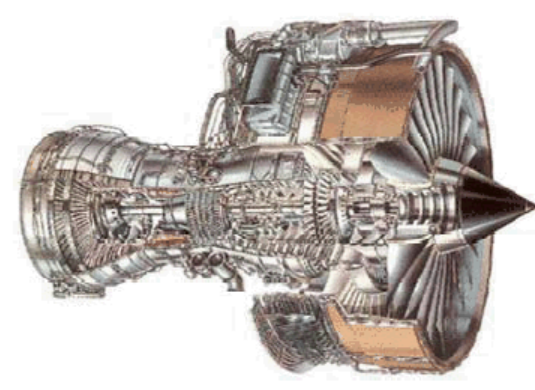
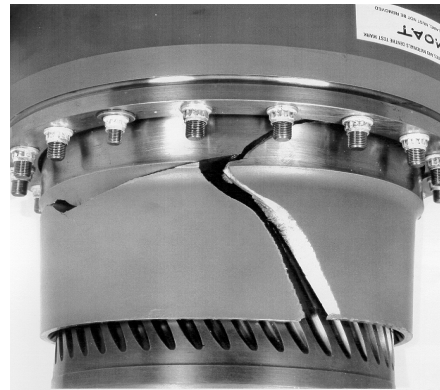
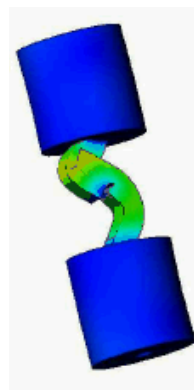
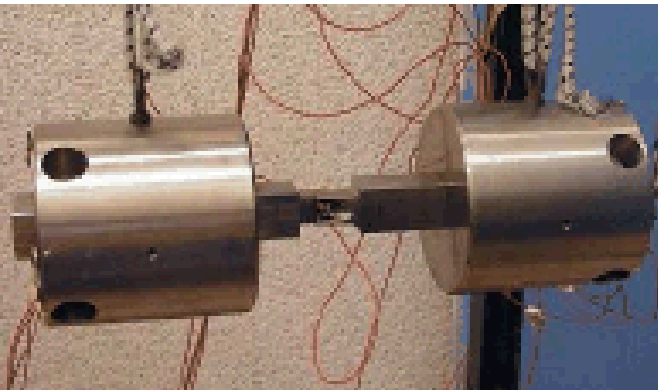


*Exceptional service in the national interest*



# Project 3: Numerical Round Robin for Prediction of Dissipation in Joints

Loic Salles, Christian Swacek, Robert Lacayo, Matt Brake

# Project 03 - Participants

- Loic Salles (France) – Imperial College, London – Research Associate
- Christian Swacek (Germany) – University of Stuttgart – Student
- Robert Lacayo (USA) – University of Wisconsin, Madison – Bachelor of Science
- Matt Brake (USA) – Sandia National Labs, Albuquerque – PhD, Mechanical Engineering

# Motivation

- Accurate prediction and measurement of single components
- Joints to put together different structures will introduce nonlinear behavior of jointed system
- Introduced level of nonlinearity may not be negligible for the structural dynamical behavior
- Different methods have been developed to model dissipation in joints.
- There is no benchmark to compare different models

# Project 03 - Numerical Round Robin

- Developing a benchmark model to compare different numerical routines
- Comparison of different methodologies to predict nonlinear dissipation behavior in joints
- Comparison of results obtained by different solvers
- Comparison of performance of different solvers
- Application of a high fidelity model and comparison to experimental data

# Numerical Methods

Stuttgart Method

Salinas

Imperial College: FORSE

# Approach Stuttgart

- Equation of Motion with nonlinear friction force


$$M\ddot{x} + D\dot{x} + Kx + f_T(\dot{x}, x) = f_{\text{exc}}$$

- Linearize friction force using Harmonic Balance Method

$$f_T(\dot{x}, x) \approx D_{\text{hbm}}\dot{x} + K_{\text{hbm}}x$$

- Linearized equation of motion in frequency domain

$$\underbrace{((K + K_{\text{hbm}}(\hat{x})) - i\omega(D + D_{\text{hbm}}(\hat{x})) - \omega^2 M)}_{H_{\text{hbm}}(\hat{x})} \hat{x} = \hat{f}_{\text{exc}}$$

