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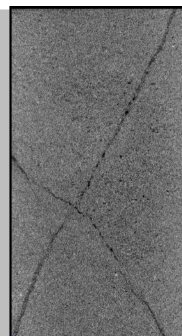

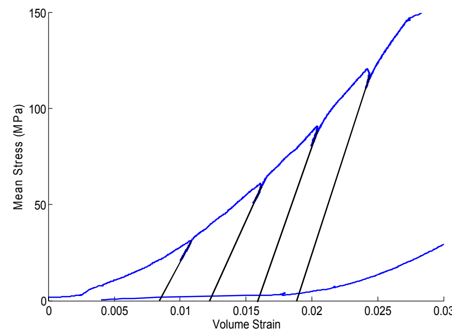

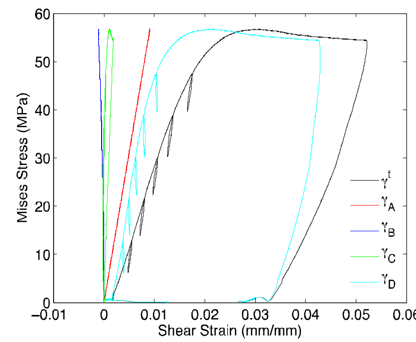

The importance of the intermediate principal stress on failure and localization behavior

M.D. Ingraham

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Outline

- Motivation
- Testing
  - Observed response
  - Failure Dependence
- Strain separation
  - Strain breakdown
  - Constitutive parameter development
- J_3 dependence
  - Failure
  - Band Angle
- Conclusions

Motivation

- The much cited Rudnicki and Rice ‘75 localization criterion lacks comprehensive experimental investigation
- Attempt to experimental illustrate intermediate principal stress dependence and by extension J_3 dependence.
- Compare experimental results from true triaxial tests to Rudnicki and Rice ‘75 predictions.

Motivation

- First invariant of stress

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

- Second invariant of deviatoric stress

$$J_2 = \tfrac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

- Third invariant of deviatoric stress

$$J_3 = \sigma_1' \sigma_2' \sigma_3'$$

- Deviatoric stress

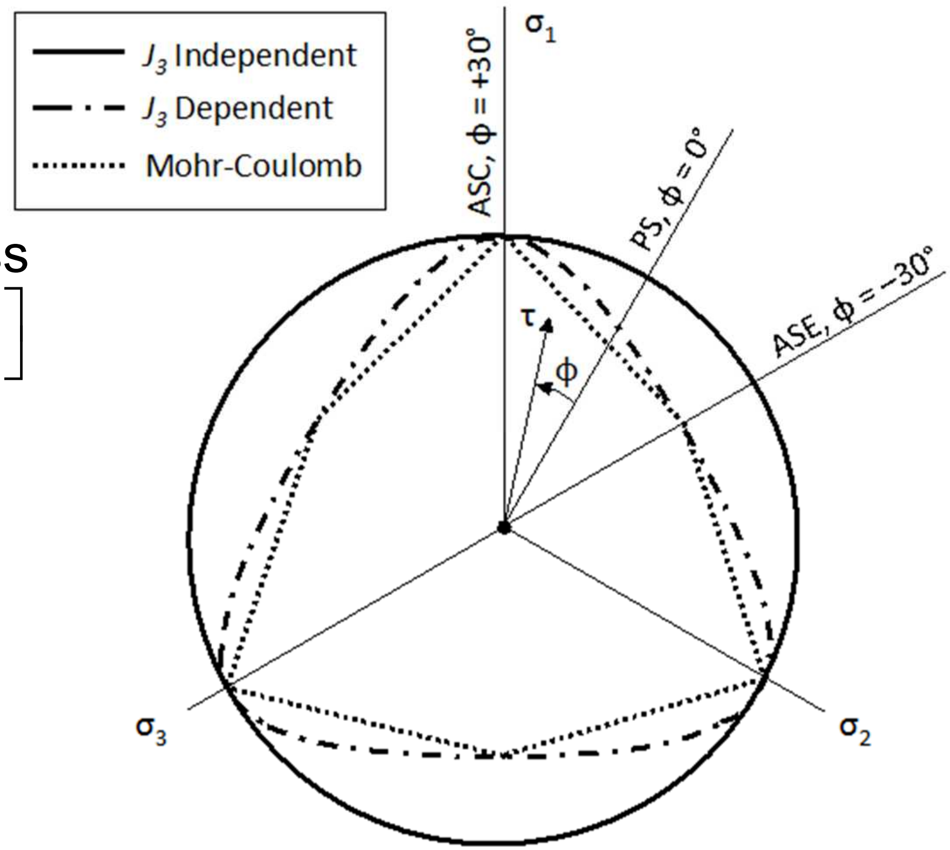
$$\sigma_{ij}' = \sigma_{ij} - \tfrac{1}{3}\sigma_{kk}\delta_{ij}$$

- Shear stress

$$\tau = \sqrt{J_2}$$

- Mean Stress

$$\sigma = -\tfrac{1}{3} I_1 = -\tfrac{1}{3}\sigma_{kk}$$



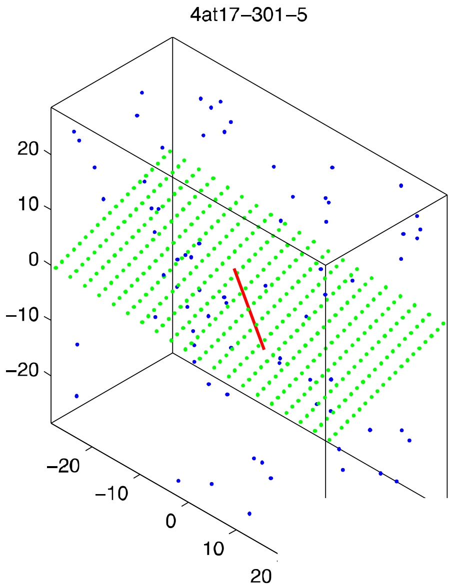
Sign convention is positive in compression

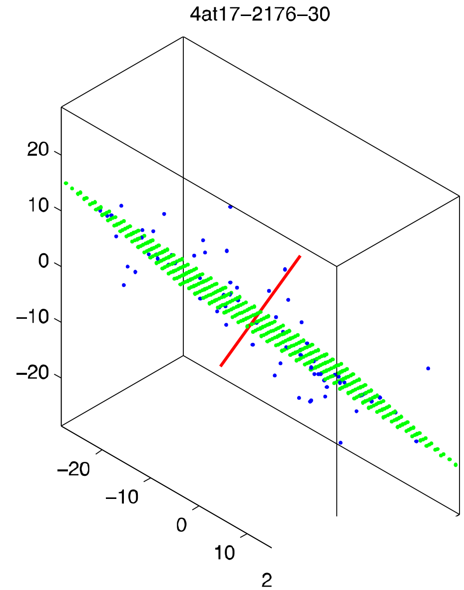
Tests Performed

- True triaxial tests performed at 5 different Lode angles
  - 30, 14.5, 0, -14.5, -30 degrees (axisymmetric compression to axisymmetric extension)
  - Tests were performed under constant mean stress conditions
  - 5 mean stresses were tested ranging from 30 to 150 MPa in 30 MPa increments in order to map out the failure/yield surface for a range of Lode angles
- Unload loops from all tests were used to develop the strain separation process.

