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Toward Using Surrogates to Accelerate Solution of Stochastic Electricity Grid Operations Problems

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Problem Statement

- **Unit Commitment** (UC): schedule thermal generating units with the objective is to minimize overall production costs
  - satisfy forecasted demand for electricity; reserve margins are universally imposed in UC to ensure that sufficient capacity is available in case demand is higher
  - respect constraints on both transmission (e.g., thermal limits) and generator infrastructure
- **Stochastic UC model** (SUC): typically minimize the expected cost across load scenarios, thus ensuring sufficient flexibility to meet a range of potential load realizations during operations.
  - reliance on reserve margins is reduced, yielding less costly solutions than deterministic UC
  - computationally difficult due to the *large number of samples* needed to achieve "converged" solutions

Stochastic Unit Commitment

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & c^u(\boldsymbol{x}) + c^d(\boldsymbol{x}) + \overline{Q}(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{X}, \\ & \boldsymbol{x} \in \{0, 1\}^{|G| \times |T|} \end{aligned}$$

- G and T: index sets of generating units and time periods
- $\mathcal{X}$ and $\boldsymbol{x}$: set of unit commitment constraints and vector of unit commitment decisions
- $c^u(\boldsymbol{x})$ and $c^d(\boldsymbol{x})$: generating unit start-up and shut-down costs
- $\overline{Q}(\boldsymbol{x})$: the expected generation cost

Classical approach, compute

$$\overline{Q}(\boldsymbol{x}) = \frac{1}{|\mathcal{S}|} \sum_{s=1}^{|\mathcal{S}|} Q_s(\boldsymbol{x})$$

using a finite number of load realizations (i.e., scenarios) $s \in \mathcal{S}$

Economic Dispatch Problem

$Q_s(\boldsymbol{x}) =$

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{q}} \quad & \sum_{t\in T}\sum_{g\in G} c_g^P(p_g^t) + \sum_{t\in T} Mq^t \\ \text{s.t.} \quad & \sum_{g\in G} p_g^t - q^t = D_s^t, \quad \forall t \in T \\ & \underline{P}_g x_g^t \le p_g^t \le \overline{P}_g x_g^t, \quad \forall g \in G, t \in T \\ & p_g^t - p_g^{t-1} \le RU(x_g^{t-1}, x_g^t), \quad \forall g \in G, t \in T \\ & p_g^{t-1} - p_g^t \le RD(x_g^{t-1}, x_g^t), \quad \forall g \in G, t \in T. \end{aligned}$$

$\boldsymbol{D}_s = \left\{D_s^1, D_s^2, \cdots, D_s^{|T|}\right\}$ is a realization (scenario) sampled from $p(\boldsymbol{D})$.

Polynomial Chaos Representation of Random Variables

In general $Q = Q(x, \mathcal{D})$. Consider the demand D a random variable, then estimate the RV Q

- Can describe a RV as a function on a probability space
- Constraining the analysis to RVs with finite variance
  - $\Rightarrow$ Represent RV as a spectral expansion in terms of orthogonal functions of standard RVs
    - Polynomial Chaos Expansion (PCE)
- Multiple uses for the PCE, including cheap evaluations of RV moments.

Polynomial Chaos Expansion (PCE)

- Model uncertain quantities as random variables (RVs)
- Given a *germ* $\boldsymbol{\xi}(\omega) = \{\xi_1, \cdots, \xi_n\}$ – a set of *i.i.d.* RVs
  - where $p(\boldsymbol{\xi})$ is uniquely determined by its moments

Any RV in $L^2(\Omega, \mathfrak{S}(\boldsymbol{\xi}), P)$ can be written as a PCE:

$$f(\boldsymbol{x}, \omega) \simeq \sum_{k=0}^{P} f_k(\boldsymbol{x}) \Psi_k(\boldsymbol{\xi}(\omega))$$

- $f_k(\boldsymbol{x})$ are mode strengths
- $\Psi_k()$ are multivariate functions orthogonal w.r.t. $p(\boldsymbol{\xi})$

With dimension n and order p: $$P + 1 = \frac{(n+p)!}{n!p!}$$

Employ Orthogonality to Compute PCE Coefficients

By construction, the functions $\Psi_k()$ are orthogonal with respect to the density of ξ

$$f_k(\boldsymbol{x}) \quad = \quad \frac{\langle f \Psi_k \rangle}{\langle \Psi_k^2 \rangle} = \frac{1}{\langle \Psi_k^2 \rangle} \int f(\boldsymbol{x}; D(\boldsymbol{\xi}))\Psi_k(\boldsymbol{\xi})\ p_{\boldsymbol{\xi}}(\boldsymbol{\xi})\ d\boldsymbol{\xi}$$

Examples:

- Hermite polynomials with Gaussian basis
- Legendre polynomials with Uniform basis, ...
- Global versus Local PC methods
  - Adaptive domain decomposition of the support of ξ

Represent Optimal Cost via PCE

Defining $D^t \overset{iid}{\sim} U(D^t_{\min}, D^t_{\max})$, then by construction

$$D^t = \xi_t \frac{D^t_{\max} - D^t_{\min}}{2} + \frac{D^t_{\max} + D^t_{\min}}{2}, \quad \forall t \in T$$