$H_{\text{hbm}}(\hat{x})$   Pseudo-Receptance Matrix

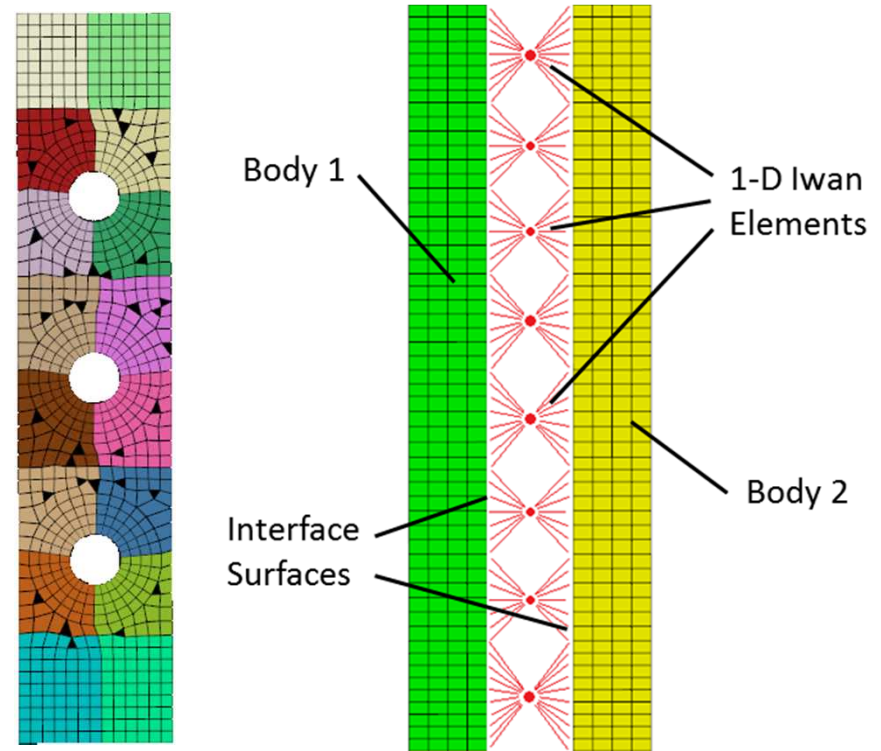
- Problem of the form

$$\hat{x} = H_{\text{hbm}}^{-1}(\hat{x}) \hat{f}_{\text{exc}}$$

# Approach: Salinas - Interface Model

## Model interface using Salinas Tied-Joint Iwan implementation

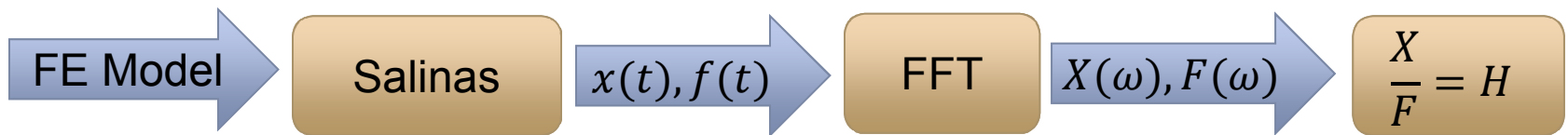
- Divide interface surfaces into smaller sections.
- Tie interfacing sections to a 1-D Iwan Element.



# Approach: Salinas - Getting the FRF

Run transient analysis using Implicit alpha-modified Newmark-Beta method.

Transform time response to frequency response using Fast Fourier Transform.



## Pros

- Segalman documents success with Iwan elements.

## Cons

- Computationally expensive.
- Time integrator may not converge.



# Harmonic Balance Method

- Truncated Fourier's Series

$$U(\tau) = \tilde{U}_0 + \sum_{n=1}^{Nh} \tilde{U}_{n,c} \cos(n\tau) + \tilde{U}_{n,s} \sin(n\tau)$$

$$f(\tau) = \tilde{f}_0 + \sum_{n=1}^{N_h-1} \tilde{f}_{n,c} \cos(n\tau) + \tilde{f}_{n,s} \sin(n\tau)$$

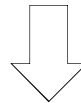
$$\tau = \omega t$$

- Galerkin's Procedure

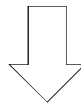
$$\int_0^{2\pi} f(\tau) d\tau = 0$$

$$\int_0^{2\pi} f(\tau) \cos(n\tau) d\tau = 0$$

$$\int_0^{2\pi} f(\tau) \sin(n\tau) d\tau = 0$$



$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}_T\mathbf{U} = \mathbf{F}_{ex}(t) - \lambda$$



$$\mathbf{Z}(\omega)\tilde{\mathbf{U}} = \tilde{\mathbf{F}}_{ex} - \tilde{\lambda}$$

# Harmonic Balance Method

- Algebraic system in frequency domain

$$\mathbf{Z}(\omega)\tilde{\mathbf{U}} = \tilde{\mathbf{F}}_{ex} - \tilde{\boldsymbol{\lambda}}$$

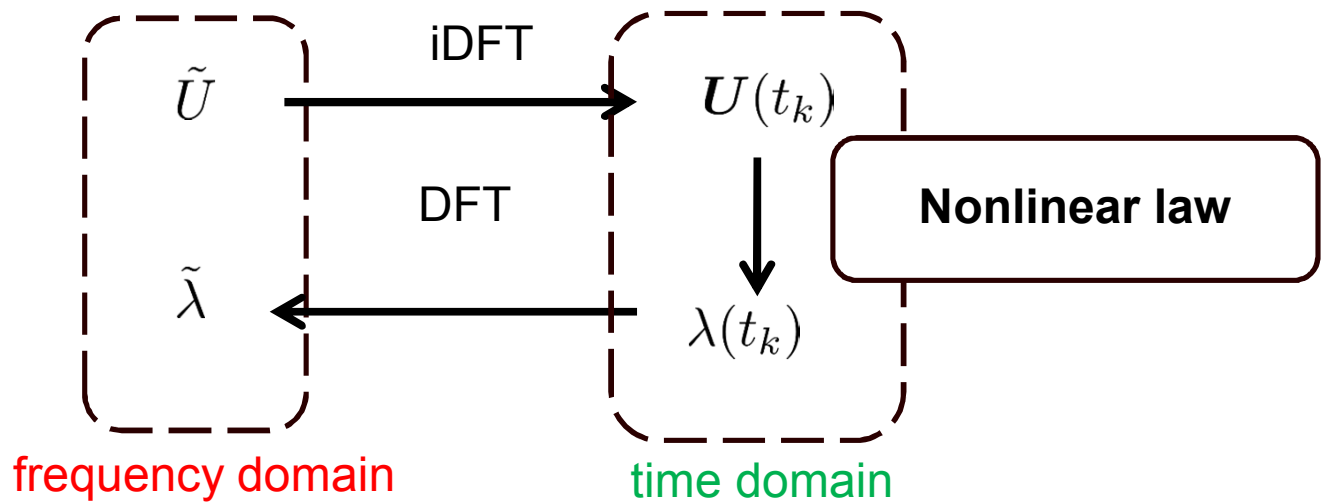
$\mathbf{Z}$  is the dynamic stiffness

$$\mathbf{Z} = \begin{bmatrix} \mathbf{K} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K} - (N_h\omega)^2\mathbf{M} & N_h\omega\mathbf{C} \\ \mathbf{0} & \mathbf{0} & -N_h\omega\mathbf{C} & \mathbf{K} - (N_h\omega)^2\mathbf{M} \end{bmatrix}$$

