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Global Solution of AC Optimal Power Flow Problems

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U.S. DEPARTMENT OF ENERGY
NNSA National Nuclear Security Administration

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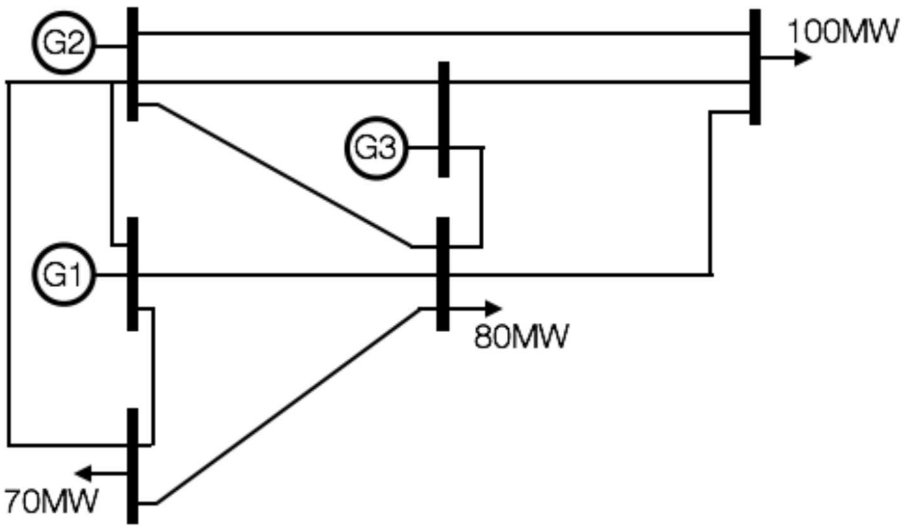
The Optimal Power Flow (OPF) Problem



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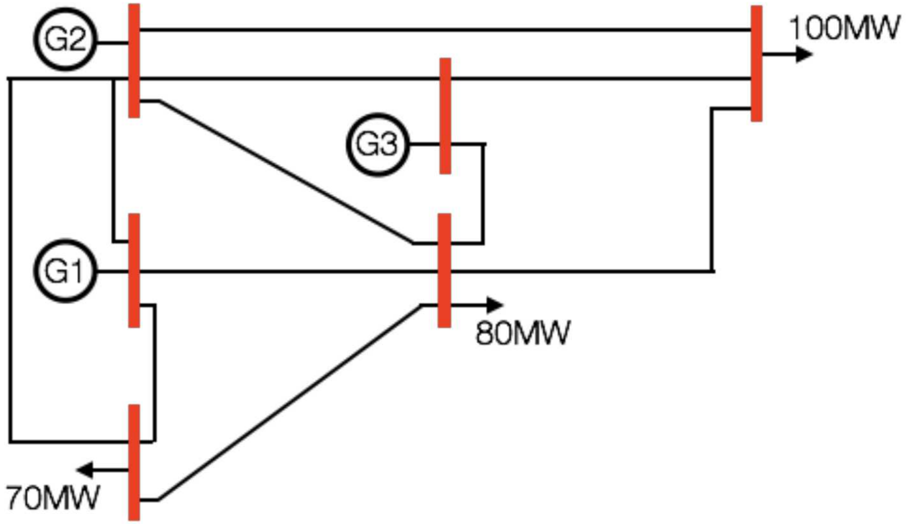
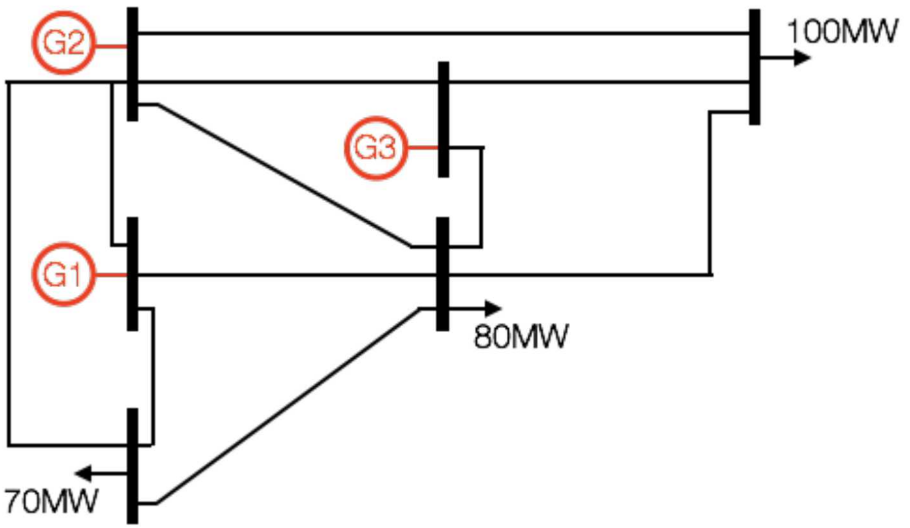


6 Buses

The Optimal Power Flow (OPF) Problem



6 Buses

3 Generators

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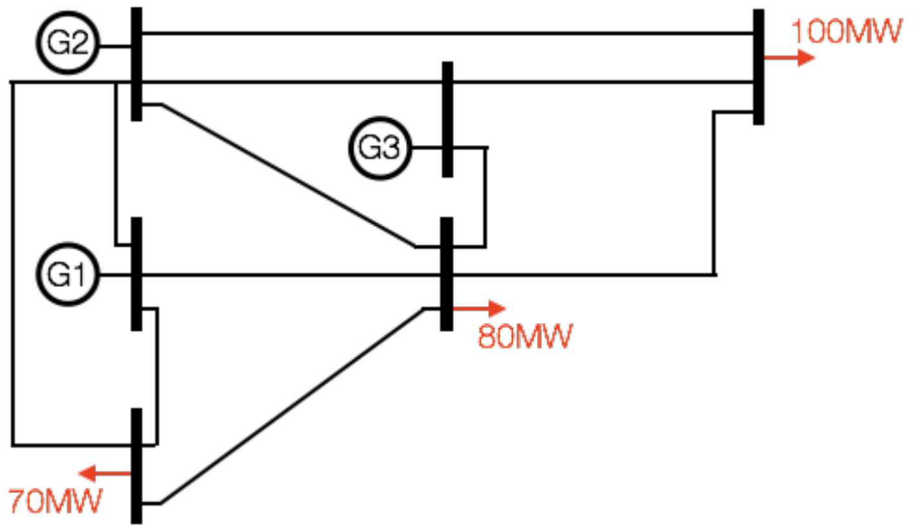

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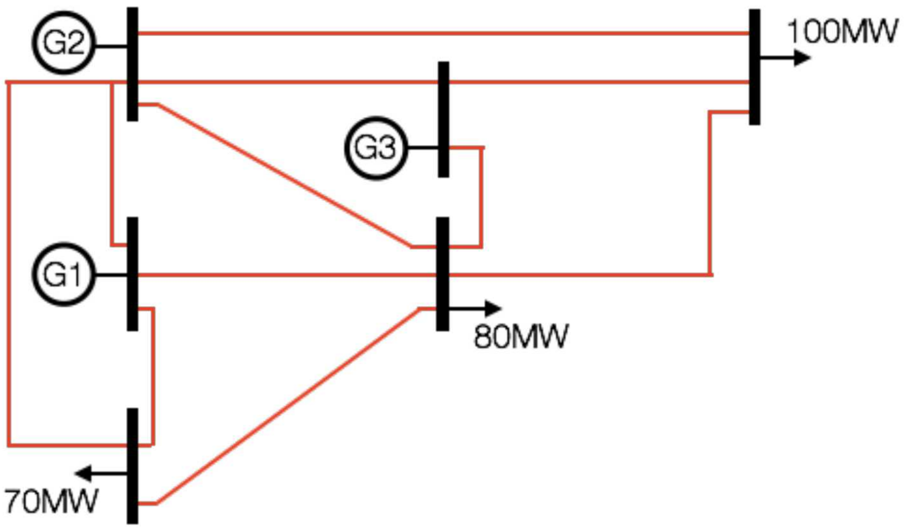

