

*Exceptional service in the national interest*



# Computational Electromagnetics at Sandia National Laboratories - Current Code Capability

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**Sandia National Laboratories – Organization 1352**

**Oct. 14, 2014**



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# Outline

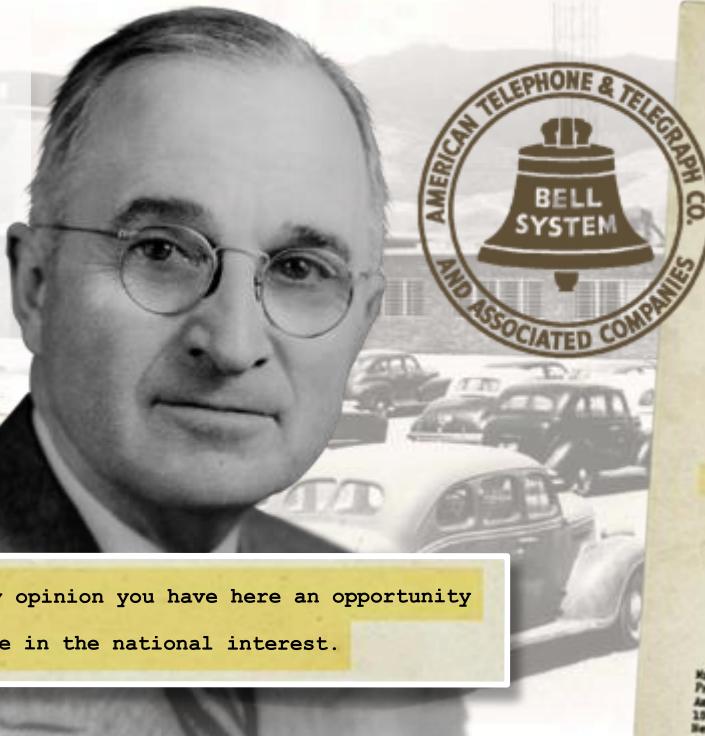
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- **Overview of Sandia National Labs**
  - Electromagnetic Theory Organization
- **Electromagnetic Environments**
- **Solution Process**
- **Computational Electromagnetics**
  - Focus on Method of Moments – EIGER
    - Thin-slot Algorithm
    - Matrix Compression
- **Conclusions / Future Work**

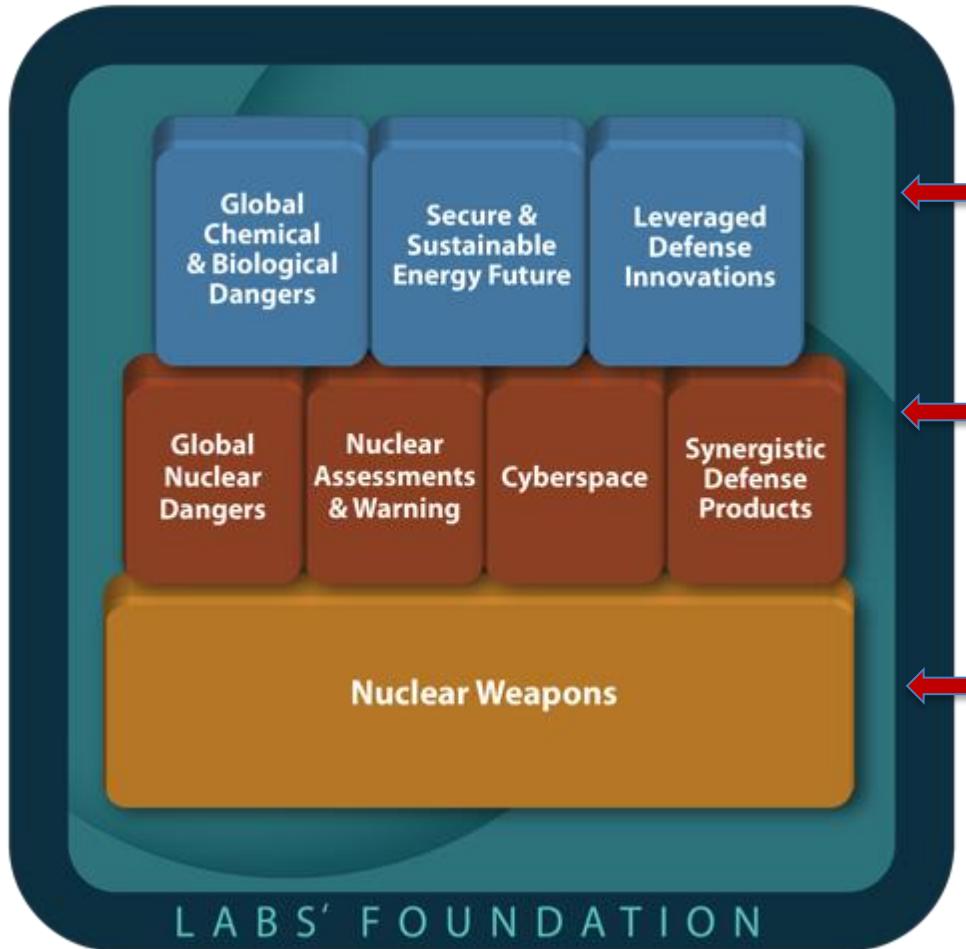
# Sandia's History

*Exceptional service in the national interest*

- July 1945: Los Alamos creates Z Division
- Nonnuclear component engineering
- November 1, 1949: Sandia Laboratory established



# National Security Mission Areas



- Top row: Critical to our national security, these three mission areas leverage, enhance, and advance our capabilities.
- Middle row: Strongly interdependent with NW, these four mission areas are essential to sustaining Sandia's ability to fulfill its NW core mission.
- Bottom row: Our core mission, nuclear weapons (NW), is enabled by a strong scientific and engineering foundation.

# Sandia's Current Nuclear Weapons Activities



## Warhead Systems Engineering and Integration



*An extensive suite of multi-disciplinary capabilities are required for Design, Qualification, Production, Surveillance, Experimentation / Computation*

## Major Environmental Test Facilities and Diagnostics



Z Machine

Light Initiated High Explosive

Annular core research reactor

## Gas Transfer systems



## Design Agency for Nonnuclear Components

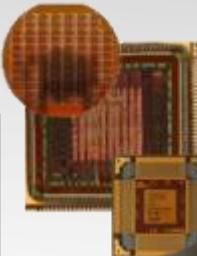


Arming, fusing, and firing systems

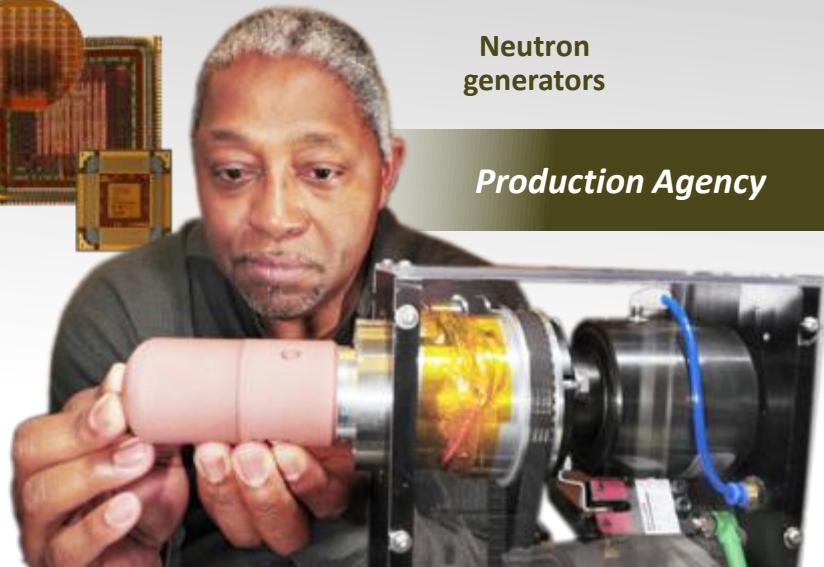
## Safety systems



## MESA Microelectronics



## Neutron generators



## Production Agency

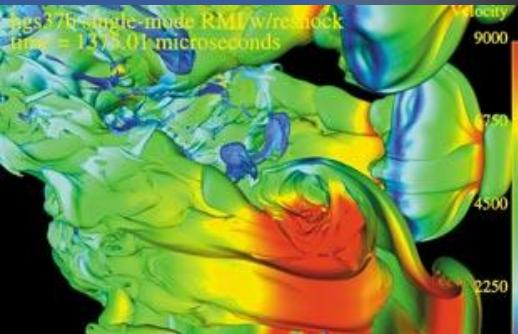
# Our Research Framework

*Strong research foundations play a differentiating role in our mission delivery*

