

# Microstructural Evolution Modeling

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# Outline

## Background

- Experimental  
Modeling

## Developed Models

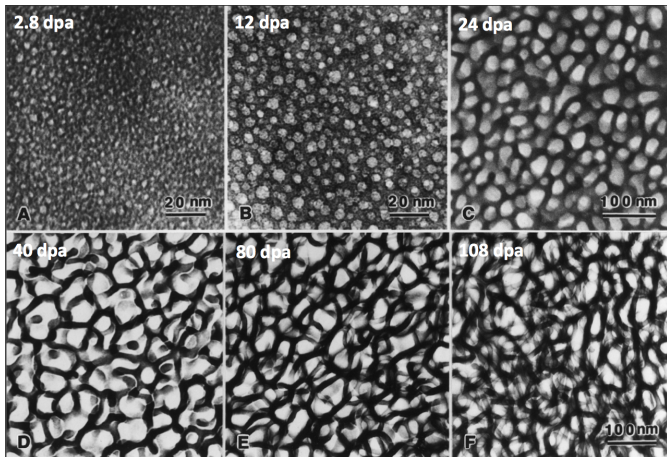
- Sharp Interface
- RIS
- Swelling

## Current State

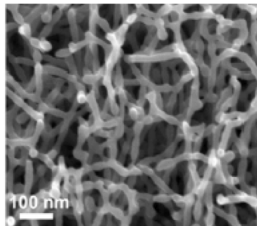
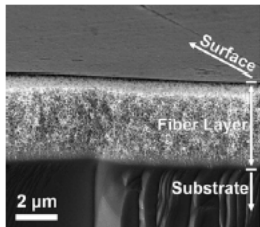
- Coupling Models



# Ion Irradiation



# On Several Semiconductors



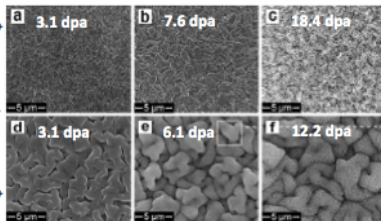
**GaSb**

**InSb**

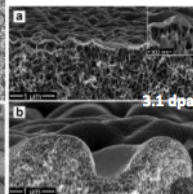
**1 MeV Au<sup>+</sup>**

Top-view

**3 MeV Au<sup>+</sup>**

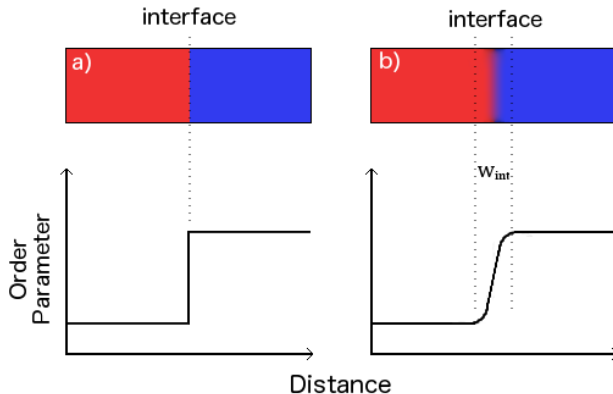


Side-view





## hPPF



$$E_{Potts} = J \sum_{j=1}^n (1 - \delta_{s(i)s(j)})$$

$$E_{pf} = \int_V (\frac{\gamma}{2} |\nabla C|^2) dV$$



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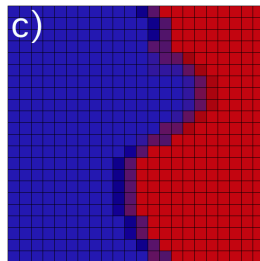
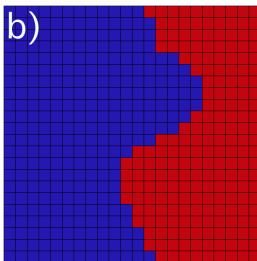
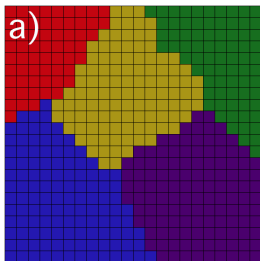
Sharp Interface  
RIS  
Swelling