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11 Transmission Lines

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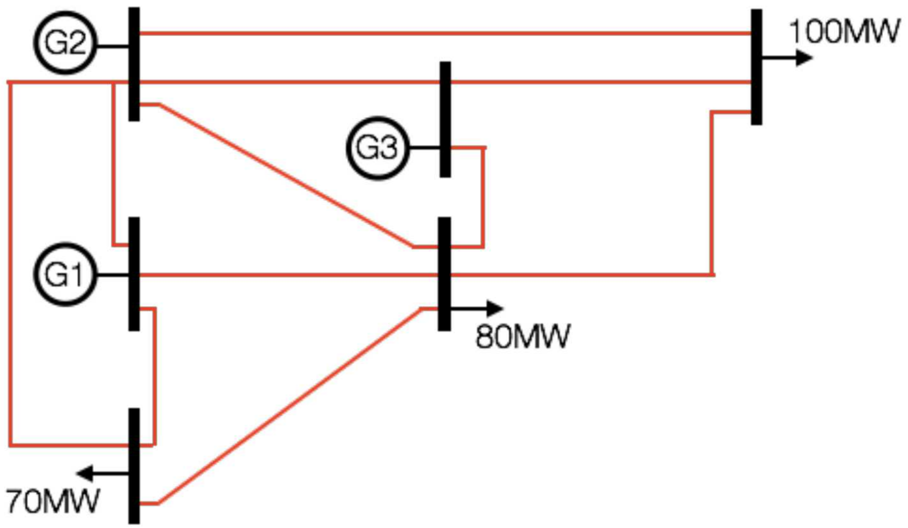

6 Buses

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11 Transmission Lines

What should the online generator production levels be to minimize cost?

Motivation for Global Optimization

Problems with discrete decisions (e.g., the unit commitment problem) typically use linearized transmission models

Linearized models do not guarantee optimality or even feasibility for the nonlinear system

Using a nonconvex nonlinear model requires global optimization

Outline

Review of ACOPF formulations

Review of convex relaxations of ACOPF formulations

Unit commitment with ACOPF

Alternative formulations

- Rectangular McCormick
- Importance of the reference bus

ACOPF Formulations

$$\min \sum_{g \in \mathcal{G}} [A_g^2 (p_g^G)^2 + A_g^1 p_g^G + A_g^0]$$

s.t.

$$\sum_{(b,k) \in \mathcal{L}_b^{in}} p_{b,k}^t + \sum_{(b,k) \in \mathcal{L}_b^{out}} p_{b,k}^f + G_b^{sh} w_b + P_b^D - \sum_{g \in \mathcal{G}_b} p_g^G = 0 \qquad \forall\, b \in \mathcal{B}$$

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$$c_{b,k}^2 + s_{b,k}^2 = w_b w_k$$

SOCR

$$c_{b,k}^2 + s_{b,k}^2 \leq w_b w_k$$

[Jabr, 2006] Second order cone (SOC) relaxation (Very efficient – scales well)

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- Cycle constraints (ensure the voltage angle differences around the cycles sum to 0)
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[Liu et al. 2017]: global solution of ACOPF using Outer Approximation (NLP ←→ MISOCP) based on SOC relaxation with "cycle constraints" and piecewise refinement of arctans and McCormick for the other side of the SOC.

Table 2
Computational performance. Note that the solver CPU time does not include overhead computational costs related to data processing, model construction, and OBBT.

Case name	Upper bound	Lower bound	Gap (%)	CPU time (s)	Iteration
nesta_case3_lmbd	5812.64	5812.64	0.00	0.25	3
nesta_case5_pjm	17551.89	17536.94	0.09	50.37	16
case6ww	3143.97	3142.55	0.02	0.07	1
nesta_case6_ww	3143.97	3143.43	0.02	0.11	1
case14	8081.52	8081.10	0.01	0.10	1
nesta_case14_ieee	244.05	244.04	0.00	0.07	1
case30	576.89	576.45	0.08	33.01	6
nesta_case30_ieee	204.97	204.78	0.09	250.02	8
case39	41864.12	41862.14	0.00	2.76	1
nesta_case39_epri	96505.52	96499.21	0.01	0.72	1
case57	41737.79	41731.17	0.02	0.92	1
nesta_case57_ieee	1143.27	1143.10	0.01	0.27	1
case118	129660.69	129562.20	0.08	53.83	3
nesta_case118_ieee	3718.64	3696.81	0.59	423.13[a]	4

[a] This case failed to solve to a gap of 0.1% in the allotted time (3600 s). The reported gap of 0.59% was achieved in 423 s.

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[Liu et al. 2018]: extended this approach to global solution of unit-commitment problems with AC power flow constraints (48 hour time horizon).
- Outer Approximation approach MIP ←→ NLP
- NLP solved using above, MIP refined with integer cuts only
- Good results on the few available test cases, but not yet "operational"

Case	Upper Bound ($)	Lower Bound ($)	Optimality Gap (%)	Wall Clock Time (s)	Iteration (k)
6-bus	101,763	101,740	0.02%	8.5	2
RTS-79	895,040	894,392	0.07%	1394	6
RTS-96	886,362	885,707	0.07%	321.0	1
IEEE-118mod	835,926	833,057	0.34%	14400*	2

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McCormick Relaxations?
- SOCR ⊆ R-McC with particular vr, vj bounds