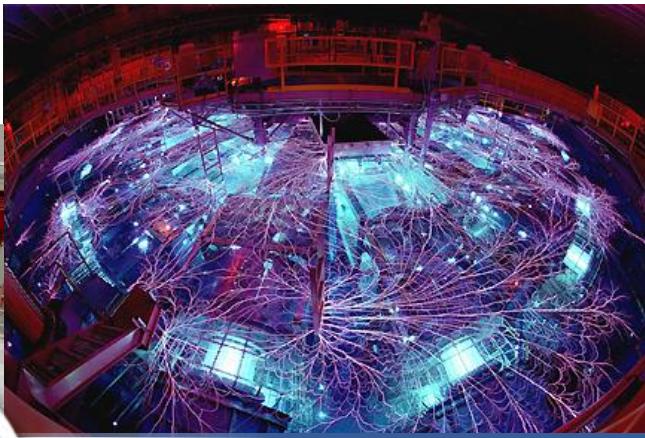
# Computing & Information Sciences



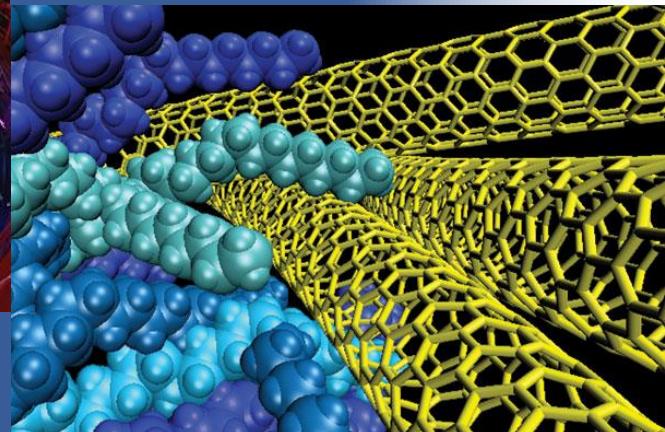
# Engineering Sciences



## Bioscience

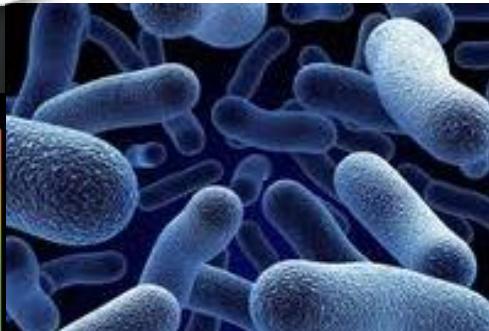


# Radiation Effects & High Energy Density Science

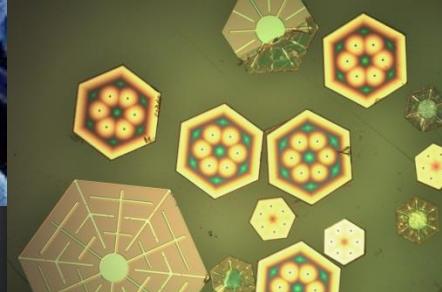


## Materials Sciences

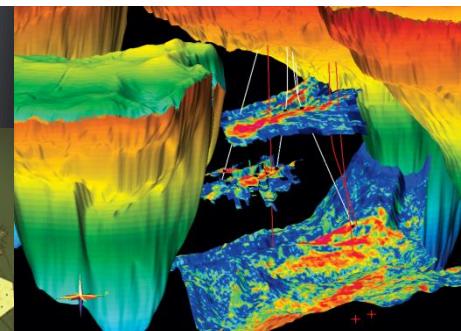
# Engineering Sciences



# Nanodevices & Microsystems



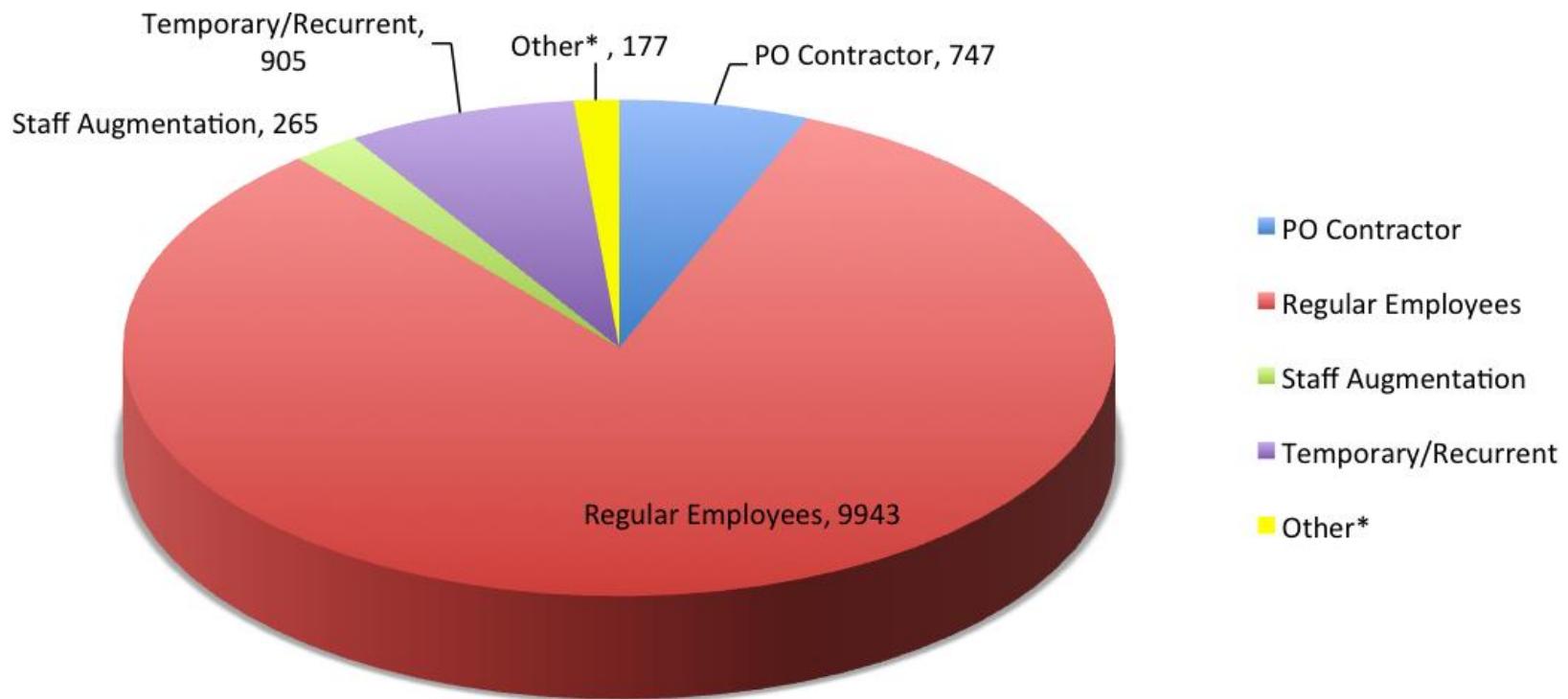
## Geoscience



# Our Workforce

- Total Sandia workforce: 12,037
- Regular employees: 9,943
- Advanced degrees: 5,703

*Data as of August 25, 2014*



# Organization 1352 - Big Picture

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- **What?**
  - Provide high-fidelity, robust, computational tools based on Maxwell's Equations.
- **Why?**
  - To aid in weapons qualification in conjunction with experiments.
  - Weapon component and subsystem modeling.
  - In addition can be used to address problems for DoD customers.
- **How?**
  - Time-domain finite element formulation.
    - EMPHASIS
  - Frequency domain boundary element formulation.
    - EIGER

# Electromagnetic Environments

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- **Electromagnetic Interference**
  - Radars, etc.
- **Lightning**
  - Nearby, direct strike
- **System Generated EMP (SGEMP)**
  - High-energy particles produce currents and fields.

# Solution Process

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- **Geometry to appropriate mesh for analysis.**
  - CUBIT – SANDIA Mesh Software
- **Boundary conditions and excitations applied**
  - Computational Electromagnetic Codes in Organization 1352
    - EMPHASIS/EIGER
    - EMPHASIS/NEVADA
    - Solver Technology
      - TRILINOS – SANDIA Solver Technology
- **Computational**
  - CIELO - LANL
  - SEQUOIA – LLNL
  - SANDIA Computational Resources

# Radiation Suite of Tools

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- RAMSES (Radiation Analysis, Modeling and Simulation for Electrical Systems) framework includes Emphasis, Xyce, Charon and Sceptre
- EIGER
  - Frequency domain, boundary element
- **EMPHASIS**
  - Transient, volumetric finite-element, particle-in-cell
- **QUICKSILVER**
  - Transient, volumetric finite-difference, particle-in-cell (Legacy)

# EMPHASIS

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- Time-domain
- Volumetric (space between parts) mesh
  - Unstructured finite-element
  - Structured finite-difference (stair-stepped)
  - Hybrid combination
- Requires truncation of simulation domain
- Formulation results in sparse matrix
  - Limited by ability to generate large mesh

# EMPHASIS Features

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- Finite-Element Time-Domain (FETD) solver
  - Full-field (no approximations)
  - Arbitrary geometry subject to meshing limitations
- Two formulations for field solve
  - **Unconditionally stable, 2<sup>nd</sup> order Helmholtz**
  - Conditionally stable, 1<sup>st</sup> order Curl-Curl (generalization of structured Finite-Difference Time-Domain (FDTD))
- Vector finite elements-Edge & Face based
  - Advantageous field-continuity and boundary properties
  - Divergence-free, avoiding spurious solutions
- Sub-element algorithms for slots and wires

# Emphasis – Solution FETD

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- Newmark-Beta approximation for time derivatives.
- Implicit solution
  - Matrix solve each time step
    - Symmetric positive definite
      - Conjugate gradient can be used
  - Unconditionally stable
    - Theoretically independent of  $\Delta t$

# Common Emphasis - EIGER Features

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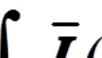
- General purpose based on Maxwell's equations (3D).
- Full-wave formulations.
- Include separate electrostatic components.
- Massively parallel capable.
- Slot, thin wire, and lumped element models.
  - Sub-cell models
- Wide variety of sources and boundary conditions.

- Frequency-domain method of moments solution
  - Steady state solution
  - F90 code – Object Oriented Design
- Boundary element formulation
  - Mesh surfaces of parts – interface between regions
- Exact radiation boundary condition
  - Due to Green's function
- Formulation results in dense (fully populated) matrix
  - Simulations can be limited by available memory
  - Entries are double precision complex

# EIGER – Basic Formulation

$$\bar{E}^{scatt} = -[j\omega \bar{A} + \nabla \phi]$$

$$\bar{A}(\mathbf{r}) = \mu \int \bar{J}(\mathbf{r}') g(R) ds' \quad \phi(\mathbf{r}) = \frac{1}{\epsilon} \int \sigma(\mathbf{r}') g(R) ds'$$



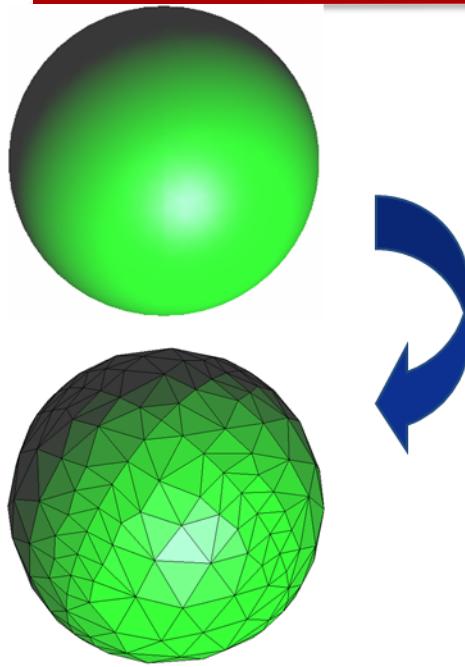

**Surface Current** **Surface Charge**

## Relationship between current and charge

$$j\omega\sigma = -\nabla \cdot \bar{J} \quad \text{Free space Green's Function} \quad g(R)$$

**Note : example is for the electric field integral equation**

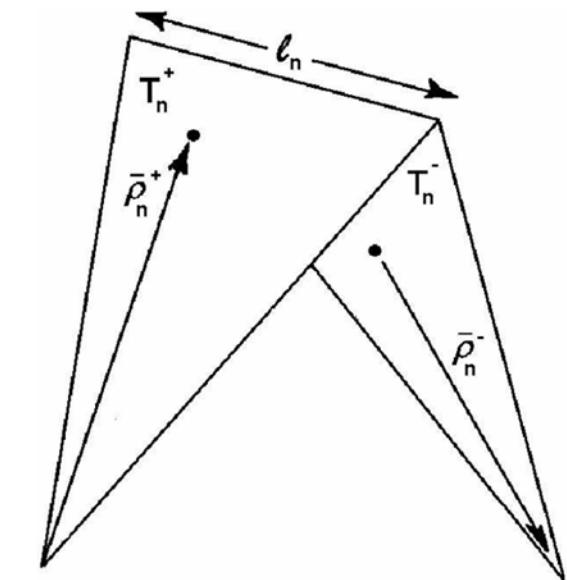
# EIGER – Numerical Implementation



$$\bar{J}(\mathbf{r}) = \sum_{j=1}^N I_j \bar{f}_j(\mathbf{r})$$

Discretize object

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \frac{l_n}{2A_n^+} \rho_n^+ & \mathbf{r} \in T_n^+ \\ \frac{l_n}{2A_n^-} \rho_n^- & \mathbf{r} \in T_n^- \\ 0 & \text{otherwise} \end{cases}$$



Div-conforming Rao-Wilton-Glisson (RWG) basis functions

# EIGER – Numerical Implementation

- The integral equation is (on the surface):

$$T(\bar{J}) = \bar{E}_{scatt} \Big|_{\tan} = \bar{E}_{incident} \Big|_{\tan}$$

- The currents are expanded in terms of the Rao-Wilton-Glisson expansion functions ( $\sim 10$  per wavelength) :

$$\bar{J}(r) = \sum_n I_n \bar{f}_n(r)$$

- Test the integral equation with the basis functions:

$$\langle \bar{f}_m, T(\bar{J}) \rangle \Rightarrow \int_{surface} \bar{f}_m \cdot T(\bar{J}) ds$$

# EIGER – Numerical Implementation

- The integral equation through discretization has become a matrix equation:

$$\bar{\bar{Z}} \bar{I} = \bar{V}$$

- Implications:
  - The matrix Z is fully populated – a dense matrix.
  - This method is memory limited.