For experimental description see Ingraham et al. 2013

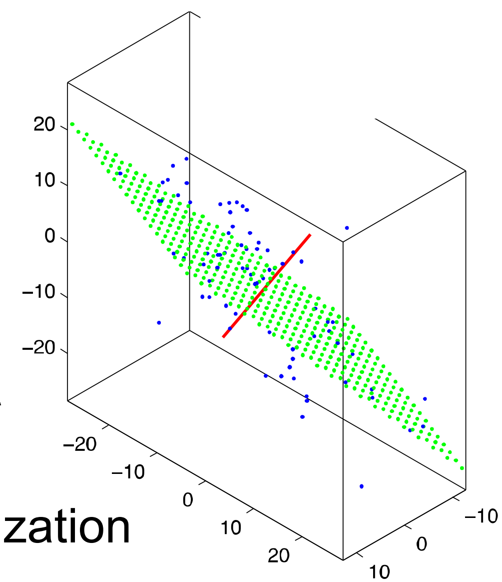
Shear Band



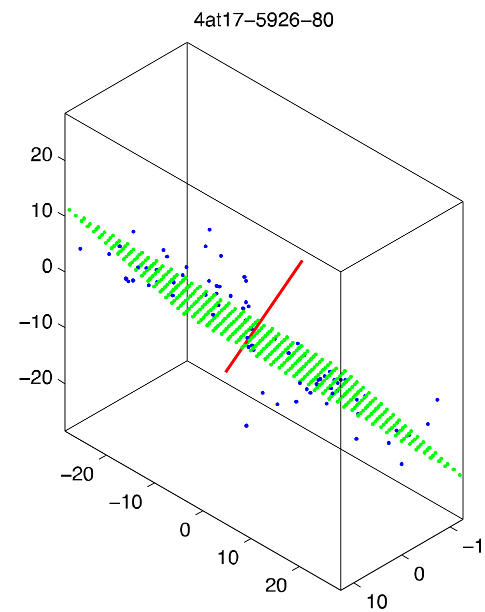


$$\phi = 0$$

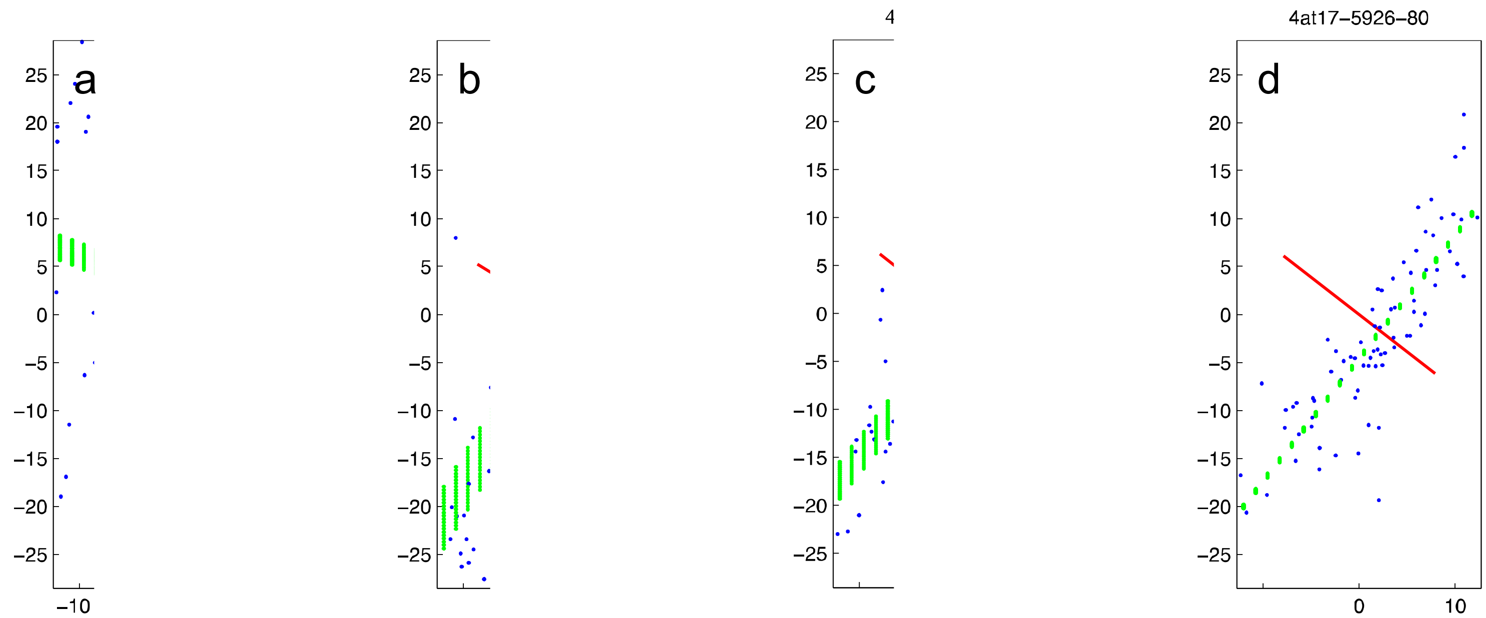
$$\sigma = 30$$

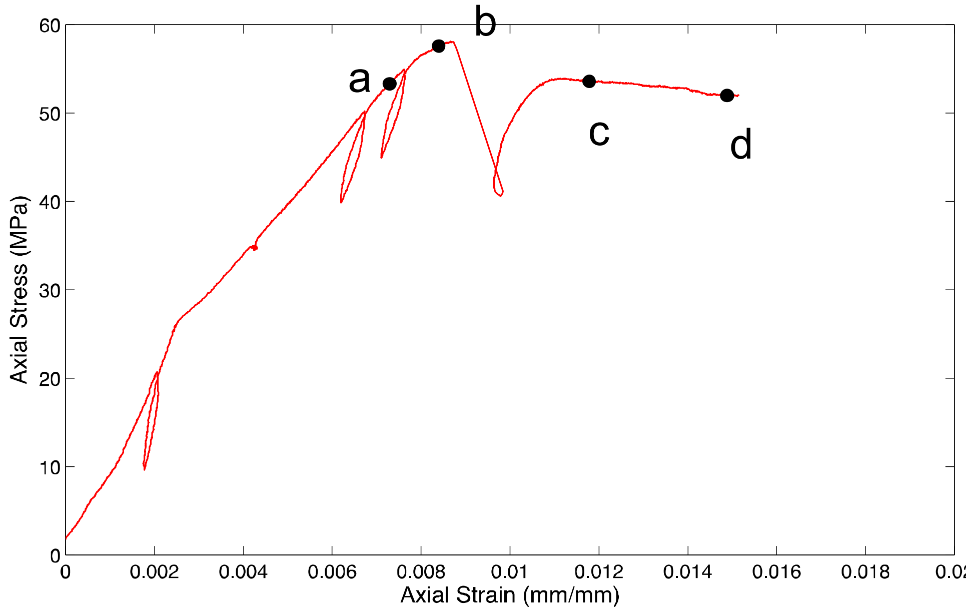


Localization



Shear Band

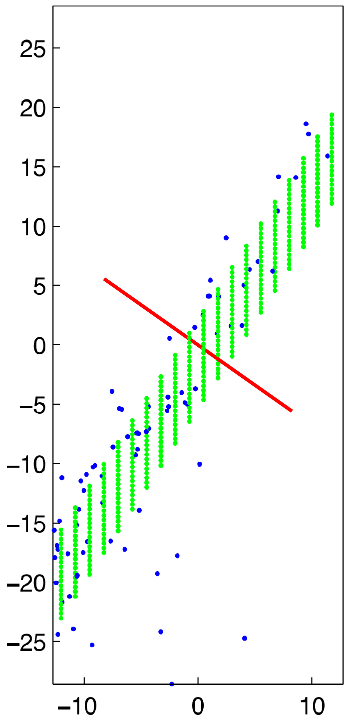


