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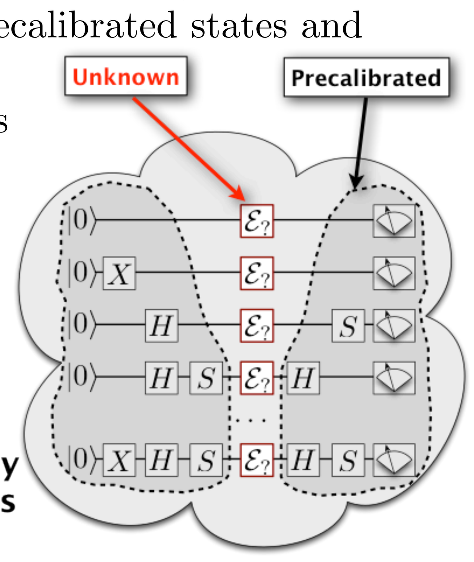
Gate Set Tomography (GST):
Robust, accurate, full characterization of quantum logic gates

Robin Blume-Kohout, John K. Gamble, Erik Nielsen (Sandia)

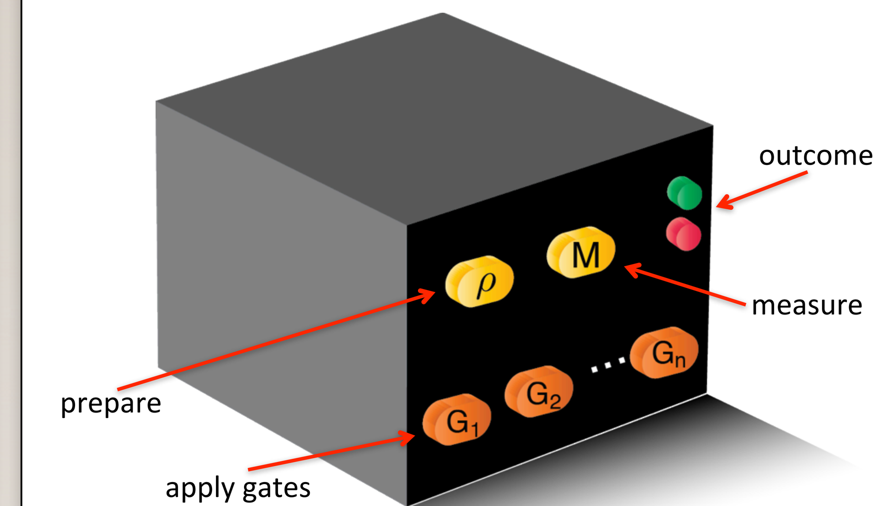
Standard process tomography (the old tool for full characterization of gates) relies on a *reference frame* of precalibrated states and operations, which prepare known inputs and apply known measurements, to characterize the unknown gate[s].



Summary Of Results

- Process tomography is unreliable, depending on precalibrated gates.
- We developed GST as a robust, calibration-free alternative.
- Use of structured data allows a *closed-form* linear inversion algorithm, which (unlike MLE) is guaranteed to give a pretty good answer.
- Within GST we can also use and analyze *long circuits* involving many gates. These amplify errors, and yield very accurate estimates.
- GST supports a rich error analysis, and detects non-Markovian noise.

GST treats the QIP (e.g. qubit) as a black box. Precalibrated states/gates are not available, and GST therefore does not use or rely upon them.



The 1st stage of GST is *Linear GST*. Simple, efficient, and 100% reliable, LGST provides a rough estimate that can be iteratively improved:

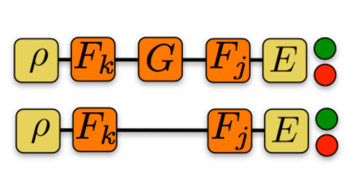
(1) Choose a set of *fiducial sequences:*

$$\{F_1, F_2, F_3, F_4\} \stackrel{\text{e.g.}}{=} \{G_1, G_2, G_3, G_2^2\}$$

(2) Do "tomography" by measuring:

$$(\tilde{G}_i)_{j,k} = \langle\langle E|\, F_j\, G_i\, F_k\, |\rho\rangle\rangle$$

and $\tilde{\mathbb{1}}_{j,k} = \langle\langle E|\, F_j\, F_k\, |\rho\rangle\rangle$



(3) Use linear algebra to get the estimate:

$$\hat{G}_i = \tilde{\mathbb{1}}^{-1}\tilde{G}_i, \text{ etc.} \iff \hat{G}_i = B^{-1}G_iB,\ \langle\langle\hat{E}| = \langle\langle E|B,\ |\hat{\rho}\rangle\rangle = B^{-1}|\rho\rangle\rangle$$

(4) Transform the estimated gateset by a *gauge transformation* to make it as close as possible to the desired target gateset:

$$\hat{G}_i \to S^{-1}\hat{G}_iS \text{ and } \langle\langle\hat{E}| \to \langle\langle\hat{E}|S \text{ and } |\hat{\rho}\rangle\rangle \to S^{-1}|\hat{\rho}\rangle\rangle$$

***All* forms of GST contain a gauge freedom. It's annoying but unavoidable.**

Every observable probability

$$p(\bullet) = \langle\langle E|G_{s_1}\dots G_{s_L}|\rho\rangle\rangle$$

is totally invariant under the **gauge** transformation

$$G_i \to B^{-1}G_iB \text{ and } \langle\langle E \to \langle\langle E|B \text{ and } |\rho\rangle\rangle \to B^{-1}|\rho\rangle\rangle$$

This can change the gates a *lot*. But it doesn't change *anything* observable. So these gatesets are *equivalent*.