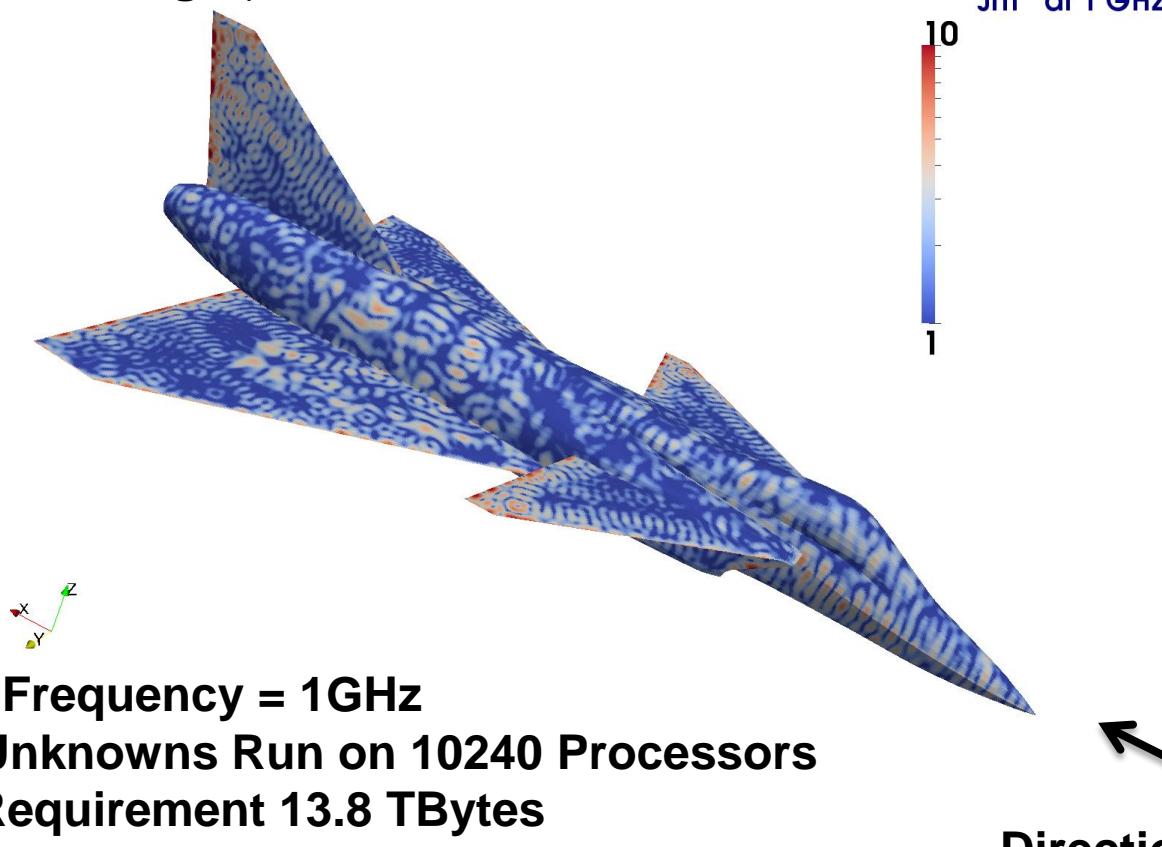
- The boundary element code EIGER:
  - Validated with:
    - Measurements (slot in a box)
    - Analytical solutions (sphere)
  - Used for weapon qualification
  - Additional customers
- The method is memory limited.
  - Limits size of the problem ( with respect to frequency)
- Path Forward -> Compression techniques:
  - Relaxes the memory limit issue.
    - Increases the size of the problem (with respect to frequency) that now can be solved.

# Results – External Problem

## (Direct Solve on CIELO)

### External Problem

VFY 218 (50.6 ft. length)



# Direct Solve Information

Geometry	Max Frequency (GHz)	Unknowns	Solve Time (per frequency)	Number of Processors	Flop Rate (Gflops per Processor)
#1	5.	389756	13529	1600	7.3
#1	12.	1855082	65875	40000	6.5
#2	16.5	2474989	83575	80000	6
#3	2.	226647	7271	640	6.7
#3	12.5	858826	33375	7600	6.7
#3	12.5	858826	30364	8000	6.9

# EIGER Thin-Slot Formulation

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- This modeling feature enables the incorporation of potential penetration points on a structure that couple fields into a cavity without gridding the slot explicitly.
- Based on research by Warne and Chen.
  - Slot is modeled by a wire (carrying magnetic current) whose effective radius depends of the depth and width of the slot.
    - Note the length of the slot  $\gg$  depth, width
  - Incorporated into EIGER, EMPHASIS, and used by other investigators.
  - Validated
    - Compared to analytic and experimental results.

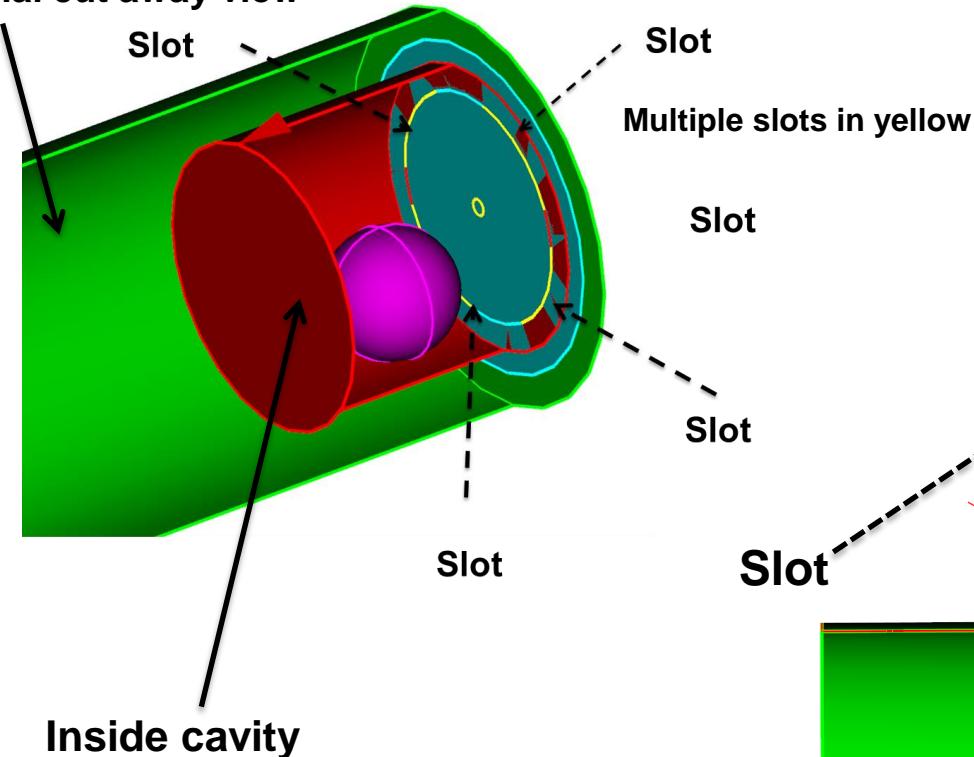
# EIGER Thin-Slot Formulation

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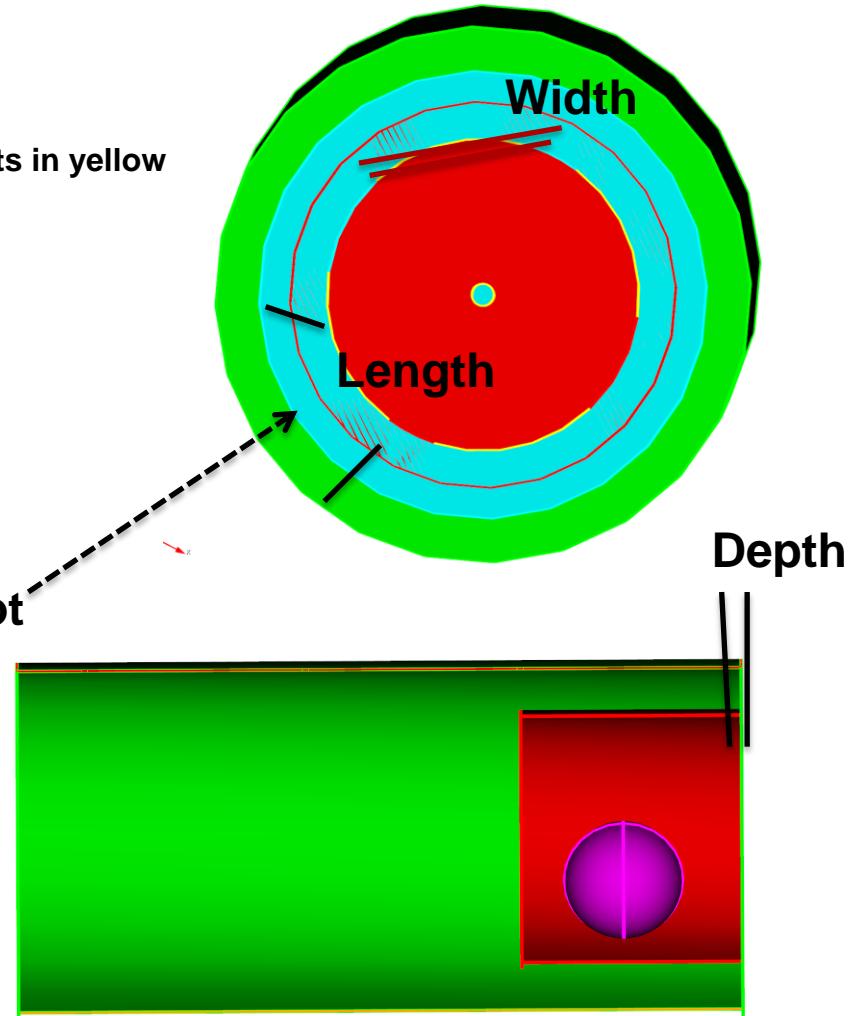
- Key features
  - Integral equation for the exterior surface current and slot current (magnetic current)
  - Integral equation for the interior surface current and slot current (magnetic current).
  - Two contributions
    - Green's function
    - Non-Green's function
- Implications
  - The exterior unknowns do not interact with the interior unknowns.
  - Coupling of the exterior to the interior is through the slot contribution.
    - Matrix has blocks with zero elements – no coupling.

