

# Molecular dynamics simulation of quench, stress relaxation and fracture in silica melts and glasses

J. Matthew D. Lane

Sandia National Laboratories,  
Albuquerque, NM



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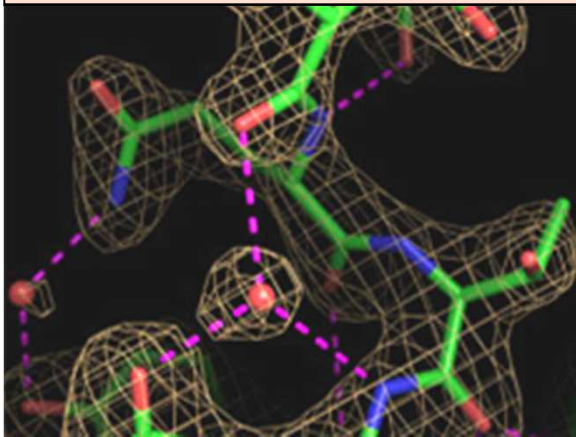


# Hierarchy of modeling approaches

Fracture response in glass depends on processes and structure at several length scales:

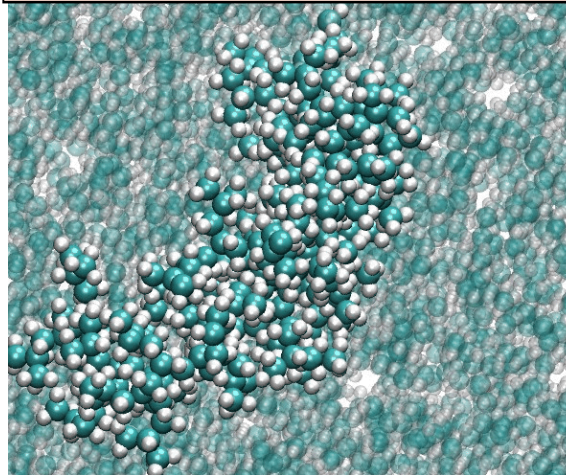
## Quantum scale

fs times & Å lengths



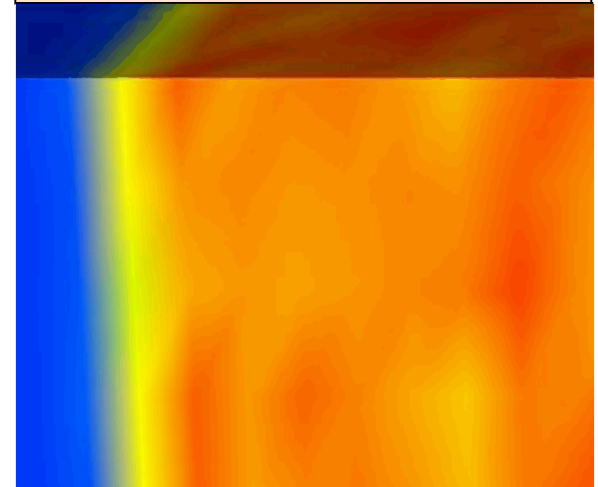
## Molecular scale

ns times & 10s nm lengths



## Continuum scale

>μs times & >mm lengths



Increasing time and length scales

# Molecular Dynamics (MD) simulations

- Solve Newton's equation...

$$m_i \frac{d^2 \mathbf{r}_i}{dt^2} = \mathbf{F}_i$$

$$\mathbf{F}_i = -\nabla V_i$$

## Mathematical Formulation

Classical Mechanics

Atoms are Point Masses:  $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N$

Positions, Velocities, Forces:  $\mathbf{r}_i, \mathbf{v}_i, \mathbf{F}_i$

Potential Energy Function =  $V_i(\mathbf{r}^N)$

## Sandia's LAMMPS code is spatially parallel

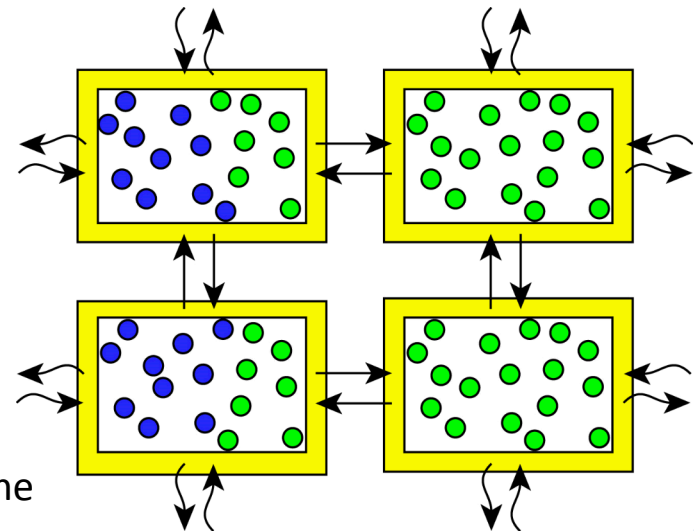
Large-scale Atomic/Molecular Massively Parallel Simulator

Problem:  $\Delta t$  tied to the fastest atomic motion  $t$  is the timescale of physical interest