## Current State

Coupling Models



# Microstructural Representation



# Thermodynamics

The EoS is,

$$F = \int_V f_0 dV + \int_S \gamma dS = \int_V f_0 dV + \frac{\gamma_{CH}}{2} \int_V |\nabla C|^2 dV + \gamma_{Potts} \int_S dS,$$

which can be discretized

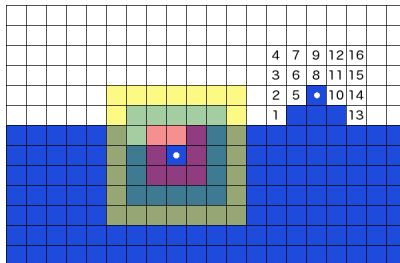
$$F_{CH,i} = \sum_{i=1}^N \left( f_0(q_i, C_i) + \frac{\gamma_{CH}}{2} |\nabla C_i|^2 + J \sum_{j=1}^n (1 - \delta_{s(i)s(j)}) \right)$$

$$F_{Potts,i} = \sum_{i=1}^N \left( f_0(q_i, C_i) + J \sum_{j=1}^n (1 - \delta_{s(i)s(j)}) \right)$$

$$\frac{\delta F}{\delta C} = \gamma \frac{\delta S}{\delta C} = \gamma \frac{\delta V}{\delta C} \frac{\delta S}{\delta V} = \gamma \Omega \frac{8\pi R \delta r}{4\pi R^2 \delta r} = \frac{2\gamma \Omega}{R} \equiv \gamma \Omega \kappa$$



# Curvature and Phenomenological Eqns



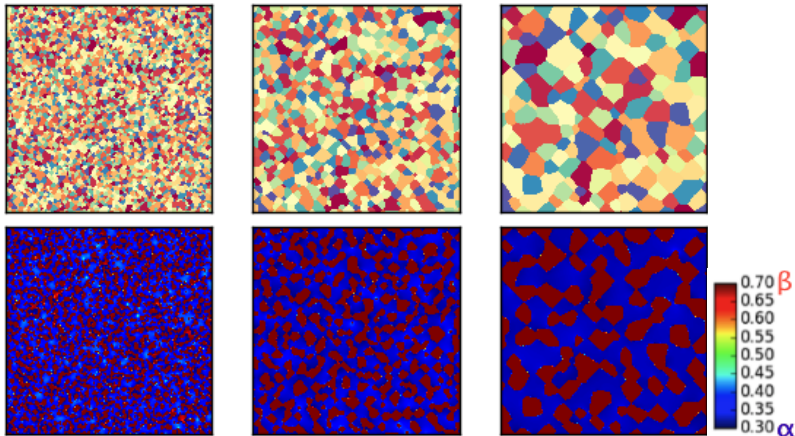
$$\kappa_i = \frac{\sum_{j=1}^{n_s} (1 - \delta_{q(i)q(j)}) - n_{base}}{n_s}$$

$$\frac{\partial C}{\partial \tilde{t}} = \tilde{\nabla} \cdot \left[ \tilde{\nabla} \left( \frac{\partial \tilde{f}_0}{\partial C} - \tilde{\nabla}^2 C \right) \right]$$