- Nonlinear Forces
  - Forces cannot be explicitly handled in the frequency domain
  - The AFT procedure is applied to calculate the coefficient of Fourier's series of forces in time domain

# AFT procedure

## Scheme of AFT procedure



# Contact Modelling

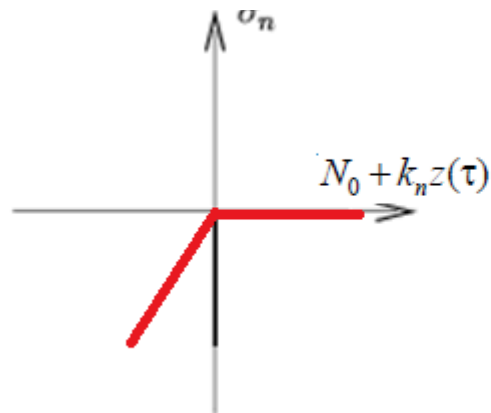
- Tangential stresses

$$\sigma_t = \begin{cases} \sigma_t^0 + k_t(x(\tau) - x_0) & \text{for stick} \\ \pm \sigma_n(\tau) & \text{for slip} \\ 0 & \text{for separation} \end{cases}$$

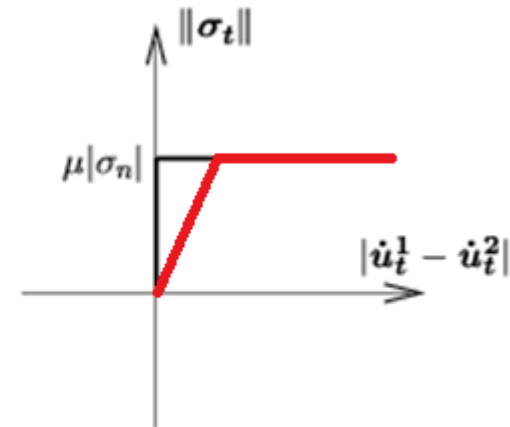
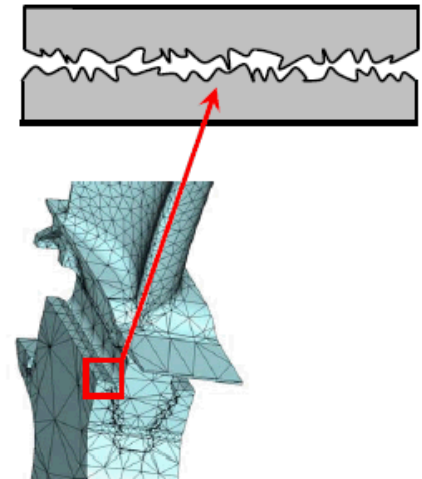
- Normal stresses

$$\sigma_n = \begin{cases} N_0 + k_n z(\tau) & \text{for contact} \\ 0 & \text{for separation} \end{cases}$$