$$nsteps = t / \Delta t = 10^6 \dots 10^{15}$$

6N coupled ODEs

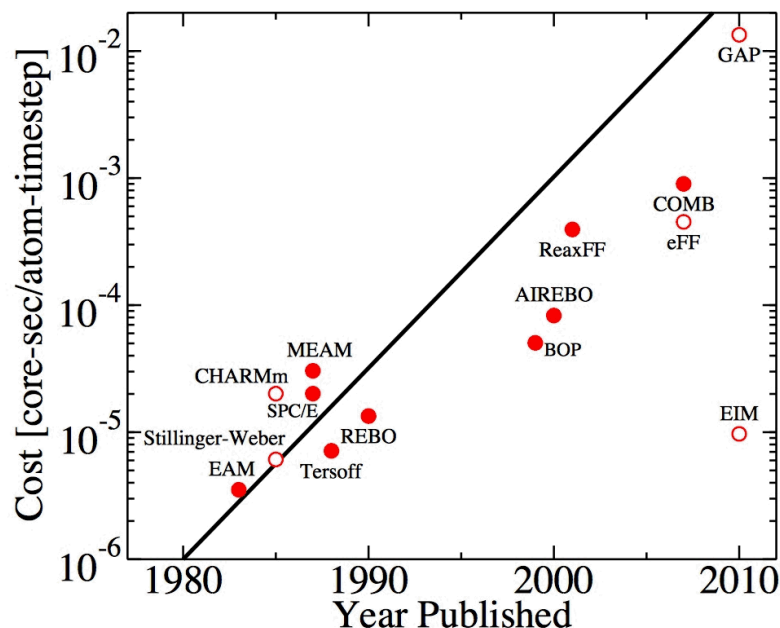
$$\begin{aligned} \mathbf{v}_i^{(l+\frac{1}{2})} &= \mathbf{v}_i^l - \frac{\mathbf{F}_i^l}{2m_i} \Delta t \\ \mathbf{r}_i^{(l+1)} &= \mathbf{r}_i^l + \mathbf{v}_i^{(l+\frac{1}{2})} \Delta t \\ \mathbf{v}_i^{(l+1)} &= \mathbf{v}_i^{(l+\frac{1}{2})} - \frac{\mathbf{F}_i^{(l+1)}}{2m_i} \Delta t \end{aligned}$$



# Strengths, weaknesses and trends

## Strengths:

- Atomistic detail  
Nanoscale Dynamics,  
Chemistry,  
Structure / Property
- Scales on parallel processors



Plot courtesy Aidan Thompson, Sandia

## Weaknesses:

- Finite set of interatomic potentials
- Atom counts  $\ll 10^{23}$
- Short (generally ns) timescales

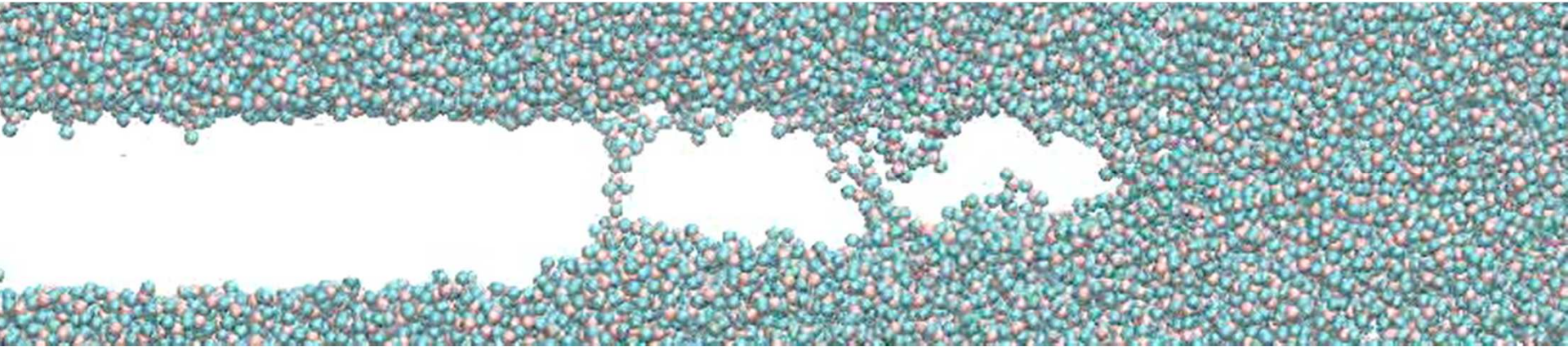
## Trends in molecular dynamics:

- Complex interatomic potentials  
Reactive & polarizable  
Multi-elemental
- Computing for scale  
New algorithms for new  
machines  
Increased spatial parallelism
- Computing for fidelity  
DFT-based training sets

Quantitative studies benefit from larger systems with more complex chemistry



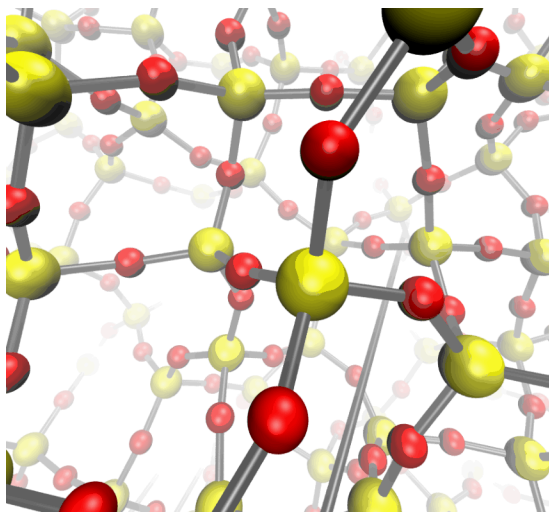
# Motivation: “long-time” dynamics



We’re interested in “long-time” dynamics in molecular dynamics for the study of fracture in glass. Particularly, we have noted that fracture response in glasses is loading rate dependent.

We’d like to take advantage of time-temperature superposition to accelerate the loading relaxation and its effect on glass structure, before crack initiation. This would allow us to investigate much slower effective loading rates.