[Hijazi et al. 2017] convex quadratic relaxation of the polar form

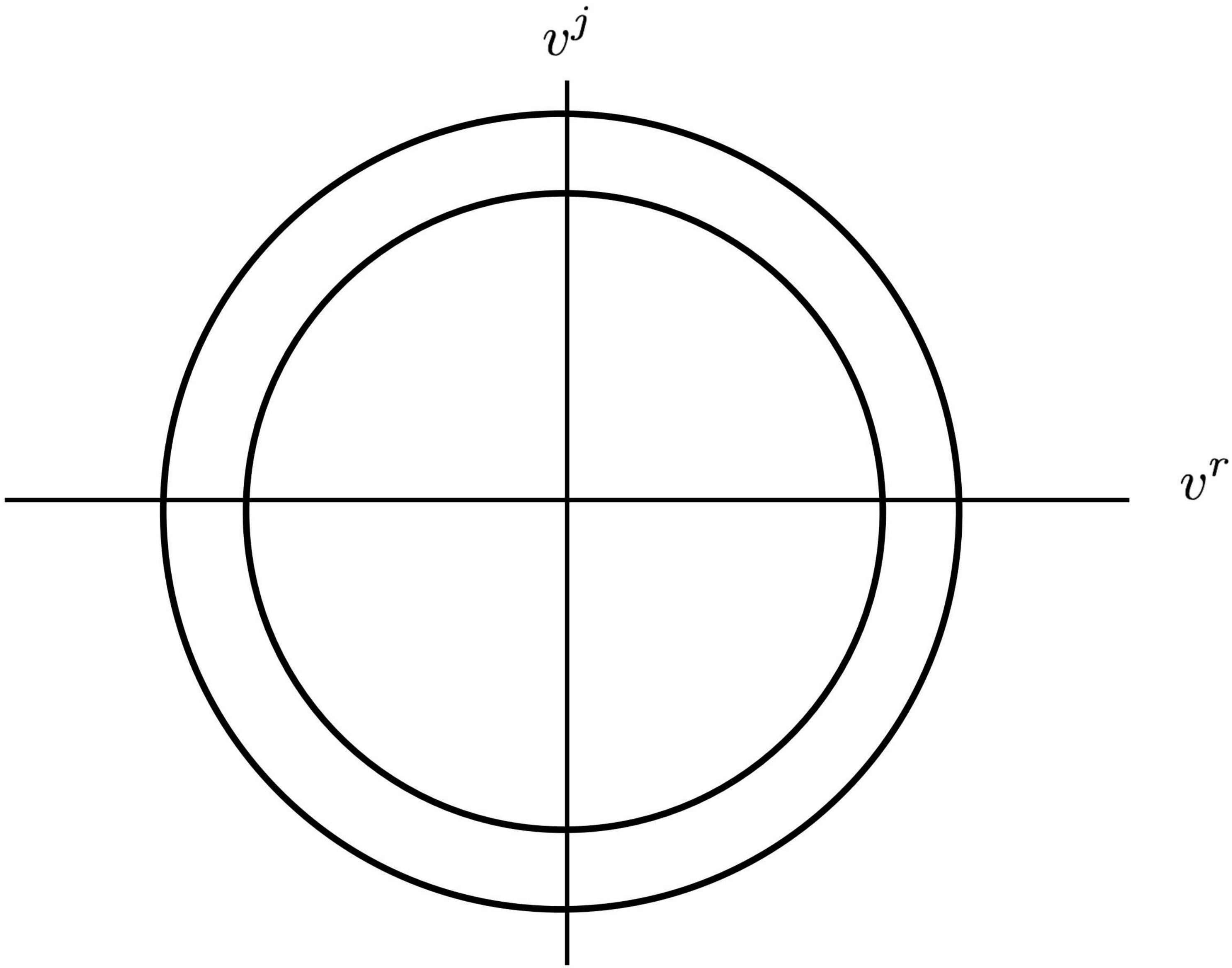
* [Lavaei & Low, 2012] SDP relaxations

Reference Bus

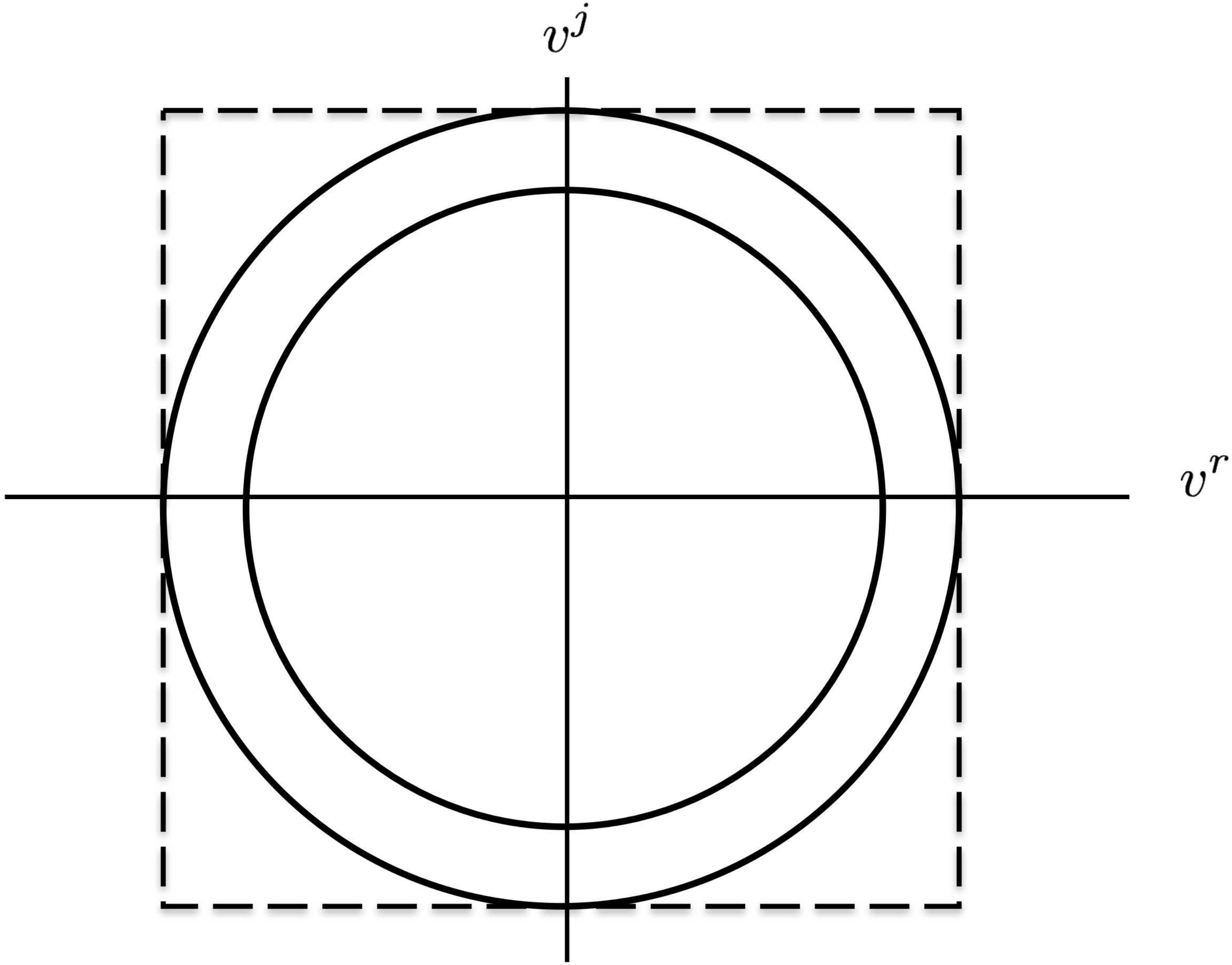
The reference bus is used to specify the voltage angle at one bus.

The solution to the ACOPF problem is not unique without a reference bus.

