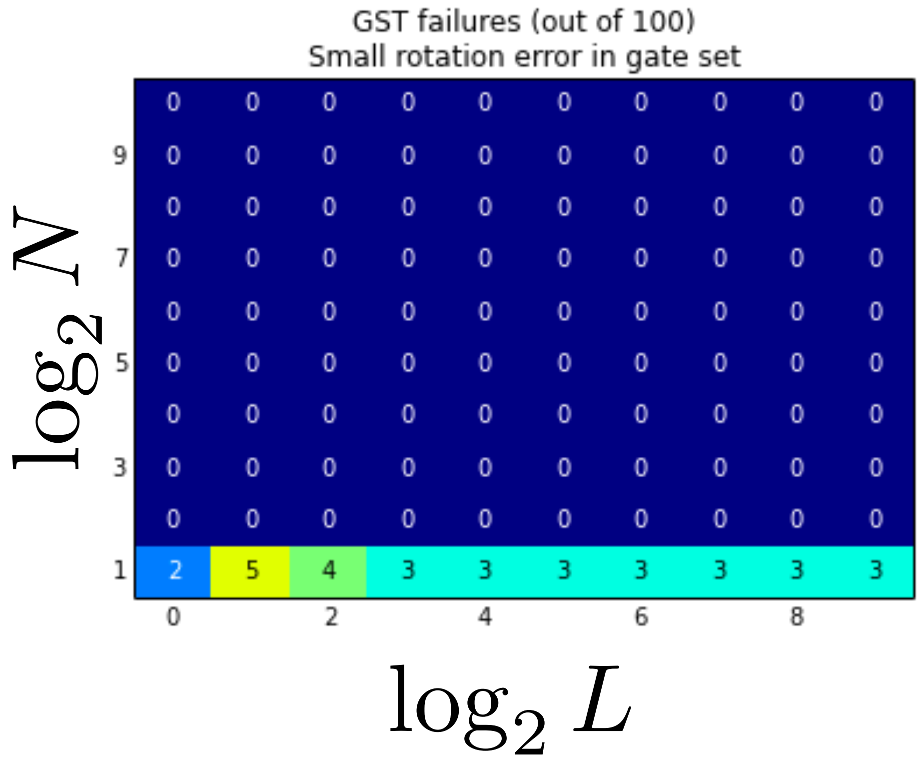
Success probability

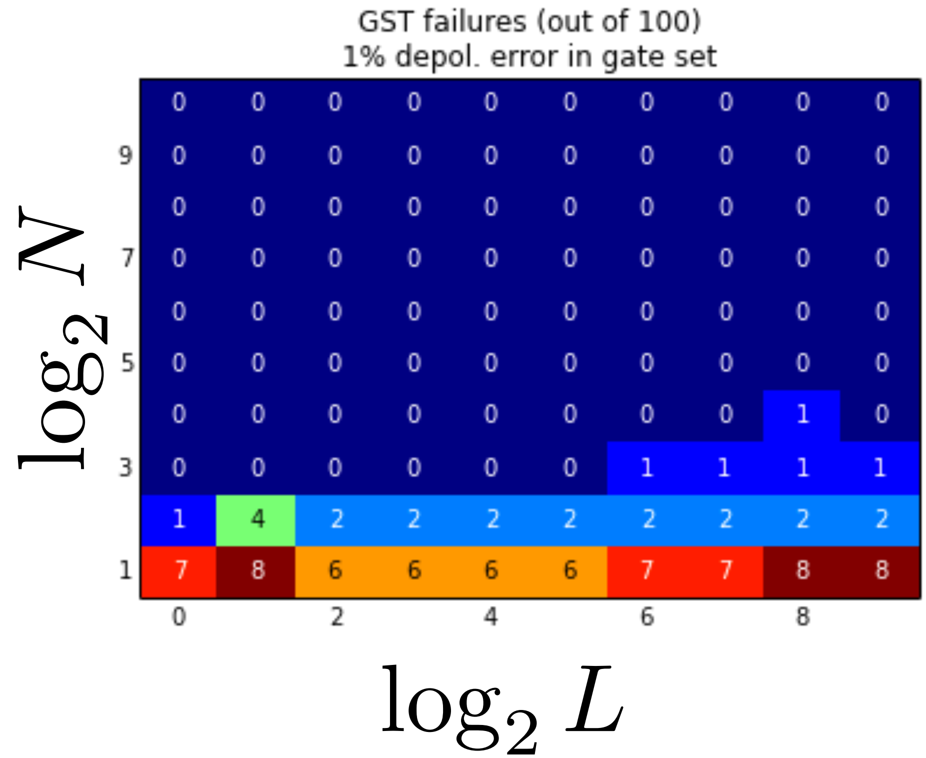




$$\text{Sample } \color{red}{\text{N}} \text{ times : } \langle E | G_k (G_i G_j)^{\color{red}{L}} G_m | \rho \rangle$$