# Silica glass structure and methodology

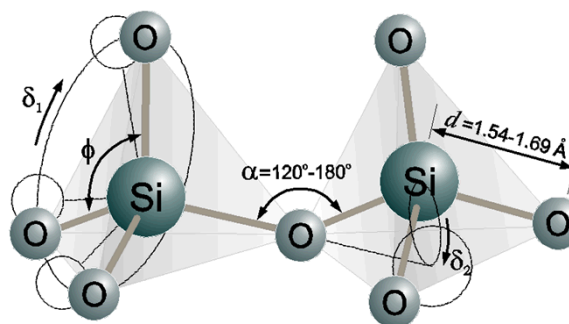
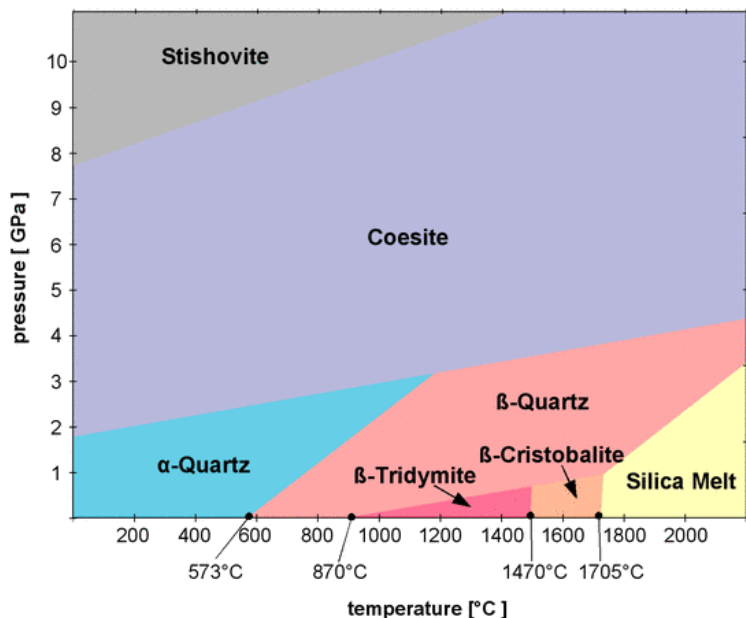


- Silica, is a strong random network glass former with
- Tetrahedral cores about the **silicon**
  - Bridging **oxygen** linking tetrahedra (Si-O-Si-O)

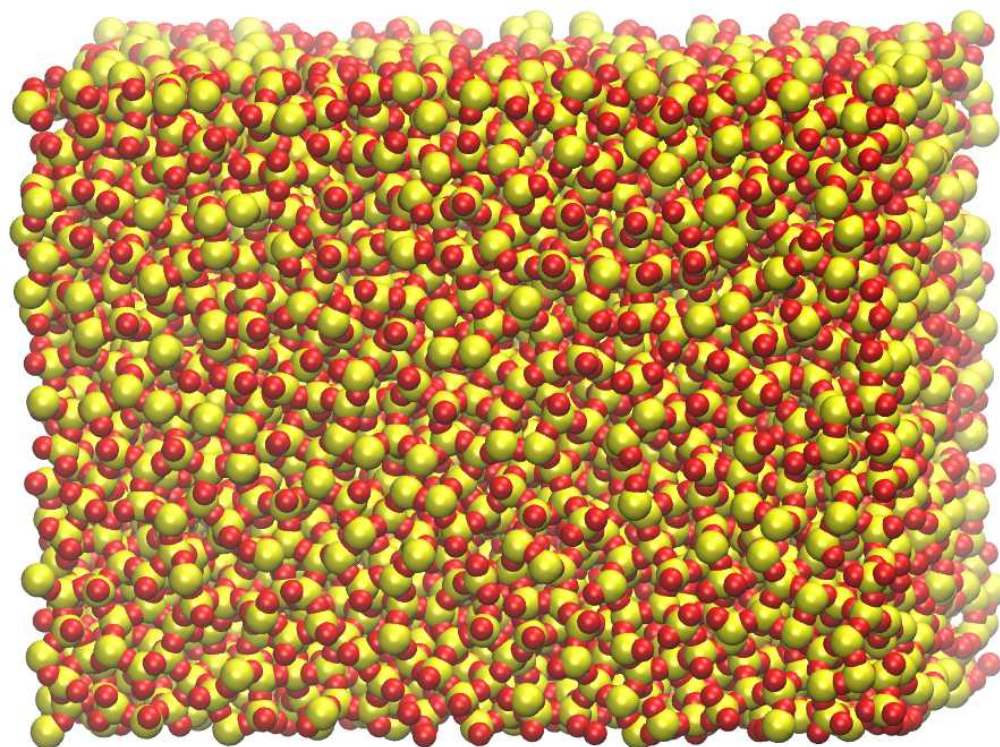
Well ordered on short length scales, disordered on longer length scales

SiO<sub>2</sub> has several metastable phases at room temperature, leading to easy structure frustration during rapid cooling from melt

Glasses are formed by a melt/quench process with experimental quench rates of a few Kelvin/minute



# Simulation methods and models



10 Å

Van Beest, Kramer, and Van Santen. *Phys. Rev. Lett.* **64**, 1955 (1990).

Vollmayr, Kob and Binder, *PRB*, **54**, 15808 (1996)

Sandia's LAMMPS molecular dynamics code <http://lammps.sandia.gov>

BKS potential, modified to prevent core collapse at high temperatures  
With long-range coulomb interaction

Thermostat and barostat with Nose-Hoover, NPT and NVT in LAMMPS

45 x 25 x120 Angstrom cell with periodic boundaries with 13824 atoms

Formation from  $\beta$ -cristobalite crystal melted to 8000 K, then cooled at a linear rate to 300 K



