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Constitutive Theory for Velocity Dispersion in Rock with Dual Porosity

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ABSTRACT: The high frequency behavior of the bulk modulus of fluid-saturated rock can be obtained from a double-porosity constitutive model, which is a direct conceptual extension of Biot's (1941) constitutive equations and which provides additional stiffening due to unrelaxed induced pore pressures in the soft porosity phase. Modeling the stiffening of the shear modulus at high frequency requires an effective medium average over the unequal induced pore pressures in cracks of different orientations. The implicit assumptions are that pore fluid equilibration does not occur between cracks of different orientations and between cracks and porous matrix. The correspondence between the constitutive equations of Berryman and Wang (1995) and Mavko and Jizba (1991) is explicitly noted.

INTRODUCTION

Biot's theory of wave propagation in a fluid-saturated, porous medium (Biot 1962a,b) tends to predict smaller amounts of velocity dispersion and wave attenuation in rock than is observed from measurements at ultrasonic frequencies (Winkler 1985; 1986). The discrepancy has usually been attributed to rock microstructural effects. Fluid flowing in cracks that connect pores adds a grain-scale, 'local' flow mechanism to account for the discrepancy. Mavko and Jizba (1991) developed a method to estimate the change in bulk and shear modulus between low and high frequency based on a model in which the stiffening is the result of unrelaxed induced pore pressures. The behavior at low frequencies is the same as for dry (drained) cracks and, at high frequencies, it is the same as for isolated (undrained) fluid-filled cracks. The model inputs are the pressure-dependence of the drained (dry) moduli.

In this paper we show the equivalence of Mavko and Jizba's formulation for bulk modulus to our phenomenological extension of Biot's quasistatic constitutive equations to a double porosity medium (Berryman and Wang 1995). A problem arises, however, for the shear modulus because no pore pressure changes are induced by shear stress in Biot theory. The solution requires addition of an effective medium component to the theory (Berryman and Wang 2001). Our focus is to show the one-to-one correspondence between the constants appearing in the two approaches.

CONSTITUTIVE EQUATIONS

Berryman and Wang's (1995) constitutive theory for a fluid-saturated, double-porosity medium extends formally Biot's single porosity theory (Biot 1941). The main postulate is the linearity of the dependence of strains and fluid-mass content on all the applied stresses and internal pore pressures. Initially it will be sufficient to describe the constitutive equations for a confining pressure p_c as the applied stress. A separate pore pressure can be maintained in each phase, designated $p_f^P(1)$ and $p_f^{(2)}$. Generally, phase (1) is considered to be a matrix containing stiff pores and phase (2) is interpreted to consist of soft cracks. Then

$$\begin{pmatrix} e \\ -\zeta^{(1)} \\ -\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} -p_c \\ -p_f^{(1)} \\ -p_f^{(2)} \end{pmatrix}, \qquad (1)$$

where e is volumetric strain, p_c is confining pressure (equal to the negative of the mean stress with the convention that extensional stresses are positive), $\zeta^{(1)}$ is the increment of fluid content in the matrix, and $\zeta^{(2)}$ is the increment of fluid content in the fracture. The coefficient matrix is symmetric. The coefficients $-a_{12}$ and $-a_{13}$ can be interpreted as *poroelastic expansion coefficients*, analogous to thermal expansion, because they represent the volumetric strain induced as a result of a fluid pressure increment in the stiff and soft porosity, respectively, while holding the external stress constant and the opposite phase drained.

The submatrix elements a_{22}, a_{33}, and a_{23} are *storage coefficients* of the matrix. Berryman and Wang (1995) argued that the cross-storage coefficient a_{23} may often be considered to be negligible.