Reference Bus

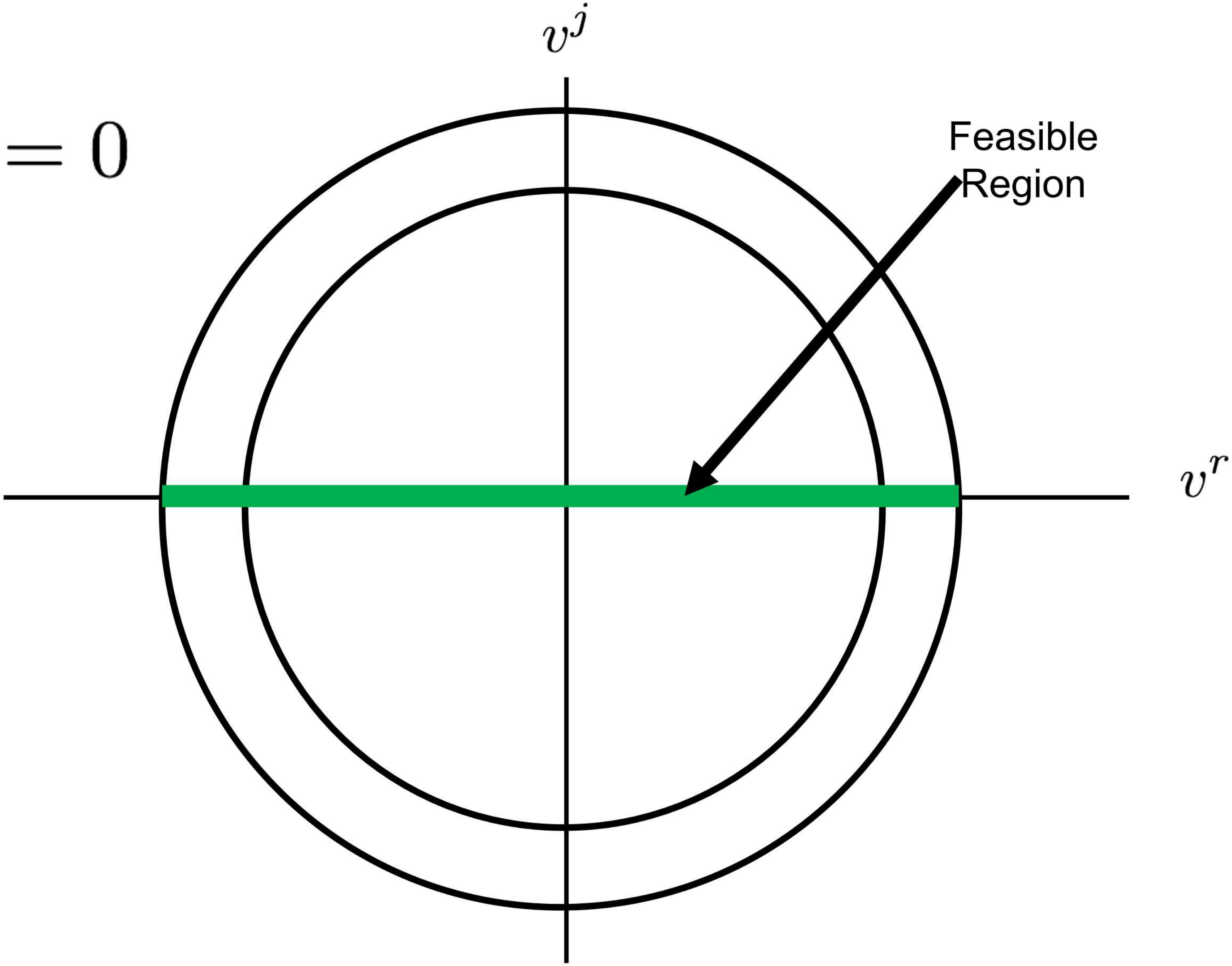


Reference Bus



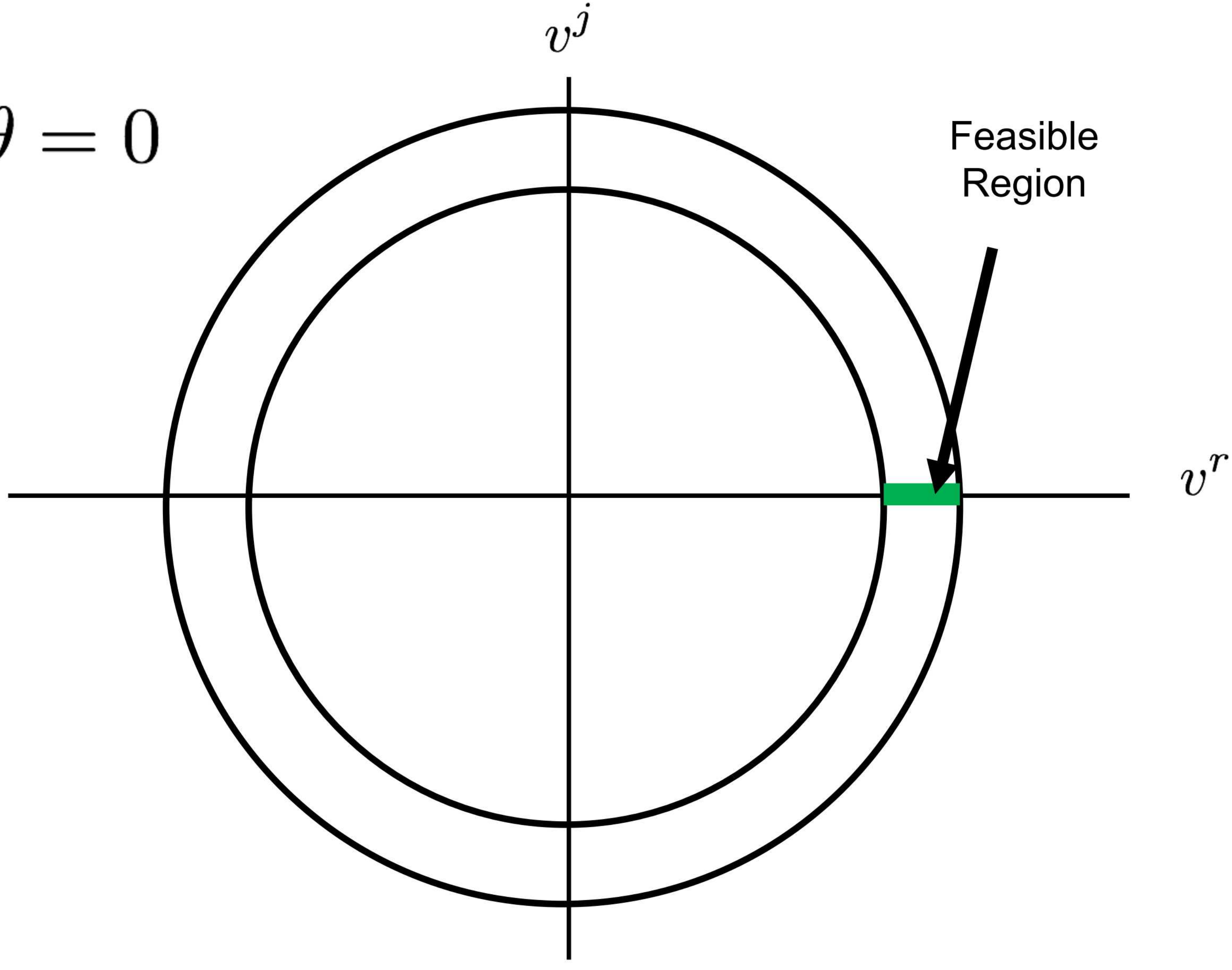
Reference Bus

$$v^j = 0$$



Reference Bus

$\theta = 0$



Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \leq \widehat{v_b^r} \leq (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \leq \widehat{v_b^j} \leq (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \leq v_b^r, v_b^j \leq V_b^{\max}$$

$$v_{ref}^j = 0$$

$$V_b^{\min} \leq v_{ref}^r \leq V_b^{\max}$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \leq \widehat{v_b^r} \leq (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \leq \widehat{v_b^j} \leq (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \leq v_b^r, v_b^j \leq V_b^{\max}$$

$$v_{ref}^j = 0$$

$$V_b^{\min} \leq v_{ref}^r \leq V_b^{\max}$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \leq \widehat{v_b^r} \leq (V_b^{r,\max} + V_b^{r,\min})v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \leq \widehat{v_b^j} \leq (V_b^{j,\max} + V_b^{j,\min})v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \leq v_b^r, v_b^j \leq V_b^{\max}$$