# Thin-Slot Parameters

External cut away view



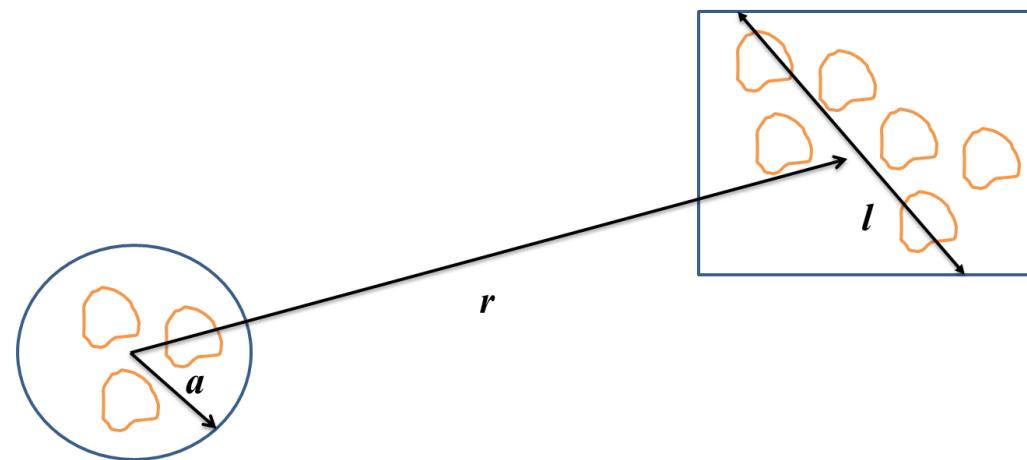
For most problems  
Width, depth vary from .5 to 3mm



# Compression Techniques

- These are techniques that no longer store the full matrix but a lower rank version of the matrix.
- Based on work by Bucci and Francescetti
  - “On the Degrees of Freedom of Scattered Fields” IEEE AP, July 1989

$$N_{dof} = \frac{4la}{r\lambda}$$



# Compression Techniques

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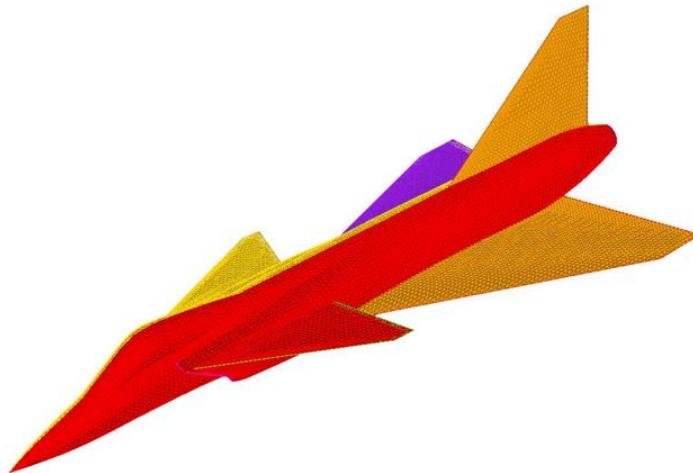
- **Fast Multipole Method (FMM)**
  - **Compression achieved through Green's function simplification:**
    - Factorization
    - Use of the addition theorem
    - Diagonalization
    - Results in low-rank approximation of matrix blocks
- **Adaptive Cross Approximation (ACA)**
  - **Compression achieved:**
    - Low-rank approximation of matrix blocks.
    - Done on the fly
      - Compressed matrix blocks never fully populated.
    - Since the process only operates on matrix blocks it is independent of Green's function simplification.

# Compression Techniques

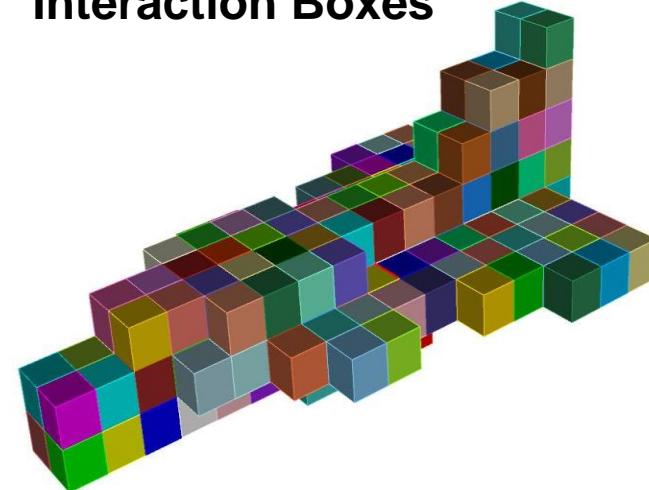
- Identification of all matrix blocks
  - Discretized object (meshed) is encased in a oct-tree structure

VFY 218

Meshed Object



Interaction Boxes



All compression techniques use this step in the solution process



# ACA Matrix Compression

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- Each box contains elements with current unknowns on the elements.
  - Can be compared to a 1-level fast multipole algorithm
- 2 boxes interact to form a matrix block.
- The distance between boxes, size of the boxes, and wavelength determine if a reduced or low-rank approximation can be used.
  - Not all blocks can be compressed.
    - Compression criterion :
      - Distance between the center of boxes  $> 2 * (\text{box radius})$

# ACA Matrix Compression

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- The matrix  $\bar{\bar{Z}}$  is given by:

$$\bar{\bar{Z}} = \sum_{j=1}^{MOM\_blocks} Z_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{Z}_i^{com}$$

***MOM\_Blocks – Moment method matrix blocks (full matrix blocks)***

***COM\_Blocks – Compressed matrix blocks (low-rank approximation)***

# ACA Matrix Compression

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- Approximate matrix description:

$$\tilde{Z}^{m \times n} = U^{m \times r} V^{r \times n} = \sum_{i=1}^r u_i^{m \times 1} v_i^{1 \times n}$$

- The key step is the determination of the sub-matrices  $u$  and  $v$ .

# Solution of the Compressed System

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- The matrix equation to be solved is :

$$\bar{\bar{\mathbf{Z}}} \bar{\mathbf{I}} = \bar{\mathbf{V}}$$

- The matrix is not completely available but is stored as:

$$\bar{\bar{\mathbf{Z}}} = \sum_{j=1}^{MOM\_blocks} \mathbf{Z}_j^{mom} + \sum_{i=1}^{COM\_blocks} \tilde{\mathbf{Z}}_i^{com}$$

- Therefore a iterative solution approach needs to be used.
  - Generalized Minimum residual method(GMRES)
    - Saad and Schultz 1986
  - Transpose Free Quasi Minimum Residual (TFQMR)
    - Freund 1993

# Solution of the Compressed System

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- The Iterative solution technique of choice is the TFQMR method.
  - Based on heuristic numerical experiments performed on electromagnetic problems.
  - Extended for use on parallel platforms.
- On a parallel machine each processor does not have all the matrix blocks – they are partitioned on different processors for load balancing and memory balancing.
  - No processor can have more or less than one block than any other processor.
  - Processors have both MOM and COM blocks.

# Solution of the Compressed System



Sandia  
National  
Laboratories

## Using the TFQMR Method

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- In all iterative methods a matrix vector product is needed during the solution process.
  - This is performed in parallel (each processor has a portion of the compressed and MOM blocks).
- In the original algorithm (used here) the residual norm is not available.
  - However an estimate is computable.
  - The convergence curves show two values
    - The normalized initial residual norm
    - The estimate to the norm.
- A solution tolerance of 5 e-3 was used in all problems.
  - Will affect accuracy.

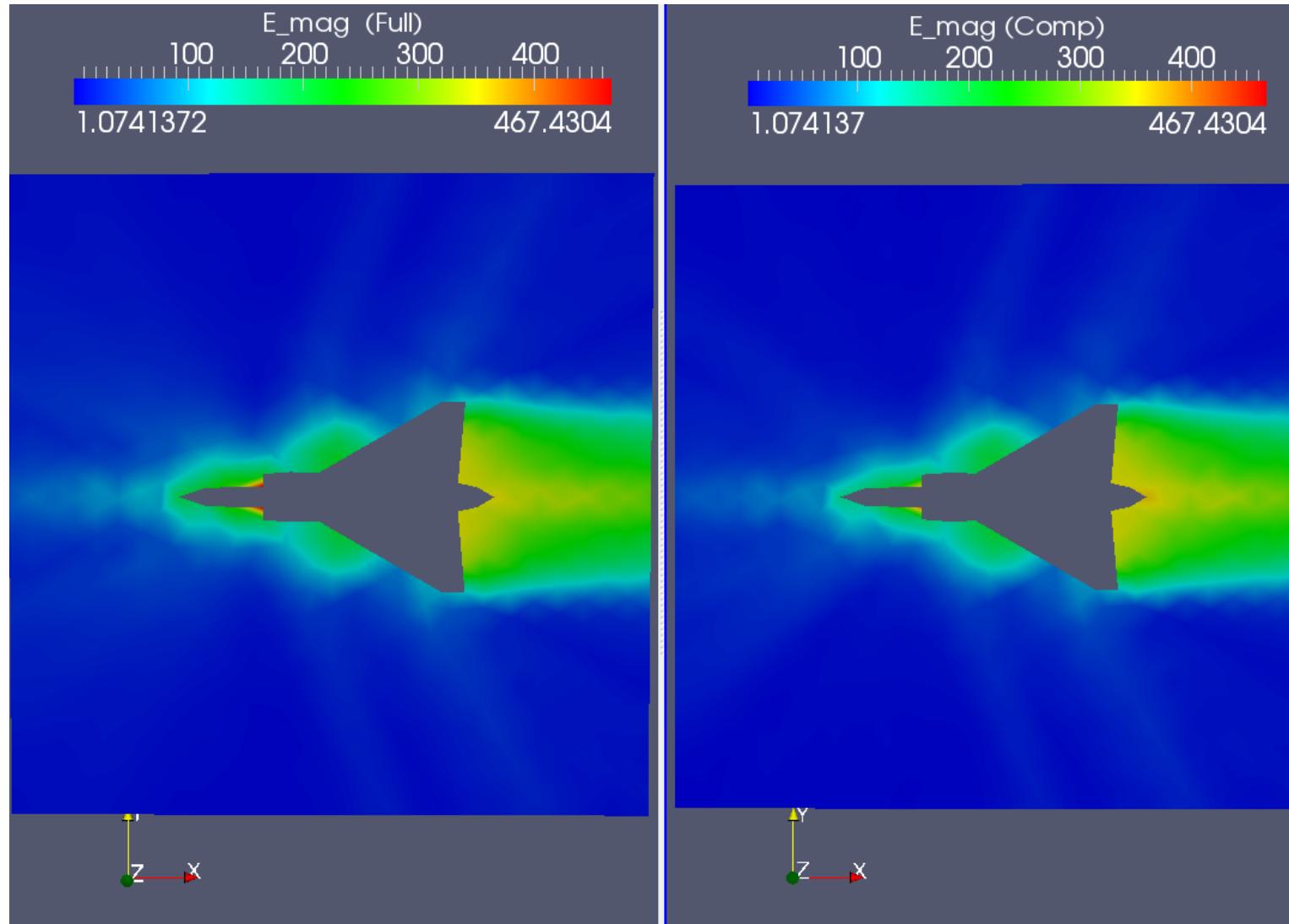
# VFY-218 Compression Results

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- 15 meter long aircraft.
- Frequency 1 GHz
- Number of unknowns 934128
  - 2500 iterations
  - 256 Processors
  - 70,826 sec.
- Epsilon 4.e-02
- Memory
  - Full matrix 16 \*(872) GBytes
  - Compressed 16\*(19 + 7.7) GBytes
  - ~ 97 % compressed.