Success probability

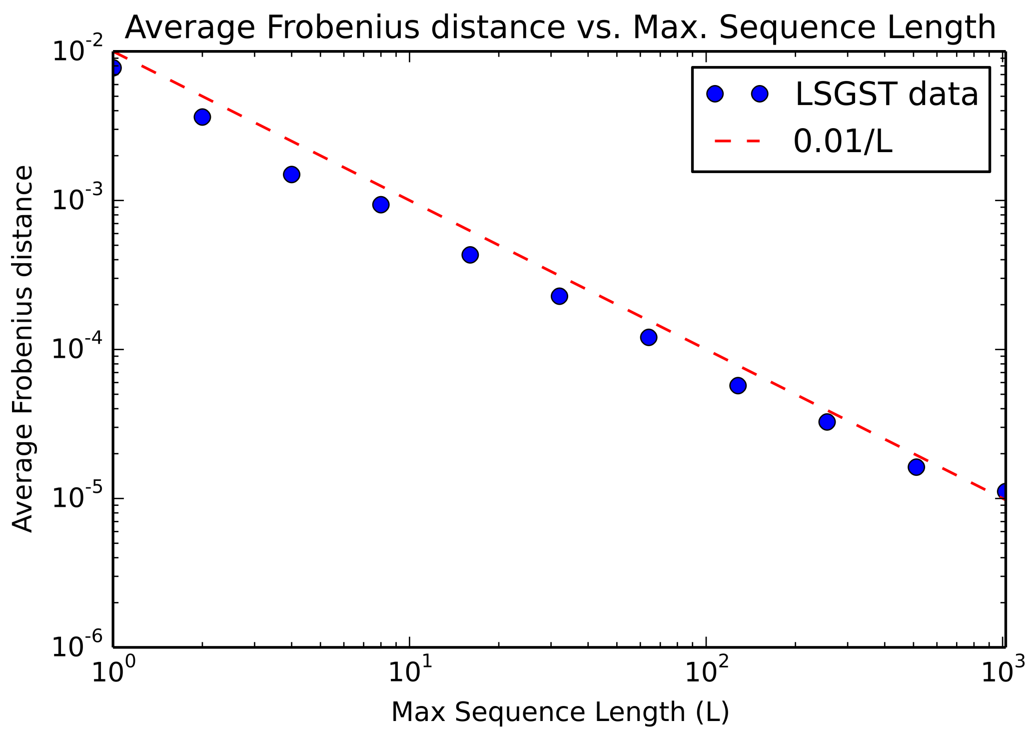




$$\text{Sample } \color{red}{\text{N}} \text{ times} : \langle E|G_k(G_iG_j)^{\color{red}{L}}G_m|\rho\rangle$$

SUCCESS!

