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Efficient Traffic Grooming in SONET/WDM BLSR Networks[1]

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Abstract

In this paper, we study traffic grooming in SONET/WDM BLSR networks under the uniform all-to-all traffic model with an objective to reduce total network costs (wavelength and electronic multiplexing costs), in particular, to minimize the number of ADMs while using the optimal number of wavelengths. We derive a new tighter lower bound for the number of wavelengths when the number of nodes is a multiple of 4. We show that this lower bound is achievable. All previous ADM lower bounds except perhaps that in [1] were derived under the assumption that the magnitude of the traffic streams (r) is *one* unit ($r = 1$) with respect to the wavelength capacity granularity g. We then derive new, more general and tighter lower bounds for the number of ADMs subject to that *the optimal number of wavelengths is used*, and propose heuristic algorithms (circle construction algorithm and circle grooming algorithm) that try to minimize the number of ADMs while using the optimal number of wavelengths in BLSR networks. Both the bounds and algorithms are applicable to any value of r and for different wavelength granularity g. Performance evaluation shows that wherever applicable, our lower bounds are at least as good as existing bounds and are much tighter than existing ones in many cases. Our proposed heuristic grooming algorithms perform very well with traffic streams of larger magnitude. The resulting number of ADMs required is very close to the corresponding lower bounds derived in this paper.

Index Terms — SONET/WDM networks, traffic grooming, BLSR networks, wavelength lower bounds, ADM lower bounds.

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1 Introduction

Wavelength division multiplexing (WDM) networks have emerged as the choice for the next generation backbone networks due to its high capacity (on the order of Tb/s per fiber) and many other features. It is evident that the number of traffic demands is likely to be much larger than the number of wavelengths available and that individual traffic demand is likely to require a smaller bandwidth than that of a full wavelength channel. Both factors call for multiplexing low-speed traffic requests onto a wavelength to efficiently utilize network resources. The multiplexing of lower rate traffic streams in current technologies employs time-division multiplexing (TDM) that requires electro-optic conversions. It has been recognized that the cost of electro-optic equipment such as SONET add-drop multiplexers (ADMs) is one of the dominant network cost metrics [1, 2]. These factors give rise to the concept of traffic grooming that is defined as the techniques of multiplexing lower speed traffic streams onto appropriate wavelength channels in order to minimize the cost metric and/or to optimize the throughput [1–3].

Much work [1, 2, 4–17] has focused on traffic grooming in SONET/WDM ring networks. Previous work has considered many aspects of traffic grooming, including minimizing the number of wavelengths, minimizing the number of ADMs, considering different traffic models, using different network architectures, incorporating switching capability, wavelength conversion, transceiver tunability and so on. Modiano et al. [7] and Wan et al. [10] have proved that the general traffic grooming problem is $\mathcal{NP}$-complete. The authors in [11, 14, 16, 18] formulate the traffic grooming problem as an integer linear programming (ILP) based optimization problem. The limitation of the ILP approach is that the numbers of variables and equations increase explosively as the size of the network increases. High computational complexity makes this approach unattractive in many practical cases.

The bounds on the number of ADMs needed for traffic grooming in SONET/WDM ring networks have been addressed in previous work including [1, 2, 5–8, 13, 17]. For uniform all-to-all traffic, lower bounds on the numbers of ADMs required for BLSR/2 rings with sub-wavelength traffic have been formulated in [1]. The bounds assume the availability of wavelength converters in the network and are rather loose. The work by Simmons et al. [5] considers all-to-all uniform and distance-dependent traffic models for BLSR networks. Expressions on approximate number (not necessarily lower bounds) of ADMs based on *super-node* approximation were derived for odd number of nodes only. No algorithms for grooming traffic streams were presented. Lower bounds on the number of ADMs have been calculated algorithmically for both unidirectional and bidirectional rings in [2] for all-to-all uniform traffic model. However, no lower bound expressions were given. Heuristic algorithms for grooming traffic have been presented. The grooming algorithm developed in [2] for all-to-all uniform traffic is based on traffic circles constructed using algorithms developed in [19]. To the best of our understanding, the circle construction algorithm of [19] for even number of nodes (N) **does not** include all traffic streams in one direction of the ring for all to-all uniform traffic model in BLSR networks, and therefore is not entirely correct for BLSR networks. Consequently, the number of circles constructed by that algorithm for even N is $\lceil \frac{N^2}{8} \rceil$ which is less than the lower bound on the number of circles we derive in Section 3 when $N = 4m, m \in Z^+$. Wan et al. [10] studied the grooming of arbitrary traffic in BLSR networks. General lower bounds that are claimed to be better than the bounds of [4] were derived for arbitrary traffic in BLSR networks. A second lower bound, more suited for all-to-all uniform traffic model has also been derived. Various approximation algorithms were proposed and their approximation ratio were analyzed. Modiano et al. [7] and Qiao et al. [2] have shown through examples that it is not always possible to minimize the number of wavelengths and the number of ADMs simultaneously. It has also been shown in [4] that minimizing the number of ADMs and the number of wavelengths are intrinsically different problems and that there exist cases where the two minima cannot be achieved simultaneously.

In this work, we study traffic grooming in BLSR networks under the uniform all-to-all traffic model with an objective to reduce total network costs (wavelength and electronic multiplexing costs), in particular, to minimize the number of ADMs *while using the optimal number of wavelengths*. All previous ADM lower bounds except perhaps that in [1] were derived under the assumption that the magnitude of the traffic streams (r) is *one* unit ($r = 1$) with respect to the wavelength capacity granularity g. We then derive new, more general and tighter lower bounds for the number of ADMs subject to that *the optimal number of wavelengths is used*, and propose heuristic algorithms (circle construction algorithm and circle grooming algorithm) that try to minimize the number of ADMs while using the optimal number of wavelengths in BLSR networks. Both the bounds and algorithms are applicable to any value of r and for different wavelength granularity g. Performance evaluation shows that wherever applicable, our lower bounds are at least as good as existing bounds and are much tighter than existing ones in many cases. Our proposed heuristic grooming algorithms perform very well with traffic streams of larger magnitude. The resulting number of ADMs required is very close to the corresponding lower bounds derived in this paper. Similar studies for UPSR networks have been reported in the complementary work [20].

The rest of the paper is organized as follows. Section 2 addresses the lower bounds on the number of traffic circles and wavelengths needed. Section 3 proposes algorithms to construct the optimal number of traffic circles for odd and even number of nodes. In Section 4, expressions for new, tighter lower bounds on the number of ADMs are derived. In Section 5, an efficient algorithm for grooming circles in BLSR networks is described. Results from simulations, derived lower bound expressions, and other work are compared and explained in Section 6. Finally, Section 7 concludes the paper.

2 New Bounds on the Number of Wavelengths

The lower bounds on the number of wavelengths necessary for traffic grooming in both SONET/WDM UPSR and SONET/WDM BLSR/2 networks were studied in previous work including [1, 2, 6]. In this section, we derive tighter lower bounds on the number of wavelengths needed in BLSR networks under uniform traffic (full-duplex and all-to-all). We also show that all the bounds presented are achievable.

In the sequel, we use N, g, and r to represent the number of nodes, the granularity of a wavelength, and the size of a inter-node traffic stream in terms of low rate tributary streams (e.g., OC-3), respectively. We also take into account the impact of traffic splitting on the wavelength lower bounds. By traffic splitting, we mean that a traffic stream at a source node may be decomposed into several smaller streams, and each smaller stream may be groomed onto a different wavelength.

In SONET/WDM BLSR networks, a traffic stream from node i to node j is carried through the shortest path, i.e., with the minimum number of hops [21]. The maximum hop count of a path is, therefore, limited to $\frac{N}{2}$ for an N-node ring. However, each wavelength in a BLSR/2 ring can carry only up to 50% of its full capacity. The rest of its capacity is reserved for carrying protection traffic [1]. We calculate the wavelength lower bounds for BLSR networks for odd and even number of nodes individually. We first compute the number of "circles" that are required to support uniform all-to-all traffic of r low rate streams in **one direction** of the ring. A circle is formed by inter-node traffic streams of the same size in the same direction. The traffic in the reverse direction is carried through another fiber in exactly the same way. A *closed* or *full* circle is one that is constructed out of several traffic streams with overlapping end-nodes such that there is no "gap" between a pair of nodes. A circle that has one or more gaps between pairs of nodes is called an *open-ended* circle. After constructing the circles, the number of wavelengths is calculated in terms of r, g, and the number of circles.

2.1 Circles in BLSR Networks with Odd Number of Nodes

In a BLSR network with odd number (N) of nodes, there is always a single shortest path from a source to a destination. The maximum number of hops between any pair of nodes is $\frac{N-1}{2}$. Traffic from a source node s to every other destination node d goes through $1, 2, \cdots \frac{N-1}{2}$ hops. We observe the following properties and utilize them to determine the number of circles in a BLSR network with odd number of nodes.

