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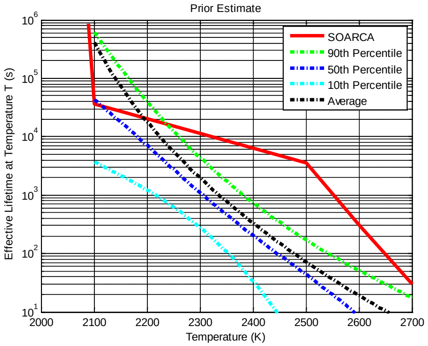

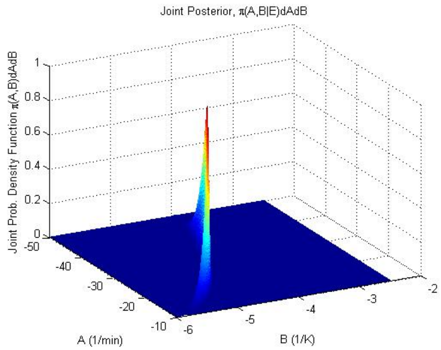

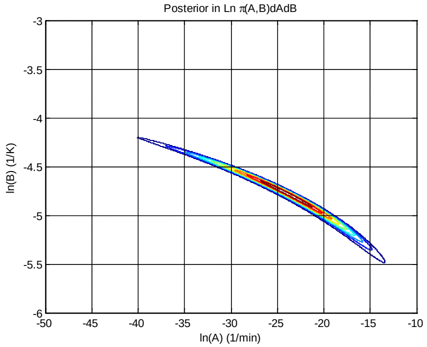

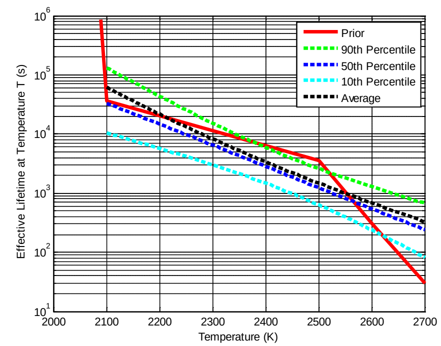

Fuel Rod Structural Integrity - Time at Temperature
Code Name: Shark-Fin

Matthew Denman
06231 - Risk and Reliability Analysis

U.S. DEPARTMENT OF ENERGY
NNSA National Nuclear Security Administration

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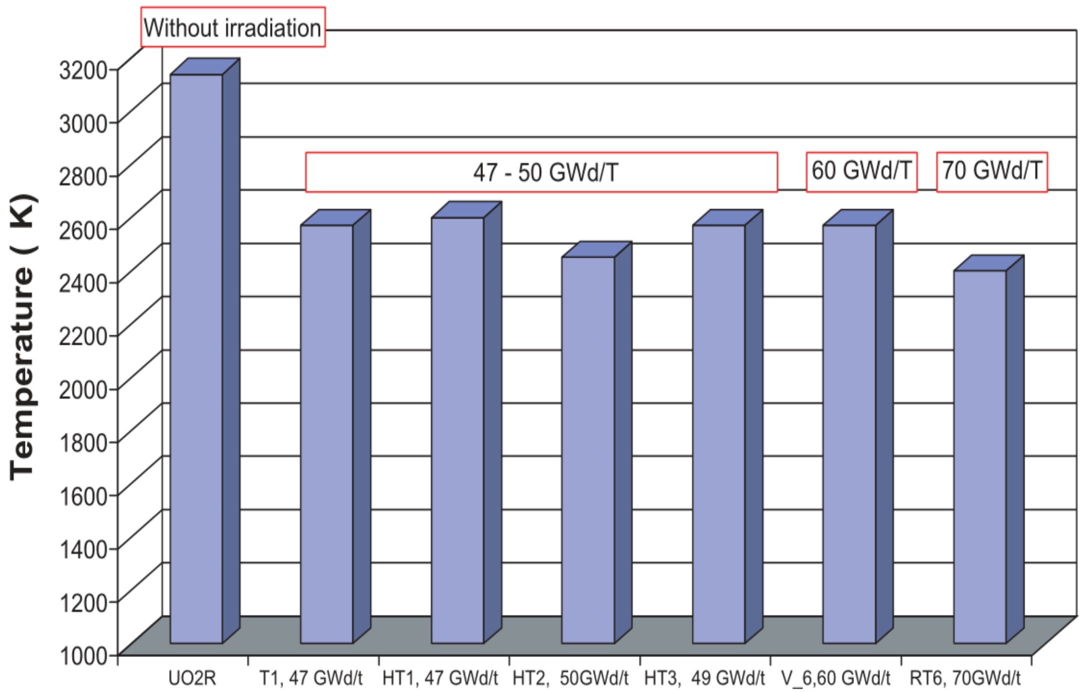
Objective