# Glass formation and cooling-rate dependence

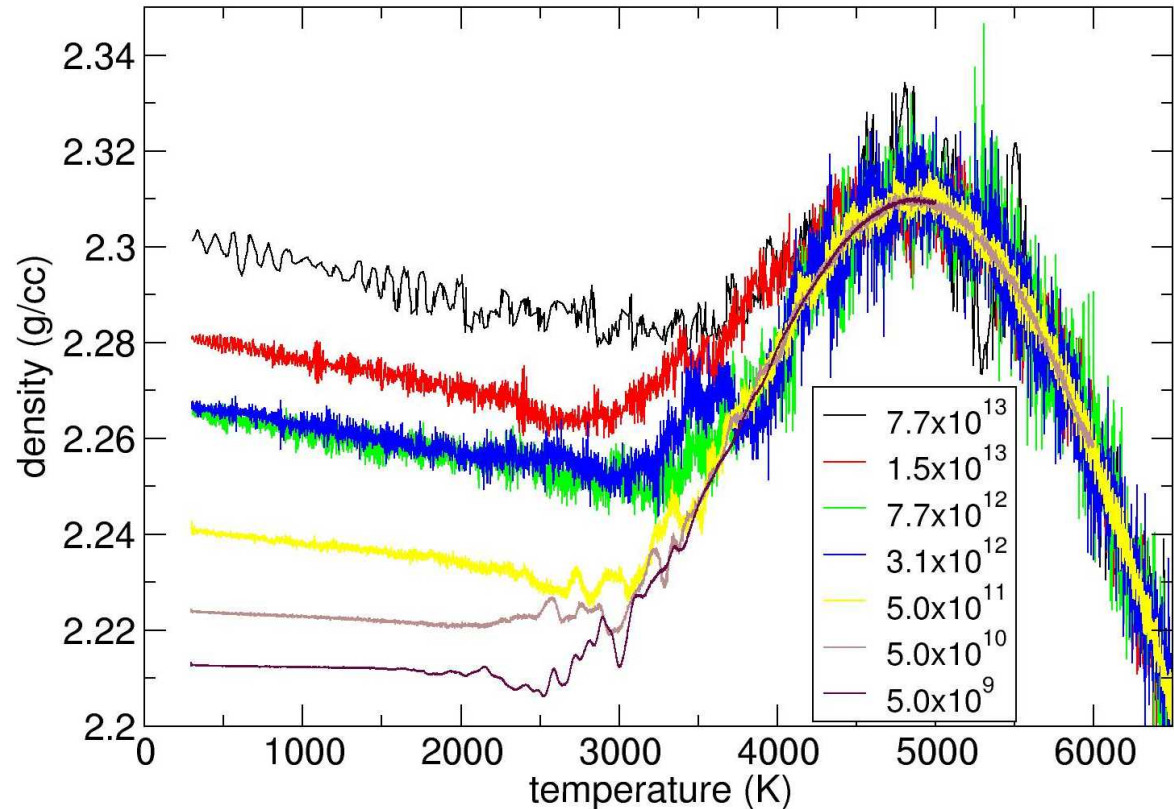
Both experiment and model silica has an anomalous density maximum in temperature near  $T_g$

Strong (~5%) density dependence on cooling rate

MD cooling rates are many orders of magnitude larger than experiments

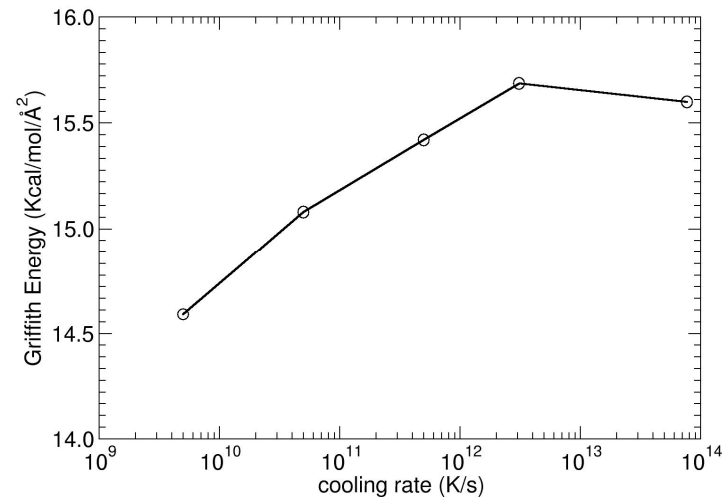
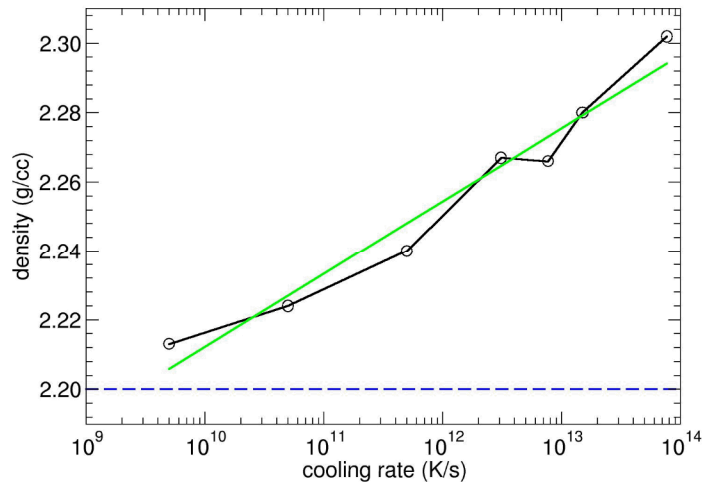
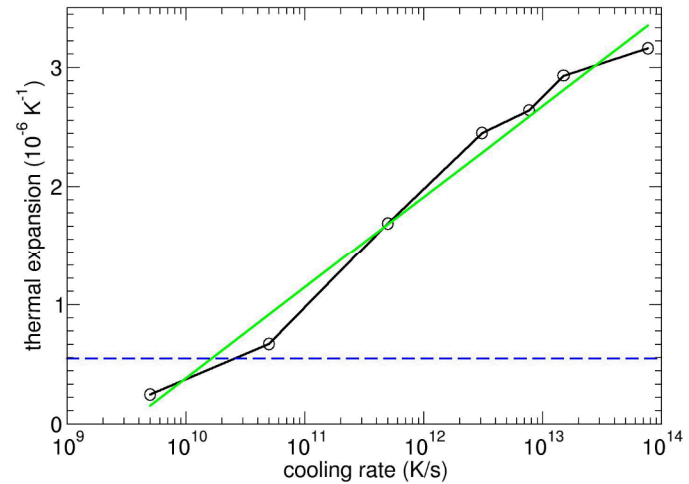
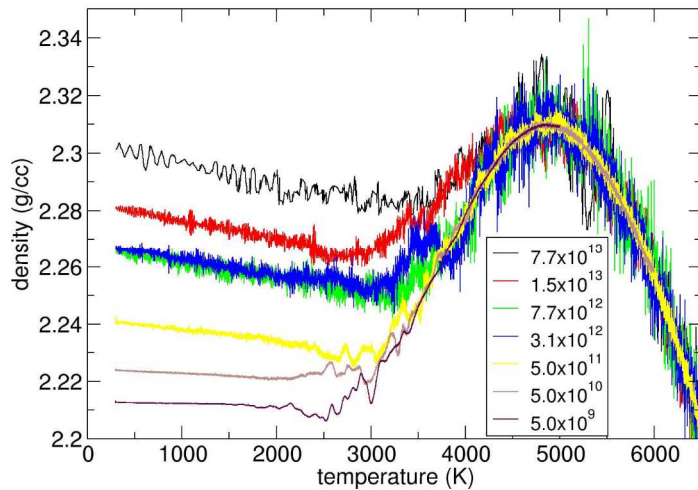
Thermal expansion properties of final glass appear to depend on quench rate.