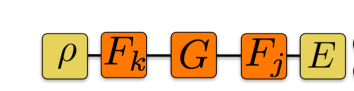
So we can choose the gauge to best match our target gates.

Or to get as close as possible to a set of CPTP gates.

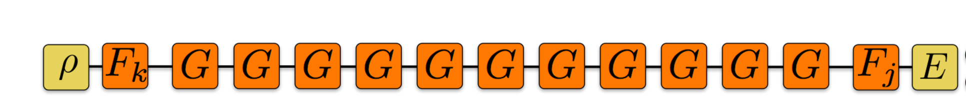
Argh...

GST is (necessarily) capable of analyzing *nonlinear* data -- i.e., experiments in which the unknown gate is used multiple times.

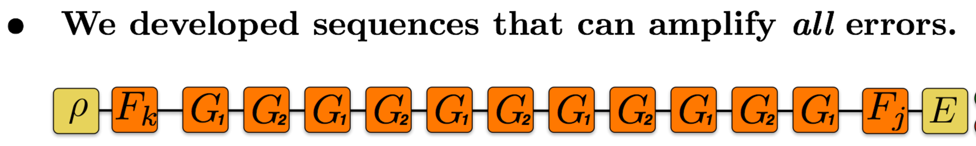
- **We care deeply about very small errors (0.01%) in gates. These are very hard to detect in a single use of the gate.**



- **By using the unknown gate many times in an experimental test circuit, we can *amplify* errors**



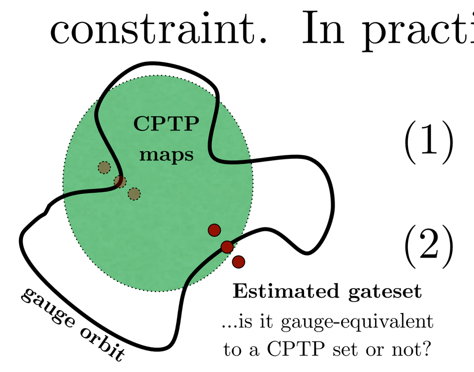
- **We developed sequences that can amplify *all* errors.**



Brute-force maximum likelihood GST (and why we don't do it any more)

In principle, GST could be done on data from *any* circuits by numerically maximizing $\mathcal{L} = Pr\,(\text{data}|\{\mathbf{G_k}\})$

But nonlinearity makes the likelihood horrible. And the gauge plays very badly with the CPTP constraint. In practice, brute-force MLE fails:



(1) It takes about a day to analyze typical single-qubit data!
(2) Even with sophisticated codes, we don't reliably find global maxima.

Glossary: Hilbert-Schmidt notation

1. States (ρ) and measurement outcomes (E) live in the *Hilbert-Schmidt space* of Hermitian $d\times d$ matrices.

$$|\rho\rangle\rangle = \begin{pmatrix}\rho_{\mathbb{1}}\\ \rho_X\\ \rho_Y\\ \rho_Z\end{pmatrix}$$

2. Gates (processes) are $d^2\times d^2$ linear *superoperators* that act on states:

$$|\rho_{\text{out}}\rangle\rangle = \mathbf{G}|\rho_{\text{in}}\rangle\rangle$$

3. Observable probabilities are inner products:

$$Pr = \langle\langle E|\rho\rangle\rangle \quad \text{or} \quad Pr = \langle\langle E|\mathbf{G}_1\mathbf{G}_2\dots\mathbf{G}_L|\rho\rangle\rangle$$