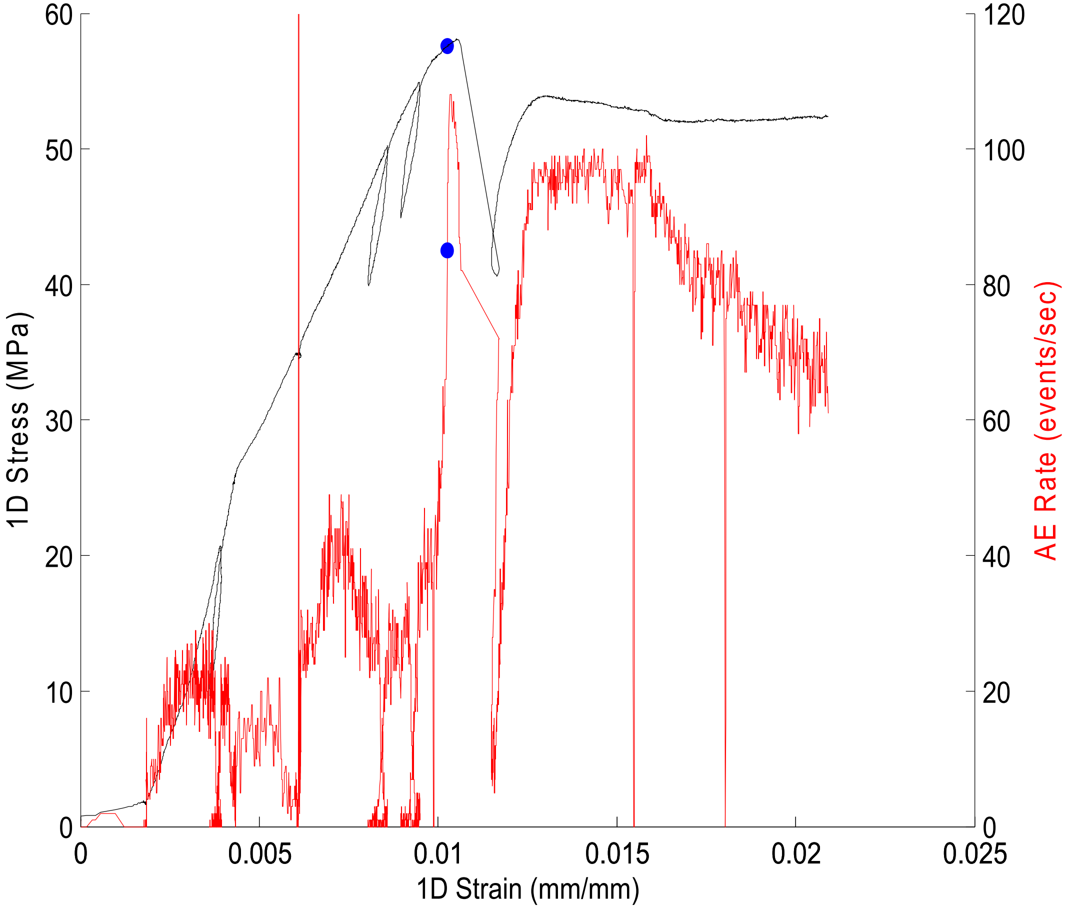


$$\phi = 0$$

$$\sigma = 30$$

Shear Band

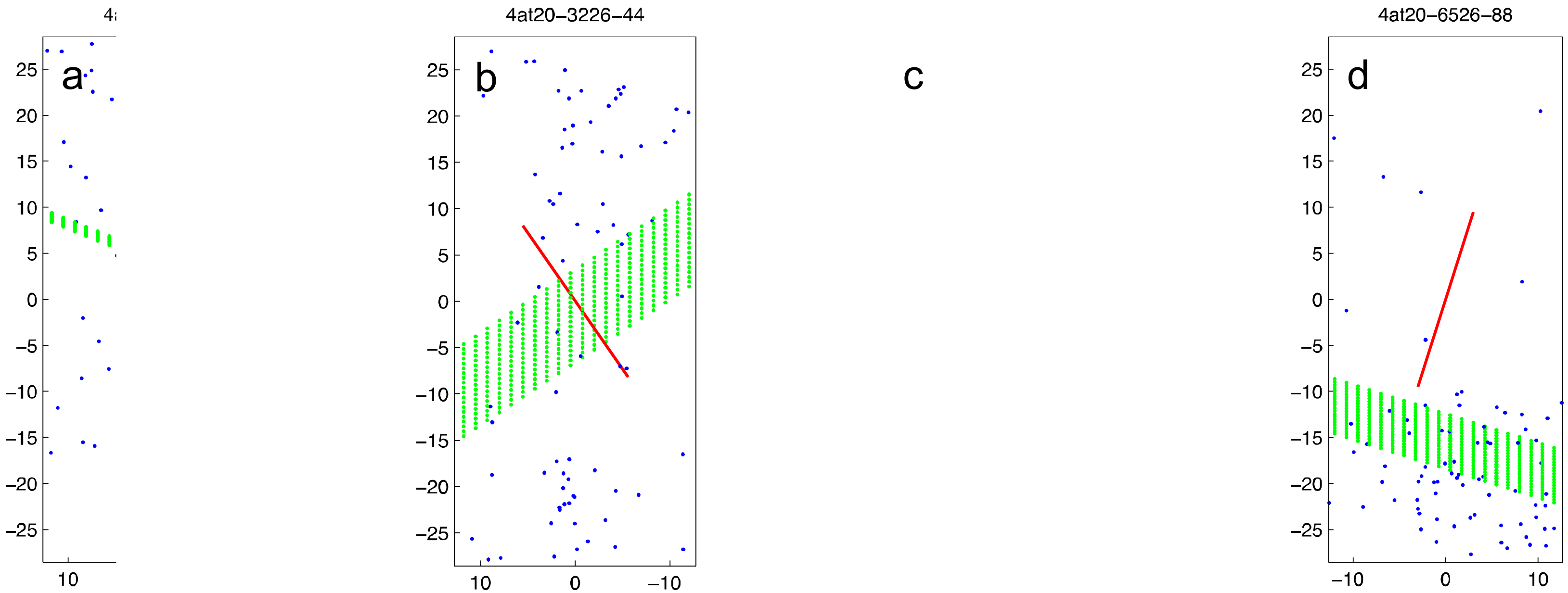


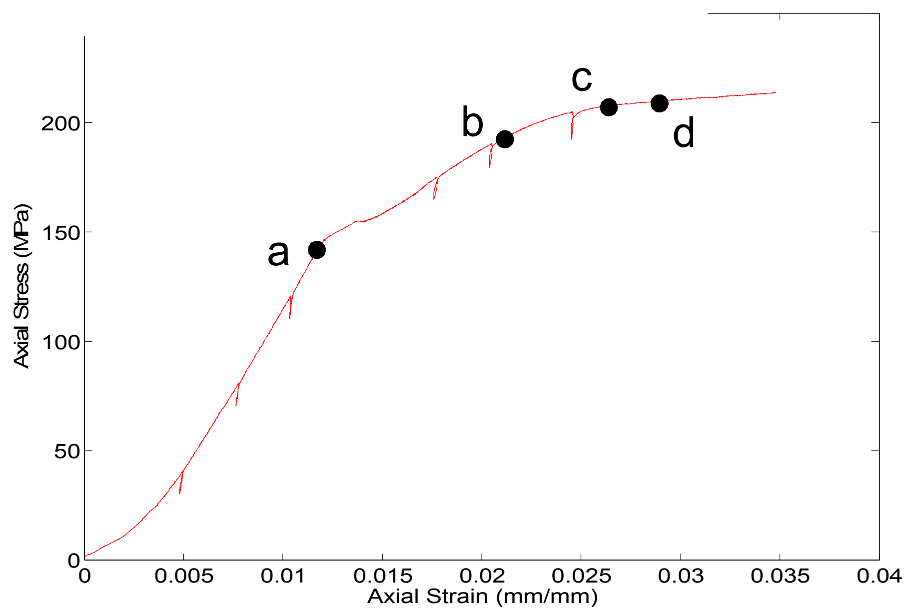


$$\phi = 0$$

$$\sigma = 30$$

Compaction Localization

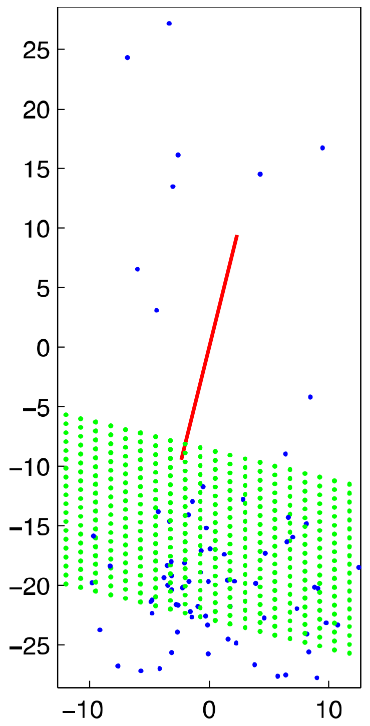




$$\phi = 0$$

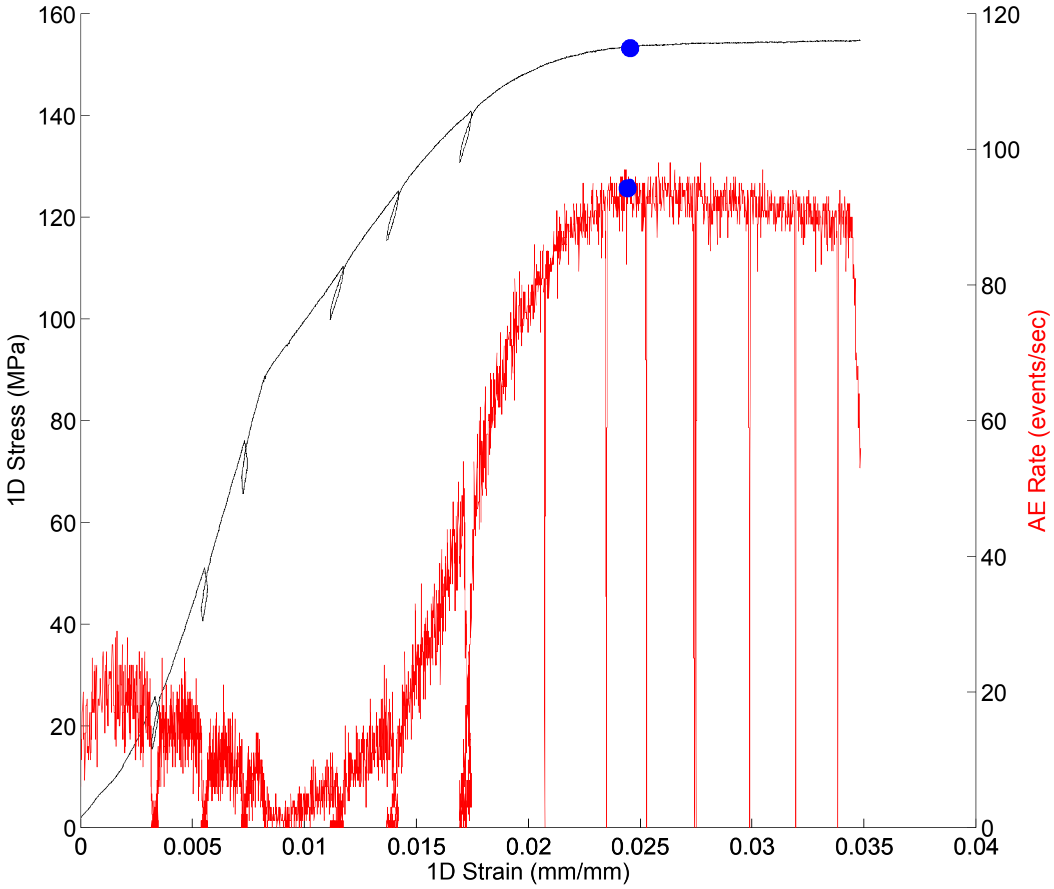
$$\sigma = 150$$