UNRELAXED BULK MODULUS

Mavko and Jizba (1991) considered the pore space of the rock to consist of a distribution of crack shapes, which they divided into sets labeled i. To a first approximation, many rocks can be idealized by a bimodal pore shape distribution in which cracks constitute one porosity and equant pores constitute the second porosity. Mechanically, cracks are compliant, or 'soft,' and pores are relatively incompressible, or 'stiff.' Hydraulically, cracks can be isolated. Mavko and Jizba attributed the high frequency values of bulk and shear moduli to unrelaxed (undrained) induced pore pressures in the thinnest (crack) porosity. Fig. 1 shows schematically various combinations of fluid-pressure relations between the two types of porosity. The stiffest combination is when no local or global flow occurs in either the stiff or soft porosity, i.e., the induced fluid pressures are completely unrelaxed. Mavko and Jizba have made one choice among these alternatives, each of which can be treated explicitly in the context of the double-porosity formulation.

To show the equivalence between results obtained from our constitutive equations with the results of Mavko and Jizba (1991), we calculate the compressibility defined for a state in which cracks are undrained ($\delta\zeta^{(2)} = 0$) while the matrix is drained ($\delta p_f^{(1)} = 0$). These conditions are also illustrated schematically in Fig. 1. Application of the Mavko and Jizba conditions leads to [see Berryman and Wang 1995, Eqns. (36)-(38)]:

$$e = -a_{11}p_c - a_{13}p_f^{(2)} \tag{2}$$

$$-\zeta^{(1)} = -a_{21}p_c - a_{23}p_f^{(2)} \tag{3}$$

$$0 = -a_{31}p_c - a_{33}p_f^{(2)} \tag{4}$$

Then, Eqn. (4) directly leads to the pore pressure buildup coefficient for the undrained pore pressure in the soft (crack) porosity phase:

$$B[u^{(2)}] \equiv \left.\frac{\partial p_f^{(2)}}{\partial p_c}\right|_{\delta\zeta^{(2)}=\delta p_f^{(1)}=0} = -\frac{a_{31}}{a_{33}}. \tag{5}$$

And the effective bulk modulus for an undrained crack porosity phase is

$$\frac{1}{K[u^{(2)}]} \equiv -\left.\frac{\partial e}{\partial p_c}\right|_{\delta\zeta^{(2)}=\delta p_f^{(1)}=0} = a_{11} + a_{13}B[u^{(2)}], \tag{6}$$

where $a_{11} = 1/K$ is the drained bulk compressibility and $-a_{13}$ is the poroelastic expansion coefficient for the crack phase, i.e., the volumetric strain due to an increment in the fluid pressure in the crack phase while maintaining the external stress constant and drained conditions in the stiff porosity phase. Eqn. (6) carries the physical interpretation that the partially undrained strain is the difference between the totally drained strain and the poroelastic expansion due to the induced undrained pore pressure in the soft porosity phase.

Eqn. (6) can be expressed in the alternative form

$$\frac{1}{K[u^{(1)}]} + a_{12} = (a_{11} + a_{12} + a_{13}) + (a_{13} + a_{33})B[u^{(2)}]. \tag{7}$$

The sum $[-(a_{11} + a_{12} + a_{13})]$ is readily observed from the top row of Eqn. (1) to be the unjacketed compressibility, $1/K_s$, defined by the conditions that $\delta p_c = \delta p_f^{(1)} = \delta p_f^{(2)}$. Eqn. (7) is exact, but an approximate expression, based on the reasonable assumption, that $a_{23} = 0$ is given by [see Berryman and Wang 1995, following their Eqn. (77)]:

$$a_{12} \approx -v^{(1)}\frac{\alpha^{(1)}}{K^{(1)}} = v^{(1)}\left[\frac{1}{K^{(1)}} - \frac{1}{K_s^{(1)}}\right], \tag{8}$$

where $v^{(1)}$ is the volume fraction of phase 1 (approximately one), $\alpha^{(1)}$ is the Biot-Willis parameter of phase 1 alone, $K^{(1)}$ is the bulk modulus of phase 1 alone, and $K_s^{(1)}$ is the unjacketed bulk modulus of phase 1 alone. These values for the matrix phase are obtainable from a typical laboratory experiment in which elastic moduli of a dry sample are measured as a function of confining pressure (e.g., Coyner 1984) by obtaining the slope of the strain with confining pressure at a pressure above which the cracks are closed (25 MPa).