Property 1: In an N-node BLSR network where N is odd, the number of inter-node traffic with k hops, $k = 1, 2, \cdots, \frac{N-1}{2}$ is equal to the number of nodes N in the network.

Property 2: In an N-node BLSR network where N is odd, the number of circles consisting of inter-node traffic with k hops is always k, where $k = 1, 2, \cdots \frac{N-1}{2}$.

For example, only one circle is required to carry all inter-node traffic between adjacent nodes in one direction. Two circles are required to carry traffic streams between nodes that are two hops apart, and so on. The total number of circles is therefore given by: $1 + 2 + 3 + \cdots \frac{N-1}{2} = \frac{N^2-1}{8}$. This is the minimum number of circles that include all traffic streams. The lower bound on the number of circles for odd N is thus,

$$\mathcal{C}^o_{LB} = \frac{N^2 - 1}{8} \tag{2.1}$$

2.2 Circles in BLSR Networks with Even Number of Nodes

In a SONET/WDM BLSR network with even number of nodes, the maximum number of hops between any pair of nodes is $\frac{N}{2}$. Traffic from a source node s to every other destination node d goes through $1, 2, \cdots \frac{N}{2}$ hops. There is always a single shortest path between a pair of nodes at a hop distance less than or equal to $\frac{N-2}{2}$. However, there are two shortest paths between a pair of nodes that are at a hop distance $\frac{N}{2}$, of which only **one** is used for traffic in one direction of the ring [21]. We observe the following properties and utilize them to determine the number of circles in a BLSR network with even number of nodes.

Property 3: In an N-node BLSR network where N is even, the number of inter-node traffic with k hops is $N, k = 1, 2, \cdots \frac{N-2}{2}$.

Property 4: In an N-node BLSR network where N is even, the number of circles consisting of inter-node traffic with k hops is always $k, k = 1, 2, \cdots \frac{N-2}{2}$.

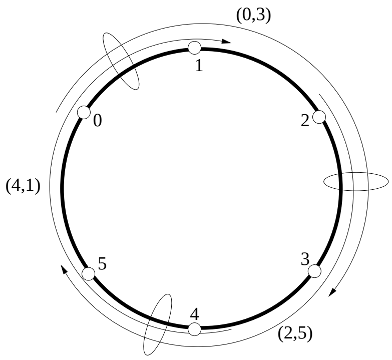


Figure 1: In a BLSR network with even number of nodes (6 nodes in this example), three 3-hop paths ({(0, 3), (2, 5), (4, 1)}) can be accommodated on two wavelengths if nodes are capable of wavelength conversion. This scenario, however, can be avoided using an intelligent circle construction as will be shown in Section 3.

Property 5: In an N-node BLSR network where N is even, the number of inter-node traffic necessary to include all traffic at distance $\frac{N}{2}$ is $\frac{N}{2}$.

As stated in *Property* 5, the number of necessary paths between a pair of nodes at hop-distance $\frac{N}{2}$ is $\frac{N}{2}$. We note that there are two paths between a pair of nodes that are apart by a hop distance of $\frac{N}{2}$. But only one path in one direction of the ring is required. By carefully assigning each traffic stream to a circle, it is possible to accommodate them in $\lfloor \frac{N}{4} \rfloor + 1$ circles. Fig. 1 shows how such a circle may be constructed. We note that this assignment (Fig. 1) appears to require that nodes have wavelength conversion capabilities. However, wavelength conversion is actually not needed if some circles are constructed by combining one path with a hop distance $\frac{N}{2}$ and two other carefully selected paths with a hop distance ($< \frac{N}{2}$). That is, the scenario depicted in Fig. 1 can be avoided all together and will be shown in Section 3 using an intelligent circle construction algorithm. The total number of circles is then given by: $(1+2+3+\cdots \frac{N-2}{2} + \lfloor \frac{N}{4} \rfloor + 1) = \frac{N(N-2)}{8} + \lfloor \frac{N}{4} \rfloor + 1$. This is the minimum number of circles required to include all traffic streams. The lower bound on the number of circles for even N can be derived from above expression as,

$$\mathcal{C}^e_{LB} = \begin{cases} \frac{N^2}{8} + 1 & \text{if } N \text{ is a multiple of 4,} \\ \lceil \frac{N^2}{8} \rceil & \text{if } N \text{ is even, not a multiple of 4.} \end{cases} \tag{2.2}$$

Combining Eqs. (2.1) and (2.2), we have the lower bounds on the number of circles for BLSR networks under the uniform all-to-all traffic model as:

$$\mathcal{C}_{LB} = \begin{cases} \frac{N^2-1}{8} & \text{if } N = 2m+1, \\ \frac{N^2}{8} + 1 & \text{if } N = 4m, \\ \lceil \frac{N^2}{8} \rceil & \text{if } N = 4m+2, \end{cases} \tag{2.3}$$

where $m \in Z^+$. In addition, the above lower bounds are all achievable. In particular, the lower bound when $N = 4m$ is new and tighter than any previous lower bounds reported and is achievable as shown in Section 3.

2.3 Lower Bounds on Number of Wavelengths in $BLSR/2$ Rings

Two scenarios can be pictured depending on whether the traffic originating from a node can be split onto more than one wavelength. Note that a node must have more than one SONET ADM to groom the split traffic to different wavelengths. If r and g are not multiples and traffic is not allowed to be split across multiple wavelengths, then a part of every wavelength will remain unused resulting in more necessary wavelengths. Whether r and g are multiples or not has an impact on the number of wavelengths required.

The minimum number of wavelengths required in a SONET/WDM BLSR/2 network in various scenarios can be formulated in terms of the number of circles determined above.

- *Case 1*: If traffic splitting is allowed or $g/2$ is a multiple of f ($f = r$ mod $g/2$), $W_{min} =$

$$\begin{cases} \frac{h(N^2-1)}{8} + \lceil\frac{(N^2-1)f}{4g}\rceil & \text{if } N = 2m+1, \\ \frac{h(N^2+8)}{8} + \lceil\frac{(N^2+8)f}{4g}\rceil & \text{if } N = 4m, \\ h\lceil\frac{N^2}{8}\rceil + \lceil\frac{2\lceil\frac{N^2}{8}\rceil f}{g}\rceil & \text{if } N = 4m+2, \end{cases} \tag{2.4}$$

  where $m \in Z^+$, $h = r$ div $g/2$ and $f = r$ mod $g/2$.

- *Case 2*: If traffic splitting is not allowed and $g/2$ is not a multiple of f, $W_{min} =$

$$\begin{cases} \frac{h(N^2-1)}{8} + \lceil\frac{N^2-1}{8\lfloor\frac{g}{2f}\rfloor}\rceil & \text{if } N = 2m+1, \\ \frac{h(N^2+8)}{8} + \lceil\frac{(N^2+8)}{8\lceil\frac{g}{2f}\rceil}\rceil & \text{if } N = 4m, \\ h\lceil\frac{N^2}{8}\rceil + \lceil\frac{\lceil\frac{N^2}{8}\rceil}{\lfloor\frac{g}{2f}\rfloor}\rceil & \text{if } N = 4m+2, \end{cases} \tag{2.5}$$

  where $m \in Z^+$, $h = r$ div $g/2$ and $f = r$ mod $g/2$.

The above bounds can be easily extended to calculate the lower bounds for BLSR/4 networks by replacing $g/2$ with g, and is omitted here.

3 Circle Construction Algorithm

Unlike in UPSR networks [2, 17], construction of circles in BLSR networks is not straightforward. We propose two separate algorithms for constructing circles for networks with odd and even number of nodes, respectively.

3.1 Algorithm I – Constructing Circles for *Odd N*

In Section 2, we have seen that the minimum number of circles for networks with odd number of nodes N is $\frac{N^2-1}{8}$. We propose an algorithm that constructs exactly $\frac{N^2-1}{8}$ full circles in polynomial time to include all the traffic streams in a all-to-all traffic model in BLSR networks. The pseudo code of Algorithm I is given in Fig.(2). Initially, we construct full circles that have either three or four traffic streams. We also point out

1. Procedure *construct_Circles_Odd* () {
2. // Construct circles with three traffic streams
3. for $i = 0$ to $(N-1)/2 - 1$ {
4. construct full circle with nodes
5. $\{i, i+1, (N+1)/2 + i\}$
6. } //end i loop
7. // Construct circles with four traffic streams
8. for $i = 0$ to $(N-1)/2 - 1$ {
9. for $s = (N-1)/2$ downto $i + 2$ {
10. construct full circle with nodes
11. $\{i, s, (N+1)/2 + i, N + i + 1 - s\}$
12. } //end s loop
13. } //end i loop
14. } // end

Figure 2: Algorithm I for circle construction in BLSR networks with odd N.

that the circles are so constructed that they can be modified to have more traffic streams (e.g., five, six, etc.) if necessary for efficient traffic grooming in later phases. The nodes are numbered $0 \ldots N-1$.