Compaction Localization

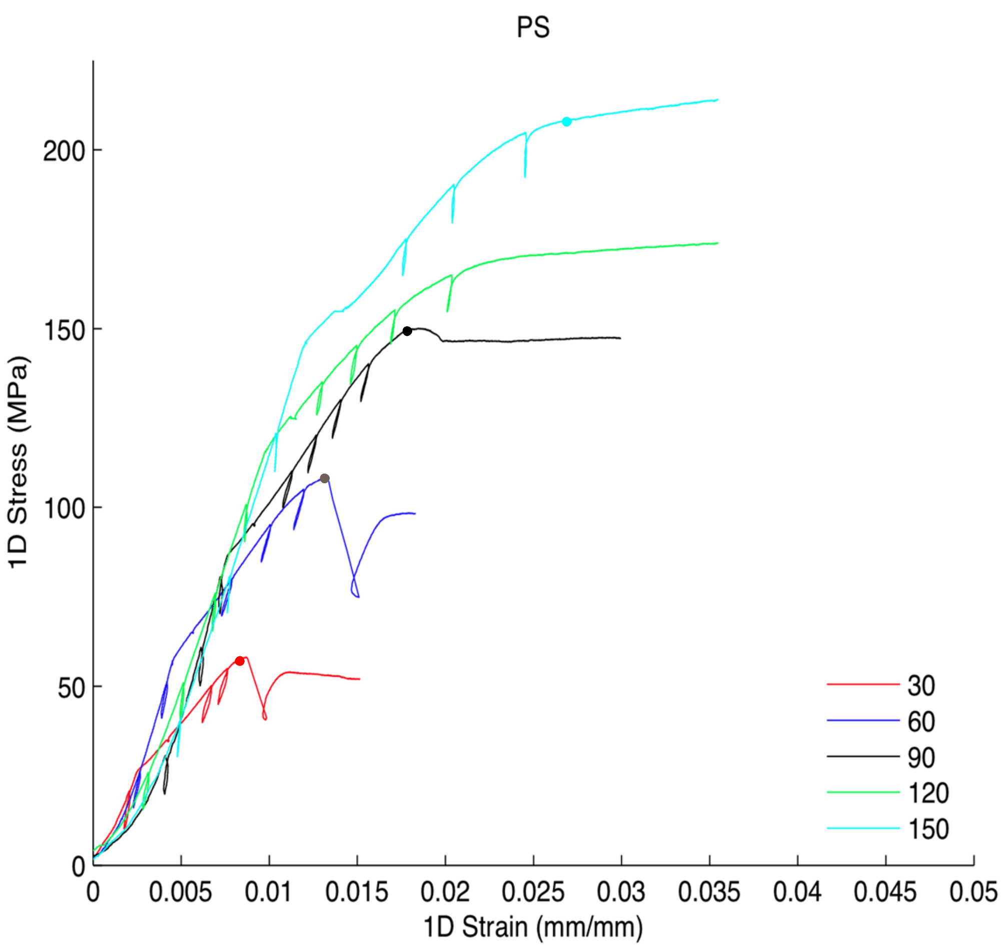


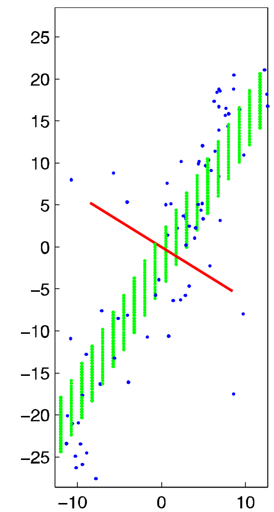
$$\phi = 0$$

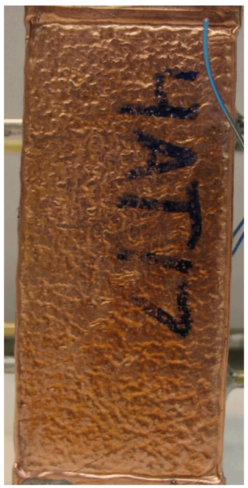
$$\sigma = 150$$

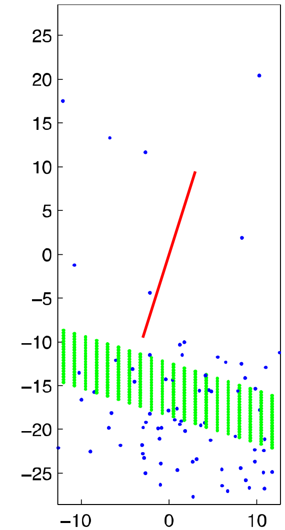


Effect of Mean stress

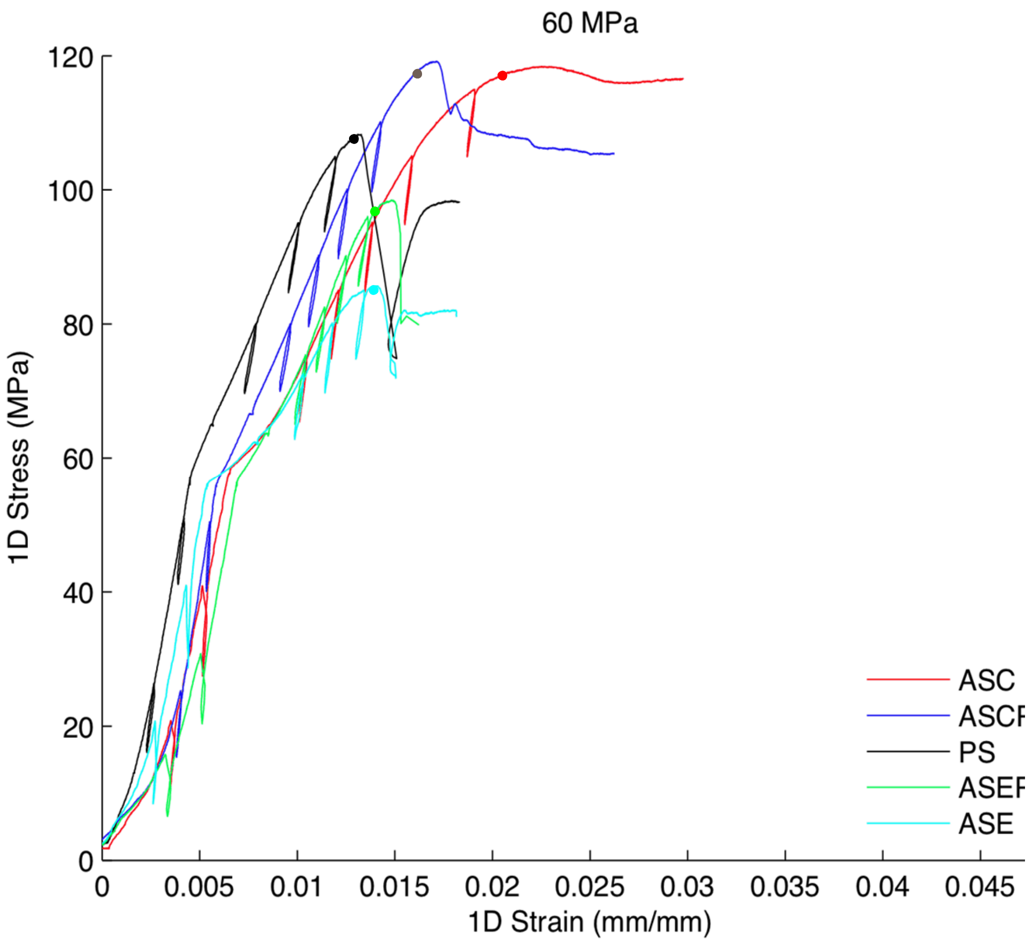


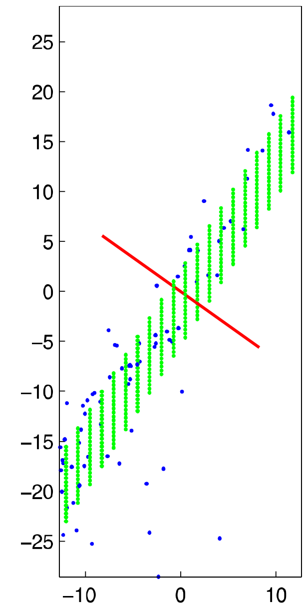