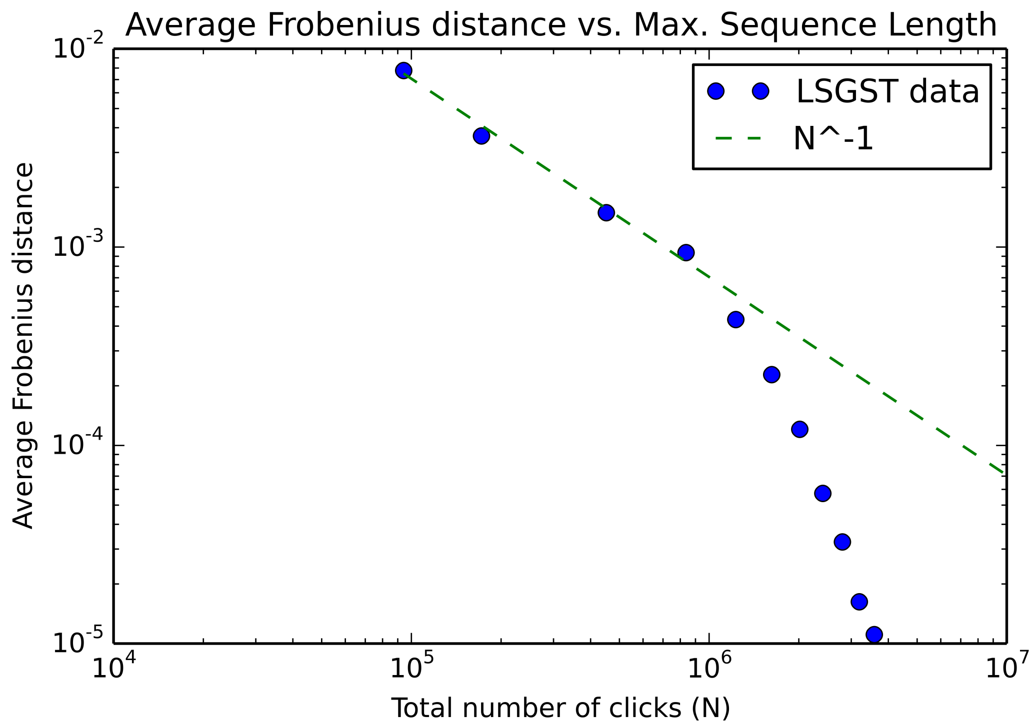
Can we beat $\frac{1}{\sqrt{N}}$?



Error scales as $\sim \frac{1}{L}$!

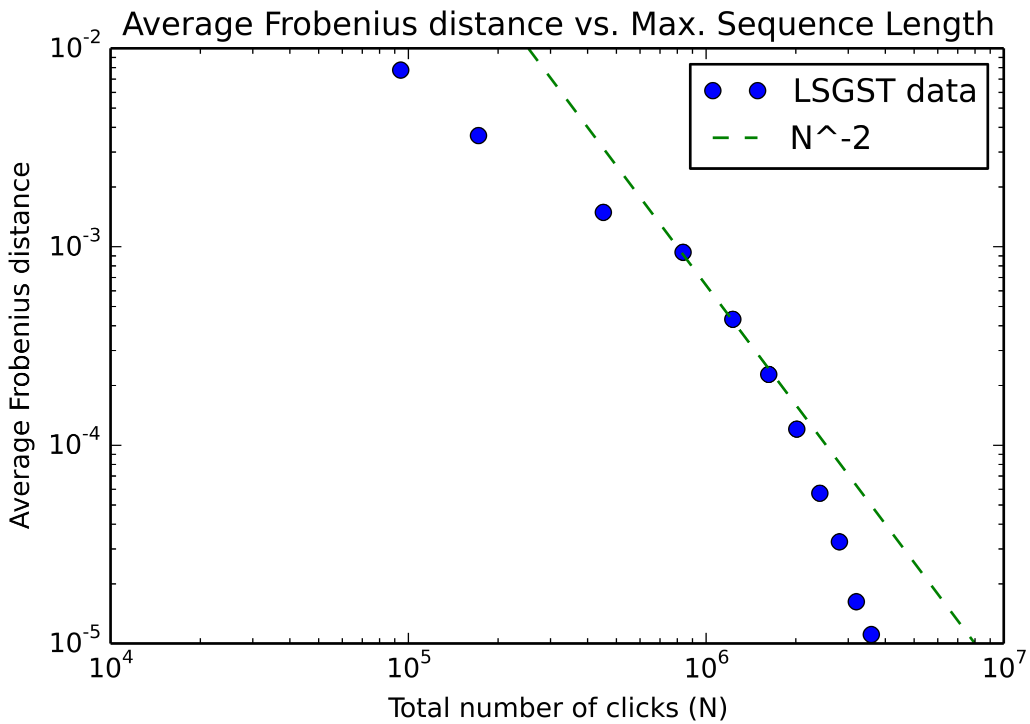
Can we beat $\frac{1}{\sqrt{N}}$?