The first *for* loop (lines 3-6) constructs $\frac{N-1}{2}$ full circles, each of which has 3 traffic streams. The second nested *for* loop (lines 8-14) constructs $\frac{N-1}{2} \times ((\frac{N-1}{2} - 1) + (\frac{N-1}{2} - 2) + \ldots + 2 + 1) = \frac{(N-1)(N-3)}{8}$ full circles, each of which has 4 traffic streams. The total number of full circles is exactly $\frac{N^2-1}{8}$ and these circles include all the traffic streams of a uniform all-to-all traffic model in BLSR networks. Therefore, the algorithm is optimal. An algorithm for constructing the optimal number of circles for odd N is given in [19]. However, our approach is different from theirs. Circles constructed in our algorithm can be restructured to contain number of traffic streams other than three or four as is shown in Example 1.

In the sequel, we represent a circle by the end-nodes of each traffic stream separated by a dash "$-$" and enclosed by a pair of parentheses "()". A representation that contains one or more commas is an open-ended circle while a representation that does not contain any comma and that has the same ending node as the starting node in the representation is a closed circle. The following example illustrates how Algorithm I constructs the circles.

Example 1: $N = 17$ **(**$2m+1, m = 8$ **)**
In this case, the number of circles that have 3 traffic streams is eight (Eq. (4.4)) and that have 4 traffic streams is twenty eight (Eq. (4.5)). Accordingly, the first *for* loop constructs the following eight 3-traffic stream circles. $(0-1-9-0), (1-2-10-1), (2-3-11-2), (3-4-12-3), (4-5-13-4), (5-6-14-5), (6-7-15-6), (7-8-16-7)$. The second nested *for* loop constructs the following twenty eight 4-traffic stream circles. $(0-8-9-10-0), (0-7-9-11-0), (0-6-9-12-0), (0-5-9-13-0), (0-4-9-14-0), (0-3-9-15-0), (0-2-9-16-0), (1-8-10-11-1), (1-7-10-12-1), (1-6-10-13-1), (1-5-10-14-1), (1-4-10-15-1), (1-3-10-16-1), (2-8-11-12-2), (2-7-11-13-2), (2-6-11-14-2), (2-5-11-15-2), (2-4-11-16-2), (3-8-12-13-3), (3-7-12-14-3), (3-6-12-15-3), (3-5-12-16-3), (4-8-13-14-4), (4-7-13-15-4), (4-6-13-16-4), (5-8-14-15-5), (5-7-14-16-5), (6-8-15-16-6)$. The circles generated above include all the traffic streams and are optimal. In addition, circles constructed by Algorithm I can be restructured to contain number of traffic streams other than three or four. For example,

consider two 4-traffic stream circles $c_1 = (0-8-9-10-0)$ and $c_2 = (1-8-10-11-1)$. Traffic stream $8-10$ from c_2 can be exchanged with two traffic streams $(8-9-10)$ from c_1 while keeping both the circles closed. The resulting circles are $(0-8-10-0)$ and $(1-8-9-10-11-1)$, one of which has three traffic streams and the other has five traffic streams. The latter two circles contain the same traffic streams as the former two circles. Such a feature can be useful for efficient grooming of circles. □

3.2 Algorithm II – Constructing Circles for *Even N*

The minimum number of circles for networks with even number of nodes N is $\frac{N^2}{8}+1$ when $N=4m$ and is $\lceil\frac{N^2}{8}\rceil$ when $N=4m+2$, $m\in Z^+$. In either case, the number of full circles is $\frac{N(N-2)}{8}$. Therefore, the number of open-ended circles is $\lfloor\frac{N}{4}\rfloor+1$. We propose an algorithm that constructs exactly the above numbers of circles in polynomial time to include all the traffic streams. The nodes are numbered $0\ldots N-1$. The algorithm is outlined in Fig. 3.

The first *for* loop, Loop 1 (lines 4-7) is applicable only when $N=4m$ (N completely divisible by 4). This loop constructs $\frac{N}{4}$ full circles. Each such circle contains four traffic streams of stride $\frac{N}{4}$. Loop 2 (lines 10-13) constructs $\lceil\frac{N}{4}\rceil$ full circles, each of which has four traffic streams. Loop 3 (lines 15-20), a nested loop, constructs $(\lceil\frac{N}{4}\rceil-2)(\frac{N}{2})$ full circles, each of which has four traffic streams. Loop 4 (lines 22-25) forms $\lfloor\frac{N}{4}\rfloor$ full circles, each of which has three traffic streams. Loop 5 (lines 27-30) forms $\lfloor\frac{N}{4}\rfloor$ *open-ended* circles, each of which has two consecutive traffic streams involving three nodes. Finally, Loop 6 constructs the last open-ended circle. This circle contains $\lceil\frac{N}{4}\rceil$ traffic streams with all non-overlapping nodes so that the number of nodes involved in the circle is $2\cdot\lceil\frac{N}{4}\rceil$.

By examining the algorithm closely, we see that the total number of full circles, when $N=4m$, is $(\frac{N}{4}+\lceil\frac{N}{4}\rceil+\frac{N}{2}(\lceil\frac{N}{4}\rceil-2)+\lfloor\frac{N}{4}\rfloor)=\frac{N(N-2)}{8}$, and when $N=4m+2$, is $\lceil\frac{N}{4}\rceil+\frac{N}{2}(\lceil\frac{N}{4}\rceil-2)+\lfloor\frac{N}{4}\rfloor)=\frac{N(N-2)}{8}$. The total number of open-ended circles in either case is $\lfloor\frac{N}{4}\rfloor+1$. We consider two examples, $N=10$ and $N=12$ to illustrate the two cases ($N=4m$ and $N=4m+2$).

Example 2: N=10 ($4m+2, m=2$)
The first $\lceil\frac{10}{4}\rceil=3$ full circles (Loop 2) are $(1-2-6-7-1),(3-4-8-9-3),(4-5-9-0-4)$. The next $\frac{10}{2}(\lceil\frac{10}{4}\rceil-2)=5$ full circles (Loop 3) are $(0-2-5-7-0),(1-3-6-8-1),(2-4-7-9-2),(3-5-8-0-3),(4-6-9-1-4)$. The next $\lfloor\frac{10}{4}\rfloor=2$ full circles (which contain 3 traffic streams) are $(0-5-6-0),(2-7-8-2)$ (Loop 4). Finally, the $\lfloor\frac{10}{4}\rfloor+1=3$ number of open-ended circles are, $(1-5,6-1),(3-7,8-3),(0-1,2-3,4-9)$ (Loop 5 and 6). □

Example 3: N=12 ($4m, m=3$)
The first $\frac{12}{4}=3$ full circles (Loop 1) are $(0-3-6-9-0),(1-4-7-10-1),(2-5-8-11-2)$. The next $\lceil\frac{12}{4}\rceil=3$ full circles (Loop 2) are $(1-2-7-8-1),(3-4-9-10-3),(5-6-11-0-5)$. The next $\frac{12}{2}(\lceil\frac{12}{4}\rceil-2)=6$ full circles are $(0-2-6-8-0),(1-3-7-9-1),(2-4-8-10-2),(3-5-9-11-3),(4-6-10-0-4),(5-7-11-1-5)$. The next $\lfloor\frac{12}{4}\rfloor=3$ full circles (which contain 3 traffic streams) are $(0-6-7-0),(2-8-9-2),(4-10-11-4)$. Finally, the $\lfloor\frac{12}{4}\rfloor+1=4$ number of open-ended circles are, $(1-6,7-1),(3-8,9-3),(5-10,11-5),(0-1,2-3,4-5)$. □

The circle construction algorithm [19] used for grooming uniform traffic in BLSR networks in [2] does

Procedure *construct_Circles_Even* () {
//Loop 1: Construct N/4 full circles when N=4m
if(N==4m)
for $i = 0$ to $(N-1)/2-1$ {
construct full circle with nodes
$\{i, i+N/4, N/2+i, 3N/2+i, i\}$
} //end i loop
} //end if
//Loop 2: Construct $\lceil \frac{N}{4} \rceil$ full circles
for $i = 1$ to $N/2-1$, step 2 {
construct full circle with nodes
$\{i, i+1, (N/2)+i, (N/2+i+1)\%N, i\}$
} //end i loop
//Loop 3: Construct $\frac{N}{2}(\lceil \frac{N}{4} \rceil - 2)$ full circles
for $s = 2$ to $\lceil \frac{N}{4} \rceil - 1$ {
for $i = 0$ to $N/2-1$ {
construct full circle with nodes
$\{i, i+s, N/2+i, (N/2+i+1)\%N, i\}$
} //end i loop
//end s loop
//Loop 4: Construct $\lfloor \frac{N}{4} \rfloor$ full circles
for $i = 0$ to $\lfloor \frac{N}{4} \rfloor - 1$ {
construct full circle with nodes
$\{i, N/2+i, (N/2+i+1)\%N, i\}$
} //end i loop
// Loop 5: Construct $\lfloor \frac{N}{4} \rfloor$ open-ended circles
for $i = 0$ to $\lfloor \frac{N}{4} \rfloor - 1$ {
construct open-ended circle using two traffic streams
$(i+N/2, i), (i, N/2+i-1)$
} //end i loop
//Loop 6: Construct the last open-ended circle
for $i = 0$ to $N/2-2$, step 2 {
construct open-ended circle by adding traffic streams
add traffic $(i, i+1))$
if $N == 4m+2$ {
add traffic $(N/2-1, N-1))$
} //end if
} //end i loop
} // end