$$v_{ref}^j = 0$$

$$V_b^{\min} \leq v_{ref}^r \leq V_b^{\max}$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min}) v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min}) v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$v_{ref}^j = 0$$

$$V_b^{\min} \le v_{ref}^r \le V_b^{\max}$$

Rectangular McCormick Relaxation

$$w_b = \widehat{v_b^r} + \widehat{v_b^j}, \quad c_{b,k} = \widehat{v_b^r v_k^r} + \widehat{v_b^j v_k^j}, \quad s_{b,k} = \widehat{v_b^j v_k^r} - \widehat{v_b^r v_k^j}$$

$$(v_b^r)^2 \le \widehat{v_b^r} \le (V_b^{r,\max} + V_b^{r,\min}) v_b^r - V_b^{r,\max} V_b^{r,\min}$$

$$(v_b^j)^2 \le \widehat{v_b^j} \le (V_b^{j,\max} + V_b^{j,\min}) v_b^j - V_b^{j,\max} V_b^{j,\min}$$

$$\widehat{v_b^r v_k^r} \in MCC(v_b^r, v_k^r), \quad \widehat{v_b^j v_k^j} \in MCC(v_b^j, v_k^j)$$

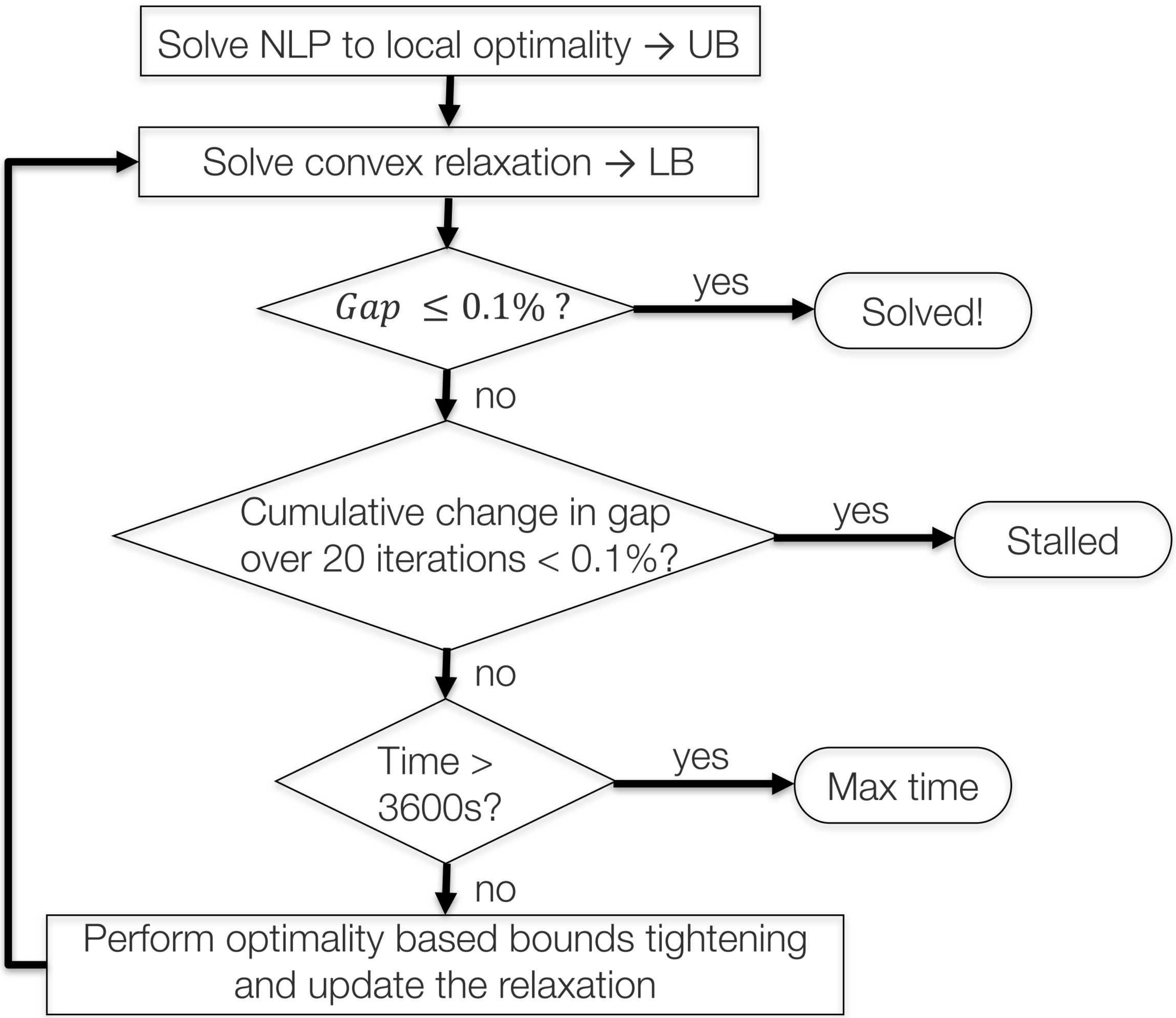
$$\widehat{v_b^j v_k^r} \in MCC(v_b^j, v_k^r), \quad \widehat{v_b^r v_k^j} \in MCC(v_b^r, v_k^j)$$

$$-V_b^{\max} \le v_b^r, v_b^j \le V_b^{\max}$$

$$v_{ref}^j = 0$$

$$V_b^{\min} \le v_{ref}^r \le V_b^{\max}$$

Computational Tests



Optimality Based Bounds Tightening (OBBT)

$$\min / \max \quad [variable]$$

$$\text{s.t.}$$

$$[convex\ relaxation]$$

$$\sum_{g \in \mathcal{G}} [A_g^2 (p_g^G)^2 + A_g^1 p_g^G + A_g^0] \leq UB$$

Computational Tests

Comparison of relaxations for the ACOPF problem

- SOC relaxation
- McCormick relaxation of the rectangular form
  - With and without the reference bus constraint
- QC relaxation of the polar form [Hijazi et al. 2017]
- With and without UB in OBBT

Performed OBBT in parallel

- 24 nodes, 16 cores per node (2.6 GHz Intel SB) – 12 tasks per node

NESTA Archive (std, api, sad) of case300 and smaller [Coffrin et al. 2014]

Pyomo Tools

Pyomo provides a rich programming environment through Python

- Python environment for building "meta-algorithms"
- Expression interrogation, manipulation, and transformation framework
- Higher-level user-defined modeling constructs

In development: Package for computational MINLP research

- Parallel bounds tightening, parallel branch & bound framework, piecewise refinement management, MIP solver callbacks
- Structure to inherit relaxations/cuts, McCormick, Piecewise McCormick and piecewise univariate, SOCP, sin/cos, arctan, polyhedral cuts (edge-concave), ...

In development: EGRET – package for optimization of power systems

- Modular model building
- DCOPF, DCOPF with losses
- ACOPF
- Unit Commitment
- ACOPF relaxations – SOCP, QCP, etc.