The second result we will use, which also follows from the assumption that $a_{23} = 0$, is that [see Berryman and Wang 1995, their Eqns. (60) and (76), and discussion immediately following Eqn. (77)]:

$$a_{13} + a_{33} \approx \frac{v^{(2)}}{K_f}, \tag{9}$$

where $v^{(2)}$ is the volume fraction of phase 2 (small compared with one) and K_f is the fluid bulk modulus. Therefore, Eqn. (7) becomes

$$\frac{1}{K[u^{(2)}]} - \frac{1}{K_s} = v^{(1)}\left[\frac{1}{K^{(1)}} - \frac{1}{K_s^{(1)}}\right] + \frac{v^{(2)}}{K_f}B[u^{(2)}]. \tag{10}$$

Eqn. (10) is essentially the same result as Mavko and Jizba's Eqn. (8) for the case of bimodal porosity (cf. comparison of notation in Table 1), if we identify their term $(d\Phi/d\sigma)_{\text{drained}}$ with the first term on the right-hand side of Eqn. (10), [cf. statement immediately preceding their Eqn. (9)].

UNRELAXED SHEAR MODULUS

In the double-porosity formulation, the shear modulus is still independent of the properties of the saturating fluids, i.e., undrained (wet) and drained (dry) shear moduli remain identical at higher frequencies, barring chemical effects (Berryman and Wang 2001). But experimentally and in effective medium theories, the presence of the liquid results in an increase in the shear modulus, even though the liquid shear modulus is zero. Why is that? The reason is that in an inhomogeneous medium, when we apply stress or strain at the macroscopic scale, that stress or strain gets resolved locally in a complicated way because of the inhomogeneities. An applied external compression can produce a shear field locally, and an applied external pure shear can produce a compression locally. That is the physical source of the effect. If we apply an external shear to a porous medium containing liquid, it matters that the liquid is present and not replaced by air. It matters because the external shear can be resolved into local compression in some regions containing the liquid. In these regions, the liquid can support the compression (but not a shear), and therefore the liquid stores some of the energy applied to the system by the external shearing force. On the other hand, if the liquid has enough time (and finite permeability permits it) to move out of the way, it can relax to a state that does not support any of the local compression, and then we have Gassmann's result (Gassmann 1951).

Mavko and Jizba (1991) derived an expression for shear stiffening at high frequencies by examining the consequence of the soft porosity being randomly oriented. Different pore pressures are induced in soft porosity with different orientations because a crack is compliant in the direction normal to its plane but stiff in the plane parallel to the crack. Mavko and Jizba assumed that each compliant crack has a direction of maximum compression and that perpendicular to this direction compressibility is negligible, that is, a crack is compressible only perpendicular to its plane and is infinitely stiff to any shear and normal stress in the plane of the crack. Thus, the soft porosity has the characteristics of penny-shaped cracks with very small aspect ratios. Mavko and Jizba estimated the *change in* shear modulus with saturation as a function of crack orientation and averaged over all solid angles. They found that the difference between the reciprocals of drained and undrained shear modulus is proportional to the difference between drained and undrained compressibility:

$$\frac{1}{G^{dry}} - \frac{1}{G^{sat}} = \frac{4}{15}\left(\frac{1}{K^{dry}} - \frac{1}{K^{sat}}\right). \tag{11}$$

The factor of 4/15 was the result of volume averaging over a uniform distribution of crack orientations. We note that Eqn. (11) carries the assumption that the different induced pore pressures in cracks of different orientation do not equilibrate.