$F_{NL}(U_0)$

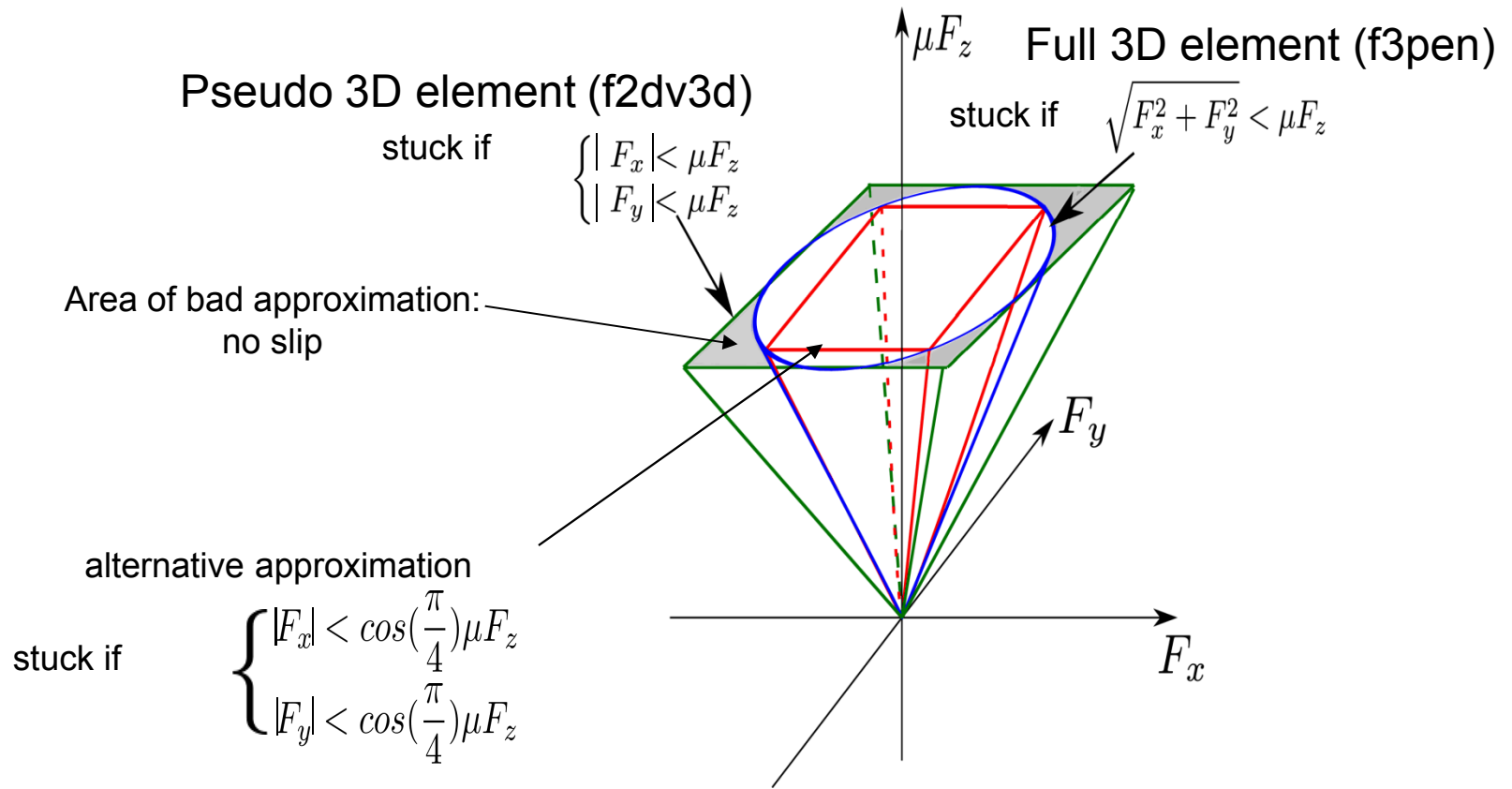


## Contact of rough surface



# 3D contact: friction cone

## Coulomb's friction cone



# Separation of problems

- Equation of motion

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{F}_{int}(\mathbf{U}) = \mathbf{F}_s + \mathbf{F}_{ex}(t) - \mathbf{F}_{NL}(\mathbf{U}, \dot{\mathbf{U}})$$

- Dynamic problem around static equilibrium position  $\mathbf{U}_0$

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{U}}_d + \mathbf{C}\dot{\mathbf{U}}_d + \mathbf{K}_T(\mathbf{U}_0)\mathbf{U}_d + \mathbf{K}_T(\mathbf{U}_0)\mathbf{U}_0 \\ = \mathbf{F}_s + \mathbf{F}_{ex}(t) - \mathbf{F}_{NL}(\mathbf{U}_0) - \Delta\mathbf{F}_{NL}(\mathbf{U}_d, \dot{\mathbf{U}}_d)\end{aligned}$$

- Separation of the static and dynamic problems

$$\begin{aligned}\mathbf{K}_T(\mathbf{U}_0)\mathbf{U}_0 &= \mathbf{F}_s - \mathbf{F}_{NL}(\mathbf{U}_0) \\ \mathbf{M}\ddot{\mathbf{U}}_d + \mathbf{C}\dot{\mathbf{U}}_d + \mathbf{K}_T\mathbf{U}_d &= \mathbf{F}_{ex}(t) - \Delta\mathbf{F}_{NL}(\mathbf{U}_d, \dot{\mathbf{U}}_d)\end{aligned}$$

# Reduced Order Modelling

$$\mathbf{Z}(\omega)\tilde{\mathbf{U}} = \tilde{\mathbf{F}}_{ex} - \tilde{\boldsymbol{\lambda}}$$

- Frequency response function

$$\tilde{\mathbf{X}}_c = \underbrace{\mathbf{B}_c \mathbf{Z}^{-1}}_{\mathbf{H}(\omega)} (\tilde{\mathbf{F}}_{ex} - \tilde{\boldsymbol{\lambda}})$$

- Modal synthesis

$$\mathbf{H}(\omega) = \mathbf{B}_c \mathbf{Z}^{-1} = \sum_{j=1}^{Nddl} \frac{\boldsymbol{\Phi}_j \boldsymbol{\Phi}_j^t}{(1 + i\eta_j)\omega_j^2 - \omega^2}$$

- Truncated modal synthesis with static correction

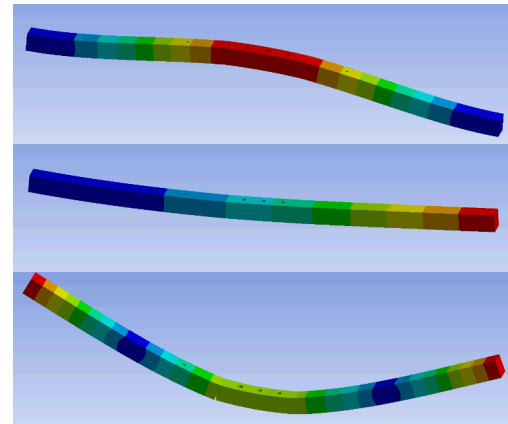
$$\mathbf{H}(\omega) \approx \sum_{j=1}^{Nr} \frac{\boldsymbol{\Phi}_j \boldsymbol{\Phi}_j^t}{\omega_j^2 - \omega^2} + \underbrace{\sum_{j=1}^{Nddl} \frac{\boldsymbol{\Phi}_j \boldsymbol{\Phi}_j^t}{\omega_j^2 - \omega_0^2}}_{(\mathbf{K} - \omega_0^2 \mathbf{M})^{-1}} - \sum_{j=1}^{Nr} \frac{\boldsymbol{\Phi}_j \boldsymbol{\Phi}_j^t}{\omega_j^2 - \omega_0^2}$$