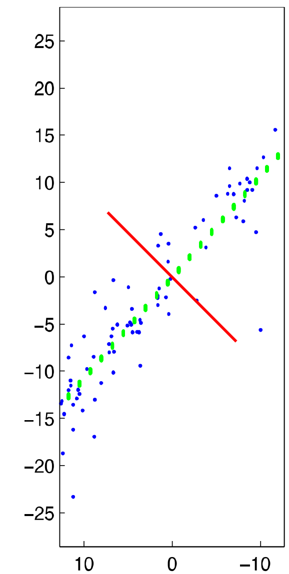




Effect of Stress State

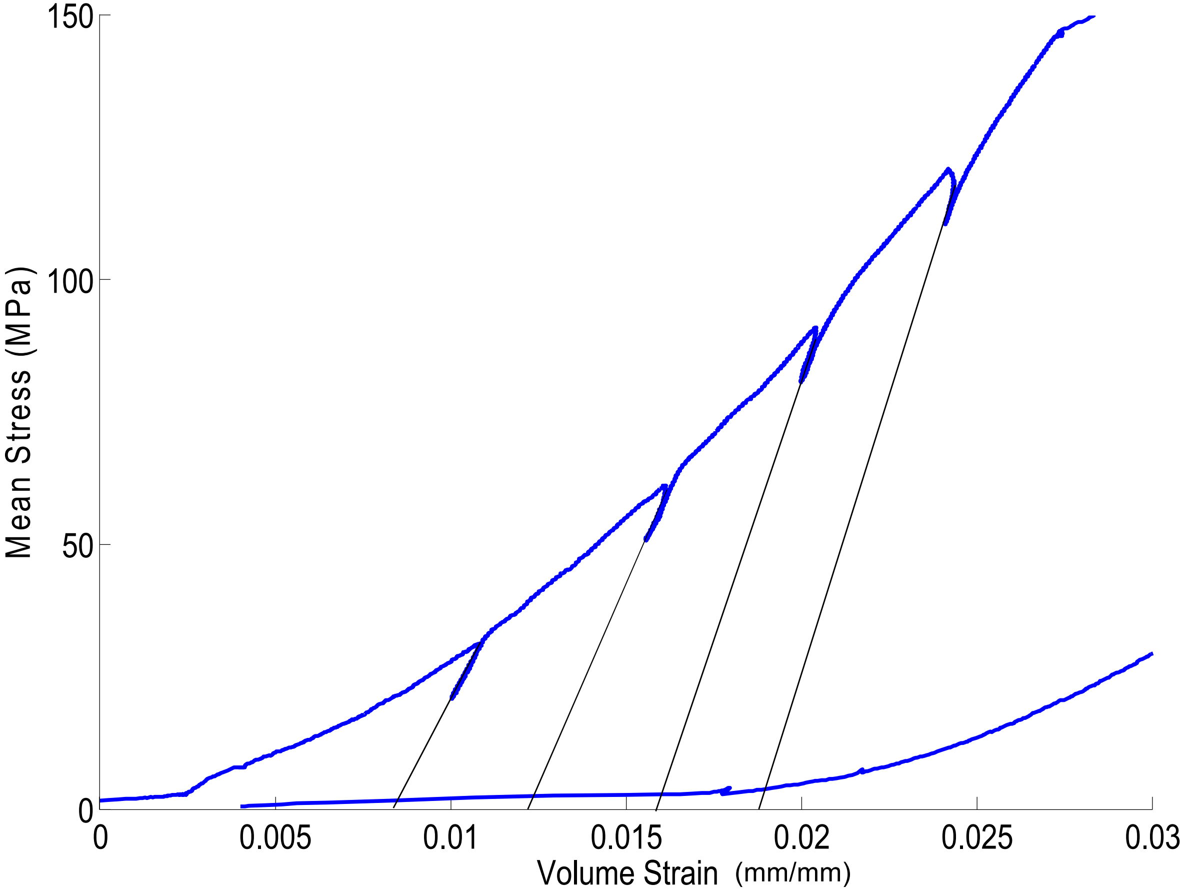






Strain Separation

To Determine the inelastic increment of strain



$$G(\tau, \gamma_P)$$

$$K(\sigma, \varepsilon_P)$$

Strain Separation: Constitutive Laws

- Starting with a common elastic-plastic constitutive model:

