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A Modal Craig-Bampton Substructure for Experiments, Analysis, Control and Specifications

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IMAC XXXIII

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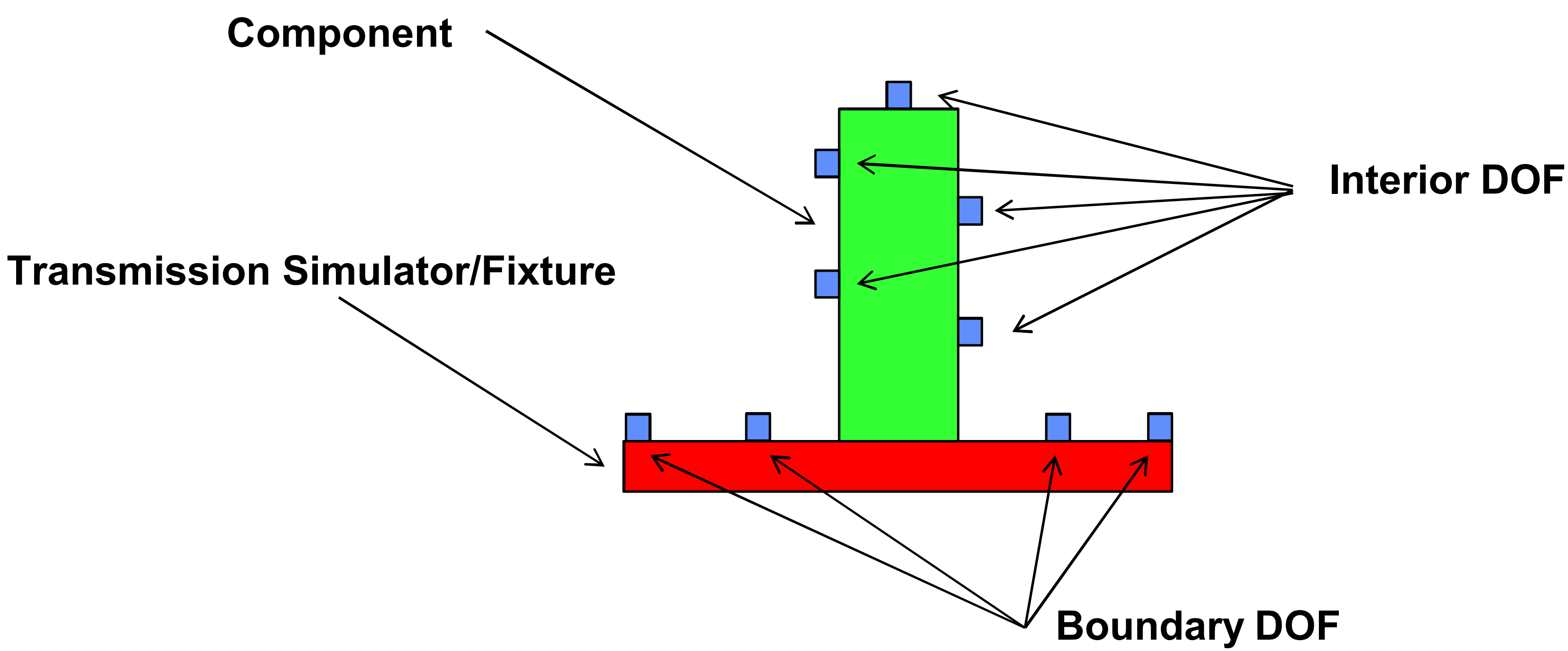
Outline of Presentation

- **This work was based on experimental dynamic substructuring using the transmission simulator method**
- **A *modal* Craig-Bampton-like (MCB) substructure is developed**
- **By accident, rather than intent, it was discovered that the substructure has some useful properties**
  - **The impedance of the boundary condition for a structural model is quantified**
  - **The model can be utilized for SDOF and 6DOF shaker control**
  - **Energy based qualification specification can be derived with the model to reduce over-conservatism but still guarantee conservatism**