Excellent previous work studied glass behavior down to  $4.44 \times 10^{12}$  K/s. They extrapolated out an experimental glass transition temperature  $T_g = 2525$  K compared to experimental values of 1446 K. Vollmayr, Kob and Binder, *PRB*, **54**, 15808 (1996)





# Glass formation and cooling-rate dependence

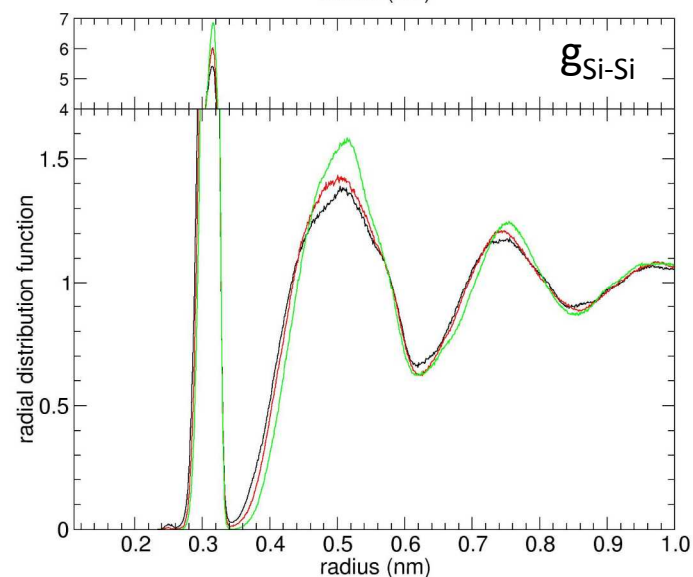
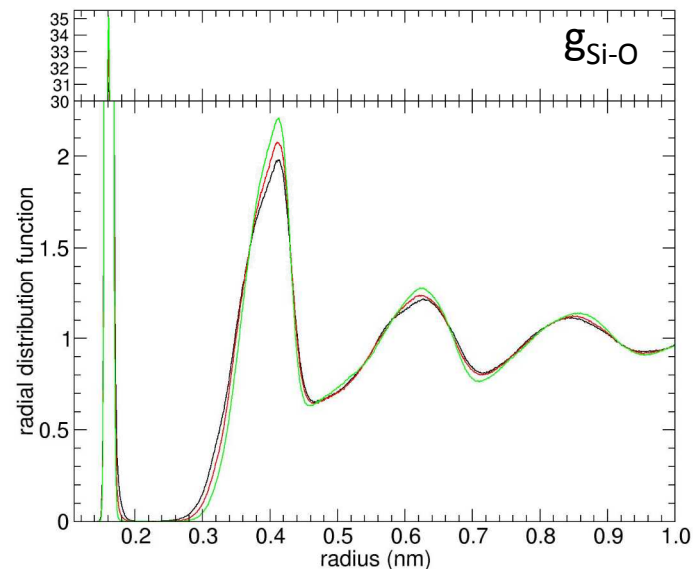
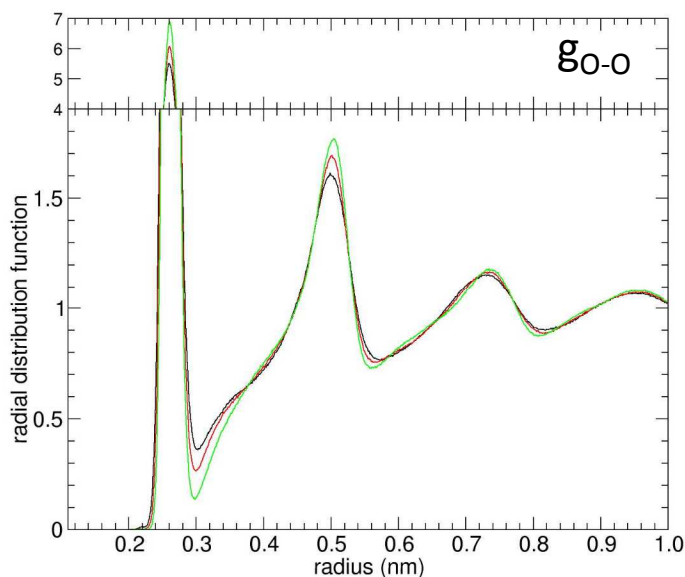


# Cooling-rate dependence - structure

Radial distribution function analysis of the glass structure from **fast**  $10^{13}$ , **moderate**  $10^{12}$  and **slow**  $10^9$

Significant additional structural relaxation occurs with reduced cooling rate. More ordered structures result when more time is allowed to cool.

The 2nd neighbor structure is most significantly impacted with lesser effects of 1st and 3rd neighbors, indicating ordering of tetrahedra rather than bonds.