$$N^{-1}$$



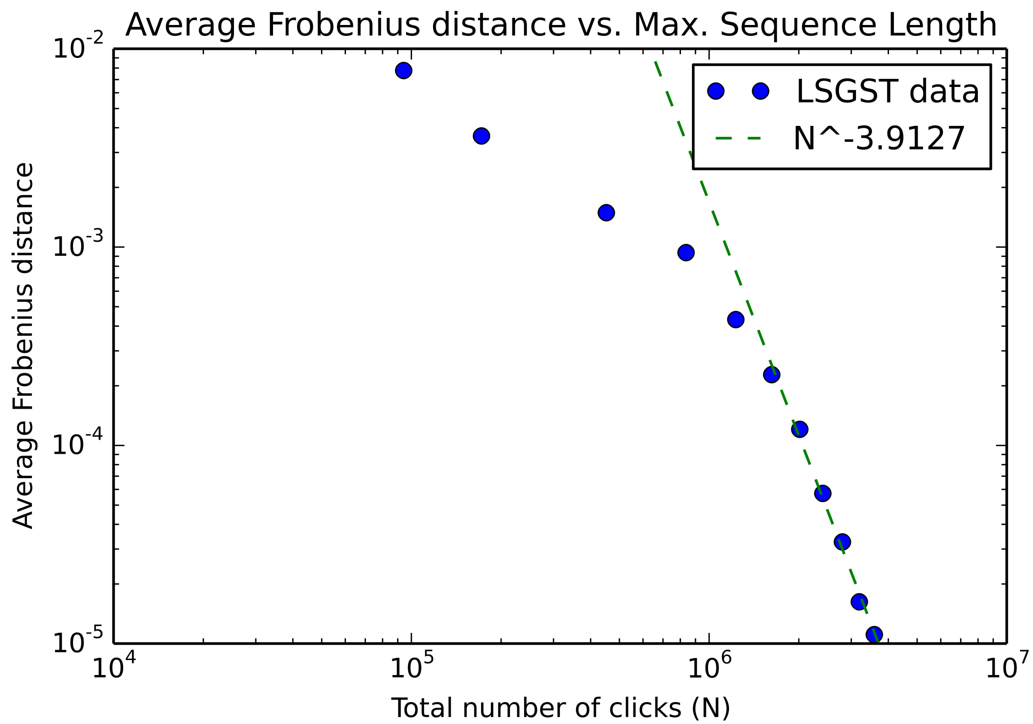
Can we beat $\frac{1}{\sqrt{N}}$?

N^{-2}



Can we beat $\frac{1}{\sqrt{N}}$?

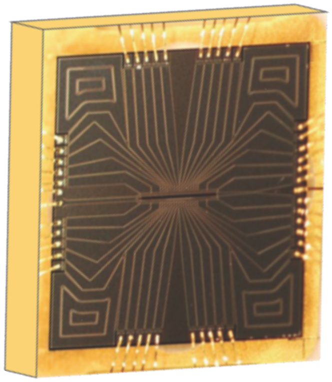
$$N^{-3.9}$$



Scaling *at least* as good as $N^{-3.9}$!

Maybe as good as e^{-N}?

Experimental implementation?