Figure 3: Algorithm II for circle construction with even N.

not account for all traffic streams. In particular, half of the traffic streams between pairs of nodes that are $\frac{N}{2}$ (N is even) hops apart are not included in the algorithm for circle construction presented in [19]. The algorithm based on Complementary Assembling with Dual Strides (CADS) [19] when N is even, describes the first step that constructs $\frac{N}{4}$ circles as:
Step 1: for $s = \frac{N}{2}$ (a special case)
for $i = 0, 1, \ldots \frac{N}{4} - 1$
assemble $(i, \frac{N}{2})$ and $(\frac{N}{2}+i, \frac{N}{2})$ in one circle
Therefore, the algorithm does not account for the traffic streams between node pairs $(\frac{N}{4}, \frac{3N}{4}), (\frac{N}{4}+1, \frac{3N}{4}+1) \ldots (\frac{N}{2}-1, N-1)$. Consequently, the number of circles constructed by that algorithm [19] for even N is $\lceil \frac{N^2}{8} \rceil$

which is less than the lower bound on the number of circles that is derived in Eq. (2.2) when $N = 4m, m \in Z^+$.

4 Traffic Grooming and New ADM Lower Bounds

In this section we derive new general and tighter lower bounds on the number of ADMs for traffic grooming in BLSR networks under the all-to-all uniform traffic model. To the best of our knowledge, our bounds are tighter and more general than all lower bounds derived earlier. The derived lower bounds are applicable to any integral traffic stream magnitude r. Under the all-to-all uniform traffic model, the number of traffic streams in one direction (on one fiber) is $N_p = N(N-1)/2$. The number of circles (partial and full) in one direction has been derived in Eq. (2.3).

4.1 Preliminaries

When circles are constructed out of traffic streams, the average number of traffic streams per circle can be expressed as,

$$\frac{N(N-1)}{2}\frac{8}{N^2-1} = \frac{4N}{N+1} \tag{4.1}$$

for odd N and

$$\frac{N(N-1)}{2}\frac{8}{N^2} = \frac{4(N-1)}{N} \tag{4.2}$$

for even N. In either case, the average number of traffic streams carried by each circle tends toward 4 as N increases. The number of circles that can be bundled together onto one wavelength depends on the magnitude of the traffic stream r and is equal to $\lfloor \frac{g}{r} \rfloor$. To determine the appropriate number of traffic streams in a circle and the best way of packing the circles onto wavelengths are the main challenges for optimal traffic grooming. Intuitively, constructing circles with open ends may result in inefficient traffic grooming. Such circles pose obstacles to attain optimal solutions in terms of the minimum number of wavelengths and/or the minimum number of ADMs required. Note, however, that one or more open ended circles are unavoidable when N is even as shown in the previous section. Having too many traffic streams in some circles may force other circles to have too few traffic streams, and sometimes may lead to open ended circles. Any open ended circle in networks with odd number of nodes is suboptimal in terms of the number of wavelengths and could also be suboptimal in terms of the number of ADMs as well. In light of the above observations, we investigate the properties of a SONET/WDM BLSR network and formulate several lemmas.

Lemma 1: A closed circle must have at least three end-points in a BLSR network with odd or even number of nodes.

Proof: To construct a closed circle with two end-points, we can have at most two traffic streams in the circle such that one end node of a traffic stream terminates at the start node of the other traffic stream. The longest possible stride of a traffic stream in a BLSR network is $\frac{N-1}{2}$ for odd N. Two such traffic streams span over exactly $N-1$ strides (arcs), leaving a single stride (between the start node of the first traffic stream and the end node of the second traffic stream) open, and therefore forcing the circle to have at least three end-points.

The case for even number of nodes (N) is somewhat subtle. The longest possible stride of a traffic stream in a BLSR network is $\frac{N}{2}$. Apparently, two such traffic streams span over exactly N strides, forming a circle with two end-points. However, since the two traffic streams in the circle are between the same pair of nodes and for traffic in **different** directions, they cannot be on the same fiber (by the definition of a BLSR network [21]). Therefore, one traffic stream in the circle must be replaced by at least two traffic streams *on the same fiber* with shorter strides, thereby resulting in a circle with at least three end-points. This proves the claim. $\square$

Given that the average number of traffic streams per circle is $\frac{4N}{N+1}$ for odd N and $\frac{4(N-1)}{N}$ for even N, constructing circles with four or three traffic streams tends to make the number of traffic streams per circle balanced over all circles. Moreover, as N increases the *relative* number of circles with four traffic streams should increase and that of circles with three traffic streams should decrease. By carefully constructing the circles with four or three traffic streams, it is possible to accommodate all traffic streams using optimal number wavelengths as shown in the previous section. The following lemma quantifies the numbers of circles with four or three traffic streams as functions of N.

Lemma 2: When full circles with four or three traffic streams are constructed for traffic in one direction in a BLSR network with an odd number of nodes, N, all the circles can be full such that the number of circles is optimal.

Proof: Let $n_3^o(N)$ and $n_4^o(N)$ be the numbers of circles with three and four traffic streams, respectively. Then $n_4^o(N) = N_c^o(N) - n_3^o(N)$, where $N_c^o(N)$ is the total number of circles for odd N. The total number of traffic streams is $\frac{N(N-1)}{2}$. Therefore,

$$3 \cdot n_3^o(N) + 4 \cdot (N_c^o(N) - n_3^o(N)) = \frac{N(N-1)}{2} \tag{4.3}$$

Substituting $N_c^o(N) = \frac{N^2-1}{8}$ and solving for $n_3^o(N)$, we have

$$n_3^o(N) = \frac{N-1}{2}, \tag{4.4}$$

and then solving for $n_4^o(N)$ we have,

$$n_4^o(N) = \frac{(N-1)(N-3)}{8}. \tag{4.5}$$

The right hand side of Eq. (4.4), $\frac{N-1}{2}$ is always an integer if N is odd. Similarly, The right hand side of Eq. (4.5), $\frac{(N-1)(N-3)}{8}$ is also always an integer if N is odd. Therefore, the circles in both categories can all be *full* and closed. The total number of circles constructed this way is $\frac{N-1}{2} + \frac{(N-1)(N-3)}{8} = \frac{N^2-1}{8}$, which is the optimal number of circles for odd N shown in Eq. (2.3). The algorithm in Fig. 2 constructs exactly $\frac{N-1}{2}$ and $\frac{(N-1)(N-3)}{8}$ full circles of three or four traffic streams respectively when N is odd. $\square$

Lemma 3: The number of *full* circles that can be constructed for traffic streams in one direction in a BLSR network is at least $\frac{N(N-2)}{8}$ when N is even and the number of *open-ended* circles is $\lfloor \frac{N}{4} \rfloor + 1$ if the total number of circles constructed is optimal.

Proof: From Property 4, we see that when N is even the number of circles consisting of inter-node traffic with k hops is always $k, k = 1, 2, \cdots \frac{N-2}{2}$. By limiting k from 1 through $\frac{N-2}{2}$, we exclude the traffic streams between pairs of nodes that are $\frac{N}{2}$ hop apart. Therefore, the total number of circles is $1 + 2 + \cdots + \frac{N-2}{2} = \frac{N(N-2)}{8}$, which is always an integer when N is even. Thus, the circles can be all *full*. The number of traffic streams that are accommodated in these circles is $\frac{N(N-1)}{2} - \frac{N}{2} = \frac{N(N-2)}{2}$. The number of remaining traffic streams is $\frac{N}{2}$, which can be accommodated at most in $\frac{N}{2}$ circles (one in each circle) and at least in $\lfloor \frac{N}{4} \rfloor + 1$ circles (optimal) as shown in Fig. 1. This proves the lemma. Thus,

$$n_f^e(N) = \frac{N(N-2)}{8}, \tag{4.6}$$

where $n_f^e(N)$ is the number of full circles when N is even. Algorithm II in Fig. 3 constructs exactly $\frac{N(N-2)}{8}$ full circles consisting of three or four traffic streams when N is even. □