# Time-Temperature Superposition

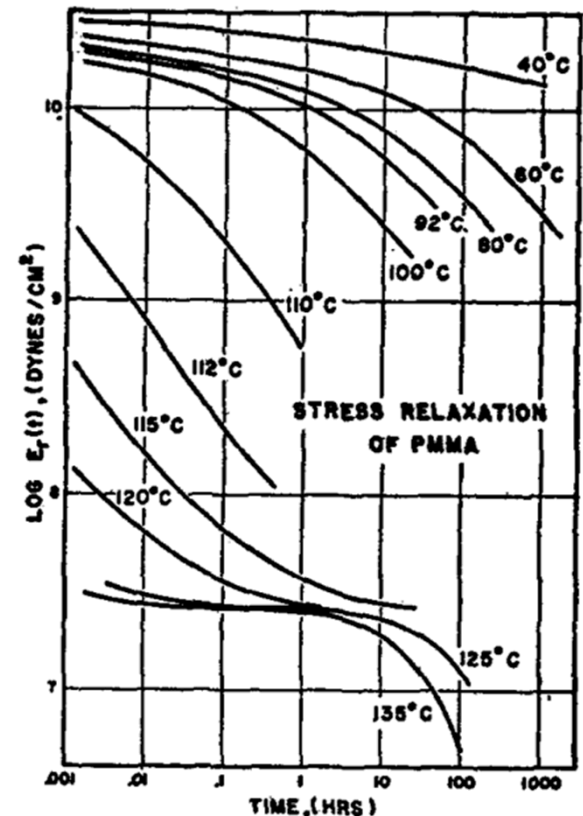
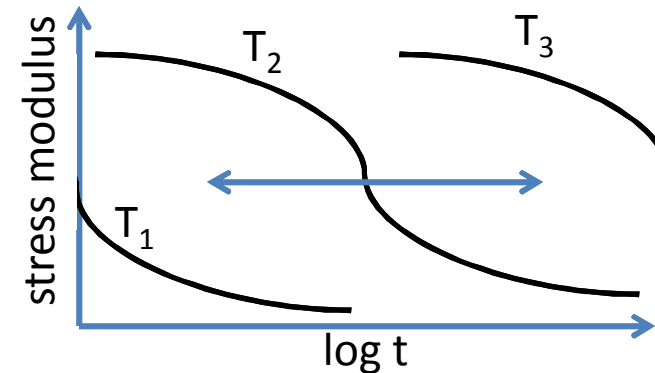
**Time-temperature superposition** is a property of viscoelastic glass forming liquids which simply states that the relaxation dynamics follow a master curve which can be shifted in  $\log(\text{time})$  to account for response at different temperature.

Moreover, the  $\log(\text{time})$  shift factor for all temperatures can be determined from the measurement of only a few.

Once time-temperature superposition is demonstrated then the master curve can be used to infer response times which are inaccessible (too long or too short)

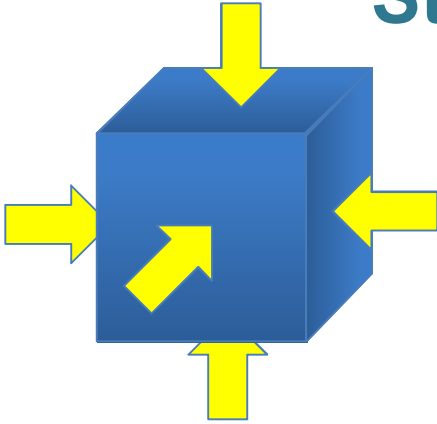
This has been used successfully in polymer experiment since the 1940s.

Plot from A.V.Tobolsky, *J. Appl. Phys.*, **27**, 627, (1956).

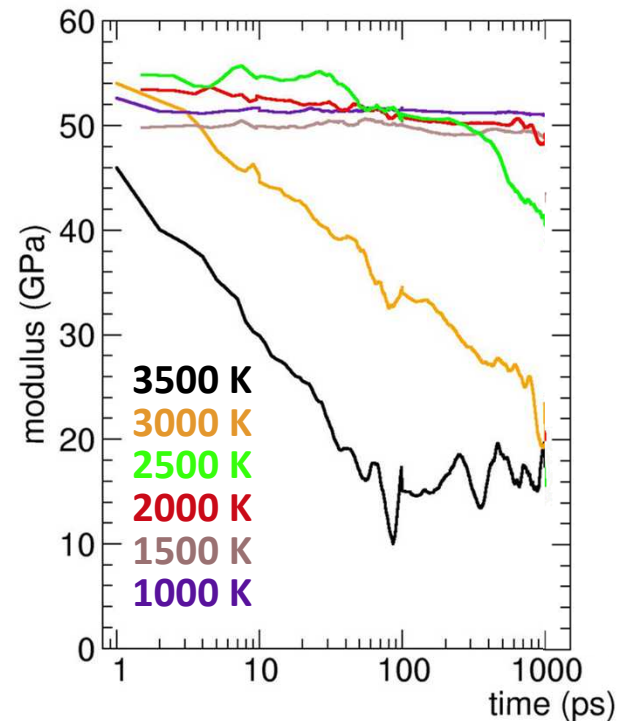




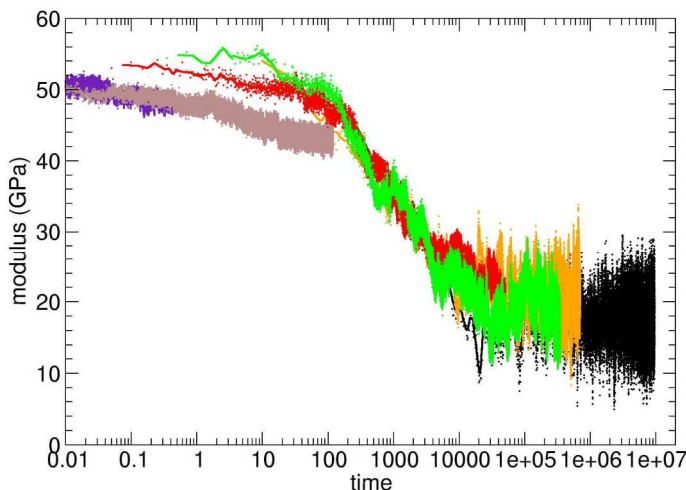
# Stress relaxation results



- Imposed and held a 3% volumetric strain imposed on equilibrated glass
- Measured modulus relaxation times at various temperatures