$$\varepsilon_{ij}^{t} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p}$$

  - Isotropy and usual invariant definitions provide the common elastic strain models

$$\gamma^{e} = \frac{\tau}{G} \qquad \varepsilon^{e} = \frac{\sigma}{K}$$

  - Assuming stress and plastic strain dependence in incremental form and expanding the total derivative

$$d\gamma^{t} = d\left(\frac{\tau}{G(\tau,\gamma^{p})}\right) + d\gamma^{p} \qquad d\varepsilon^{t} = d\left(\frac{\sigma}{K(\sigma,\varepsilon^{p})}\right) + d\varepsilon^{p}$$

$$d\gamma^{t} = \frac{d\tau}{G} - \frac{\tau}{G^{2}}\left(\frac{\partial G}{\partial \tau}d\tau + \frac{\partial G}{\partial \gamma^{p}}d\gamma^{p}\right) + d\gamma^{p}$$

$$d\varepsilon^{t} = \frac{d\sigma}{K} - \frac{\sigma}{K^{2}}\left(\frac{\partial K}{\partial \sigma}d\sigma + \frac{\partial K}{\partial \varepsilon^{p}}d\varepsilon^{p}\right) + d\varepsilon^{p}$$

Strain Separation: Breakdown

$$d\gamma^{t} = \frac{d\tau}{G} - \frac{\tau}{G^2}\left(\frac{\partial G}{\partial \tau} d\tau + \frac{\partial G}{\partial \gamma^{p}} d\gamma^{p}\right) + d\gamma^{p}$$

$$d\gamma_{A} = \frac{d\tau}{G}$$

$$d\gamma_{B} = -\frac{\tau}{G^2}\left(\frac{\partial G}{\partial \tau} d\tau\right)$$

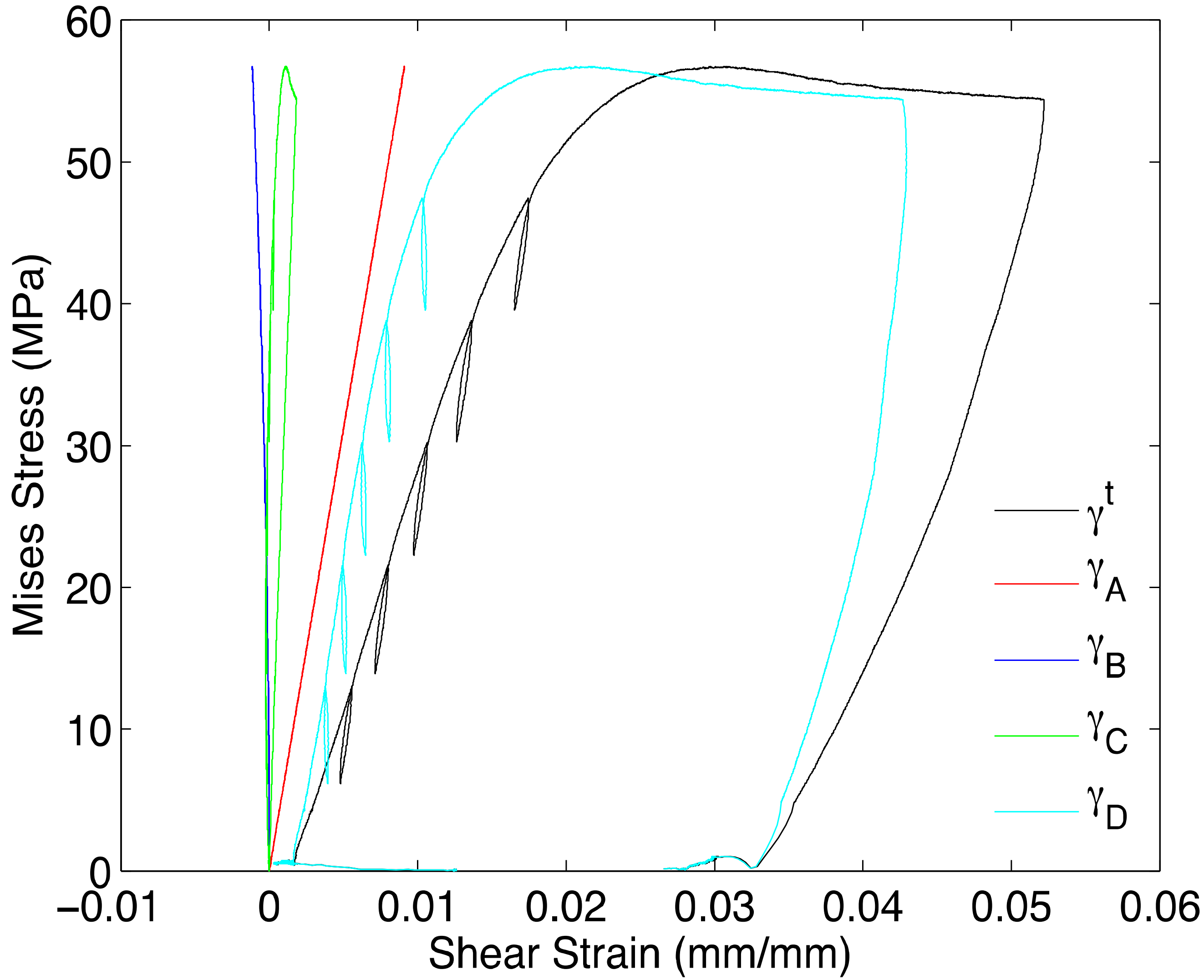
$$d\gamma_{C} = -\frac{\tau}{G^2}\left(\frac{\partial G}{\partial \gamma^{p}} d\gamma^{p}\right)$$

$$d\gamma_{D} = d\gamma^{p}$$

$$\beta = -\frac{d^{p}\varepsilon}{d^{p}\gamma} \qquad d^{p}\gamma = d\gamma_{C} + d\gamma_{D}$$

- Strain is separated into 4 forms: Elastic, elastic stress dependent, plastic strain dependent, plastic
- A,B,C are recovered upon unloading γ^{e}, however C and D are the inelastic increment of strain needed for localization theory

Separated Strains



Application

$$\theta = \frac{\pi}{4} + \frac{1}{2}\arcsin\left[\frac{\frac{2}{3}(1+\nu)(\beta+\mu) - N_{II}(1-2\nu)}{\sqrt{4-3N_{II}^2}}\right]$$