# Compression Results VFY-218

Magnitude of the near field full and compressed matrix solution.

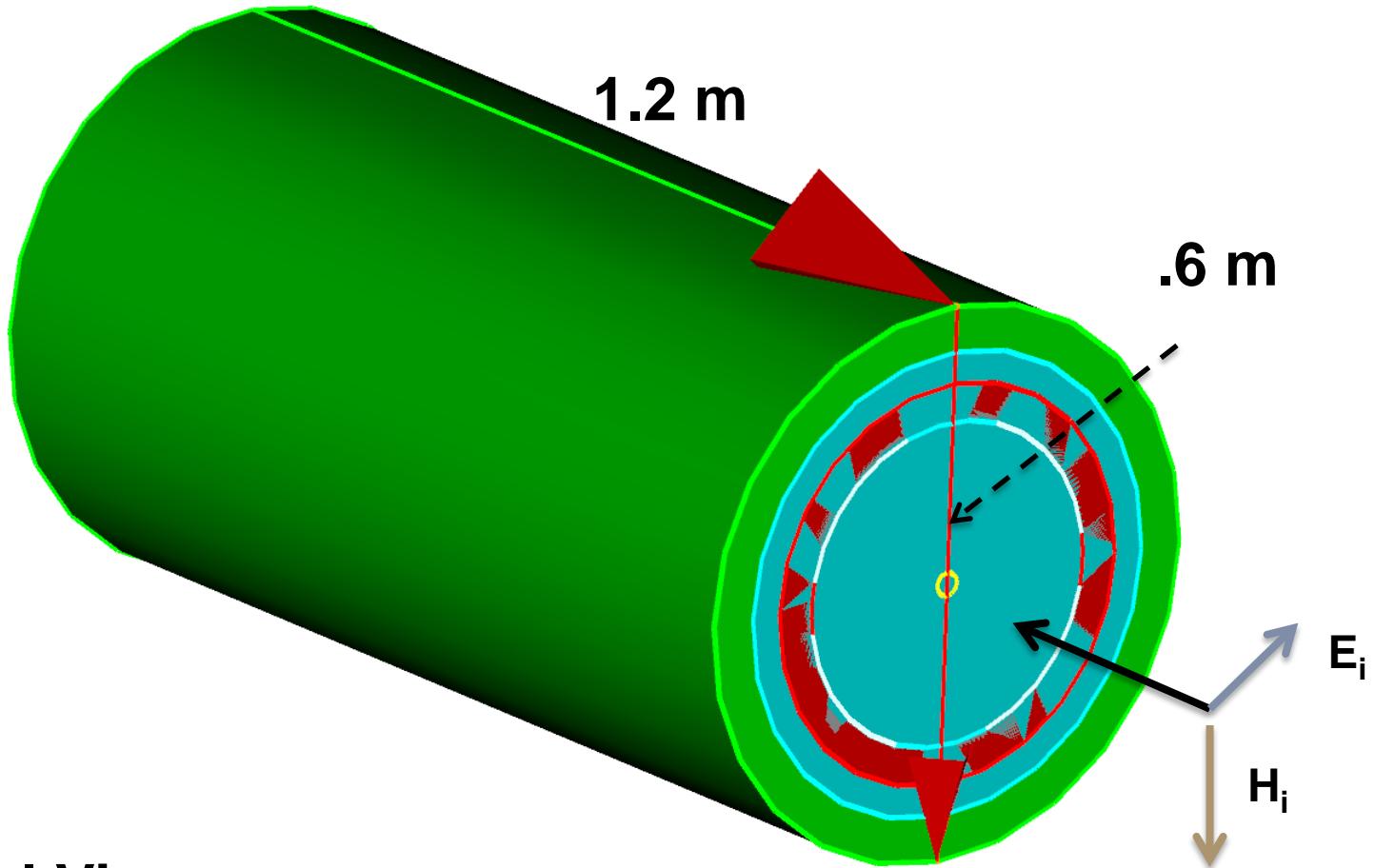


# Compression applied to an object with slots

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- Referred to as D\_cavity.
- A number of different mesh densities considered.
  - Increases the useful upper frequency limit for the model.
- Contains essential features to exercise the compression algorithm on an problem with slots.