with

$$Nr \ll Nddl$$

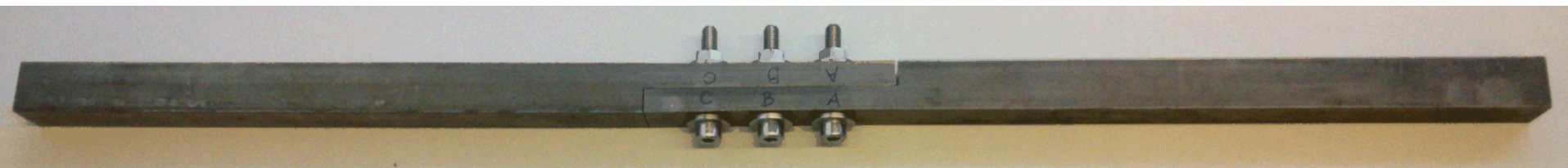
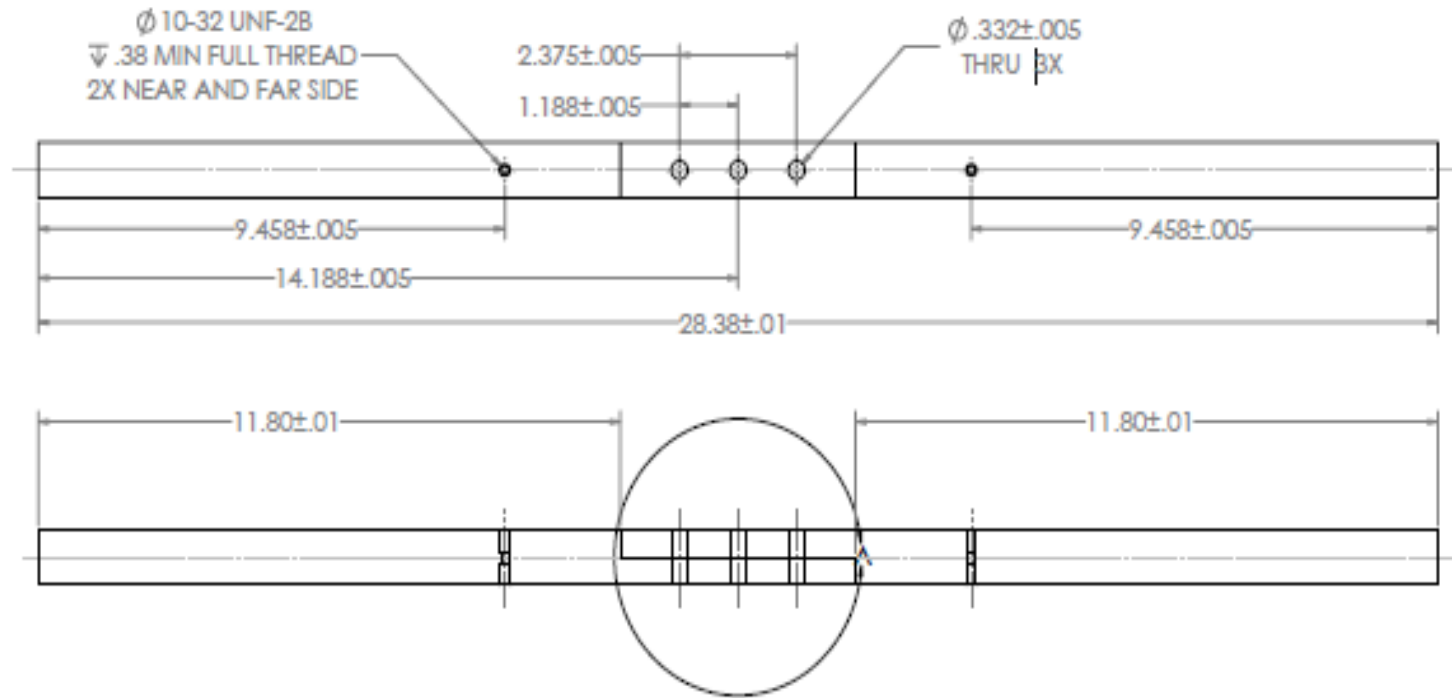
# Strategy of Round Robin

- Identification of benchmark model
- Set initial conditions, material properties and joint coefficients
- Boundary Conditions:
  - Fixed – Fixed
  - Fixed – Free
  - Free – Free
- Comparison to experimental data (Project 01)