Experimental Substructure Concept – Component/Fixture

- Perform a free modal test of a component attached to fixture
- The free modes of the fixture are known



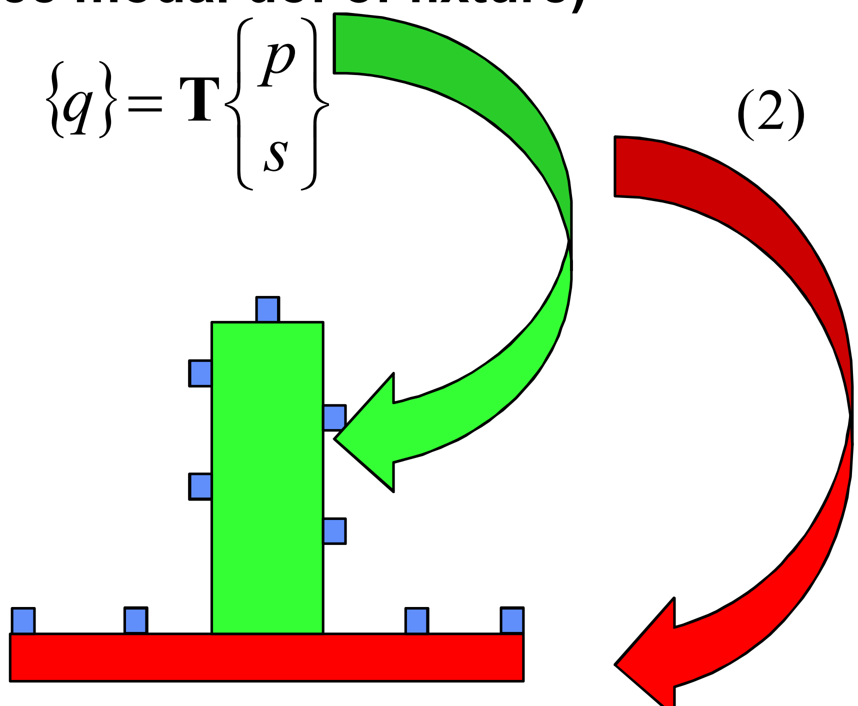
Equations for Transmission Simulator Modal CB approach

Beginning with the experimental model as

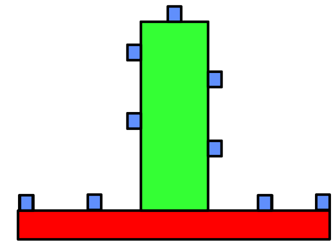
$$\left[\omega^2{}_{free} - \omega^2 I\right]\{q\} = \{0\} \qquad (1)$$

Find a square transformation $\mathbf{T}$ that relates *q* to *p* (fixed base modal dof) and s (free modal dof of fixture)

$$\{q\} = \mathbf{T}\begin{Bmatrix} p \\ s \end{Bmatrix} \qquad (2)$$



Equations for Transmission Simulator Modal CB approach



The transformation is derived in the paper as

$$\mathbf{T} = \left[\mathbf{L}_{fix}\boldsymbol{\Gamma}_{fix} \quad \Phi_b^{+}\Psi_b\right] \qquad (3)$$

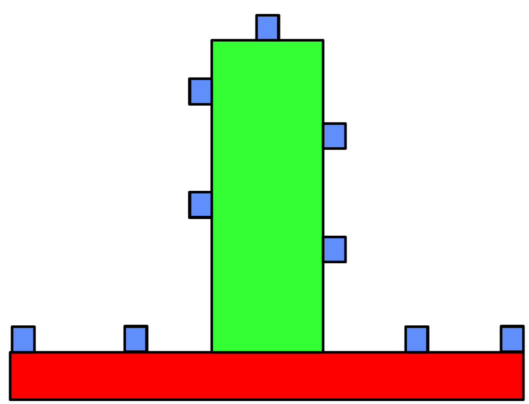
A property of a transformation is that it does not change the results of the eigenvalue analysis.