In this context, we represent $Q(\boldsymbol{x}, D(\boldsymbol{\xi}))$ with a truncated LU PCE

$$Q_{\mathrm{PC}}(\boldsymbol{x}, D(\boldsymbol{\xi})) = \sum_{k=0}^{P} c_k(\boldsymbol{x})\Psi_k(\boldsymbol{\xi}),$$

The PCE coefficient are evaluated by quadrature,

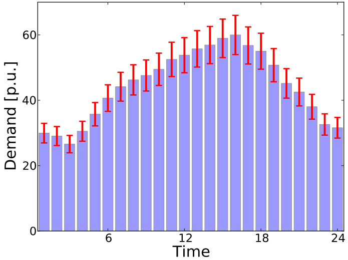
$$c_k(\boldsymbol{x}) = \frac{\langle Q\Psi_k \rangle}{\langle \Psi_k^2 \rangle} = \frac{1}{\langle \Psi_k^2 \rangle} \int_{[-1,1]^n} Q(\boldsymbol{x}, \boldsymbol{\xi})\Psi_k(\boldsymbol{\xi}) d\boldsymbol{\xi}.$$

and the expectation of the cost, $\overline{Q}(\boldsymbol{x})$

$$\overline{Q}(\boldsymbol{x}) = \mathrm{E}_{\boldsymbol{\xi}}[Q(\boldsymbol{x}, \boldsymbol{\xi})] = \langle Q(\boldsymbol{x}, \boldsymbol{\xi}) \rangle = c_0,$$

Economic Dispatch - Test Cases

- 9-bus system [J. H. Chow, Ed., 1982]
- IEEE 118-bus system

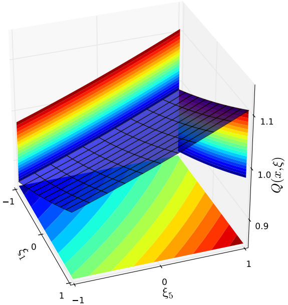


Order	Sparse Quadrature			
	L2, 85p	L3, 389p	L4, 1457p	L5, 4865p
1	1.62e-05	2.90e-05	2.15e-05	2.18e-05
2	-	7.48e-07	2.17e-07	7.83e-08
3	-	-	1.92e-07	5.36e-08
4	-	-	-	2.10e-08

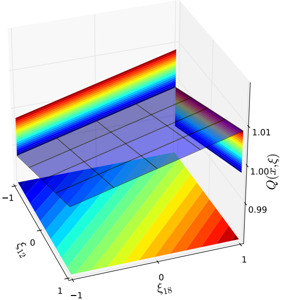
Relative L_2 error at training points for several PCE surrogates and sparse quadrature levels. Power generation cost discretized using 10 segments.

PCE Representations for $Q(\boldsymbol{x}, D(\boldsymbol{\xi}))$

9-bus system
6-dimensional PCE



118-bus system
24-dimensional PCE

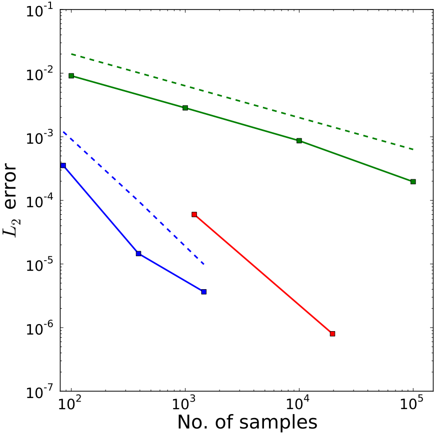


Slice through multi-dimensional PCE models for $Q(\boldsymbol{x}, D(\boldsymbol{\xi}))$

- The transparent surfaces shows the dependence of Q on the corresponding loads. Filled contours provide a qualitative view of the dependence on each load.

Convergence of $\overline{Q}(\boldsymbol{x}) = \mathbb{E}_\xi Q(\boldsymbol{x}, \xi(\boldsymbol{D}))$

- Monte Carlo (MC) results (solid green line) vs vs PCE.
  - 9-bus, 6-dimensional PCE (solid blue line)
  - 118-bus, 24-dimensional PCE (solid red line).
  - Theoretical convergence rates of $1/2$ (dashed green) and 2 (dashed blue).



The PCE results are typically one-two orders of magnitude cheaper compared to MC results.

Summary

We present an approach to reduce the computational cost associated with stochastic unit commitment and economic dispatch, by reducing the number of required forecast samples.

- The approach is based on treating uncertain demands as random variables and representing them via Polynomial Chaos Expansions.
- We present for 9-bus and 118-bus test cases. For both of these cases, quadratic PCE models for the generation cost showed global L_2 errors less than 1% throughout the uncertain demand space compared to the full model
- For the examples considered in this paper, the Polynomial Chaos approach is typically one to two orders of magnitude cheaper compared to Monte Carlo evaluation of the expected generation cost.