We use Lemma 3 and Algorithm II (outlined in Section 3) to determine the number of circles of each type. In essence, Algorithm II proves the claim in Lemma 3 through simulation and constructs exactly $\frac{N(N-2)}{8}$ full circles and $\lfloor \frac{N}{4} \rfloor + 1$ open-ended circles. Note that a full circle with 3 traffic streams involves three nodes, and an open-ended circle with two consecutive traffic streams (that have one overlapping node) also involve three nodes. Therefore, a 3-traffic stream full circle and a 3-node 2-traffic stream open-ended circle are the same in terms of the number of required ADMs per circle for traffic grooming. However, the ADM efficiency or utilization (defined as the number of ADMs needed per traffic stream) is better for full circles. We therefore classify the circles created in Loop 4 and Loop 5 in Algorithm II as 3-node circles (that contain either two or three traffic streams) and quantify as, $n_3^e(N) = \lfloor \frac{N}{4} \rfloor + \lfloor \frac{N}{4} \rfloor = 2\lfloor \frac{N}{4} \rfloor$. More specifically,

$$n_3^e(N) = \begin{cases} \frac{N}{2} & \text{if } N = 4m, \\ \frac{N}{2} - 1 & \text{if } N = 4m+2, \end{cases} \tag{4.7}$$

where $m \in Z^+$. The last open-ended circle generated in Loop 6 of Algorithm II involves $2 \cdot \lceil \frac{N}{4} \rceil$ nodes which is ≥ 4 for $N > 4$. In calculating the ADM lower bound, we will be underestimating the number of required ADMs if we consider the circle as a 4-node circle (thereby making our ADM bound a little loose). Subtracting Eq. (4.7) from Eq. (2.3) for an even N, the number of 4-node circles can be expressed as:

$$n_4^e(N) = \lceil \frac{N^2}{8} \rceil - \frac{N}{2} + 1 \tag{4.8}$$

Let $n_3(N)$ and $n_4(N)$ be the number of circles involving three and four nodes respectively such that the total number of circles for N nodes is optimal. Then combining Eqs. (4.4) and (4.7), we have,

$$n_3(N) = \begin{cases} \frac{N-1}{2} & \text{if } N = 2m+1, \\ \frac{N}{2} & \text{if } N = 4m, \\ \frac{N}{2} - 1 & \text{if } N = 4m+2, \end{cases} \tag{4.9}$$

where $m \in Z^+$. Similarly, combining Eqs. (4.5) and (4.8), we have,

$$n_4(N) = \begin{cases} \frac{(N-1)(N-3)}{8} & \text{if } N = 2m+1, \\ \lceil \frac{N^2}{8} \rceil - \frac{N}{2} + 1 & \text{if } N = 2m, \end{cases} \tag{4.10}$$

where $m \in Z^+$.

It follows from Eqs. (4.4) and (4.7) that the number of circles with three nodes increases linearly in terms of N. Similarly, it follows from Eqs. (4.5) and (4.8) that the number of circles with four nodes increases as fast as N^2.

4.2 ADM Lower Bounds

In this section we derive lower bounds on the number of ADMs subject to that the *optimal* number of wavelengths are used for both BLSR/2 and BLSR/4 networks based on the results of previous subsection. Let g' be the effective bandwidth of a wavelength so that,

$$g' = \begin{cases} \frac{g}{2} & \text{for BLSR/2 networks,} \\ g & \text{for BLSR/4 networks.} \end{cases} \tag{4.11}$$

We assume that traffic streams cannot be split and groomed onto different wavelengths. Therefore, a wavelength may or may not be fully utilized and no more traffic circle can be packed onto the wavelength when g' is not an exact multiple of r. We call this wavelength a *fully packed wavelength*. Otherwise, a wavelength is called a *partially packed wavelength* if more traffic circles can be packed onto it. We use the notation $(x_i \times i : x_j \times j)$ to denote the number of circles of each type to be groomed in a wavelength in which x_i denotes the number of circles involving i nodes, and x_j denotes the number of circles involving j nodes, given that the total number of circles packed onto the wavelength is $x_i + x_j$.

We notice that not all circles can be packed onto a wavelength using a single grooming strategy. To characterize the traffic grooming process, we introduce two terms: *grooming scheme* and *grooming class*. Consider packing a number of circles onto a wavelength, a grooming scheme determines an appropriate combination of circles (each of which may have a different number of nodes) that can be packed on the wavelength. A grooming class C is defined as the number of traffic circles that can be packed onto a wavelength (fully or partially packed) and therefore depends on the magnitude r of the traffic streams and the value of g'. For each grooming class C, our studies show that it is possible to groom all traffic circles by applying two different *grooming schemes* (S_1^C, S_2^C) for fully packed wavelengths and a third *grooming scheme* (S_p^C) for partially packed wavelengths if any. These grooming schemes will be specified below.

A few definitions of notations are in order:

- $G \in \{S_p^C, S_1^C, S_2^C\}$ denotes the *grooming scheme*. Specially, $G = S_p^C$ corresponds to the scheme used for grooming traffic circles onto partially packed wavelength if any while $G = S_1^C$ and $G = S_2^C$ denote schemes used for grooming traffic circles onto fully packed wavelengths for grooming class C.
- $C \in \{i : 1 \leq i \leq g'\}$ denotes the grooming class. For fully packed wavelengths, $C = \lfloor \frac{g'}{r} \rfloor$. For partially packed wavelengths, $C =$ the number of traffic circles remained to be groomed after no more wavelengths can be fully packed.
- $W_{G,C}(N)$ is the number of wavelengths onto which full traffic circles are packed using grooming scheme S_1^C or S_2^C for grooming class C.

- $p_C(N)$ is the number of circles to be groomed onto the partially packed wavelength using S_p^C for grooming class C. Thus, $p_C(N) \in \{i : 0 \leq i \leq C-1\}$.

- $p_{k,C}(N)$ is the number of circles, each of which has k traffic streams to be groomed onto the partially packed wavelength for grooming class C, where $k \geq 3$ and $\sum_k p_{k,C}(N) = p_C(N)$.

- $n_{k,C}(N)$ is the number of circles on the fully packed wavelengths, each of which has k traffic streams.

- $d_{G,C}$ is the number of ADMs per wavelength for grooming class C when grooming scheme G is used.

Let u be the minimum number of nodes involved in grooming C circles, each of which involves three or four nodes, onto a wavelength. Then based on Eq. (2.3), we have,

$$C \leq \begin{cases} \frac{u^2-1}{8} & \text{if } u \text{ is odd,} \\ \frac{u^2}{8}+1 & \text{if } u \text{ is a multiple of 4,} \\ \lceil \frac{u^2}{8} \rceil & \text{if } u \text{ is even, not a multiple of 4.} \end{cases} \tag{4.12}$$

Intuitively, the best utilization of ADMs can be achieved from full circles than open-ended circles. By examining Eqs. (2.3), (4.4) and (4.5), we know that when u is odd, all circles are full, each of which involves three or four nodes. It is also evident from Eq. (4.6) that if u is incremented by one and made even, the number of *full* circles still remains exactly the same (and the additional circles are all *open-ended*). Therefore, to determine the value of u, it is sufficient to consider only odd u. Eq. (4.12) can be written for odd u for $C \geq 0$ as,

$$C \leq \frac{u^2-1}{8} \tag{4.13}$$

It is obvious that when $u = 0, C = 0$. Therefore, the above inequality can be solved for an odd integer value of u, and we have,

$$u(C) = \begin{cases} Odd(\lceil \sqrt{1+8C} \rceil) & \text{if } C > 0, \\ 0 & \text{if } C = 0 \end{cases} \tag{4.14}$$

where $Odd(l)$ is the smallest odd integer that is equal to or greater than l.

Let $v_3(u(C))$ and $v_4(u(C))$ be the numbers of 3-node and 4-node circles respectively to be groomed onto a wavelength for grooming class C. In order to achieve better grooming performance, we first groom all 4-node circles. Then the remaining 3-node circles are groomed. From Eqs. (4.5) and (4.4), we have

$$v_4(u(C)) = \frac{(u(C)-1)(u(C)-3)}{8}, \tag{4.15}$$

and,

$$v_3(u(C)) = C - \frac{(u(C)-1)(u(C)-3)}{8}. \tag{4.16}$$

Eqs. (2.3), (4.9), (4.10), (4.14), (4.15) and (4.16) among others will be used as the basis for deriving the ADM lower bounds for different possible values of r ($1 \leq r \leq g'$) and C ($1 \leq C \leq g'$). For ease of exposition, we divide the range of r into two cases, namely, $\frac{g'}{2} < r \leq g'$ and $1 \leq r \leq \frac{g'}{2}$.

Case 1: $\frac{g'}{2} < r \leq g'$ and $C = 1$ Since we are not considering splitting of a single traffic stream across multiple wavelengths, a full wavelength is required for each traffic stream r in the range $\frac{g'}{2} < r \leq g'$ which is the case where $C = 1$. In this case, there is no partially packed wavelength and hence $p_{k,C}(N) = 0, W_{S_p^C,C}(N) = 0$ and $d_{S_p^C,C}(N) = 0$. Eqs. (4.9) and (4.10) include all the circles with three and four nodes respectively. Let the 3-node and and the 4-node circles be groomed using $W_{S_1^C,C}(N)$ and $W_{S_2^C,C}(N)$ wavelengths, respectively. Therefore, $W_{S_1^C,C}(N) = n_3(N), W_{S_2^C,C}(N) = n_4(N), d_{S_1^C,C}(N) = 3$, and $d_{S_2^C,C}(N) = 4$.