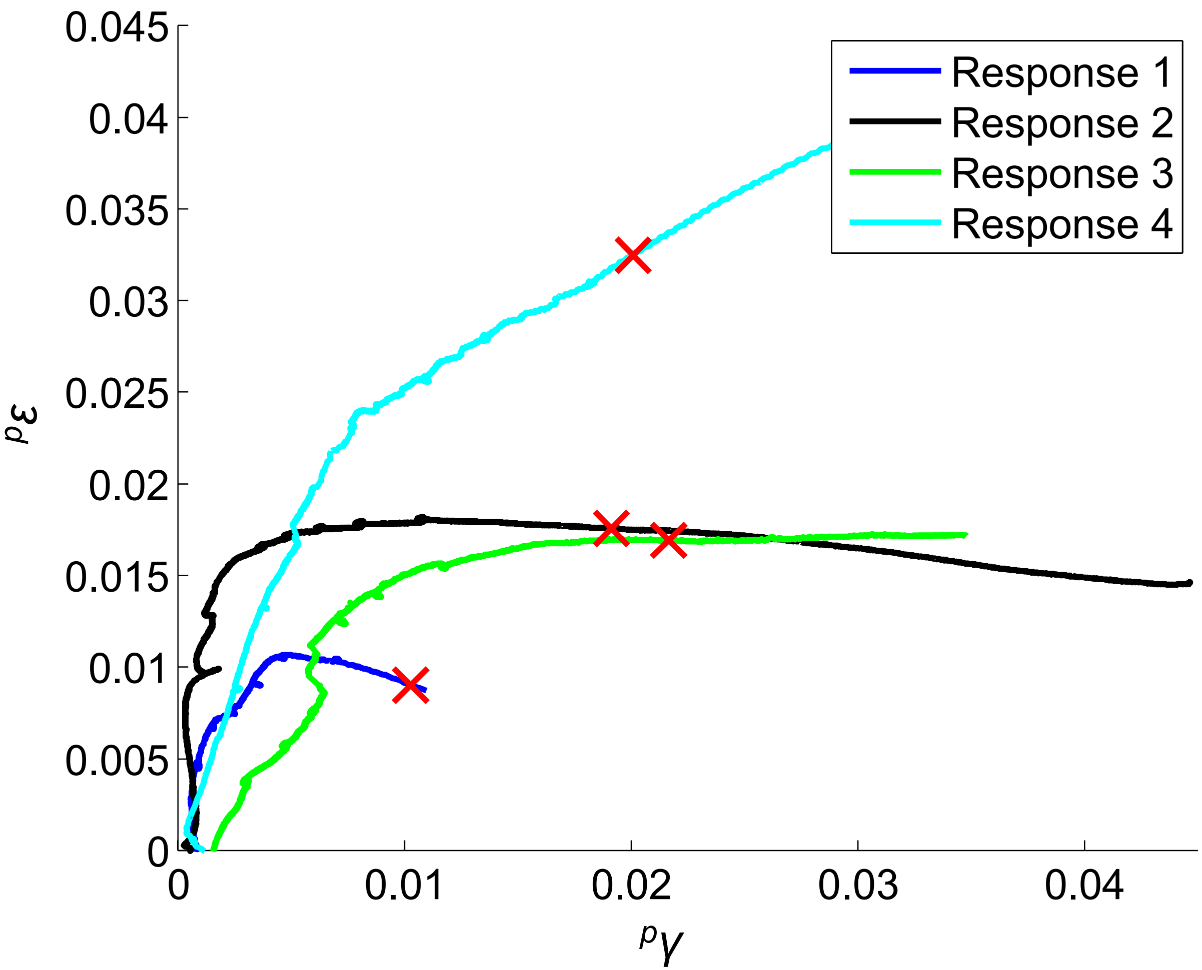
When:

$$-\frac{3(N_I + \nu N_{II})}{1+\nu} \le \beta + \mu \le -\frac{3(N_{III} + \nu N_{II})}{1+\nu}$$

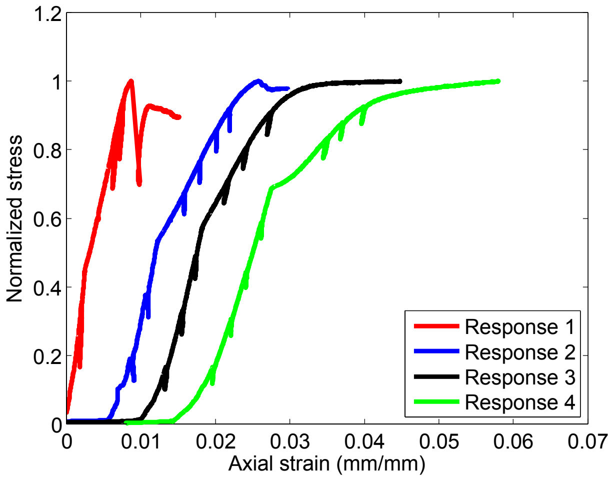
$$N_I = {(\sigma - \sigma_3)}/{\tau}, N_{II} = {(\sigma - \sigma_2)}/{\tau}, N_{III} = {(\sigma - \sigma_1)}/{\tau}$$

Rudnicki and Olsson (1998)

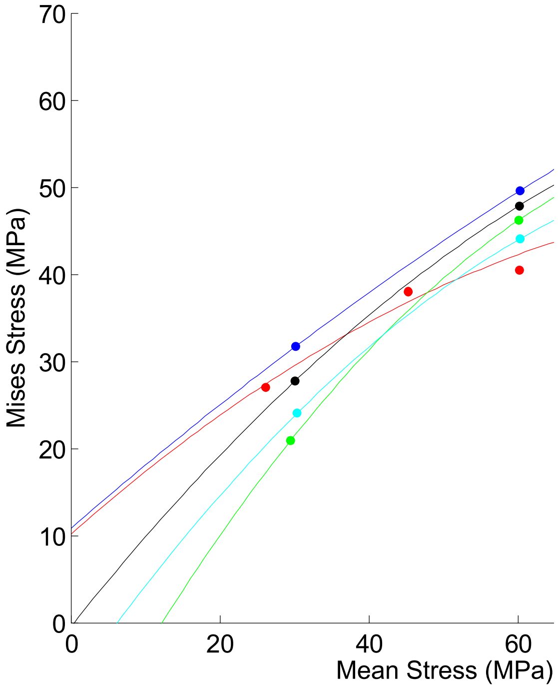
Constitutive Parameter $\boldsymbol{\beta}$