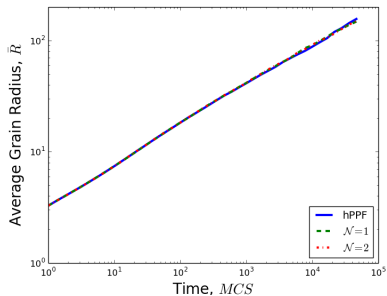
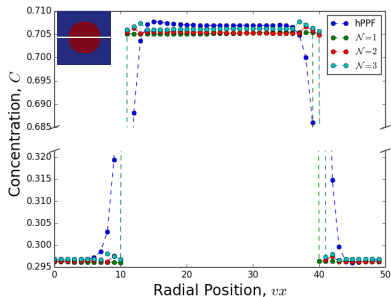
$$\frac{\partial C}{\partial \tilde{t}} = \tilde{\nabla} \cdot \left[ \tilde{\nabla} \left( \frac{\partial \tilde{f}_0}{\partial C} + \tilde{\kappa} \right) \right]$$



# Microstructural Evolution



# Grain Growth Results

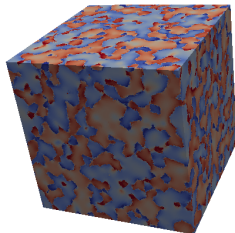


hPPF	$\mathcal{N}$		$\mathcal{N}_{3D} = 1$
0.350	1	2	0.353
0.337	0.345		

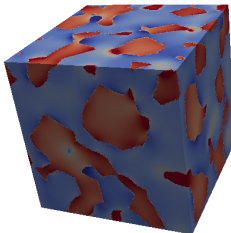


# 3D Microstructural Evolution

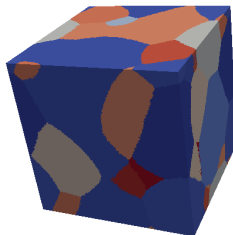
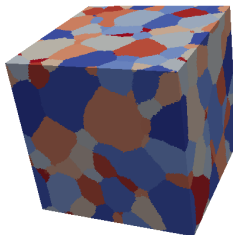
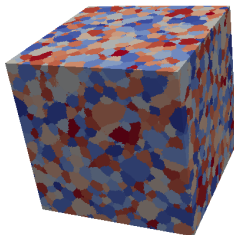
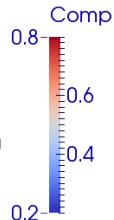
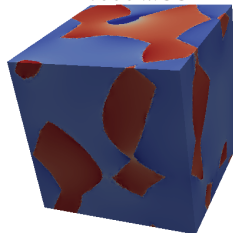
Time = 10 MCS



100 MCS



1000 MCS



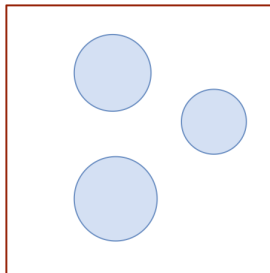
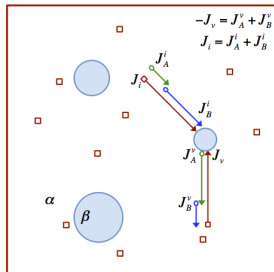


# RIS: General Description

$$J_i^a + J_i^b = J_i$$

$$J_v^a + J_v^b = -J_v$$

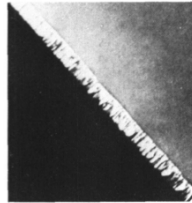
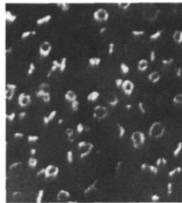
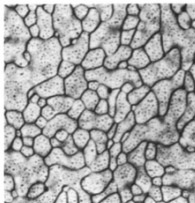
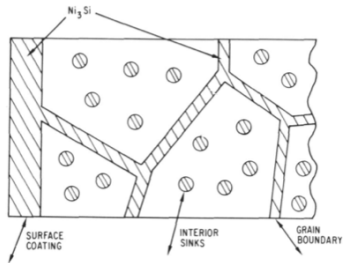
$$J_a + J_b = J_i - J_v$$