Eqn. (11) can be obtained in a more formal manner, under the same assumptions as Mavko and Jizba (1991). In Barenblatt et al.'s (1960) picture of a double porosity material (Fig. 2), we take the heavy polygonal line segments to represent cracks bounding porous matrix. Locally a rock is anisotropic due to oriented cracks, but it can still be macroscopically isotropic when volume averaged. We consider the region surrounding each line segment to be an anisotropic element in terms of its elastic properties. This local region can experience an induced pore pressure in response to a shear stress, because pure shear in an anisotropic material induces a volume change (Cheng 1997). On this very local scale, the constitutive equation has the general form:

$$\begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ -\zeta \\ e_{23} \\ e_{31} \\ e_{12} \end{pmatrix} = \mathbf{M} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ -p_f \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} \tag{12}$$

where the matrix $\mathbf{M} =$

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} & -\beta^{(1)} & & & \\ S_{21} & S_{22} & S_{23} & -\beta^{(1)} & & & \\ S_{31} & S_{32} & S_{33} & -\beta^{(1)} & & & \\ -\beta^{(1)} & -\beta^{(1)} & -\beta^{(3)} & \gamma & & & \\ & & & & \frac{1}{G_t} & & \\ & & & & & \frac{1}{G_t} & \\ & & & & & & \frac{1}{G_{dr}} \end{pmatrix}.$$

The shear stiffening occurs all as a result of the crack porosity, so the increment of fluid content and pore pressure for the porous matrix have been dropped in Eqn. (12). The Reuss averages for the reciprocals of saturated bulk and shear modulus are (Berryman and Wang 2001):

$$\begin{aligned} \frac{1}{K_R^{sat}} &= 2S_{11}^{sat} + 2S_{12}^{sat} + S_{33}^{sat} + 4S_{13}^{sat} \\ &= 2S_{11}^{dry} + 2S_{12}^{dry} + S_{33}^{dry} + 4S_{13}^{dry} \end{aligned}$$

$$-\frac{(2\beta^{(1)}+\beta^{(3)})^2}{\gamma} \qquad (13)$$

$$\begin{aligned}\frac{1}{G_R^{sat}} &= \frac{1}{15}(8S_{11}^{sat}+4S_{33}^{sat}-4S_{12}^{sat}-8S_{13}^{sat}+\\ &\quad 6S_{44}^{sat}+3S_{66}^{sat})\\ &= \frac{1}{15}(8S_{11}^{dry}+4S_{33}^{dry}-4S_{12}^{dry}-8S_{13}^{dry}+\\ &\quad 6S_{44}^{dry}+3S_{66}^{dry})-\frac{4}{15}\frac{(\beta^{(1)}-\beta^{(3)})^2}{\gamma}. \qquad (14)\end{aligned}$$

Hence, the Reuss averages for the differences between the dry and saturated reciprocals of bulk and shear modulus for an isotropic aggregate of these transversely isotropic crack elements become

$$\frac{1}{K_R^{dry}}-\frac{1}{K_R^{sat}} = -\frac{(2\beta^{(1)}+\beta^{(3)})^2}{\gamma}, \qquad (15)$$

$$\frac{1}{G_R^{dry}}-\frac{1}{G_R^{sat}} = \frac{4}{15}\frac{(\beta^{(1)}-\beta^{(3)})^2}{\gamma}. \qquad (16)$$

Eqn. (16) can be rewritten in the form

$$G_R^{sat} = \frac{G_R^{dry}}{1-\frac{4}{15}\left(\frac{\beta^{(3)}-\beta^{(1)}}{G_R^{dry}}\right)B}, \qquad (17)$$

where $B=(2\beta^{(1)}+\beta^{(3)})/\gamma$ is the Skempton coefficient. Eqn. (17) can be compared with the standard poroelastic relationship

$$K_R^{sat} = \frac{K_R^{dry}}{1-\alpha B}, \qquad (18)$$

where the Biot-Willis coefficient $\alpha = 1-K^{dry}/K_s$ and K_s is the unjacketed bulk modulus.