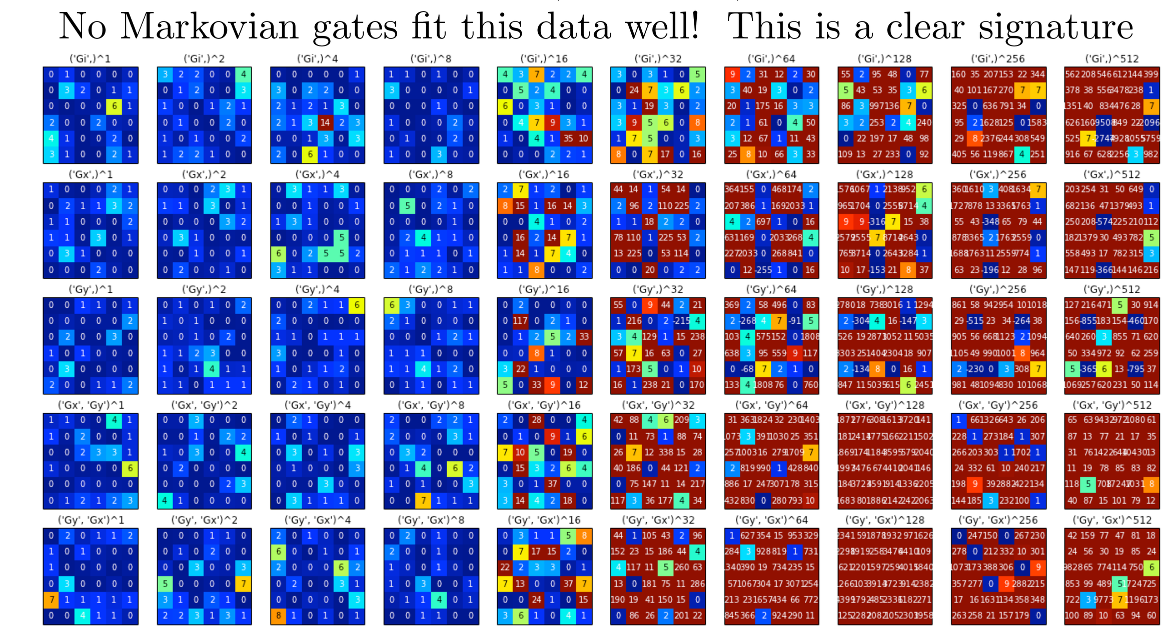
Error analysis and badness-of-fit.

When you get a tomographic estimate, it's critical that you ask how *good* it is -- e.g., can we rely on it? what are the error bars? did tomography work?

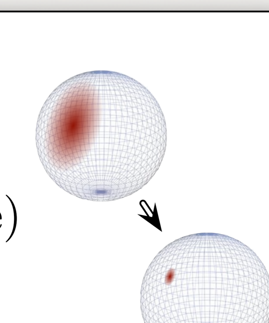
GST data is usually highly overcomplete -- O(1000) observed frequencies for O(100) gate parameters. So we can measure *badness-of-fit* by computing log-likelihood ratio or X2 statistics. These diagnose *whether* and *which data* our estimate fails to fit!

(Below): we fit the gates to data from sequences of lengths L=1,2,4,8 -- but we show the X2 b-o-f for data from L=1...512. As expected, we fit the L=1..8 data well but fail for L>16 data.

However, fits to *all* the data (not shown) fail across the board! No Markovian gates fit this data well! This is a clear signature



Efficient analysis of long circuits/sequences: *eLGST* and *least squares GST*.



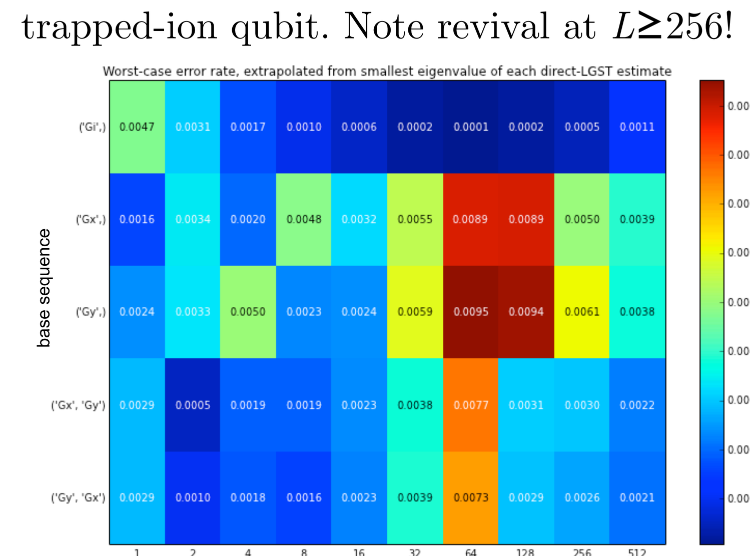
We're developing reliable, fast (1-100s running time) alternatives to MLE for analyzing long sequences.

Extended LGST (eLGST) uses LGST to directly estimate the process matrix of long periodic sequences like $(G_1G_2)^{32}$. Then, the estimates of $\{G_k\}$ are adjusted (iteratively) to fit these direct-LGST estimates of long gate sequence products.

Least squares GST (LSGST) analyzes the same structured data as eLGST, but iteratively adjusts estimates of $\{G_k\}$ by weighted least-squares fitting directly to the data.

Experimental Result:

Experiments in 3 different qubits (ion, Si-dot, superconducting) have *all* revealed significant non-Markovian noise! Figure below shows error-per-gate vs. circuit length for SNL trapped-ion qubit. Note revival at $L\geq 256$!



Conclusions

- GST is ready for use, robust, and capable of unprecedented accuracy.
- Long-sequence experiments reveal non-Markovian noise in multiple qubit techs.

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