- A atom
- B atom
- ◻ Defect



# Experimentally Observed



# Phenomenological Equations

$$\begin{aligned}\frac{\partial C_i}{\partial \tilde{t}} &= \frac{\eta_s(\vec{x})}{\Delta \tilde{t}} + \tilde{\Theta}_b \tilde{\nabla} \cdot \left( \frac{D_i}{D_b} \tilde{\nabla} C_i + C_i \tilde{\nabla} C_b \right) - \tilde{k}_{iv} C_i C_v - \tilde{S}_{fs} - \tilde{S}_{\alpha\beta} \\ \frac{\partial C_v}{\partial \tilde{t}} &= \frac{\eta_s(\vec{x})}{\Delta \tilde{t}} + \tilde{\Theta}_b \tilde{\nabla} \cdot \left( \frac{D_v}{D_b} \tilde{\nabla} C_v - C_v \tilde{\nabla} C_b \right) - \tilde{k}_{iv} C_i C_v - \tilde{S}_{fs} - \tilde{S}_{\alpha\beta} \\ \frac{\partial C_b}{\partial \tilde{t}} &= \tilde{\nabla} \cdot \left( \tilde{\nabla} \left[ \frac{\partial \tilde{G}}{\partial C_b} - \tilde{\nabla}^2 C_b \right] + \tilde{\Theta}_b C_b \left[ \frac{D_i}{D_b} \tilde{\nabla} C_i - \frac{D_v}{D_b} \tilde{\nabla} C_v \right] \right)\end{aligned}$$

with boundary condition at the *free surface*

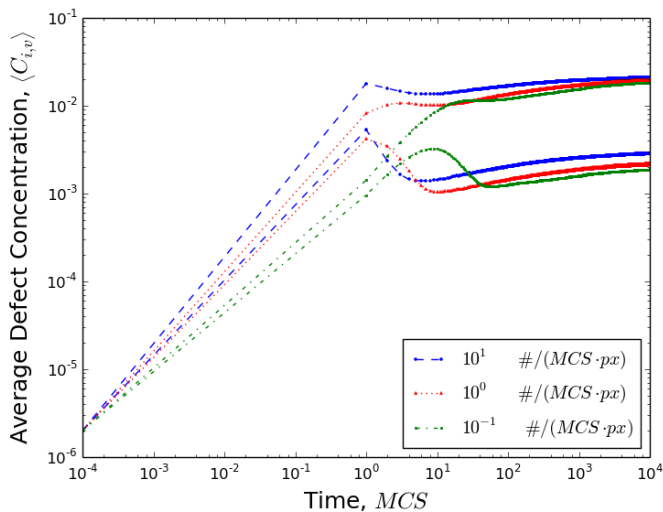
$$\mathbf{J} \cdot \mathbf{n} = 0$$

$$C_{i,v} = C_{i,v}^{eq}$$

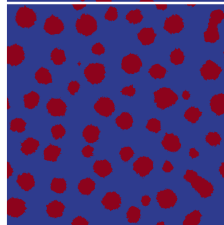
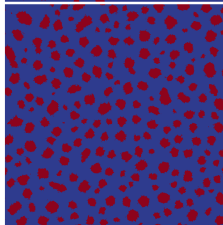
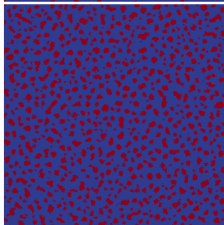
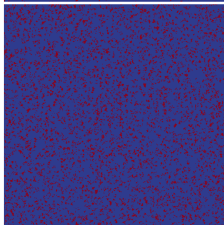
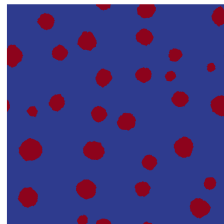
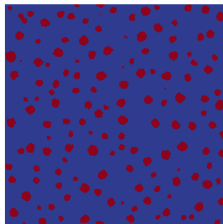
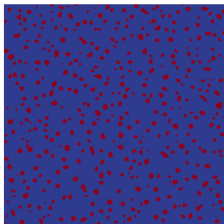
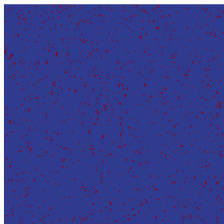