Case 2: $1 \leq r \leq \frac{g'}{2}$ and $2 \leq C \leq g'$: For this case, we first determine the parameters for grooming the traffic streams onto the partially packed wavelength and then determine the parameters for fully packed wavelengths.

Determination of $p_{k,C}(N), W_{S_p^C,C}(N)$ and $d_{S_p^C,C}(N)$ for $1 \leq r \leq \frac{g'}{2}, (2 \leq C \leq g')$: Using Eq. (2.3), the number of circles for partially packed wavelength, $p_C(N)$ can be computed as,

$$p_C(N) = N_c \bmod \lfloor \frac{g'}{r} \rfloor. \tag{4.17}$$

When using 3-node and 4-node circles, the minimum number of nodes for grooming $p_C(N)$ circles can be determined from Eq. (4.14) as,

$$u(p_C(N)) = \begin{cases} Odd(\lceil \sqrt{1 + 8p_C(N)} \rceil) & \text{if } p_C(N) > 0, \\ 0 & \text{if } p_C(N) = 0. \end{cases} \tag{4.18}$$

The number of circles with three and four traffic streams can be determined from Eqs. (4.18), (4.15), and (4.16) respectively as,

$$p_{4,C}(N) = \frac{(u(p_C(N)) - 1)(u(p_C(N)) - 3)}{8} \text{ and,} \tag{4.19}$$

$$p_{3,C}(N) = p_C(N) - \frac{(u(p_C(N)) - 1)(u(p_C(N)) - 3)}{8}. \tag{4.20}$$

The number of partially packed wavelength, $W_{S_p^C,C}(N)$ is,

$$W_{S_p^C,C}(N) = \begin{cases} 1 & \text{if } p_C(N) > 0, \\ 0 & \text{if } p_C(N) = 0. \end{cases} \tag{4.21}$$

The number of ADMs for partially packed wavelength, $d_{S_p^C,C}(N)$ is,

$$d_{S_p^C,C}(N) = \begin{cases} u(p_C(N)) & \text{if } p_C(N) > 0, \\ 0 & \text{if } p_C(N) = 0. \end{cases} \tag{4.22}$$

Determination of $W_{S_1^C,C}(N)$, $d_{S_1^C,C}(N)$, $W_{S_2^C,C}(N)$ and $d_{S_2^C,C}(N)$: We address these parameters in two categories namely, for $r = \frac{g'}{2}$ and for $1 \leq r < \frac{g'}{2}$

Case 2a: $r = \frac{g'}{2}, (C = 2)$ This is the case when $r = \lfloor \frac{g'}{2} \rfloor$ and two traffic circles can be packed onto a wavelength. A fully packed wavelength is also fully utilized if r is exactly equal to $\frac{g'}{2}$. From Eq. (4.14), $u(2) = 5$, and from Eqs. (4.15) and (4.16), $v_4(u(2)) = 1$ and $v_3(u(2)) = 1$. In other words, a $(1 \times 4 : 1 \times 3)$ circle combination would be packed onto a wavelength that requires at least 5 ADMs. The number of 4-node circles grows at a higher rate than that of 3-node circles as N grows (Eqs. (4.9) and (4.10)). The number of 3-node circles is equal to that of 4-node circles for $N = 7$. For $N > 7$, the number of 4-node circles is greater than that of 3-node circles. Therefore, a $(1 \times 4 : 1 \times 3)$ traffic combination may leave some extra 4-node circles. Since, $v_4(u(2)) = 1$ and $v_3(u(2)) = 1$, each pair of 4-node circles requires one additional ADM beyond $u(2)$, i.e. $u(2) + 1 = 6$ ADMs. However, an ADM can be saved in two pairs of 4-traffic circles by converting a suitable pair of 4-traffic circles into a 5-traffic and a 3-traffic circle, and then groom them using $(1 \times 5 : 1 \times 4)$ and $(1 \times 4 : 1 \times 3)$ combinations. This requires at least $6 + 5 = 11$ (instead of 12) ADMs. Accordingly, a 5-traffic circle can be produced for every two pairs of additional 4-traffic circles. The number of 4-traffic circles decreases by two times the number of 5-traffic circles created, while the number of 3-traffic circle increases by the number of 5-traffic circles. The following equations are obtained as the resulting numbers of circles with $5, 4$ and 3 traffic streams, respectively:

$$\begin{aligned} n_{5,2}(N) &= \begin{cases} \lfloor \frac{n_4(N) - n_3(N) - p_{3,2}(N)}{4} \rfloor & \text{if } n_4(N) > n_3(N) + p_{3,2}(N), \\ 0 & \text{otherwise,} \end{cases} \\ n_{4,2}(N) &= n_4(N) - 2n_{5,2}(N), \\ n_{3,2}(N) &= n_3(N) - p_{3,2}(N) + n_{5,2}(N). \end{aligned} \tag{4.23}$$

Let a $(1 \times 4 : 1 \times 3)$ combination constitutes grooming scheme S_1^2. The number of wavelengths for grooming scheme S_1^2 is equal to $n_{3,2}(N)$, i.e. $W_{S_1^2,2}(N) = n_{3,2}(N)$ and $d_{S_1^2,2}(N) = u(2) = 5$. The grooming scheme S_2^2 could use either $(1 \times 4 : 1 \times 5)$ or (2×4) combinations, requiring at least $u(2) + 1 = 6$ ADMs. Thus, $d_{S_2^2,2}(N) = u(2) + 1 = 6$ and

$$W_{S_2^C,C}(N) = \frac{n_{3,C}(N) + n_{4,C}(N) + n_{5,C}(N) - C \cdot W_{S_1^C,C}}{C}. \tag{4.24}$$

Since we are addressing the case for fully packed wavelengths, the *additional* 4-node circles must be in pairs. Every two pairs of 4-node circles can be groomed on two wavelengths as described above, and therefore we can have at most one wavelength left with two 4-node circles.

Case 2b: $1 \leq r < \frac{g'}{2}$ **and** $2 < C \leq g'$ The specific cases we are interested in are for $1 \leq r < \lfloor \frac{g'}{2} \rfloor$. The case $r = \lfloor \frac{g'}{2} \rfloor$ is the case for $C = 2$ discussed above. When using 3-node and 4-node circles, the minimum number of nodes for grooming C circles in a wavelength as determined from Eq. (4.14) is $u(C)$. The number of circles with three and four traffic streams to be groomed in a wavelength is given by Eqs. (4.18), (4.15), and (4.16). After grooming circles in the partially packed wavelength, the number of remaining circles with three and four nodes is determined using Eqs. (4.9), (4.10), (4.19) and (4.20) as,

$$n_{4,C}(N) = n_4(N) - p_{4,C}(N), \tag{4.25}$$

$$n_{3,C}(N) = n_3(N) - p_{3,C}(N). \tag{4.26}$$

From Eq. (4.14), (4.15), and (4.16), it can be shown that at least $u(C)$ ADMs are required to pack C circles in $(v_4(u(C)) \times 4 : v_3(u(C)) \times 3)$ combinations. Then $W_{S_1^C,C}(N)$ can be expressed as

$$W_{S_1^C,C}(N)C = \begin{cases} \lfloor \frac{n_{3,C}(N)}{v_3(u(C))} \rfloor & \text{if } v_4(u(C)) \cdot n_{3,C}(N) \leq v_3(u(C)) \cdot n_{4,C}(N), \\ \lceil \frac{n_{4,C}(N)}{v_4(u(C))} \rceil & \text{otherwise.} \end{cases} \tag{4.27}$$

In the former case, we have extra 4-node circles while in the latter we have extra 3-node circles. Therefore, in the former case S_2^C requires one more ADM than u_C. Thus,

$$d_{S_1^C,C}(N) = u(C), \tag{4.28}$$

$$d_{S_2^C,C}(N) = \begin{cases} u(C) & \text{if } v_4(u(C)) \cdot n_{3,C}(N) \geq v_3(u(C)) \cdot n_{4,C}(N), \\ u(C)+1 & \text{otherwise.} \end{cases} \tag{4.29}$$

and,

$$W_{S_2^C,C}(N) = \frac{n_{3,C}(N) + n_{4,C}(N) - C \cdot W_{S_1^C,C}}{C}. \tag{4.30}$$

The general ADM lower bounds can then be calculated by summing up ADMs used on each wavelength and is given in the following theorem.