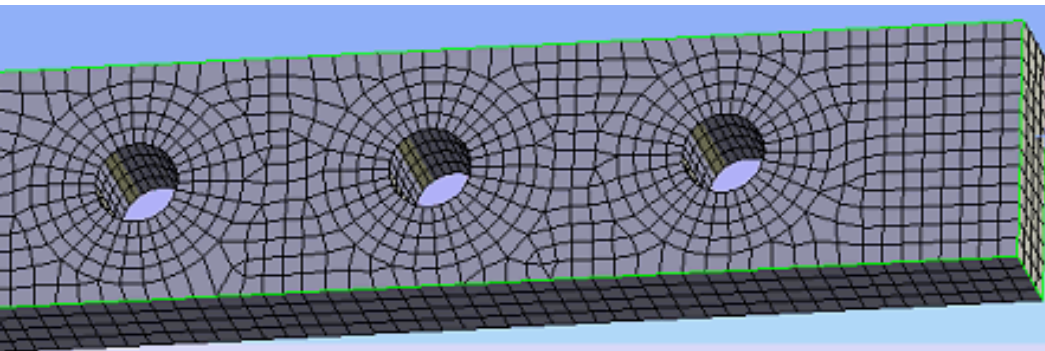
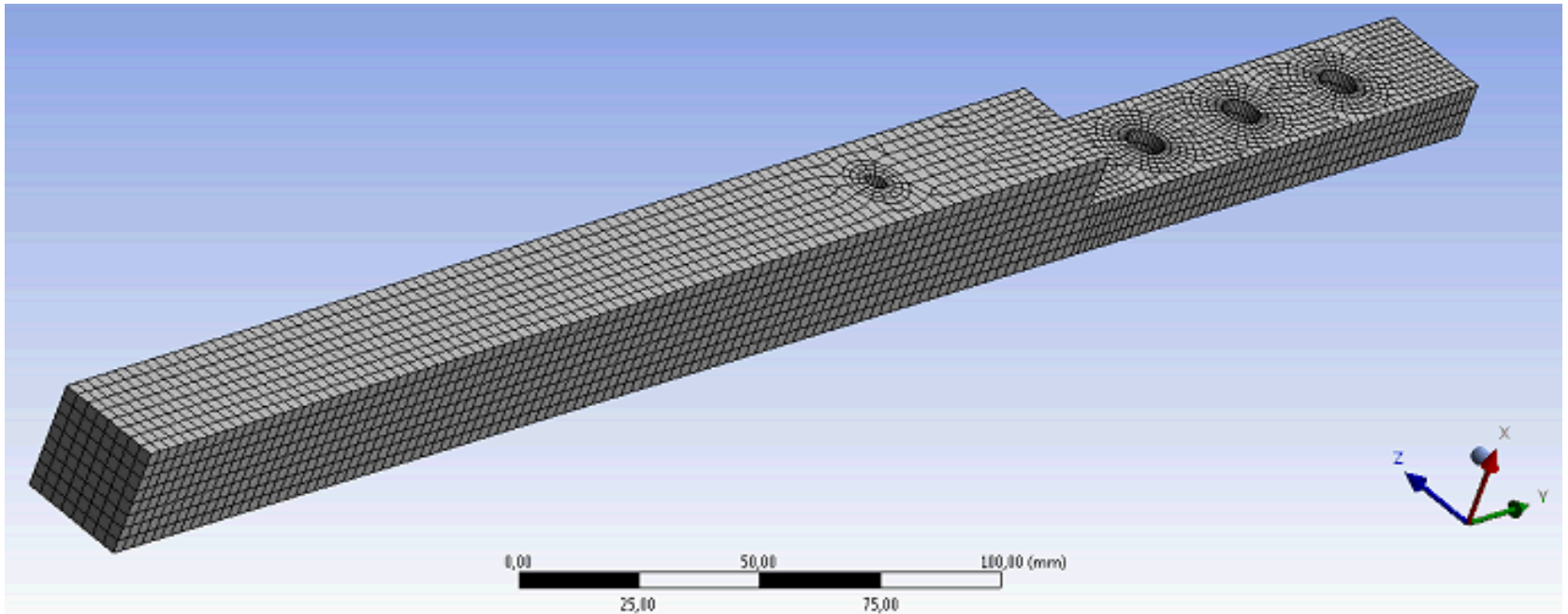




# Beam Model



# Numerical Model



Young's Modulus	$1.8248 \times 10^{11} \text{ N/m}^2$
Poissons Ratio	0.29
Density	$7900 \text{ kg/m}^3$

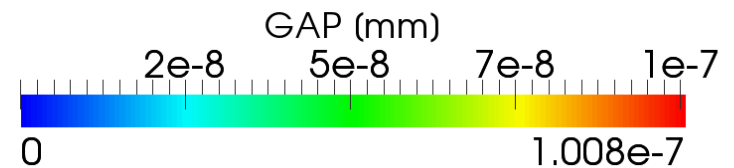
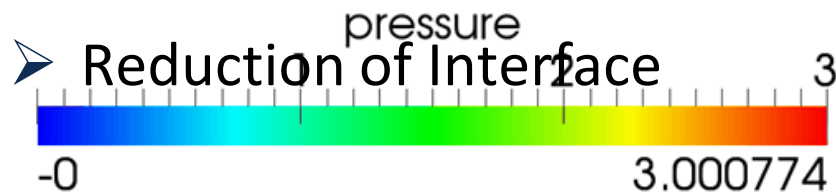
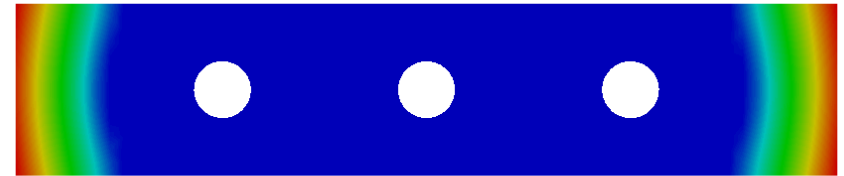
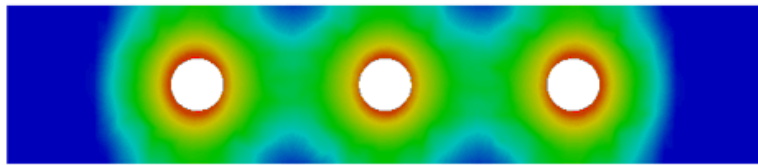
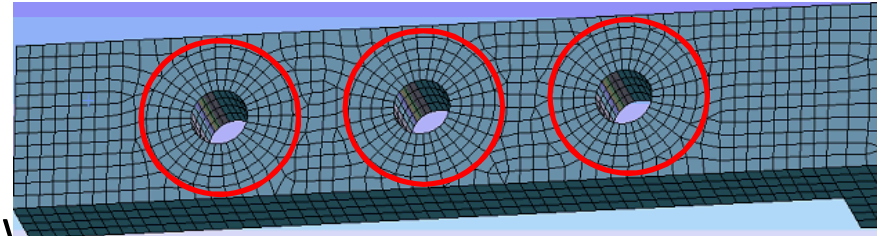
# First Steps