GST

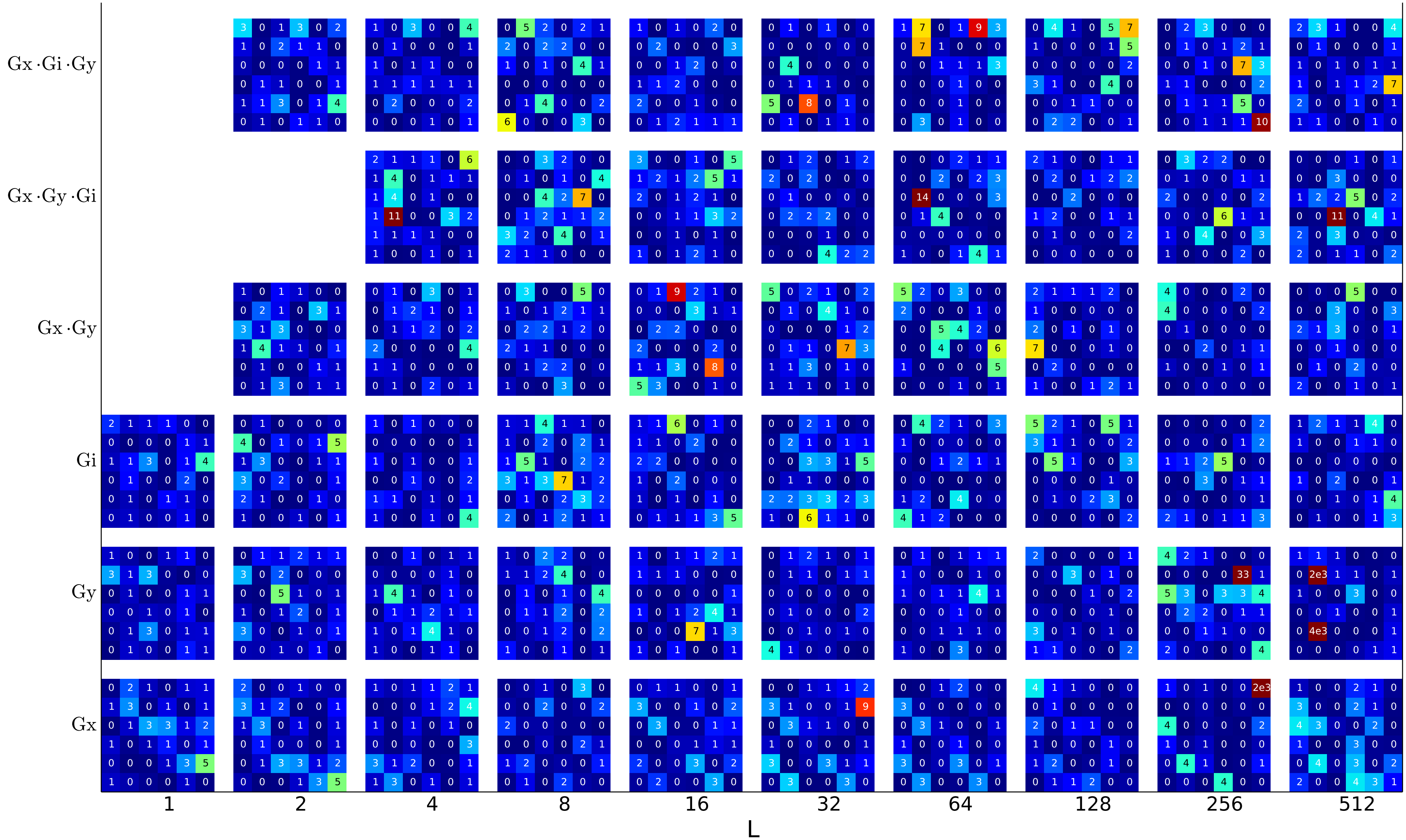
$$\{|\rho_{est}\rangle, \langle E_{est}|, G^i_{est}, G^x_{est}, G^y_{est}\}$$

Gate	Fidelity	Trace Dist.	Frobenius Dist.
Gi	0.996539	0.053209	0.020453
Gx	0.999738	0.009888	0.004233
Gy	0.999936	0.006494	0.002594