# Geometry D\_cavity

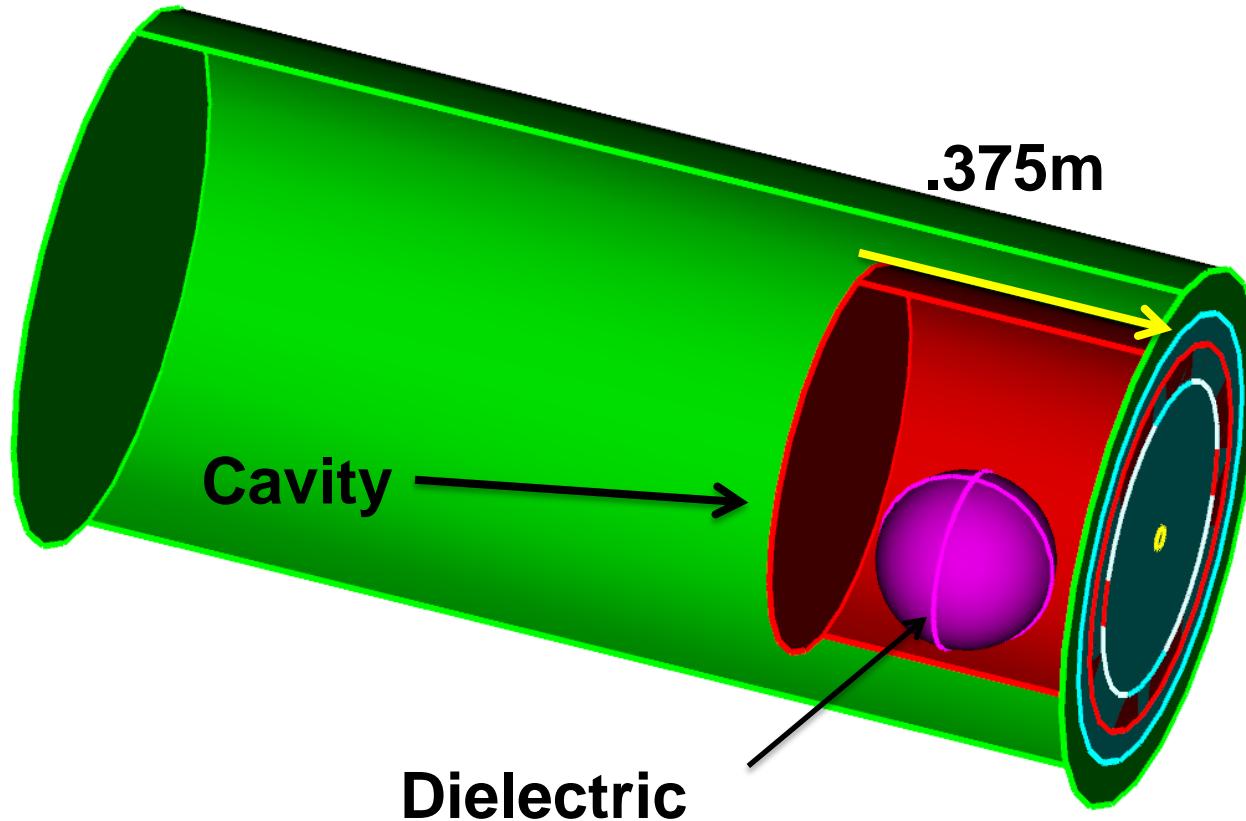


External View

# Geometry D\_cavity

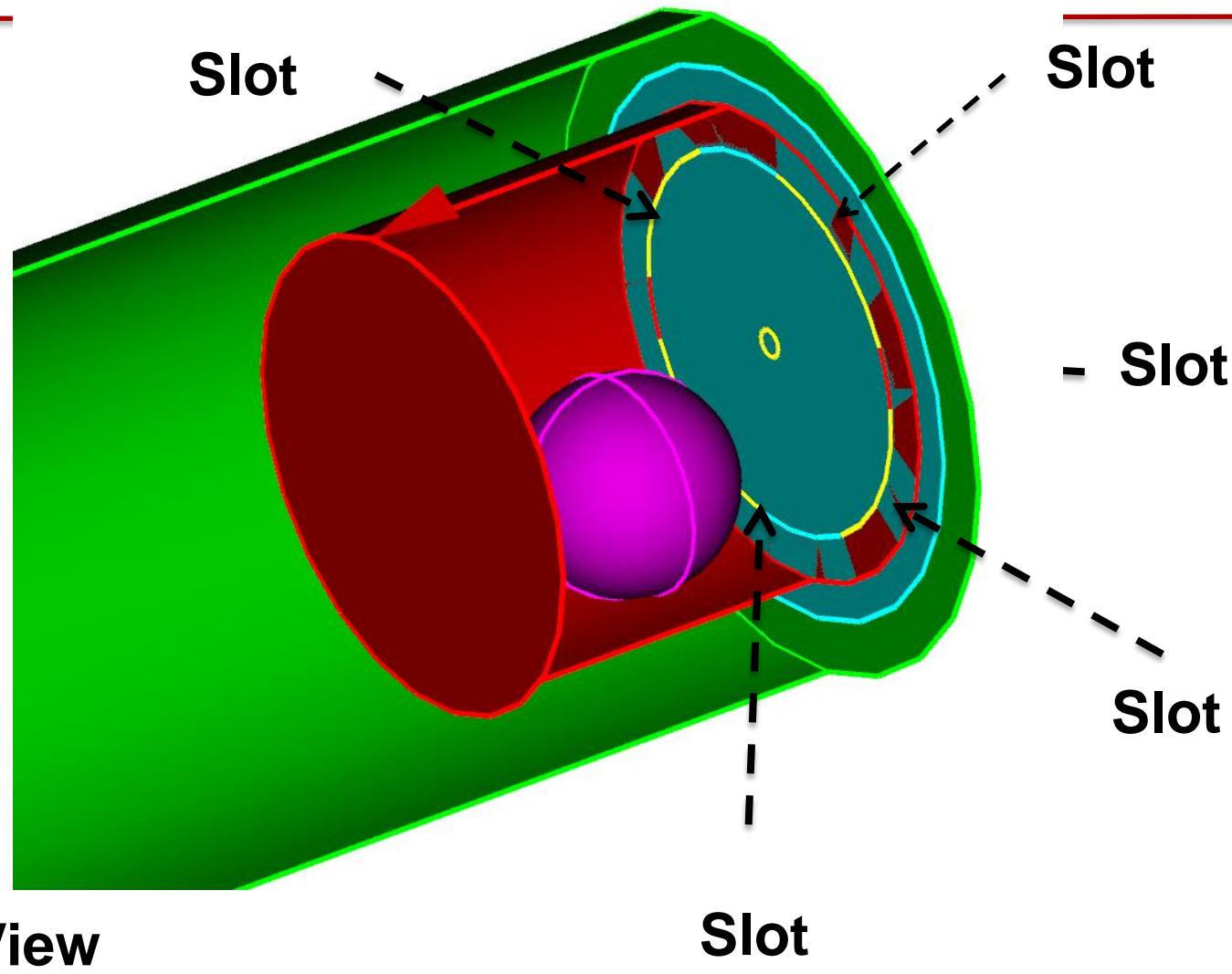
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**Internal View**