## ➤ Linear Modal Analysis and comparison of Substructures

Comparison of Eigen frequencies [Hz]				
Mode	Salinas	Code-Aster	Stuttgart-Matlab	ANSYS
1	141.644	142.224	142.224	142.22
2	143.032	143.595	143.595	143.59
3	605.463	609.419	609.419	609.42
4	738.391	741.113	741.113	741.11
5	1386.73	1394.42	1394.419	1394.40
6	1839.11	1845.14	1845.143	1845.10
7	1990.88	1991.59	1991.586	1991.60
8	2913.91	2927.57	2927.574	2927.60
9	3332.32	3333.85	3333.851	3333.90
10	3557.36	3568.74	3568.738	3568.70

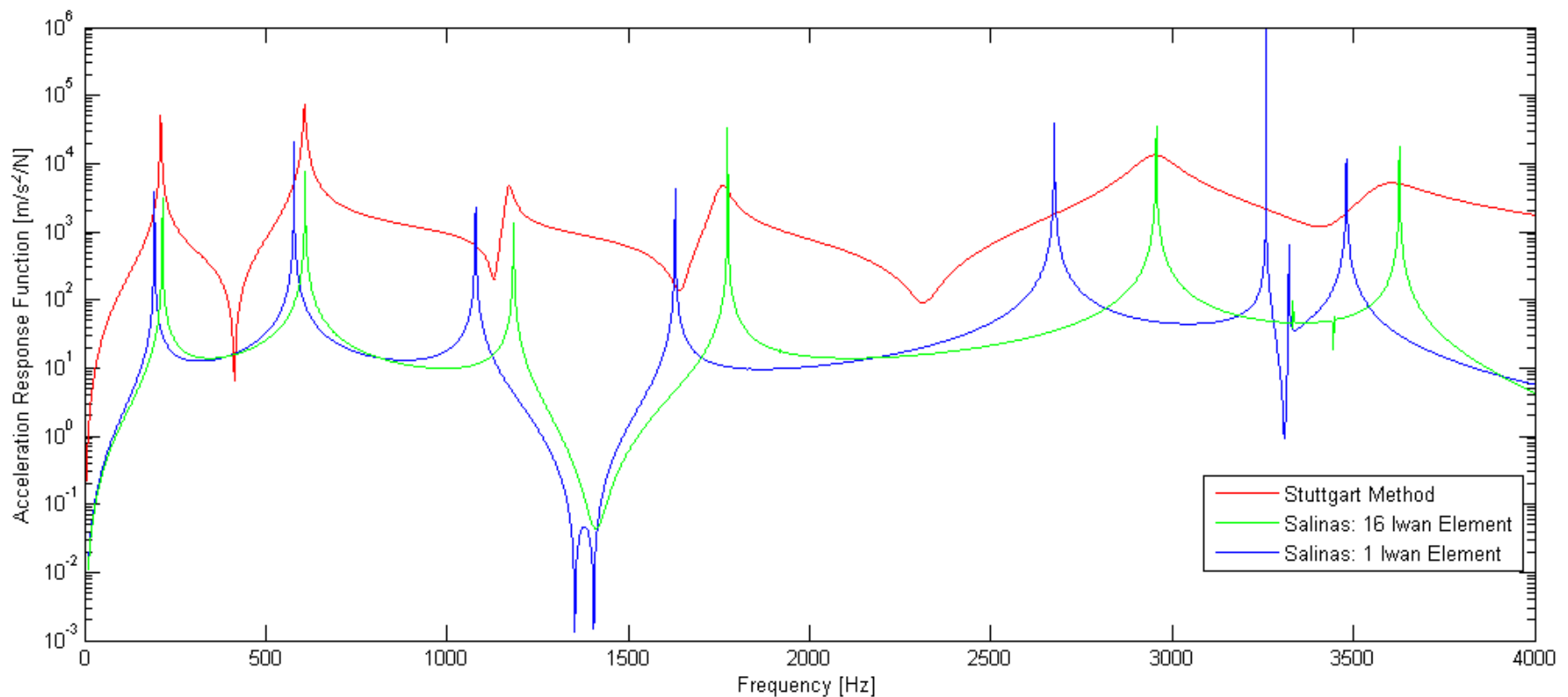
# First Steps

- Model of bolts:
  - Not modeled
  - Uniform pressure distribution on washer area
- Model of contact interface
  - Nonlinear contact static analysis



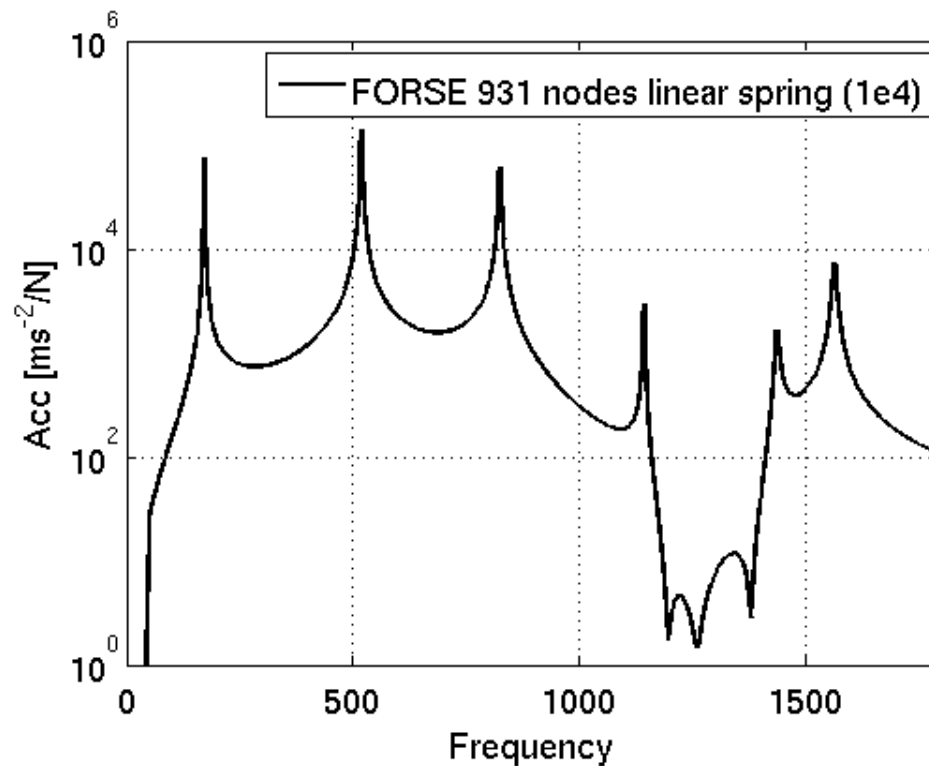
# Linear Results: Salinas

Defining multiple Iwan Tied Joints produces an FRF whose natural frequencies are more consistent with other results.