# Radiation Characteristics

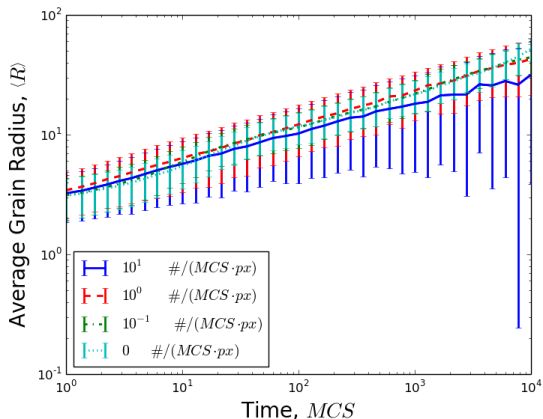
Low  $T$ /Intermediate  $\rho_s$



# Phase Evolution

 $\tau = 1 \text{ MCS}$  $\tau = 100 \text{ MCS}$  $\tau = 1000 \text{ MCS}$  $\tau = 10000 \text{ MCS}$ 

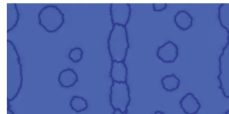
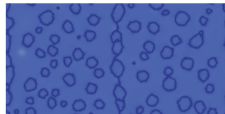
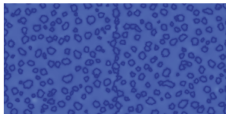
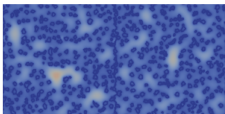
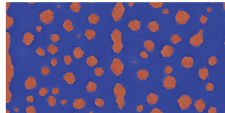
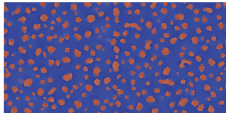
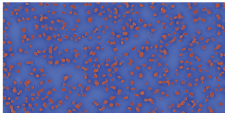
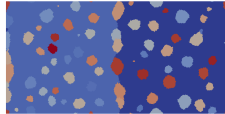
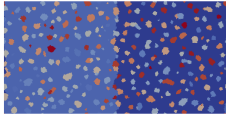
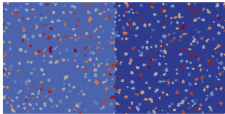
# Grain Growth Kinetics



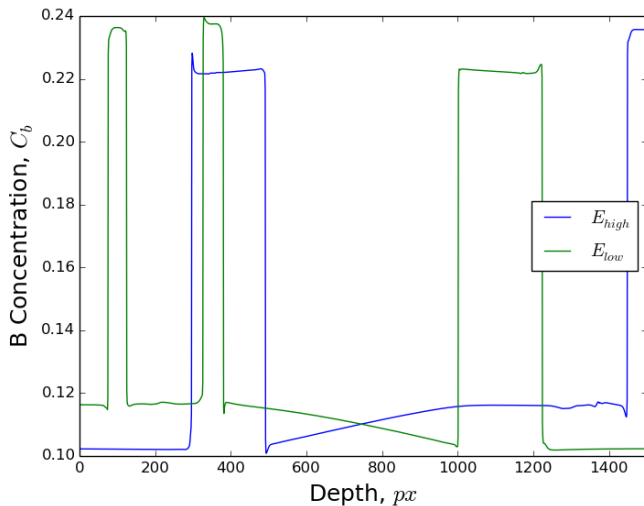
Dose [ $\#/(MCS \cdot px)$ ]	$\sim 10^1$	$\sim 10^0$	$\sim 10^{-1}$	0
$n$	4.21	3.62	3.35	3.04