- Time-temperature superposition is shown to hold for relaxation curves down to 2000 K
- Two lowest temperatures do not appear to be consistent with the master curve – not unexpected below  $T_g$



# Stress relaxation results

The shift distance along the logarithmic time axis is the time-temperature shift factor  $a_T$ ,

$$a_T = \frac{t_T}{t_0}$$

Two fits are generally made to the shift factor, the Williams-Landel\_Ferry (WLF) fit near the glass transition,

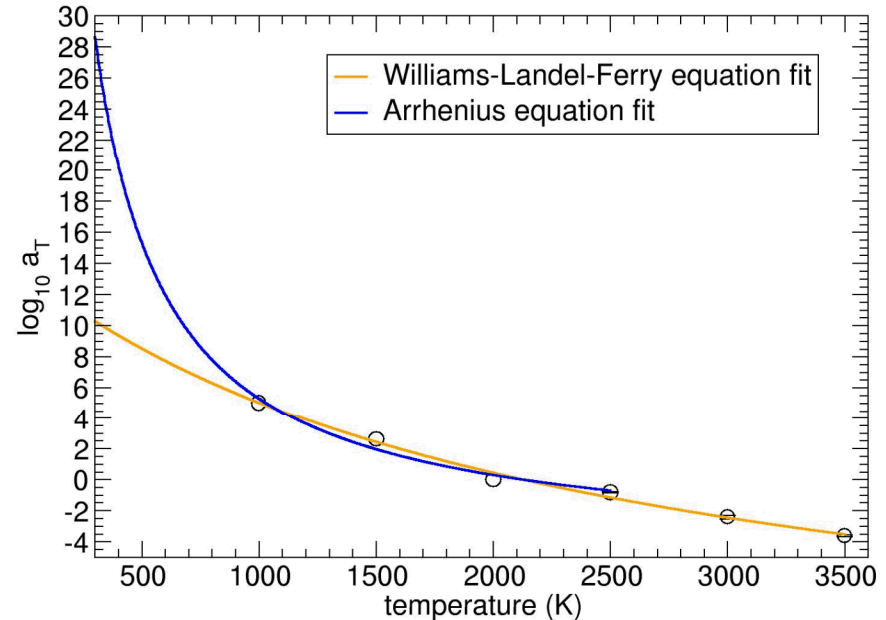
$$\log_{10} a_T = \frac{-C_1 \times (T - T_0)}{C_2 + T - T_0}$$

$$C_1 = 15.11, \quad C_2 = 4511, \quad T_0 = 2128$$

and an Arrhenius fit well below  $T_g$ .

$$\log_{10} a_T = A \left( \frac{1}{T} - \frac{1}{T_0} \right)$$

$$A = 9933, \quad T_0 = 2128$$



Full relaxation at room temperature will take simulations between  $10^{10}$  and  $10^{28}$  times longer to capture.

# Summary and conclusions

- We've studied dynamics of a simple, but realistic, glass model out to microsecond times – beginning to overlap experiment. An increase in times of 2 to 3 orders of magnitude.
- In slow quench cooling rate simulations we see significant additional relaxation from previously reported as would be expected. The thermal expansion of the set glass is significantly lower than previously reported for faster cooled glasses.
- BKS model glass demonstrates well characterized viscoelastic behavior, including time-temperature superposition which may allow well characterized accelerated stress relaxation for fracture simulations initial loadings.



# Brittle fracture models from molecular to component scale

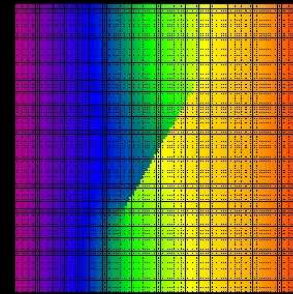
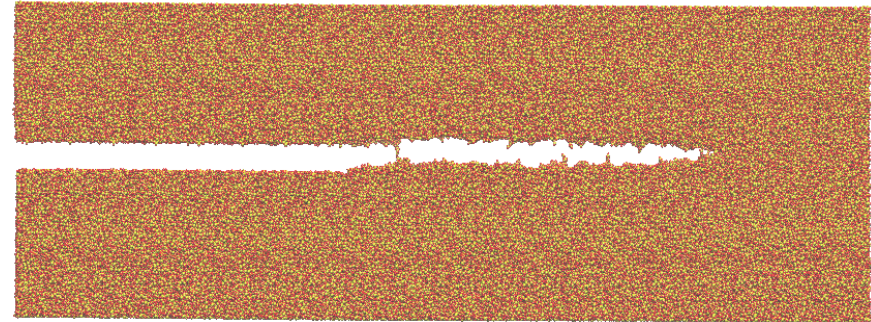
J. Matthew D. Lane, Org. 1814

Stewart Silling and David Littlewood, Org. 1444

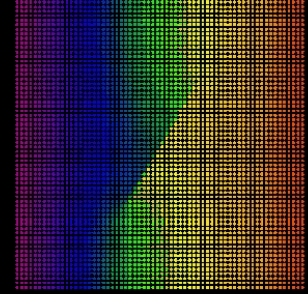
Fracture is a multiscale problem in which response at different length scales interact to determine crack propagation

Link molecular dynamics simulation to peridynamics, which has a property called upscaling to coarsen the problem to allow scaling to component geometry

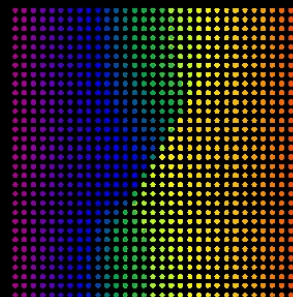
MD is used to calibrate the material model and fracture properties of the peridynamic model