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC
3_lmbd	1.32
4_gs	0.00
5_pjm	14.54
6_c	0.30
6_ww	0.63
9_wscc	0.00
14_ieee	0.11
24_ieee_rts	0.01
29_edin	0.12
30_as	0.06
30_fsr	0.39
30_ieee	15.88
39_epri	0.05
57_ieee	0.06
73_ieee_rts	0.03
118_ieee	1.83
162_ieee_dtc	4.03
189_edin	0.21
300_ieee	1.18
3_lmbd__api	3.30
4_gs__api	0.65
5_pjm__api	0.28
6_c__api	0.35
9_wscc__api	0.00
14_ieee__api	1.34
24_ieee_rts__api	20.75
29_edin__api	0.42
30_as__api	4.76
30_fsr__api	45.97

case_name	SOC
30_ieee__api	1.01
39_epri__api	2.99
57_ieee__api	0.21
73_ieee_rts__api	14.39
89_pegase__api	20.43
118_ieee__api	43.91
162_ieee_dtc__api	1.34
189_edin__api	5.67
300_ieee__api	0.71
3_lmbd__sad	4.28
4_gs__sad	4.90
5_pjm__sad	3.61
6_c__sad	1.36
6_ww__sad	0.80
9_wscc__sad	1.50
14_ieee__sad	0.06
24_ieee_rts__sad	11.42
29_edin__sad	34.68
30_as__sad	9.16
30_fsr__sad	0.62
30_ieee__sad	5.84
39_epri__sad	0.11
57_ieee__sad	0.11
73_ieee_rts__sad	8.37
89_pegase__sad	0.28
118_ieee__sad	12.77
162_ieee_dtc__sad	7.08
189_edin__sad	2.25
300_ieee__sad	1.26

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC	SOC R-McC (no RB) (with UB)
3_lmbd	1.32	1.32
4_gs	0.00	0.00
5_pjm	14.54	14.54
6_c	0.30	0.30
6_ww	0.63	0.63
9_wscc	0.00	0.00
14_ieee	0.11	0.11
24_ieee_rts	0.01	0.01
29_edin	0.12	0.12
30_as	0.06	0.06
30_fsr	0.39	0.39
30_ieee	15.88	15.88
39_epri	0.05	0.05
57_ieee	0.06	0.06
73_ieee_rts	0.03	0.03
118_ieee	1.83	1.83
162_ieee_dtc	4.03	4.03
189_edin	0.21	0.21
300_ieee	1.18	1.18
3_lmbd__api	3.30	3.30
4_gs__api	0.65	0.65
5_pjm__api	0.28	0.28
6_c__api	0.35	0.35
9_wscc__api	0.00	0.00
14_ieee__api	1.34	1.34
24_ieee_rts__api	20.75	20.75
29_edin__api	0.42	0.42
30_as__api	4.76	4.76
30_fsr__api	45.97	45.97

case_name	SOC	SOC R-McC (no RB) (with UB)
30_ieee__api	1.01	1.01
39_epri__api	2.99	2.99
57_ieee__api	0.21	0.21
73_ieee_rts__api	14.39	14.39
89_pegase__api	20.43	20.43
118_ieee__api	43.91	43.91
162_ieee_dtc__api	1.34	1.34
189_edin__api	5.67	5.67
300_ieee__api	0.71	0.71
3_lmbd__sad	4.28	4.28
4_gs__sad	4.90	4.90
5_pjm__sad	3.61	3.61
6_c__sad	1.36	1.36
6_ww__sad	0.80	0.80
9_wscc__sad	1.50	1.50
14_ieee__sad	0.06	0.06
24_ieee_rts__sad	11.42	11.42
29_edin__sad	34.68	34.68
30_as__sad	9.16	9.16
30_fsr__sad	0.62	0.62
30_ieee__sad	5.84	5.84
39_epri__sad	0.11	0.11
57_ieee__sad	0.11	0.11
73_ieee_rts__sad	8.37	8.37
89_pegase__sad	0.28	0.28
118_ieee__sad	12.77	12.77
162_ieee_dtc__sad	7.08	7.08
189_edin__sad	2.25	2.25
300_ieee__sad	1.26	1.26

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)
3_lmbd	1.32	1.32	0.49
4_gs	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00
6_c	0.30	0.30	0.10
6_ww	0.63	0.63	0.09
9_wscc	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06
24_ieee_rts	0.01	0.01	0.01
29_edin	0.12	0.12	0.10
30_as	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39
30_ieee	15.88	15.88	0.08
39_epri	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59
162_ieee_dtc	4.03	4.03	4.03
189_edin	0.21	0.21	0.21
300_ieee	1.18	1.18	1.18
3_lmbd__api	3.30	3.30	0.07
4_gs__api	0.65	0.65	0.07
5_pjm__api	0.28	0.28	0.06
6_c__api	0.35	0.35	0.05
9_wscc__api	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22
24_ieee_rts__api	20.75	20.75	2.03
29_edin__api	0.42	0.42	0.42
30_as__api	4.76	4.76	0.28
30_fsr__api	45.97	45.97	41.63

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)
30_ieee__api	1.01	1.01	0.09
39_epri__api	2.99	2.99	0.29
57_ieee__api	0.21	0.21	0.08
73_ieee_rts__api	14.39	14.39	14.39
89_pegase__api	20.43	20.43	20.20
118_ieee__api	43.91	43.91	26.87
162_ieee_dtc__api	1.34	1.34	0.98
189_edin__api	5.67	5.67	5.45
300_ieee__api	0.71	0.71	0.71
3_lmbd__sad	4.28	4.28	0.05
4_gs__sad	4.90	4.90	0.01
5_pjm__sad	3.61	3.61	0.03
6_c__sad	1.36	1.36	0.01
6_ww__sad	0.80	0.80	0.05
9_wscc__sad	1.50	1.50	0.01
14_ieee__sad	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08
29_edin__sad	34.68	34.68	2.39
30_as__sad	9.16	9.16	0.24
30_fsr__sad	0.62	0.62	0.23
30_ieee__sad	5.84	5.84	0.08
39_epri__sad	0.11	0.11	0.04
57_ieee__sad	0.11	0.11	0.10
73_ieee_rts__sad	8.37	8.37	2.59
89_pegase__sad	0.28	0.28	0.27
118_ieee__sad	12.77	12.77	5.13
162_ieee_dtc__sad	7.08	7.08	7.08
189_edin__sad	2.25	2.25	2.17
300_ieee__sad	1.26	1.26	1.26

Optimality Gap (%)

<=0.1 | 0.1 - 5 | >5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)
3_lmbd	1.32	1.32	0.49	0.03
4_gs	0.00	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00	0.09
6_c	0.30	0.30	0.10	0.01
6_ww	0.63	0.63	0.09	0.05
9_wscc	0.00	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06	0.06
24_ieee_rts	0.01	0.01	0.01	0.01
29_edin	0.12	0.12	0.10	0.08
30_as	0.06	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39	0.07
30_ieee	15.88	15.88	0.08	0.03
39_epri	0.05	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59	0.66
162_ieee_dtc	4.03	4.03	4.03	4.03
189_edin	0.21	0.21	0.21	0.21
300_ieee	1.18	1.18	1.18	1.18
3_lmbd__api	3.30	3.30	0.07	0.09
4_gs__api	0.65	0.65	0.07	0.01
5_pjm__api	0.28	0.28	0.06	0.02
6_c__api	0.35	0.35	0.05	0.01
9_wscc__api	0.00	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22	0.04
24_ieee_rts__api	20.75	20.75	2.03	0.56
29_edin__api	0.42	0.42	0.42	0.42
30_as__api	4.76	4.76	0.28	0.07
30_fsr__api	45.97	45.97	41.63	41.20