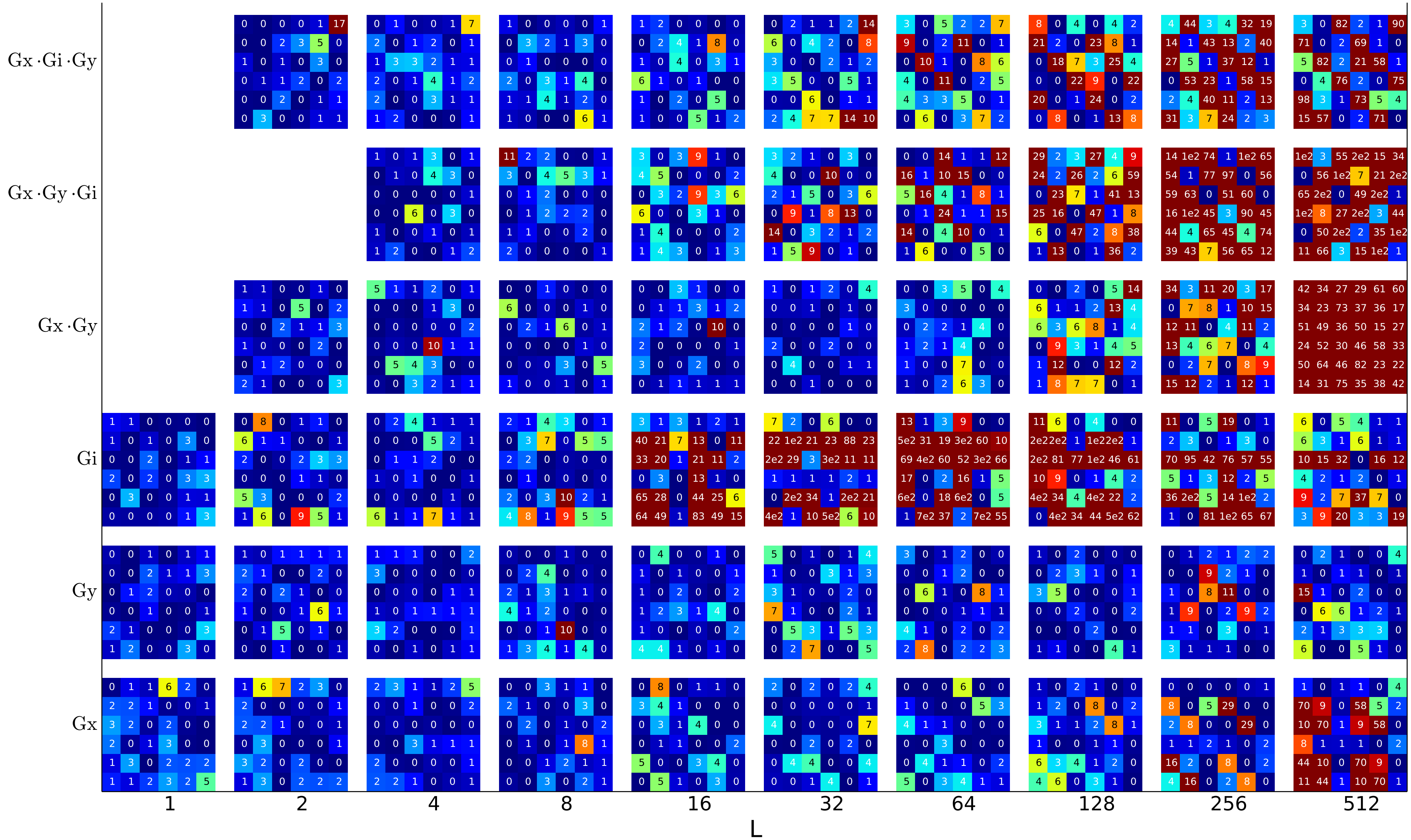
Should you believe us?

If χ^2 is small, then yes!