Theorem 1: The general ADM lower bounds in a BLSR network with N nodes under all-to-all uniform traffic model (r, g) are

$$D_{LB}(N, g, r) = \begin{cases} N & \text{if } C \geq \mathcal{C}_{LB}, \\ \sum_{C, S_p^C, S_1^C, S_2^C} d_{G,C}(N) \cdot W_{G,C}(N) & \text{otherwise,} \end{cases} \tag{4.31}$$

where $C = \lfloor \frac{g'}{r} \rfloor$ and $g' = g$ for BLSR/4 and $g' = \frac{g}{2}$ for BLSR/2 networks. $C \geq \mathcal{C}_{LB}$ is the situation when all the circles can be groomed onto a single wavelength. □

5 Proposed Circle Grooming Algorithm

This section presents heuristic algorithms for grooming uniform all-to-all traffic streams in SONET/WDM BLSR networks. We divide the traffic grooming in two phases: (1) constructing traffic circles, and (2) grooming the circles onto appropriate wavelengths. **Algorithm I** and **Algorithm II** that construct the *optimal* number of circles for odd and even number of nodes (N) are described in Section 3 and are used in the first phase. For an odd N, all circles are closed. For an even N, $\frac{N(N-2)}{8}$ circles are closed and $\lfloor \frac{N}{4} \rfloor + 1$ circles are open-ended. The time complexity of the algorithms is $O(N^2)$ in both cases.

We have seen in **Algorithm II** (for even N) that the last open-ended circle involves $2 \cdot \lceil \frac{N}{4} \rceil$ nodes, which becomes quite large for a large N, compared to the average number of nodes in a circle given by Eq. (4.2). We, therefore, develop an auxiliary algorithm, **Algorithm III**, that restructures the circles obtained in Algorithm II in order to balance the number of nodes involved in the last open-ended circle. We mentioned in Section 3 where Algorithm II is described that in the last open-ended circle every traffic stream has disjoint nodes. Two

```
Procedure adjust_LOE_Circle () {
   Loop1: while(number of nodes in s > 6) {
      i = 0;
      Loop 2: {
         ts_1 = s(i, i+1) and ts_2 = s(i+2, i+3);
         Check whether ∃t(k, k+1) such that
         t'[k] = s'[i] and t'[k+1] = s'[i+3];
         if (such traffic exists) {
            interchange ts_1 and ts_2 with t;
            goto Loop 1
         } // end if
         else increment i by 2 and goto Loop 2
      } // end Loop 2
      if(no more interchange possible)
         terminate;
   } // end Loop 1
} // end
```

Figure 4: Algorithm III for balancing the number of nodes in circles in BLSR with even N.

consecutive traffic streams in this circle involve four nodes with a "gap" between the ending node of a traffic stream and the starting node of the following traffic stream. We describe the auxiliary algorithm below.

Outline of Algorithm III:

Let m be the number of nodes involved in the last open-ended circle. Let s be the open-ended circle such that $s(i, j)$ is a traffic stream in s between its ith and jth nodes. Let t be any other 3-node or 4-node circle such that $t(k, l)$ is a traffic stream between its kth and lth nodes. Let $s'[i]$ be the ith node of s and $t'[j]$ be the jth node of t. Let ts_1 and ts_2 be two consecutive traffic streams (there may be a gap between them) in the last open-ended circle. Figure (4) outlines the algorithm using the notations just defined.

We have found several cases where the improvement on the number of ADMs is quite substantial after Algorithm III is used. The following example shows that the outcome of applying **Algorithm III** to the circles constructed by **Algorithm II**.

Example 4: N=22

We consider only those circles that are affected by **Algorithm III**. Initially, $s = (0-1, 2-3, 4-5, 6-7, 8-9, 10-21), m = 12$. In the first iteration, $t = (0-3-11-14-0)$. After the first iteration of the *while* loop, $s = (0-3, 4-5, 6-7, 8-9, 10-21), m = 10$, $t = (0-1, 2-3-11-14-0)$. In the second iteration, $t = (0-5-11-16-0)$. After the second iteration, $s = (0-5, 6-7, 8-9, 10-21), m = 8$, and $t = (0-3, 4-5-11-16-0)$. In the third iteration, $t = (6-9-17-20-6)$. After the third iteration, $s = (0-5, 6-9, 10-21), m = 6$, and $t = (6-7, 8-9-17-20-6)$. Since $m = 6$, the algorithm terminates after the third iteration.

Notice that both the last open-ended circle and the newly restructured circle have six nodes, and that the three closed circles are converted into open-ended circles. The number of ADMs obtained from running **Algorithm IV** (which is described below) without applying **Algorithm III** for case $g = 16, r = 2$ is 95, and is 84 if **Algorithm III** is applied – a savings of about 12%. □

We now propose a heuristic algorithm (termed as Algorithm IV) for grooming circles, closed or open-ended, using a simple greedy approach.

Algorithm IV Steps:
Input: A list of circles constructed using the previous algorithms.
Output: A list wavelengths (using the minimum number of wavelengths required) in which at most one wavelength could be partially packed and all other wavelengths are fully packed. Each wavelength contains information about the placement of ADMs.

- Step 1: Sort the circles in a list in descending order of the number of nodes involved in the circles;
- Step 2: Create a new wavelength channel and groom the first circle in the list onto the wavelength;
- Step 3: If the wavelength has no room for grooming another circle go to Step 8, otherwise, groom a circle, with all end-nodes overlapping the existing nodes in the wavelength channel if any, and repeat Step 3, otherwise go to Step 4.
- Step 4: Groom a circle, with one additional end-node if any, and go to Step 3 otherwise go to Step 5.
- Step 5: Groom a circle, with two additional end-nodes if any, and go to Step 3 otherwise go to Step 6.
- Step 6: Groom a circle, with three additional end-nodes if any, and go to Step 3 otherwise go to Step 7.
- Step 7: Groom a circle regardless of the number of additional end-nodes and go to Step 3.
- Step 8: If there are more circles to be groomed then go to Step 2, otherwise terminate.

Although better results are obtained in most cases when the circles are sorted in descending order (Step 1), there are a few cases where better results are obtained without sorting the circles, i.e. without applying Step 1. We run Algorithm IV with and without sorting the circles, as well as with and without applying Algorithm III (applicable only for even N) and obtain the best results as the final traffic grooming.

6 Numerical and Simulation Results

In this section, we evaluate and compare our lower bounds with those obtained in earlier work wherever applicable. We also study and evaluate the performance of the greedy algorithm for traffic grooming described in the previous section. As mentioned earlier, our bounds are very general in the sense that they can be applied for both BLSR/2 and BLSR/4 and that they are equally applicable to any integral value of g and r. In order to better compare with the results of previous work, we present our results for $g = 16$ for BLSR/4 for the same networks. We do not present the results for the case $r = g$, where there is practically no grooming and all bounds exhibit identical performance.

6.1 ADM Lower Bounds

Like our bounds, lower bounds derived in Gerstel et al. [1] are general. But they assume the availability of wavelength conversion capability in the network. Our bounds are tighter than theirs (Gerstel bounds) in

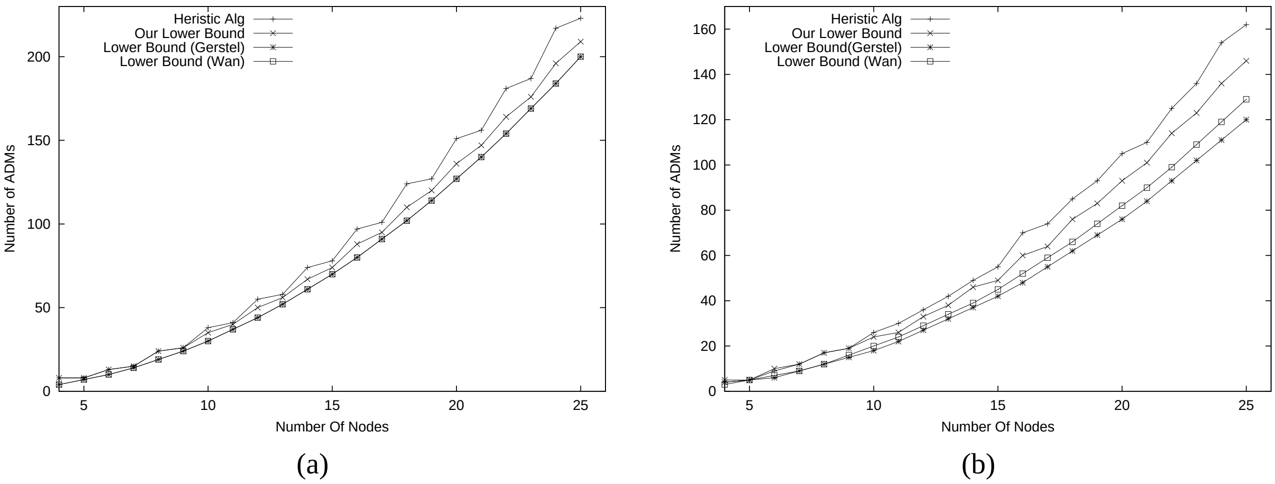


Figure 5: (a) Number of ADMs required when $g = 16$ and $r = 8$; (b) Number of ADMs required when $g = 16$, and $r = 4$.

every case in general as shown in Figs. 5-7. The bounds of [1] are equal to ours only when the total traffic in the network is less than or equal to one wavelength such that every node requires exactly one ADM. The bounds in [1] is closer to ours for traffic streams with a larger magnitude (shown in Fig. 5(a)). As the size of the traffic stream decreases (for a given g), their bounds get looser for a larger number of nodes – about 42% worse than ours for a 25-node network when $r = 1$ and $g = 16$ as shown in Fig. 7(b).