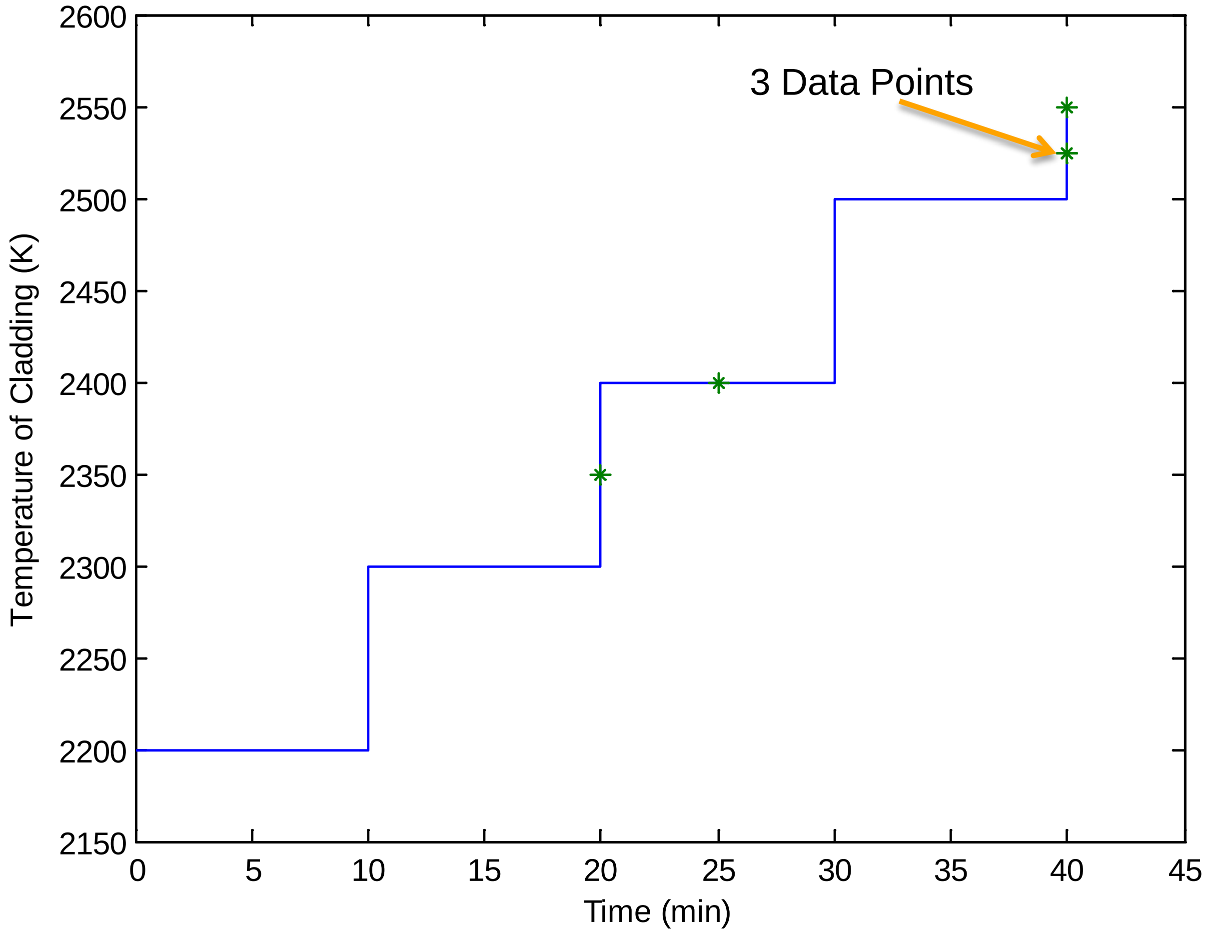
- Reframe the subjective fuel rod structural integrity time-at-temperature (TatT) curve to:
  - Incorporate high burn up collapse estimates from the VERCORS experiments.
  - Maintain engineering judgment in the shape of the time-at-temperature function.