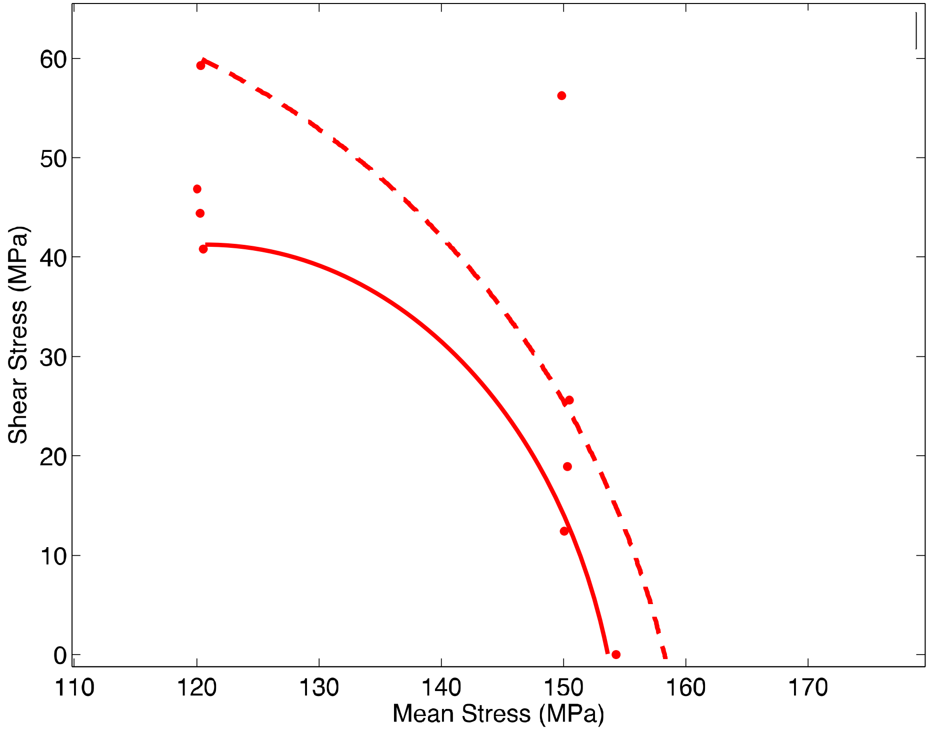


$$\beta = -\frac{d^p\varepsilon}{d^p\gamma}$$



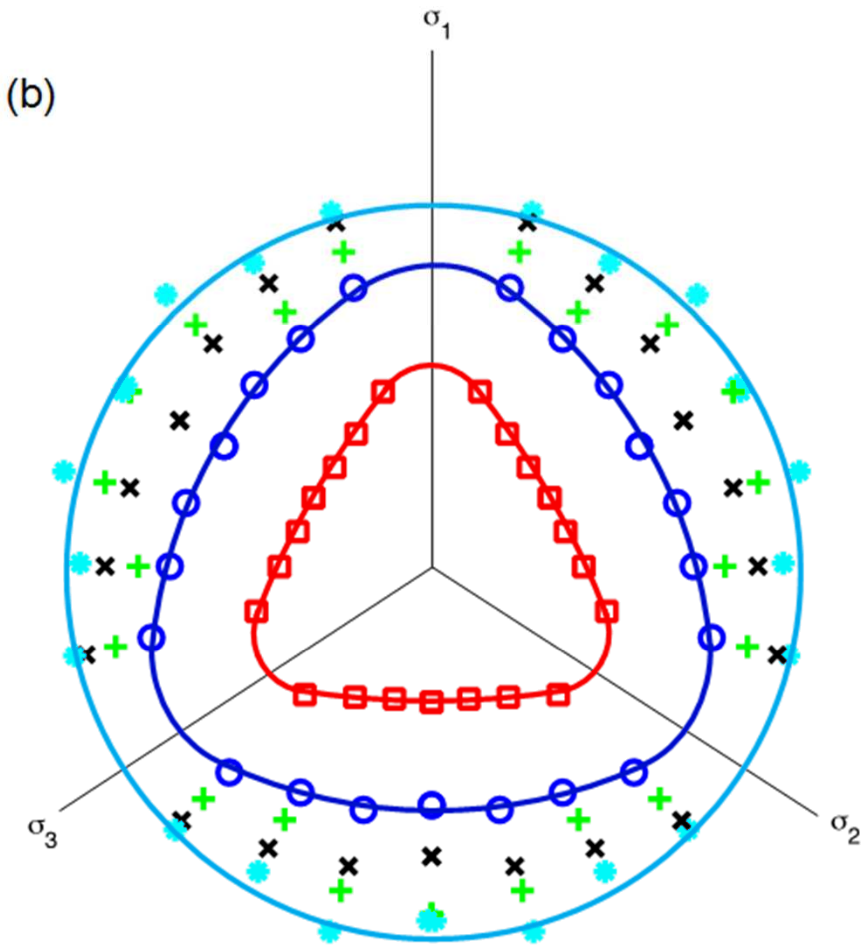
Constitutive Parameter – μ





μ – slope of a contour of constant inelastic shear/volume strain

J_3 Dependent Failure



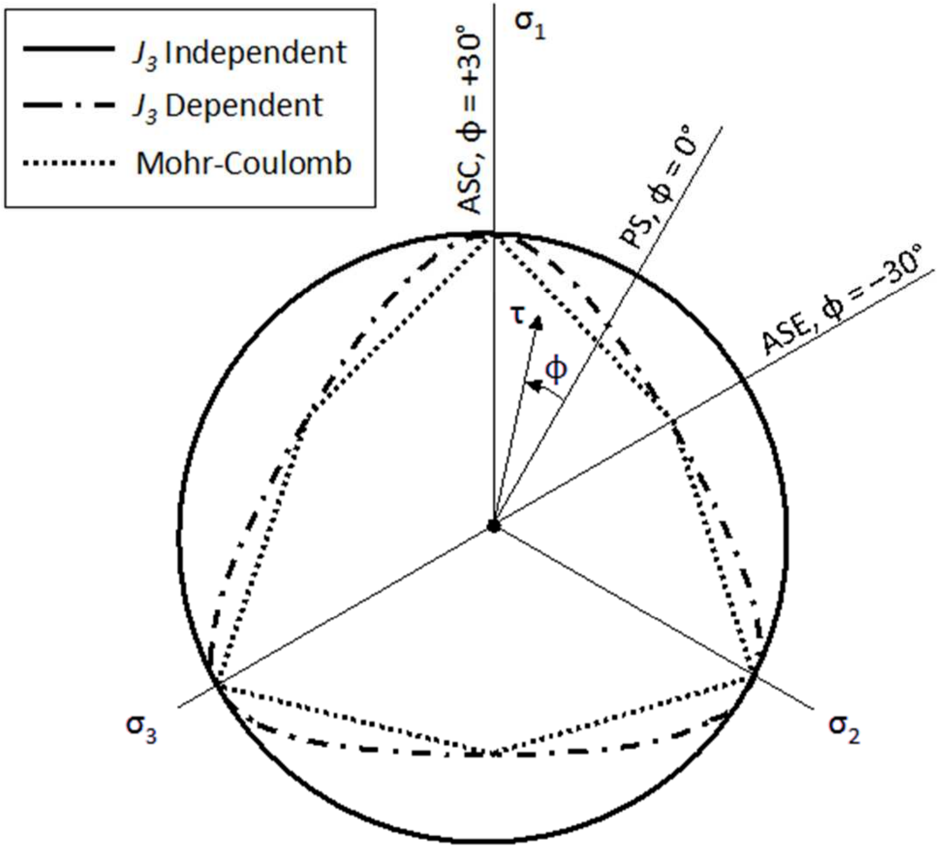


Table of Band Angles

Stress State	Mean Stress (MPa)	β	μ	Predicted θ	AE θ	Measured θ	Response Type
ASC	30	0.76	0.56	59	Conj. Bands	55-60	Shear
ASC	60	0.23	0.31	48	23	30-35	Shear
ASC	90	0.01	0.09	42	10-23	NA	CL
ASC	120	-0.29	0:-0.3	37:33	5-15	NA	CL
ASC	150	-0.66	-1.1:-3	3:0	NL	NA	NL
PS	30	0.09	0.94	57	58	61-80	Shear
PS	60	0.55	0.80	62	63	64	Shear
PS	90	0.08	0.67	54	54	58	Shear
PS	120	-0.23	0:-0.7	42:33	NL	NA	NL
PS	150	-0.75	-1.5:-4.4	15:0	16-25	NA	CL
ASE	30	0.76	0.85	80	51	65	Shear
ASE	60	0.65	0.49	68	NA*	70	Shear
ASE	90	0.04	0.13	54	41	46	Shear
ASE	120	-0.17	0:-1.9	50:23	Conj. Bands	45	Shear
ASE	150	-0.21	-1.8:-6	25:0	10-25	NA	CL

Conclusions

- Strain Separation
  - Separation of strains allows for determination of the onset of yield/yield surfaces
- Localization Predictions
  - Band angle does not appear to depend on stress state, or effects are smaller than experimental error
  - Strain separation processes can be implemented to provide localization results reasonably comparable with experimental results for high porosity sandstone.
- J_3 Dependence
  - Localization modes (shear band, compaction localization) are active at higher mean stresses when the Lode angle is low
  - Shear stress required to cause localization decreases with decreasing Lode angle

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