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)
30_ieee__api	1.01	1.01	0.09	0.09
39_epri__api	2.99	2.99	0.29	0.10
57_ieee__api	0.21	0.21	0.08	0.06
73_ieee_rts__api	14.39	14.39	14.39	14.39
89_pegase__api	20.43	20.43	20.20	20.16
118_ieee__api	43.91	43.91	26.87	26.45
162_ieee_dtc__api	1.34	1.34	0.98	0.94
189_edin__api	5.67	5.67	5.45	2.90
300_ieee__api	0.71	0.71	0.71	0.71
3_lmbd__sad	4.28	4.28	0.05	0.00
4_gs__sad	4.90	4.90	0.01	0.00
5_pjm__sad	3.61	3.61	0.03	0.01
6_c__sad	1.36	1.36	0.01	0.04
6_ww__sad	0.80	0.80	0.05	0.05
9_wscc__sad	1.50	1.50	0.01	0.01
14_ieee__sad	0.06	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08	0.04
29_edin__sad	34.68	34.68	2.39	0.55
30_as__sad	9.16	9.16	0.24	0.05
30_fsr__sad	0.62	0.62	0.23	0.04
30_ieee__sad	5.84	5.84	0.08	0.09
39_epri__sad	0.11	0.11	0.04	0.03
57_ieee__sad	0.11	0.11	0.10	0.10
73_ieee_rts__sad	8.37	8.37	2.59	1.96
89_pegase__sad	0.28	0.28	0.27	0.08
118_ieee__sad	12.77	12.77	5.13	1.47
162_ieee_dtc__sad	7.08	7.08	7.08	7.08
189_edin__sad	2.25	2.25	2.17	1.84
300_ieee__sad	1.26	1.26	1.26	1.26

Optimality Gap (%)

<=0.1	0.1 - 5	>5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)
3_lmbd	1.32	1.32	0.49	0.03	0.19
4_gs	0.00	0.00	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00	0.09	9.29
6_c	0.30	0.30	0.10	0.01	0.10
6_ww	0.63	0.63	0.09	0.05	0.01
9_wscc	0.00	0.00	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06	0.06	0.02
24_ieee_rts	0.01	0.01	0.01	0.01	0.01
29_edin	0.12	0.12	0.10	0.08	0.10
30_as	0.06	0.06	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39	0.07	0.10
30_ieee	15.88	15.88	0.08	0.03	0.05
39_epri	0.05	0.05	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59	0.66	0.47
162_ieee_dtc	4.03	4.03	4.03	4.03	0.66
189_edin	0.21	0.21	0.21	0.21	0.21
300_ieee	1.18	1.18	1.18	1.18	0.18
3_lmbd__api	3.30	3.30	0.07	0.09	0.08
4_gs__api	0.65	0.65	0.07	0.01	0.03
5_pjm__api	0.28	0.28	0.06	0.02	0.01
6_c__api	0.35	0.35	0.05	0.01	0.08
9_wscc__api	0.00	0.00	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22	0.04	0.30
24_ieee_rts__api	20.75	20.75	2.03	0.56	0.30
29_edin__api	0.42	0.42	0.42	0.42	0.09
30_as__api	4.76	4.76	0.28	0.07	0.04
30_fsr__api	45.97	45.97	41.63	41.20	2.41

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)
30_ieee__api	1.01	1.01	0.09	0.09	0.07
39_epri__api	2.99	2.99	0.29	0.10	0.05
57_ieee__api	0.21	0.21	0.08	0.06	0.02
73_ieee_rts__api	14.39	14.39	14.39	14.39	0.19
89_pegase__api	20.43	20.43	20.20	20.16	18.88
118_ieee__api	43.91	43.91	26.87	26.45	9.26
162_ieee_dtc__api	1.34	1.34	0.98	0.94	0.10
189_edin__api	5.67	5.67	5.45	2.90	0.33
300_ieee__api	0.71	0.71	0.71	0.71	0.13
3_lmbd__sad	4.28	4.28	0.05	0.00	0.04
4_gs__sad	4.90	4.90	0.01	0.00	0.01
5_pjm__sad	3.61	3.61	0.03	0.01	0.05
6_c__sad	1.36	1.36	0.01	0.04	0.03
6_ww__sad	0.80	0.80	0.05	0.05	0.00
9_wscc__sad	1.50	1.50	0.01	0.01	0.00
14_ieee__sad	0.06	0.06	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08	0.04	0.07
29_edin__sad	34.68	34.68	2.39	0.55	1.16
30_as__sad	9.16	9.16	0.24	0.05	0.06
30_fsr__sad	0.62	0.62	0.23	0.04	0.08
30_ieee__sad	5.84	5.84	0.08	0.09	0.02
39_epri__sad	0.11	0.11	0.04	0.03	0.04
57_ieee__sad	0.11	0.11	0.10	0.10	0.10
73_ieee_rts__sad	8.37	8.37	2.59	1.96	0.07
89_pegase__sad	0.28	0.28	0.27	0.08	0.08
118_ieee__sad	12.77	12.77	5.13	1.47	1.35
162_ieee_dtc__sad	7.08	7.08	7.08	7.08	0.46
189_edin__sad	2.25	2.25	2.17	1.84	0.97
300_ieee__sad	1.26	1.26	1.26	1.26	0.21

Optimality Gap (%)

<=0.1 | 0.1 - 5 | >5

case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)	SOC QC (with UB)
3_lmbd	1.32	1.32	0.49	0.03	0.19	0.01
4_gs	0.00	0.00	0.00	0.00	0.00	0.00
5_pjm	14.54	14.54	5.00	0.09	9.29	5.68
6_c	0.30	0.30	0.10	0.01	0.10	0.08
6_ww	0.63	0.63	0.09	0.05	0.01	0.01
9_wscc	0.00	0.00	0.00	0.00	0.00	0.00
14_ieee	0.11	0.11	0.06	0.06	0.02	0.01
24_ieee_rts	0.01	0.01	0.01	0.01	0.01	0.01
29_edin	0.12	0.12	0.10	0.08	0.10	0.10
30_as	0.06	0.06	0.06	0.06	0.06	0.06
30_fsr	0.39	0.39	0.39	0.07	0.10	0.08
30_ieee	15.88	15.88	0.08	0.03	0.05	0.05
39_epri	0.05	0.05	0.05	0.05	0.05	0.05
57_ieee	0.06	0.06	0.06	0.06	0.06	0.06
73_ieee_rts	0.03	0.03	0.03	0.03	0.03	0.03
118_ieee	1.83	1.83	1.59	0.66	0.47	0.08
162_ieee_dtc	4.03	4.03	4.03	4.03	0.66	0.08
189_edin	0.21	0.21	0.21	0.21	0.21	0.09
300_ieee	1.18	1.18	1.18	1.18	0.18	0.04
3_lmbd__api	3.30	3.30	0.07	0.09	0.08	0.02
4_gs__api	0.65	0.65	0.07	0.01	0.03	0.06
5_pjm__api	0.28	0.28	0.06	0.02	0.01	0.01
6_c__api	0.35	0.35	0.05	0.01	0.08	0.04
9_wscc__api	0.00	0.00	0.00	0.00	0.00	0.00
14_ieee__api	1.34	1.34	0.22	0.04	0.30	0.08
24_ieee_rts__api	20.75	20.75	2.03	0.56	0.30	0.09
29_edin__api	0.42	0.42	0.42	0.42	0.09	0.05
30_as__api	4.76	4.76	0.28	0.07	0.04	0.02
30_fsr__api	45.97	45.97	41.63	41.20	2.41	0.06