The Mavko and Jizba (1991) approximation is that the only non-zero compliance in a local coordinate system is S_{33} as is $\beta^{(3)}$, and all other $S_{ij}=0$ and $\beta^{(1)}=0$. This assumption leads directly to the proportionality [see Eqn. (11)], which can be seen to be a consequence of the extremely simple compliance tensor for a crack. Eqn. (11) is the Reuss average, which is a lower bound. Berryman and Wang (2001) calculated Voigt and Reuss bounds for an example based on Berea sandstone values, which suggested, but did *not* prove, the shear modulus dependence on the pore *liquid properties* shown by the Reuss average.

The proportionality constant of 4/15 in Eqn. (11) is the limiting case only for cracks with very low aspect ratio, because the constant tends toward zero for spherical porosity (Mavko and Jizba 1991; Goertz and Knight 1998). While the derivation leading to Eqn. (11) might make it appear that the factor of $4/15$ is an upper bound for aspect ratios approaching zero, Berryman et al. (2002) used a differential effective medium approach to show that the result depends also on the assumption of a very small crack porosity. The factor $4/15$ underestimates the ratio when significant amounts of soft porosity are present.

DISCUSSION

Rock porosity occurs as a distribution of geometric shapes with associated differences in mechanical behavior. The preceding derivations for unrelaxed bulk modulus and shear modulus are based on a double porosity approximation, which is readily generalizable to multiple porosities. The response to bulk and shear deformations can be categorized into several different frequency ranges based on the degree of fluid-pressure equilibration by local fluid flow within and between the pores and cracks (Thomsen 1995; Pointer et al. 2000). Low-frequencies are defined by locally-equilibrated fluid pressure between cracks and between cracks and adjacent pores. For this assumption to hold, the characteristic fluid diffusion distance over one period is short relative to the wavelength but long relative to the microstructural scale. At the 'moderately high frequencies' associated with ultrasonic frequencies defined by Thomsen (1995), fluid does not move at all between cracks and pores. In between is a 'squirt flow' frequency at which significant energy loss due to local fluid flow can occur (Dvorkin et al. 1995).

Mavko and Jizba (1991) assumed that the equant porosity is drained. This assumption can be satisfied either by permeability sufficiently high to drain the equant porosity over a distance scale of the wavelength or by a very low Skempton's coefficient in the porous matrix phase. At the same time they assumed that the crack porosity did not drain to the equant porosity. In general, softening occurs when excess fluid pressure in cracks is dissipated into the more voluminous equant porosity. The equant porosity can also be a conduit for fluid communication between cracks, but flow between cracks will still be rate limited by fluid diffusion within cracks. The dual porosity formulation carries the assumption that $p_f^{(1)}$ and $p_f^{(2)}$ are averages over representative elementary volumes, thereby implying pressure communication within a given porosity fraction. Hence, at this level of modeling, shear behavior is frequency independent. Therefore, to have a theory consistent with the physical problem, it is necessary to treat delays in fluid communication between individual cracks under shear deformation more realistically, thereby stiffening the shear modulus as different induced fluid pressures are obtained in individual cracks. One simple way to introduce such as effect is to consider cracks of different orientation as individual sets that are not in fluid communication.

If Skempton's coefficient in the porous matrix phase has an intermediate value, then the appropriate unrelaxed bulk modulus for the high frequency elastic behavior might be the instantaneous undrained case represented in Fig. 1. One advantage of the constitutive theory approach is that it can be extended to elastic wave propagation, incorporating the appropriate permeabilities, to account for velocity dispersion as a function of intermediate frequencies (Berryman and Wang 2000).

CONCLUSIONS

A correspondence was established between the constitutive theory for a double porosity medium by Berryman and Wang (1995) and the expressions obtained by Mavko and Jizba (1991) for the effects of fluid saturation on bulk and shear moduli. The stiffening of the shear modulus with liquid saturation required explicit consideration of the anisotropic behavior of individually oriented cracks. In particular, the derivation carried the assumption that no fluid communication occurs between cracks of different orientations and that no fluid communication occurs between cracks and equant porosity of the porous matrix.