# Geometry D\_cavity

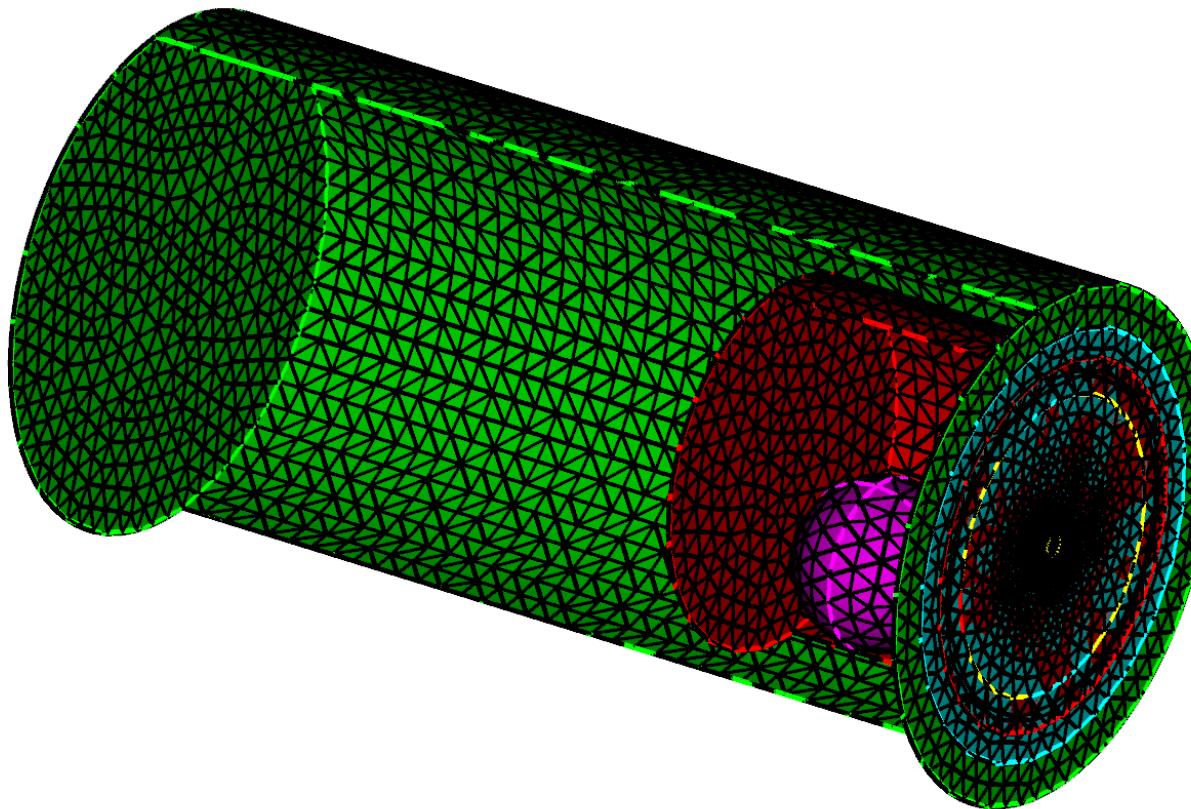


Internal View

Slot

# Geometry D\_cavity with Mesh

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Mesh 1 – 10090 elements

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15565 unknowns

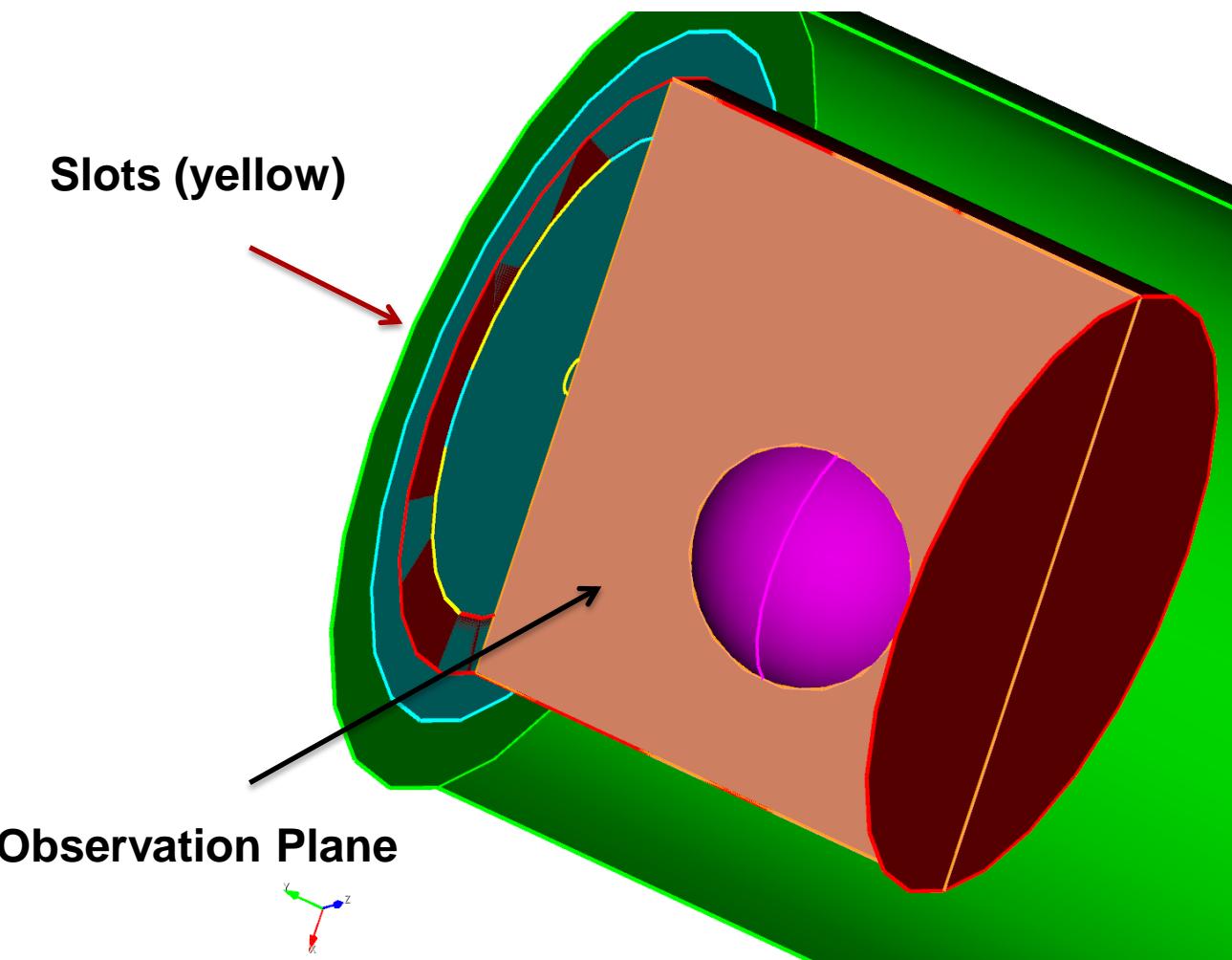
# Results

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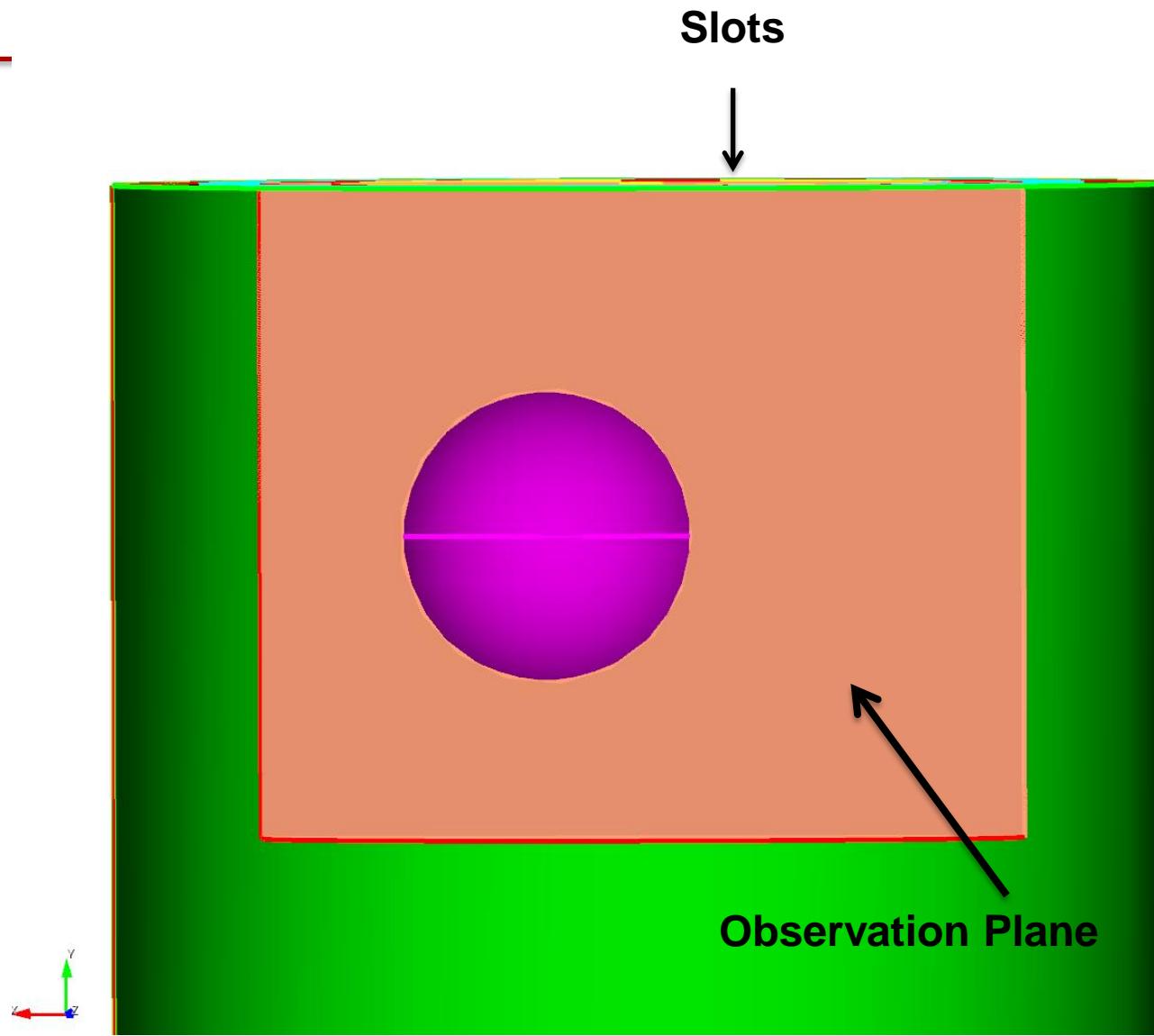
- The magnitude of the scattered electric field will be considered.
- This field value will be calculated on planes both inside the cavity and outside the cavity.
- Because of the proximity of these observation points to the object these are near field quantities.

# Data Results Observation Plane

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# Data Results Observation Plane



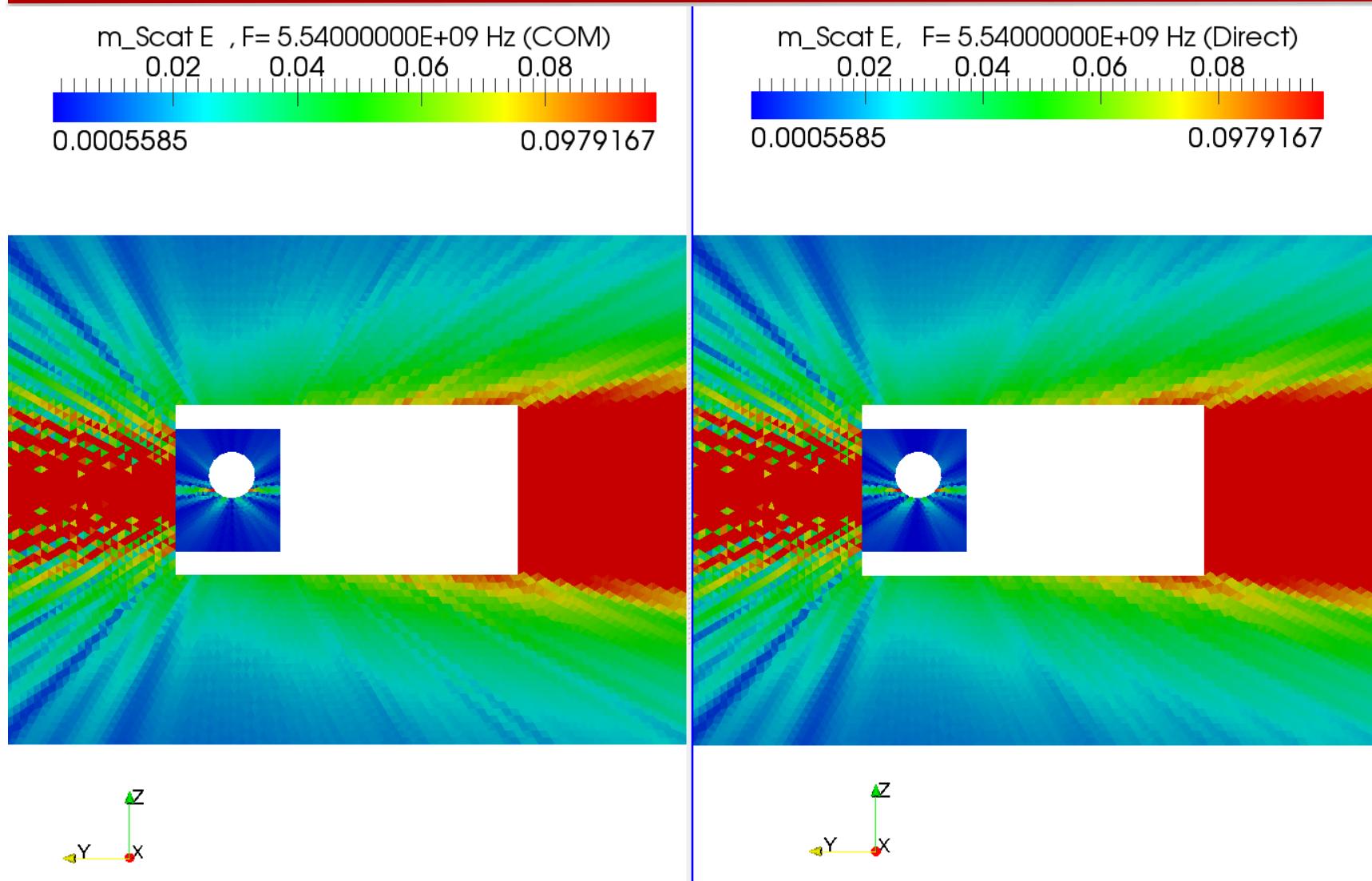
# Results - Mesh 3 D\_cavity

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- Object 1.2 m in length
- Frequency 5.5 GHz
- Number of Unknowns 247604
- Epsilon 3.1e-03
- Memory
  - Full matrix 16\*(61) GBytes
  - Compressed 16\*(36.8 + .2) Gbytes
  - ~40% compressed.

# Results - Mesh 3 D\_cavity

## Magnitude of Scattered Field



# Results - Mesh 3 D\_cavity

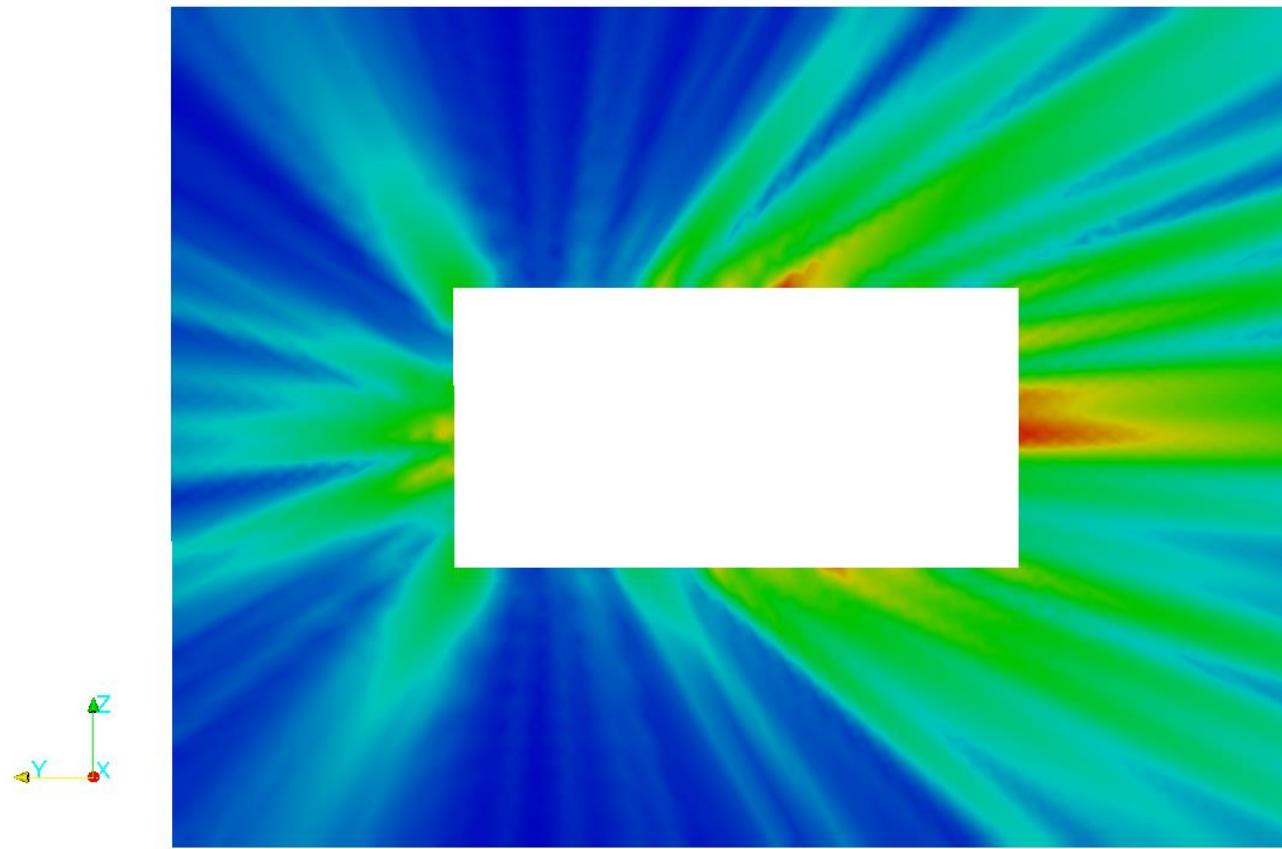
Magnitude of Scattered Field Difference between direct and compressed matrix solutions