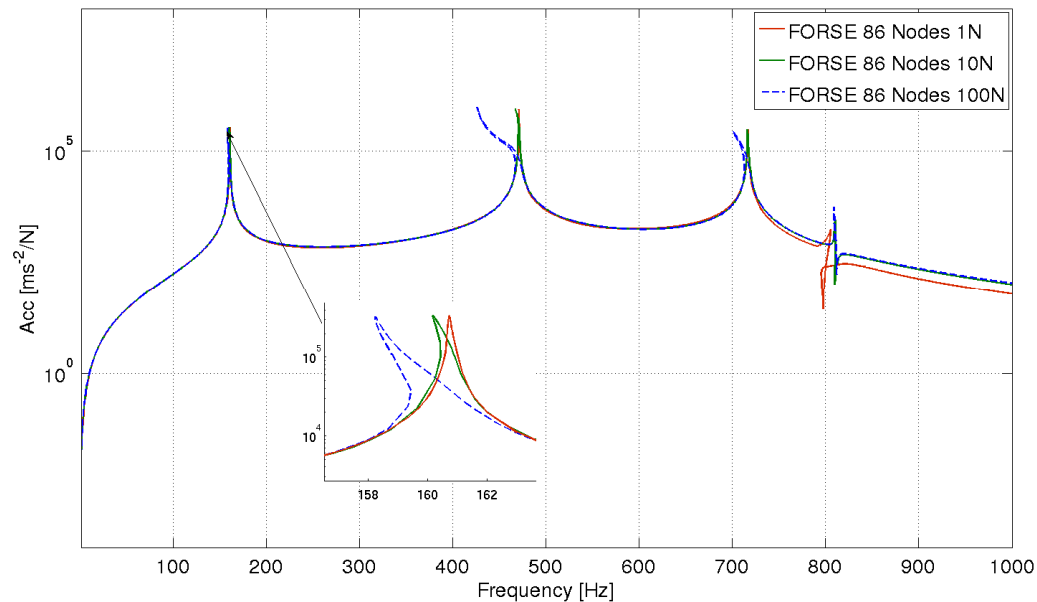
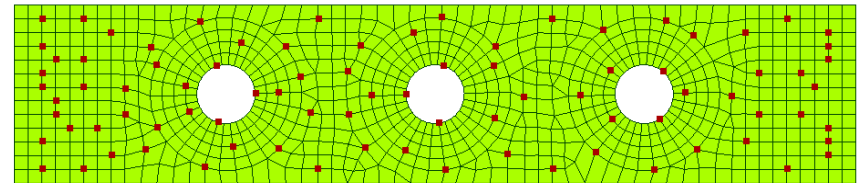
# Linear Results: FORSE

- 931 contact nodes
- Spring elements: 10000 N/mm



# Results FORSE

- 86 contact nodes
- Nodal Contact stiffness: 50000 N/mm
- Coefficient of friction: 0.3
- Full 3D contact elements



# Conclusions

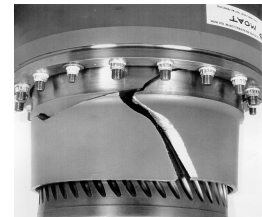
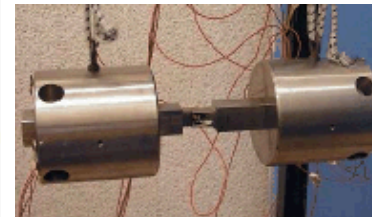
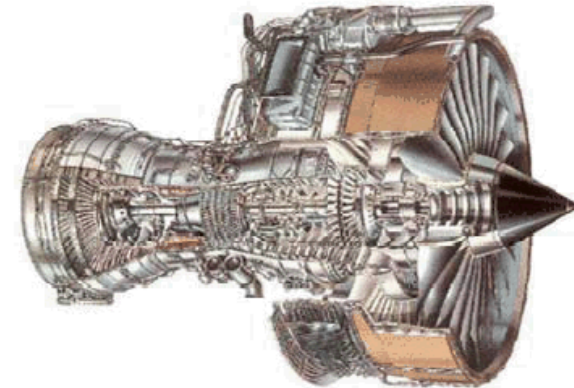
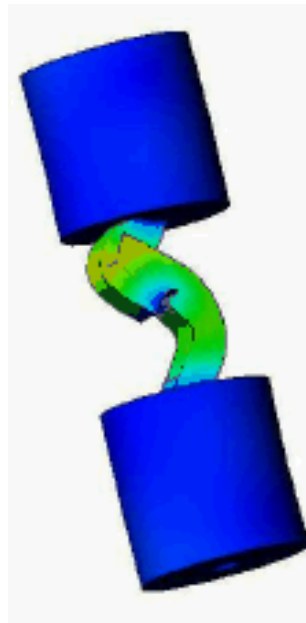
- Three methods tested
- No proper agreement between the different methods
- According to Imperial simulation not a lot of friction damping for this configuration (To verify)



Thank you for listening

Questions?

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