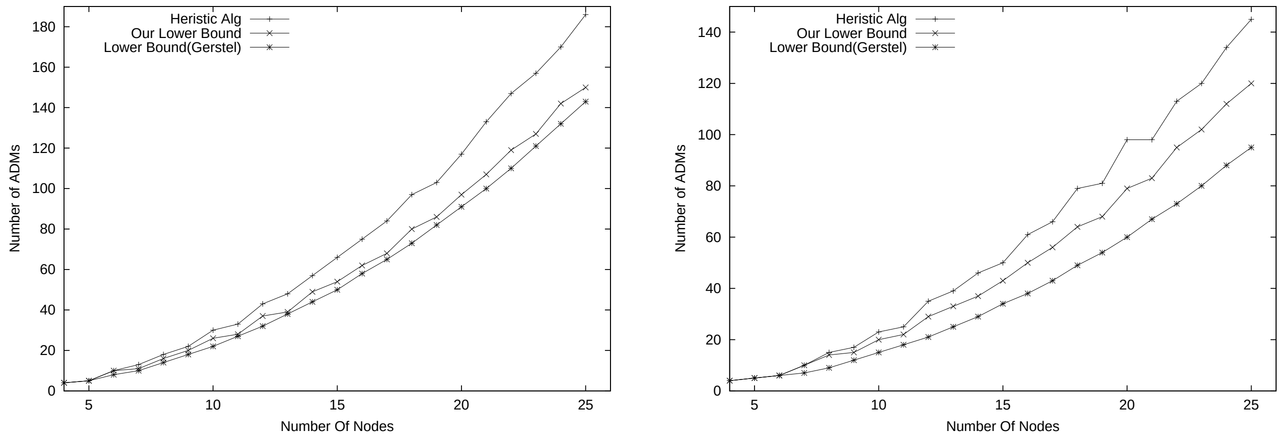
The bounds in Wan et al. [10] are derived on an implicit assumption that g is always completely divisible by r and for BLSR/4 networks. It is not clear whether their bounds can be applied to BLSR/2 networks. Fig. 5 and Fig. 7 demonstrate that our bounds are consistently tighter than the bounds of Wan et al. [10] in every case. However, the bounds of Wan et al. [10] are not applicable for cases when g is not completely divisible by r (i.e., the corresponding curve is not available in Fig. (6)). Note that the bounds of Wan et al. [10] and Gerstel et al. [1] are identical when $r = \frac{g}{2}$ as evident from Fig. 5(a).

Like Wan et al. [10], the bounds in Zhang et al. [2] are calculated for cases when g is always completely divisible by r and for BLSR/4 networks. Because no expressions are derived for the bounds, we do not have the exact values for comparison. However, it is evident from the paper's graphical presentation (Fig. 4(b) of Zhang et al. [2]) that our bounds are much tighter than those of [2] by a large margin in all cases.

Simmons et. al. [5] studied BLSR networks for odd number of nodes only. They derived an expression based on what they call "super-node approximation" to approximate the number of ADMs required and compared that with the number of ADMs obtained by manual grooming. It turns out that this approximation expression is quite loose.

6.2 Heuristic Algorithms

Algorithm IV described in Section 5 grooms uniform all-to-all traffic using the *optimal number of wavelengths*. We strongly believe (though without proof) that the number of ADMs could not be reduced further simply by using more wavelengths in BLSR networks.



(a) (b)

Figure 6: (a) Number of ADMs required when $g = 16$ and $r = 5$; (b) Number of ADMs required when $g = 16$ and $r = 3$.

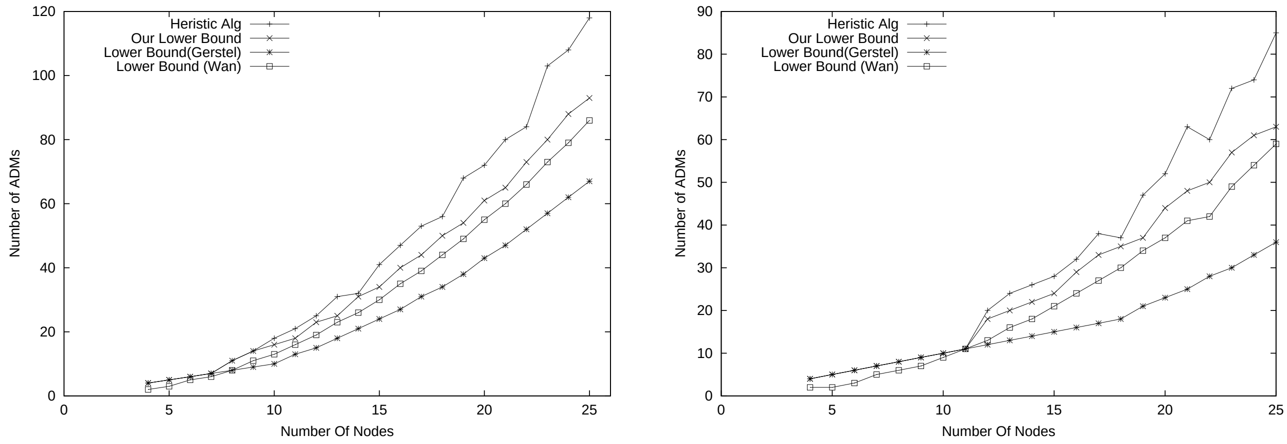
The results of circle grooming using Algorithm IV depend on the order in which circles are groomed onto wavelengths. We run Algorithm IV with and without sorting the circles (Step 1 in Algorithm IV), as well as with and without applying Algorithm III (applicable only for even N) and choose the best results as the final circle grooming. The required number of ADMs are shown in Figs. 5-7 to compare with the ADM lower bounds.

As seen in Figs. 5-7, in general, our proposed heuristic algorithm performs very well with traffic streams of larger magnitude (r). Fig. 5(a) and Fig. 5(b) show that the results obtained from the algorithm for cases $r = \frac{g}{2}$ and $r = \frac{g}{4}$ are very close to the corresponding lower bounds derived in this paper, particularly when N is odd. There are a number of cases where the algorithm generates the optimal number of ADMs for odd N. For even N, the algorithm requires relatively more ADMs because of the presence of open-ended circles. For other cases such as $r = \lfloor \frac{g}{3} \rfloor, r = \lfloor \frac{g}{5} \rfloor, r = \frac{g}{8}$, or $r = \frac{g}{16}$, the algorithm performs reasonably well as depicted in Fig. 6 and Fig. 7. Note that in cases when the total number of circles in the network can be accommodated in one wavelength, optimal results are obtained (Fig. 6).

We could not compare the performance of our algorithm with the heuristic algorithm presented in Zhang et al. [2] since the numbers are not available. In addition, their circle construction algorithm does not include all traffic streams when the number of nodes is even as pointed out earlier. However, rough visual comparison shows that our algorithm performs better than theirs. For example, our algorithm uses 127 ADMs and theirs requires about 131 ADMs (Fig. 4(b) of Zhang et al. [2]) when $N = 19$ and $r = 8$.

7 Conclusions

In this paper, we have studied traffic grooming in SONET/WDM BLSR networks under the uniform the all-to-all traffic model with an objective to reduce total network costs (wavelength and electronic multiplexing costs), in particular, to minimize the number of ADMs while using the optimal number of wavelengths. We



(a) (b)

Figure 7: (a) Number of ADMs required when $g = 16$ and $r = 2$; (b) Number of ADMs required when $g = 16$ and $r = 1$.

derive a new tighter lower bound for the number of wavelengths when the number of nodes is a multiple of 4. We show that this lower bound is achievable. We then derive new, more general and tighter lower bounds for the number of ADMs subject to that *the optimal number of wavelengths is used*, and propose heuristic algorithms (circle construction algorithm and circle grooming algorithm) that try to minimize the number of ADMs while using the optimal number of wavelengths in BLSR networks. Both the bounds and algorithms are applicable to any value of r and for different wavelength granularity g. Performance evaluation shows that wherever applicable, our lower bounds are at least as good as existing bounds and are much tighter than existing ones in many cases. Our proposed heuristic grooming algorithms perform very well with traffic streams of larger magnitude. The resulting number of ADMs required is very close to the corresponding lower bounds derived in this paper. This work can be extended in a number of avenues, e.g., consider a distance-dependent traffic model, take into account transceiver tunability, and so on.

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