m\_Scat E, F= 5.54000000E+09 Hz Difference

0.002 0.004 0.006

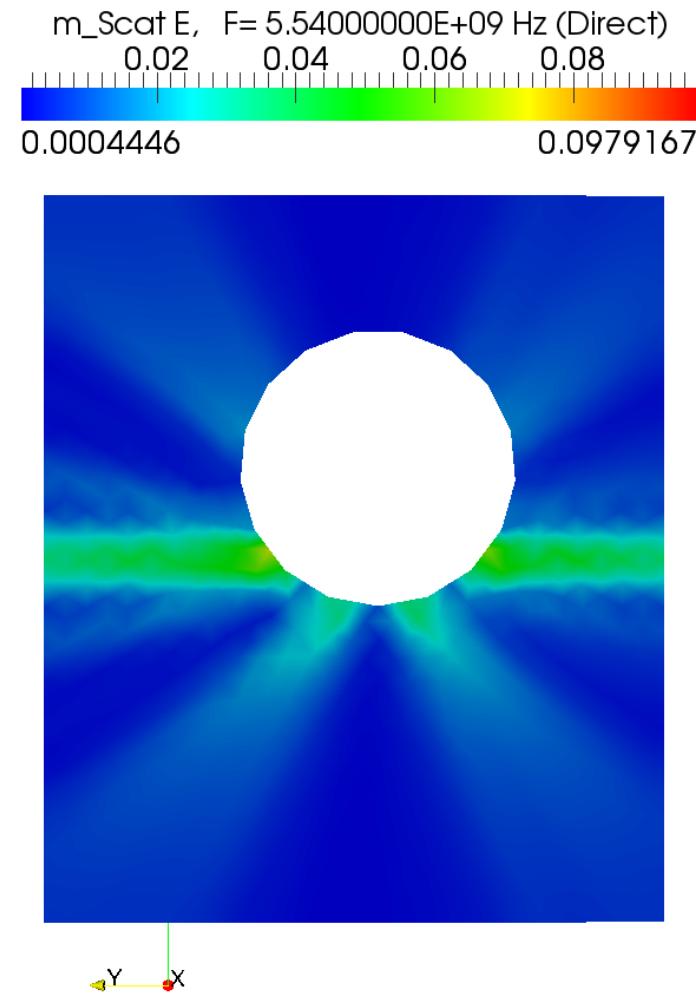
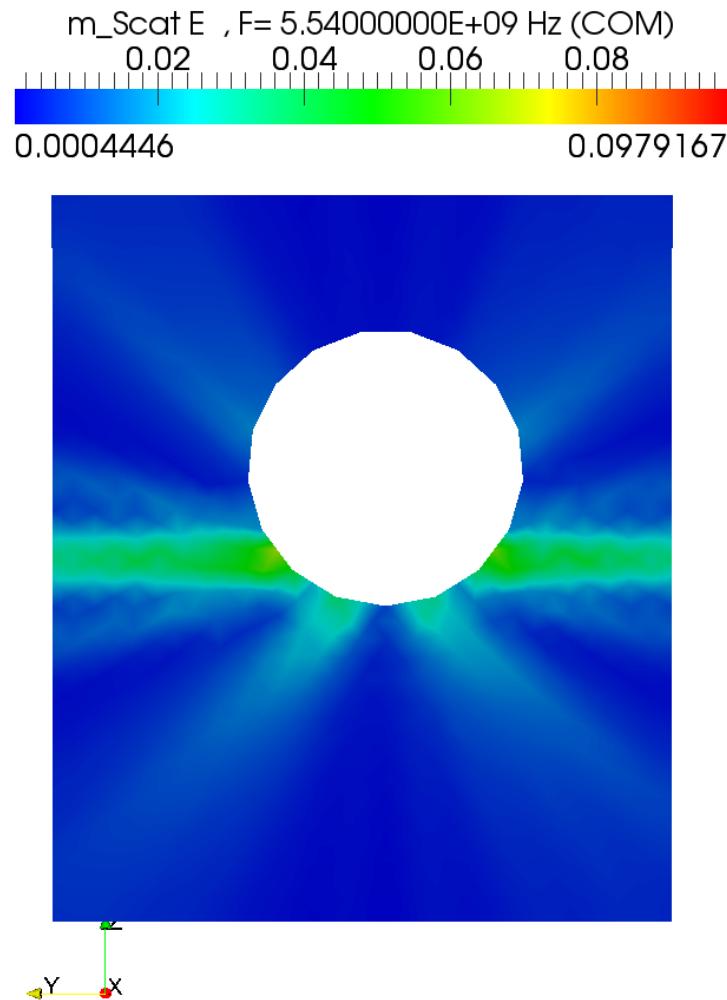
0.00013

0.007754



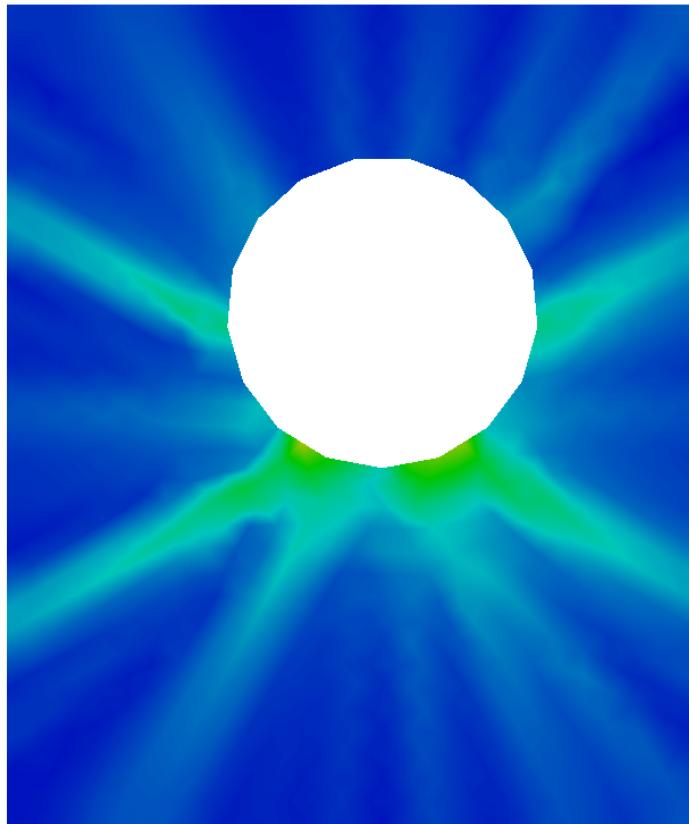
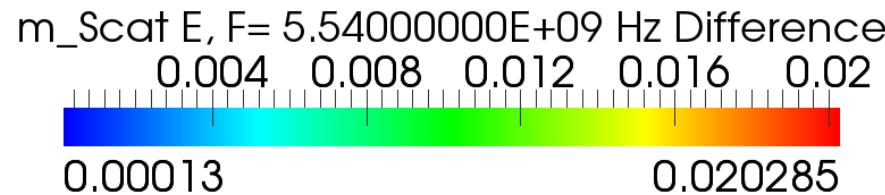
# Results - Mesh 3 D\_cavity

## Magnitude of Scattered Field



# Results - Mesh 3 D\_cavity

Magnitude of Scattered Field Difference between direct and compressed matrix solutions



# Error Norm

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- **Definition**

$$2-norm = \frac{Sqrt(\sum_{points} abs(error\_x)^2 + abs(error\_y)^2 + abs(error\_z)^2)}{Sqrt(\sum_{points} abs(e\_x)^2 + abs(e\_y)^2 + abs(e\_z)^2)}$$

# Error Norms

## 2-Norm

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### 247604 Unknown Problem

Location	2-Norm
Interior_x	.23
Interior_z	.2
Exterior_x	5.33e-03
Exterior_z	5.26e-03

**Solution tolerance 5.e-03**

# Conclusions

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- The matrix compression has been successfully integrated in EIGER.
  - For parallel machines
  - With iterative solver
- The viability of the technique has been demonstrated on a diverse group of problems.
  - Exterior problems
  - Problems with external geometry connected through slots.
    - Uses the thin-slot formulation already integrated in EIGER

# Future Work - Compression

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- **Improve the load balancing of the matrix:**
  - For the MOM blocks, by the block size not just by block number.
  - Use preprocessing to generate block matrix structure.
- **Improve solution time by reducing the iteration count**
  - Preliminary work performed by Matt Bettencourt on preconditioning revealed:
    - Standard methods ILU, Diagonal preconditioning will fail
    - Use Sparse Approximate Inverse (SAI)
    - Applied it to the two smaller problems discussed earlier.
    - Defined the algorithm to implement and tested it in MATLAB.
- **Continue testing on problems of interest to Sandia.**
  - Verify and quantify errors for a robust implementation.

# Future Work

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- **Implement alternative compression techniques**
  - Fast Multipole Method (FMM)
- **Implement cable models and interface to the EMPHASIS suite.**
  - External field to pin voltage.
- **Investigate hybrid techniques**
  - High- frequency approximations with full wave solvers.
- **Continue to validate the EMPHASIS suite**
  - Comparisons to measurements.

# Matrix Compression Backup Slides

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# ACA Matrix Compression

## Definitions

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$Z(I_1, :)$

**Row 1 of matrix Z**

$Z(:, J_1)$

**Column 1 of matrix Z**

$$\|Z\|^2 = \|Z\|_F^2 = \sum_{i=1}^m \sum_{j=1}^n |z_{ij}|^2$$

**Frobenius Norm squared of matrix Z**

# ACA Matrix Compression

## Algorithm - Initialization

---

Compute the first row of the matrix.

$$Z(I_1, :)$$

Find the maximum element of the row.

$$\max_j |Z(I_1 :, j)| = J_1$$

Normalize the row by this element.

$$v_1 = Z(I_1, :) / Z(I_1, J_1)$$

The vector  $u$  is defined.

$$u_1 = Z(:, J_1)$$

The first contribution to the Frobenius Norm squared is calculated.

$$\|\tilde{Z}^1\|^2 = \|u_1\|^2 \|v_1\|^2$$

Find the largest row value in the column  $J_1$  – is different from the previous row used.

$$\max_i |Z(i :, J_1)| = I_2 \neq I_1$$

# ACA Matrix Compression

## Algorithm – k'th Iteration

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1) Calculate next row and compute approximate matrix row.

$$\tilde{R}(I_k, :) = Z(I_k, :) - \sum_{l=1}^{k-1} (u_l)_{I_k} v_l$$

2) Find the maximum element of the row.

$$\max_j |\tilde{R}(I_k, j)| = J_k$$

3) Normalize the row by this element.

$$v_k = \tilde{R}(I_k, :) / \tilde{R}(I_k, J_k)$$

4) Adjust approximate column and compute  $u_k$ .

$$\tilde{R}(:, J_k) = Z(:, J_k) - \sum_{l=1}^{k-1} (v_l)_{J_k} u_l$$

$$u_k = \tilde{R}(:, J_k)$$

5) Compute the update to the Frobenius Norm.

$$\|\tilde{Z}^{(k)}\|^2 = \|\tilde{Z}^{(k-1)}\|^2 + 2 \sum_{j=1}^{k-1} |u_j^T u_k| \cdot |v_j^T v_k| + \|u_k\|^2 \|v_k\|^2$$

6) Find the next maximum row index.

$$\max_i |\tilde{R}(i, J_k)| = I_{k+1}$$

## Algorithm – Convergence

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- The test for convergence is (Step 5 in the previous slide) :
  - Epsilon chosen by the user

$$\|u_k\| \cdot \|v_k\| \leq \varepsilon \|\tilde{Z}^{(k)}\|$$

- The computation of a low-rank approximation to the matrix is complete.
  - Note that this was done by row and column – the full matrix was not computed and reduced.
  - The number of elements to store for this matrix is  $(n + m) \times r$ , instead of  $m \times n$ .