Pre and post multiply the mass and stiffness matrices of eqn (1) by $\mathbf{T}'$ and $\mathbf{T}$

The CB matrices separate the component and the fixture (TS)

Green matrices are fixed base modal matrices for component.
Blue matrices are the free modal matrices of fixture.

$$\left[\begin{bmatrix} \begin{matrix} \omega_{p1}^2 & 0 \\ 0 & \omega_{p2}^2 \end{matrix} & K_{ps} \\ K_{ps}^{'} & K_{ss} \end{bmatrix} - \omega^2 \begin{bmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & M_{ps} \\ M_{ps}^{'} & M_{ss} \end{bmatrix}\right] \begin{Bmatrix} p_1 \\ p_2 \\ s_{rb} \\ s_{elas} \end{Bmatrix} = \{0\} \qquad (4)$$



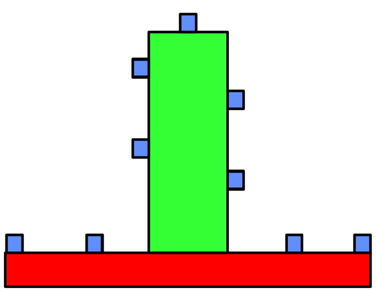
Relate fixture motion to the component

Because the p dof are uncoupled, we can consider the first row by itself, and it stands on its own.

$$\left(\omega^2{}_{fix} - \omega^2\right)p_1 - \omega^2 m_{rb} s_{rb} + (k_{elas} - \omega^2 m_{elas})s_{elas} = 0 \qquad (5)$$

$$\boxed{\left(\omega^2{}_{fix} - \omega^2\right)p_1} = \boxed{\omega^2 m_{rb} s_{rb} - (k_{elas} - \omega^2 m_{elas})s_{elas}} \qquad (6)$$

Fixed base component



Motion of fixture as forcing terms

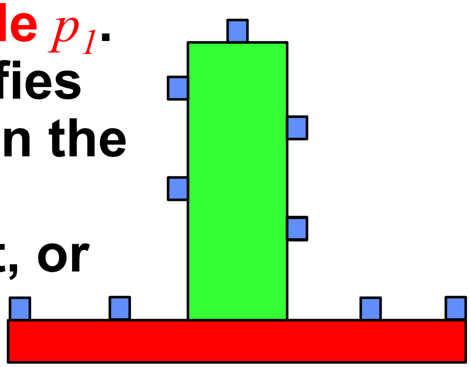
These terms quantify the effects of the fixture on the component with simple terms on a mode by mode basis

Coupling terms quantify elastic fixture effects on component, removing uncertainty of "boundary condition"

This approach quantifies the elastic effects of the fixture on the test article (this is usually a great mystery)

$$\left(\omega^2{}_{fix} - \omega^2\right)p_1 = \omega^2 m_{rb} s_{rb} - \boxed{\left(k_{elas} - \omega^2 m_{elas}\right)s_{elas}} \tag{6}$$

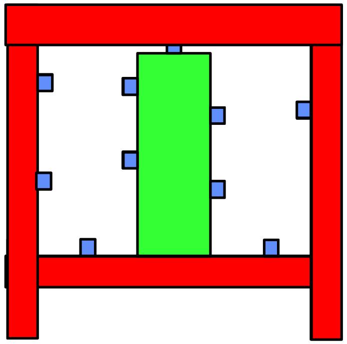
Every active fixture mode will have such a term. This model quantifies the effect of the fixture on the test article response for mode p_1. Another way to say it is, "This term quantifies the effect of the impedance of the fixture on the test article response". This can either be identified as creating overtest or undertest, or possibly corrected.