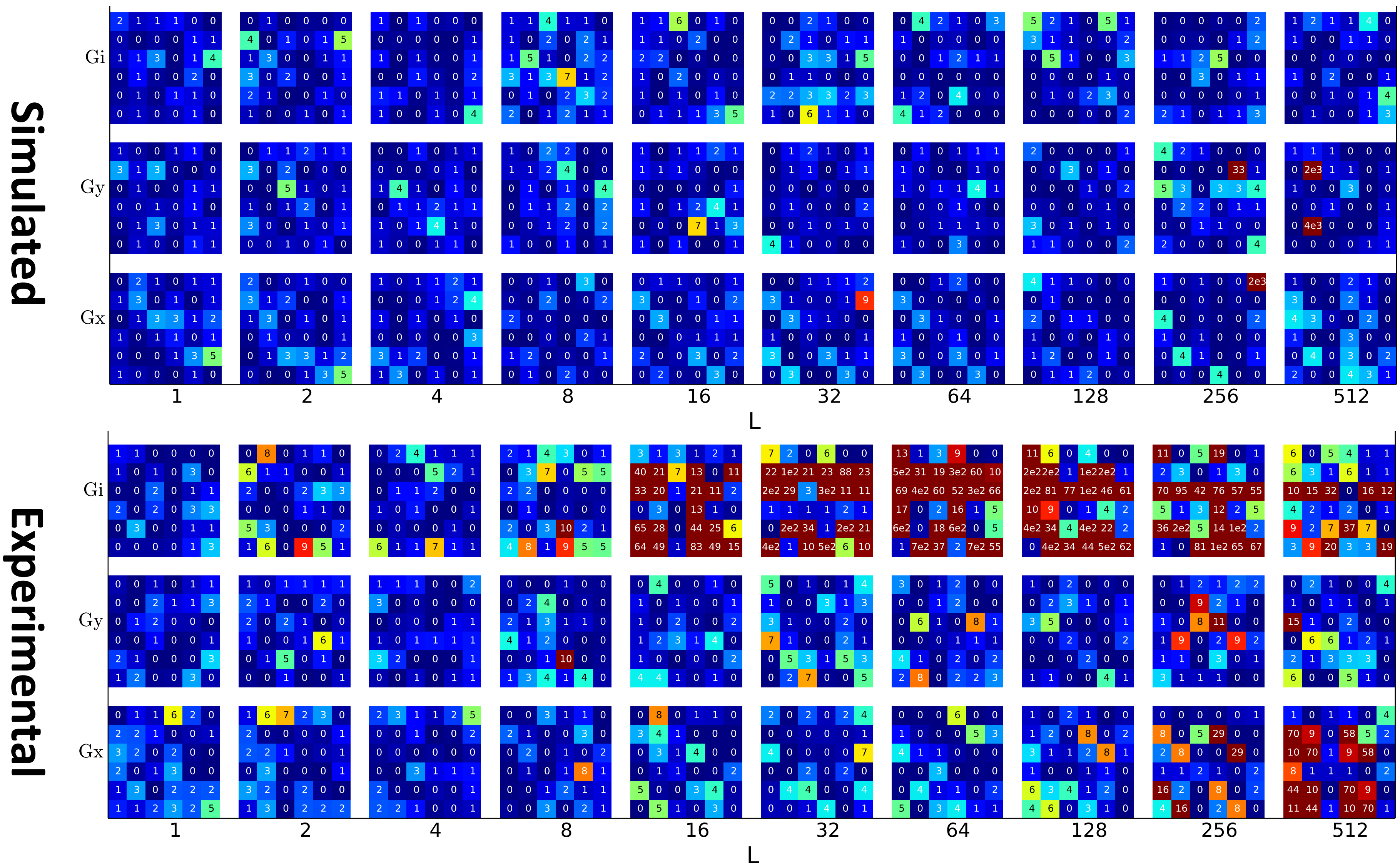
Simulated χ^2



Experimental χ^2



Simulated and Experimental χ^2



Experimental χ^2

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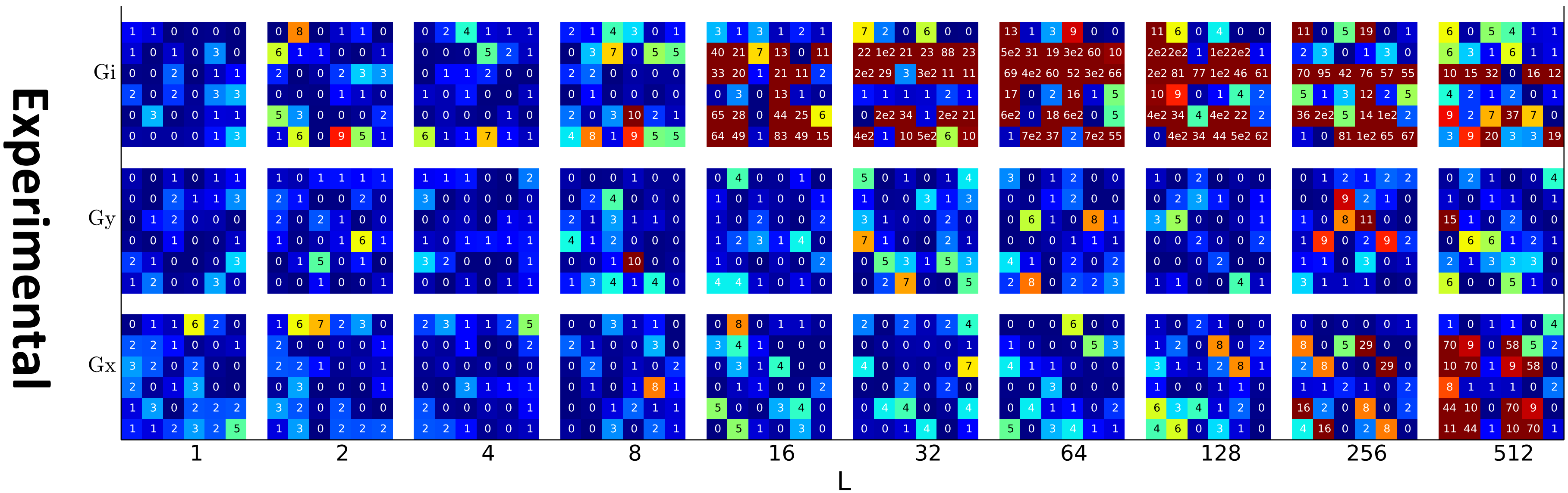
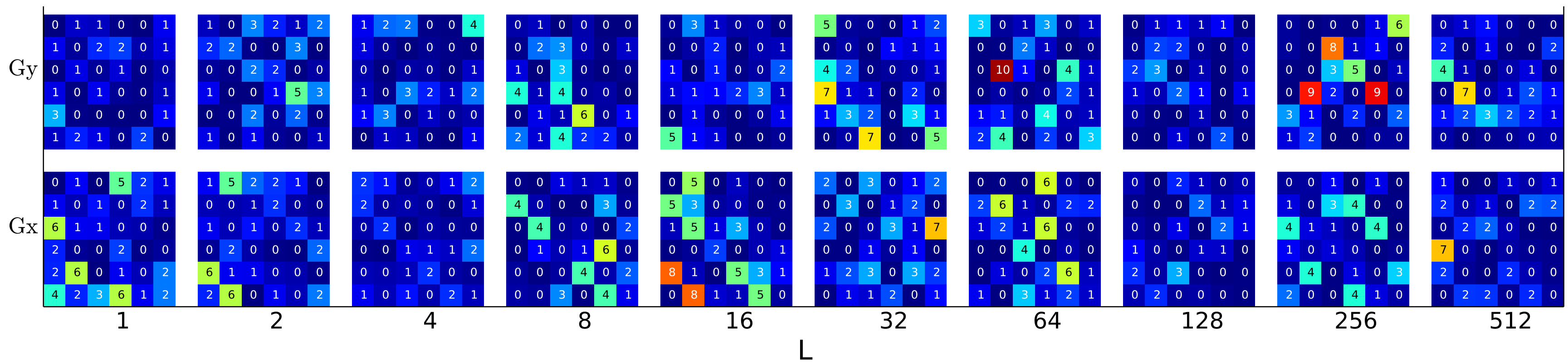


Good fit at small L, not at large L.

$\frac{d}{dt}$  $\neq 0$ **System is non-Markovian!**

Take out G_i



Gate	Fidelity	Trace Dist.	Frobenius Dist.
Gi	0.988231	0.064058	0.025256
Gx	0.999952	0.005906	0.002689
Gy	0.999955	0.005571	0.002222

We isolated non-Markovian behavior; found G_x, G_y nearly Markovian!

Are we successful?

1. Can we find "good estimate" with high probability? $> 99\%$

2. Can we find accurate estimate cheaply? (Can we beat $N^{-0.5}$?) $< N^{-3.9}$

3. Can we find estimate that fits the data well (small χ^2)? (When system is Markovian!)

Conclusions

- Can use GST to get reliable, highly accurate estimates which match experimental data; far cheaper than standard tomography.
- Can use GST to diagnose when non-Markovian noise becomes an issue.
- Can even sometimes use GST to figure out where non-Markovianity is coming from!

Future directions

- Multi-qubit systems
- Randomized benchmarking predictions
- Non-Markovian analysis
- Drift control.

- **More experimental implenetation**

  Contact me!

  kmrudin@sandia.gov

- **Thank you!**

- **Gnome image courtesy of** http://sweetclipart.com/friendly-garden-gnome-1464