# Nucleation at Grain Boundaries

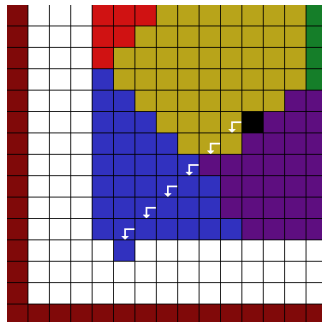
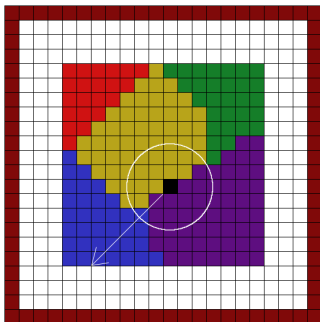
 $\tau = 10 \text{ MCS}$  $\tau = 100$  $\tau = 1000$  $\tau = 10000$ 

# Concentration Profile





# Bulk Volumetric Swelling



Swelling simulated by a site exchange mechanism  
 Choosing a random direction

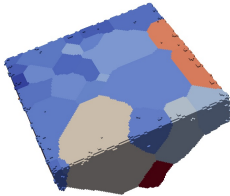
$$p_x = \cos(\phi) \sin(\theta) \quad p_y = \sin(\phi) \sin(\theta) \quad p_z = \cos(\theta)$$

Essentially, “reverse” sintering [*García-Cardona, Tikare (2011)*]

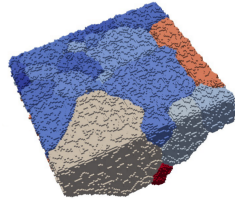


# Stages of Percolation

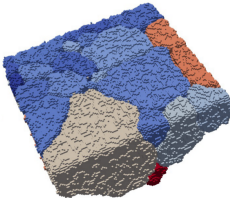
Initial Microstructure (time = 0 MCS)



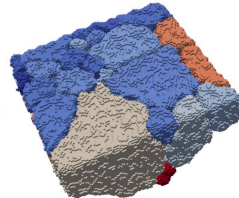
Early Stage (4503 MCS)



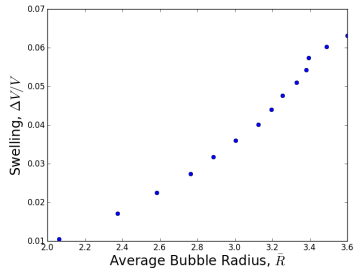
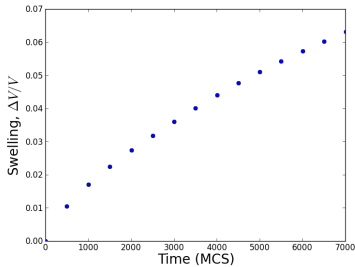
Moderate Stage (8000 MCS)



Advanced Stage (23006 MCS)

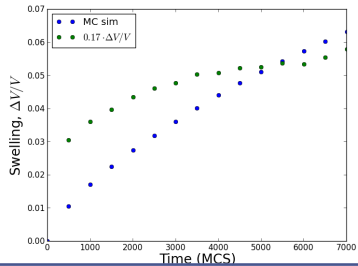


# Gas Bubble Swelling

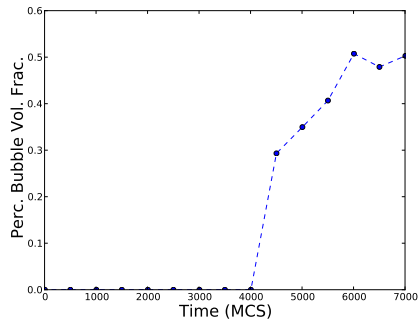
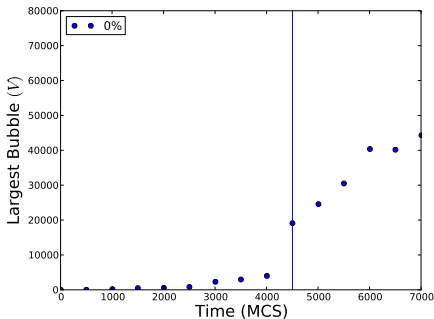


$$\frac{\Delta V}{V} = \frac{2(R_{gb}/a)(R_{gb}/\mathcal{R})^2}{1 - 2(R_{gb}/a)(R_{gb}/\mathcal{R})^2}$$