FE models can play this game, too.

We can transform a set of finite element model stiffness and mass matrices in this same way to determine system and component response to an environment. In this case, the <u>system</u> becomes the <u>transmission simulator</u> or <u>fixture</u>.

$$(6) \qquad \boxed{\left(\omega^2{}_{fix} - \omega^2\right)p_1} = \boxed{\omega^2 m_{rb} s_{rb} - (k_{elas} - \omega^2 m_{elas}) s_{elas}}$$

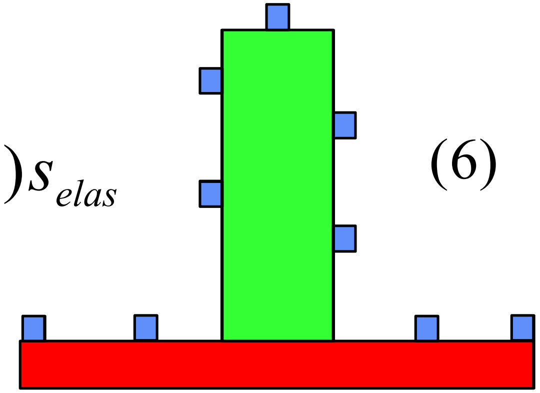


Fixed base component **Remainder of system**

The same quantification of impedance applies for the system. Now we have some way to quantify system impedance *vs* test fixture impedance.

Mass coupling terms useful for shaker control

Mass coupling terms determine the shaker input response (including 6 dof) to get the desired motion p



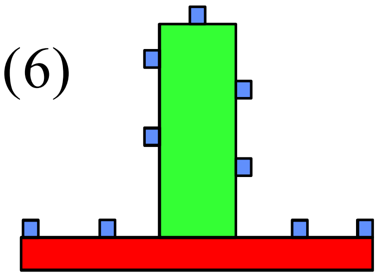
$$\left(\omega^2{}_{fix} - \omega^2\right)p_1 = \boxed{\omega^2 m_{rb} s_{rb}} - (k_{elas} - \omega^2 m_{elas})s_{elas} \qquad (6)$$

This mass coupling term is the modal participation factor. There are 6 of these terms associated with the 6 rigid body modes of the fixture. These are important terms for 1 dof or 6 dof shaker control.

This provides a possible new approach for deriving specifications that is compatible with energy methods

Develop the specification in terms of energy in mode p_1.

$$\boxed{\left(\omega^2{}_{fix} - \omega^2\right)p_1} = \omega^2 m_{rb} s_{rb} - (k_{elas} - \omega^2 m_{elas}) s_{elas} \qquad (6)$$



It may be prudent to specify the environment as a function of energy in each fixed base component mode. For example, if strain energy is the damaging potential, one might specify the required environment for this mode in terms of potential energy. This approach can guarantee conservatism but reduce over-conservatism that is common in valleys of power spectral density specifications.

$$potential\_energy = 1/2\left(\omega^2{}_{fix}\right)p_1^2$$

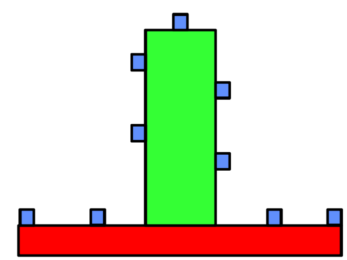
This framework is conducive to communication

This framework restructures the communication in terms of the fixed boundary modes of the test article.

1. **Vibration engineers interested in control and qualification**
2. **Environmental engineers interested in qualification or margin testing**
3. **Finite element modelers interested in response**
4. **Modal engineers interested in response**

5. **Maybe even managers**

$$\boxed{\left(\omega^2_{fix} - \omega^2\right) p_1} = \boxed{\omega^2 m_{rb} s_{rb} - (k_{elas} - \omega^2 m_{elas}) s_{elas}}$$



Fixed base component

Test fixture or system