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	Berryman and Wang (1995)	Mavko and Jizba (1991)
Equant porosity	superscript (1)	stiff porosity
Crack porosity	superscript (2)	soft porosity
Saturated bulk modulus	K_u	K_{gs}
Bulk modulus of mineral grains	K_s'	K_o
Bulk modulus of dry rock	K	K_{dry}
Bulk modulus of pore fluid	K_f	K_f
Unrelaxed frame bulk modulus	$K[u^{(2)}]$	K_{uf}
Hydrostatic stress	$-\delta p_c$	$\Delta\sigma$
Volume of unrelaxed pore space	$V^{(2)}$	v_{pi} where $i = 2$
Induced pore pressure in unrelaxed pore space	$\delta p_f^{(2)} = B[u^{(2)}](\delta p_c)$	ΔP_i
Porosity of undrained pore space	$v^{(2)} \equiv V^{(2)}/V$	$\phi^{(i)} \equiv v_{pi}/V$
Total volume	V	V
Pore compressibility at constant differential stress	$1/K_o^*$	$1/K_\phi$

Table 1: Notational correspondence between Berryman and Wang (1995) and Mavko and Jizba (1991).

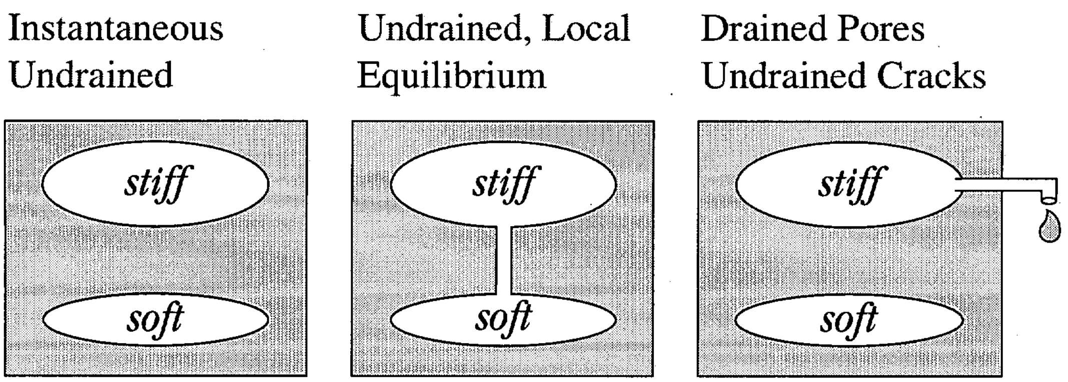

Figure 1: Three time scales following rapid compression in a double-porosity medium. (a) Totally undrained. (b) Local flow equilibrates fluid pressures between soft porosity (crack) and stiff porosity (equant pore). (c) Local flow is relatively slower than global flow through cracks, so that stiff porosity is drained and soft porosity is undrained. This case leads to the "unrelaxed bulk modulus" of Mavko and Jizba (1991).

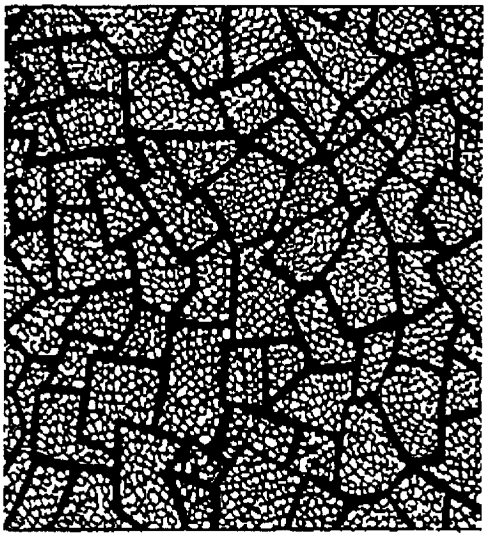

Figure 2: Barenblatt et al.'s (1960) picture of a double porosity medium. Cracks (polygonal line segments) produce local anisotropy, which can lead to induced pore pressures due to shear stress.