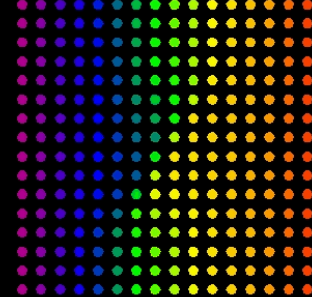
Level 0: 16384 nodes



Level 1: 4096 nodes



Level 2: 1024 nodes



Level 3: 256 nodes

# Why Multiscale with Peridynamics?

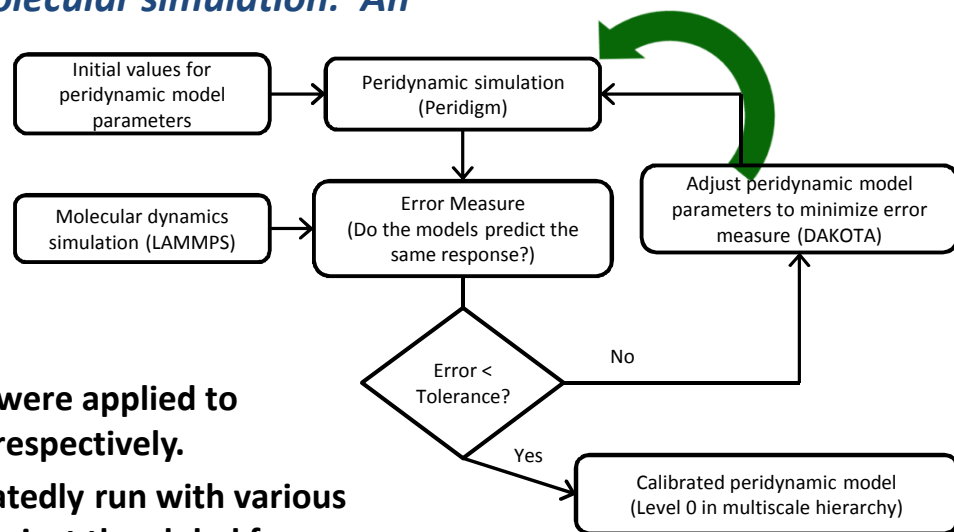
## THE KEY POINT IN THE UPSCALING IS NONLOCALITY

- **Nonlocality greatly expands the set of deformation-force relationships that can be captured in the model**
  - Nonlocal kinematic information provides a rich, vivid, highly-resolved description of deformation this is not possible in a local model
  - Nonlocal forces allow for direct incorporation of heterogeneous lower-length-scale mechanisms in the higher-scale system by explicitly modeling material point interactions over a range of length scales
- **This provides a means to incorporate lower-scale mechanisms into the higher-scale model *without smearing***
- **The response of individual bonds at the component scale is determined (hierarchically) by analysis of submodels at lower length scales**
  - Small-scale features are resolved only in the lower-scale models
  - Approach circumvents the need for full system analysis at the lower length scale
- **There is no analogy to this approach with classical local models**

# Peridynamic material models from MD

## Extracting peridynamic material models from molecular simulation: An optimization-based approach

- A generalized parameter extraction routine was developed in DAKOTA to determine parameters for the material model of amorphous silica based on molecular dynamics (MD) simulation.

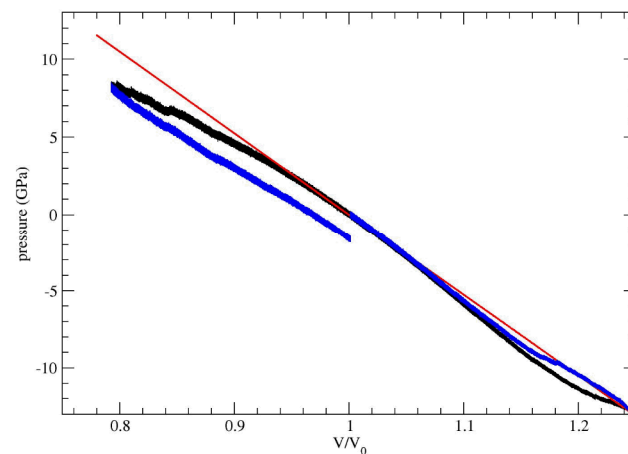


### Strain response optimization

- Both volumetric and uniaxial strains were applied to measure bulk and Young's modulus, respectively.
- Peridynamics simulations were repeatedly run with various material properties and optimized against the global force-displacement response

### Stress-strain results

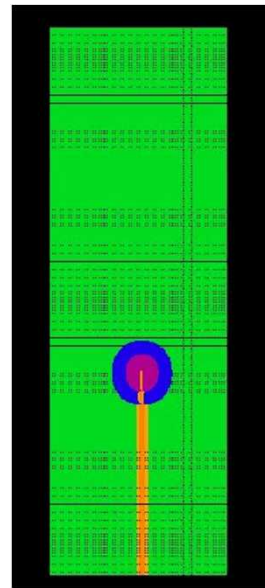
- The plot to the right shows the volumetric stress-strain response to approximately 20% strain in compression and extension. Black is the MD material response. Blue is the MD response during cycling back to zero strain. Red is the peridynamic model response.
- The glass is elastic in extension in this strain range, but nonlinear at strains over 10%. In compression, the glass deforms plastically, becoming more dense after release.





# Peridynamic fracture models from MD

- Peridynamic damage is modeled through bond breaking criteria and the suitable selection of a parameter called the horizon, which characterizes the degree of nonlocality in the model. Initial qualitative work to capture fracture properties has been promising.
- A reformulation of the peridynamic upscaling, which coarse grains the system by constrained optimization, is being evaluated in plane-strain 2D fracture geometries.



Preliminary results demonstrating qualitative agreement in glass fracture (on the left, atomistic simulation, and on the right, peridynamics modeling.)