Agrees with *Kagana*,  
 $\sim 86.3 \pm 3.9\%$  ( $\sim 83\%$  at  
 percolation) lower swelling  
 for interlinkage



# Bubble Evolution



- ▶ Substantial increase in bubble volume
- ▶ Abrupt increase in volume at  $t = t_{perc} \approx 4500$  MCS



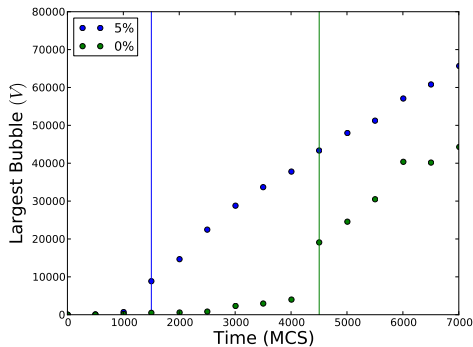
# Initial Porosity of 5%

- ▶ Initial porosity has significant consequences

- ▶ Percolation time significantly reduced

$$\frac{t_{perc}^{\rho=95\%}}{t_{perc}^{\rho=100\%}} \sim 33\%$$

- ▶ Percolation happens with smaller bubbles



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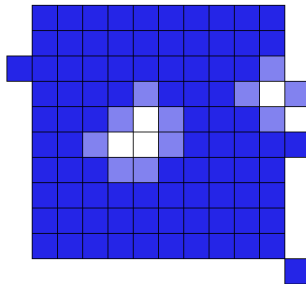
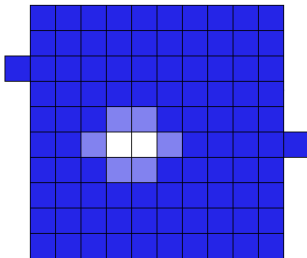
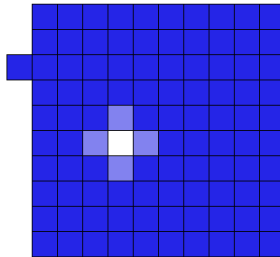
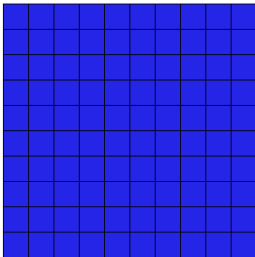
- Sharp Interface
- RIS
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## Current State

- Coupling Models



# Example of Swelling



# Alternative Surface Treatment

Using the “true” definition of a curvature, we define it as

$$\kappa_{S_f} = \nabla \cdot \hat{\mathbf{n}} = \nabla \cdot \left( \frac{\nabla S_f}{|\nabla S_f|} \right)$$

and the solid fraction evolution is proportional to the Laplacian of the curvature ( $\propto \nabla_s^2 \kappa_{S_f}$ ), where surface gradient is given by

$$\nabla_s = \nabla - \hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \nabla)$$

$$\begin{aligned} \nabla_s \kappa_{S_f} &= \nabla \kappa_{S_f} - \hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \nabla \kappa_{S_f}) \\ &= (\nabla_x \hat{i} + \nabla_y \hat{j} + \nabla_z \hat{k}) \kappa_{S_f} \\ &\quad - (n_x \hat{i} + n_y \hat{j} + n_z \hat{k}) (n_x \nabla_x + n_y \nabla_y + n_z \nabla_z) \kappa_{S_f} \\ &= A \hat{i} + B \hat{j} + C \hat{k} \end{aligned}$$