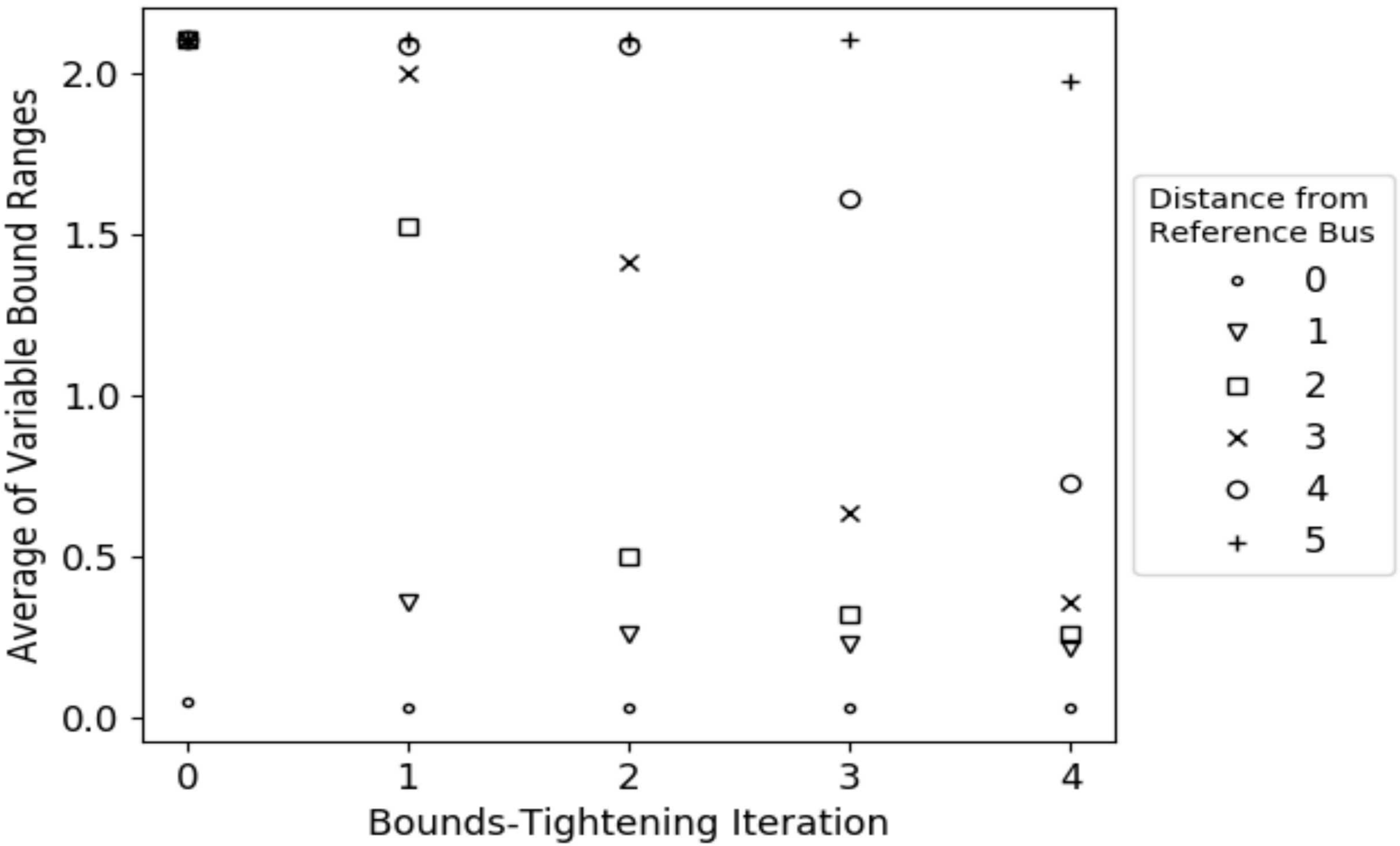
case_name	SOC	SOC R-McC (no RB) (with UB)	SOC R-McC (with RB) (no UB)	SOC R-McC (with RB) (with UB)	SOC QC (no UB)	SOC QC (with UB)
30_ieee__api	1.01	1.01	0.09	0.09	0.07	0.01
39_epri__api	2.99	2.99	0.29	0.10	0.05	0.04
57_ieee__api	0.21	0.21	0.08	0.06	0.02	0.06
73_ieee_rts__api	14.39	14.39	14.39	14.39	0.19	0.03
89_pegase__api	20.43	20.43	20.20	20.16	18.88	9.09
118_ieee__api	43.91	43.91	26.87	26.45	9.26	8.72
162_ieee_dtc__api	1.34	1.34	0.98	0.94	0.10	0.07
189_edin__api	5.67	5.67	5.45	2.90	0.33	0.13
300_ieee__api	0.71	0.71	0.71	0.71	0.13	0.04
3_lmbd__sad	4.28	4.28	0.05	0.00	0.04	0.03
4_gs__sad	4.90	4.90	0.01	0.00	0.01	0.00
5_pjm__sad	3.61	3.61	0.03	0.01	0.05	0.03
6_c__sad	1.36	1.36	0.01	0.04	0.03	0.03
6_ww__sad	0.80	0.80	0.05	0.05	0.00	0.00
9_wscc__sad	1.50	1.50	0.01	0.01	0.00	0.00
14_ieee__sad	0.06	0.06	0.06	0.06	0.06	0.06
24_ieee_rts__sad	11.42	11.42	0.08	0.04	0.07	0.03
29_edin__sad	34.68	34.68	2.39	0.55	1.16	0.49
30_as__sad	9.16	9.16	0.24	0.05	0.06	0.03
30_fsr__sad	0.62	0.62	0.23	0.04	0.08	0.02
30_ieee__sad	5.84	5.84	0.08	0.09	0.02	0.01
39_epri__sad	0.11	0.11	0.04	0.03	0.04	0.04
57_ieee__sad	0.11	0.11	0.10	0.10	0.10	0.10
73_ieee_rts__sad	8.37	8.37	2.59	1.96	0.07	0.01
89_pegase__sad	0.28	0.28	0.27	0.08	0.08	0.08
118_ieee__sad	12.77	12.77	5.13	1.47	1.35	0.10
162_ieee_dtc__sad	7.08	7.08	7.08	7.08	0.46	0.04
189_edin__sad	2.25	2.25	2.17	1.84	0.97	0.82
300_ieee__sad	1.26	1.26	1.26	1.26	0.21	0.01

Reference Bus Impact

Analyze variable bound ranges (ub – lb)

- As bounds tightening progresses
- With distance from the reference bus (shortest path length)

Reference Bus Impact



[Bynum et al. 2018]

Summary and Conclusions

SOCP / QC relaxations for ACOPF are remarkably strong

McCormick relaxations remain useful for the Rectangular formulation

- Reference bus constraints improve bounds tightening and relaxation
- With each iteration, bounds improve on nodes that are progressively further from the reference bus [Bynum et al., 2018]
- Moving the reference bus can impact relaxation tightness

A good upper bound can significantly improve the effectiveness of OBBT

- Computational performance also drastically improves
  - From 9 s to 5 s for R-McC
  - From 27 s to 17 s for QC

Between QC and R-McC, only 5 cases had more than 0.1% gap

Gaps shown achieved with bounds tightening only (no branching)

- Not performant, but illustrates quality of relaxations

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