Data – High Burnup VERCORS Tests

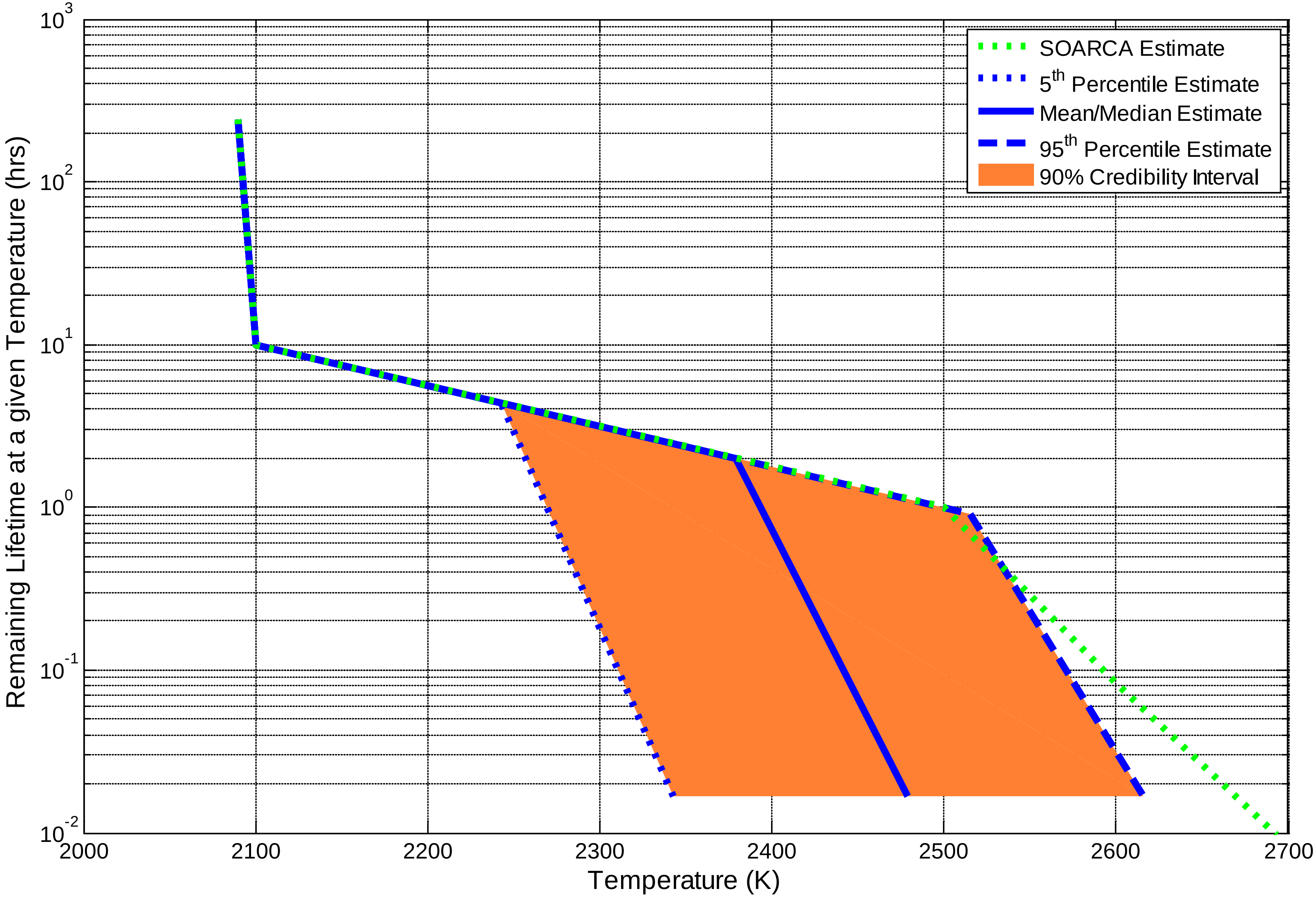
Test	*Collapse Temperature (K)*	*Driving Phenomena*
(R?)T1	2525	H_2O oxidizing atmosphere
HT1	2550	H_2 reducing atmosphere
HT2	2400	H_2O oxidizing atmosphere U–Zr–O–FP interaction
HT3	2525	H_2 reducing atmosphere
V_6 (RT4?)	2525	ZrO_2-“fuel”-FP Interaction
RT6	2350	H_2O oxidizing atmosphere
Mean	2479	
Standard Deviation	83	



Time/Temperature Profile for VERCORS Experiments



Prior Attempt to Incorporate VERCORS Data



Can we use a Bayesian Regression approach to do better?

- Assume simple damage model – Arrhenius
  - $\frac{1}{t(T)} = A * \exp(BT)$,D(t) $= \sum\left(\frac{1}{t(T)} * \Delta t\right)$
- Use prior uncertainty estimates to fit A/B values to create a prior understanding of probability of A and B
- Assume that failure of the fuel is lognormally distributed around a Damage =1.0
- Apply Bayes Theorem to create a better understanding of the relationship between A and B
  - $\pi(A, B|E, M)dAdB = \frac{L(E|A,B,M) * \pi(A,B|M)dAdB}{\int L(E|A,B,M) * \pi(A,B|M)dAdB}$

The Three Steps to Bayesian Updating

Define the Prior

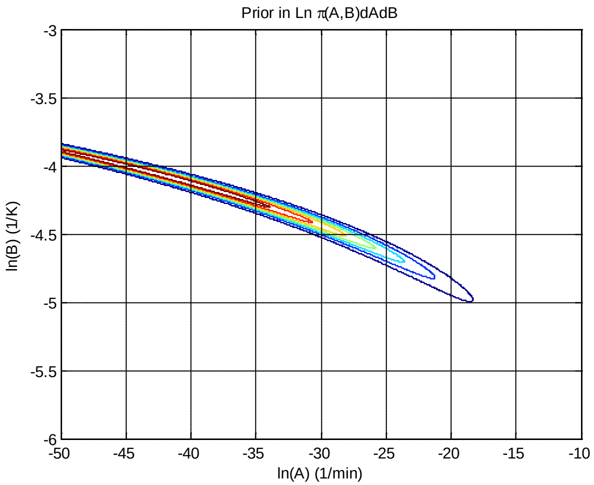
- Are there previous analyses that can be leveraged?
  - Experimental Data?
  - Expert Judgment?
- Are the parameters related?

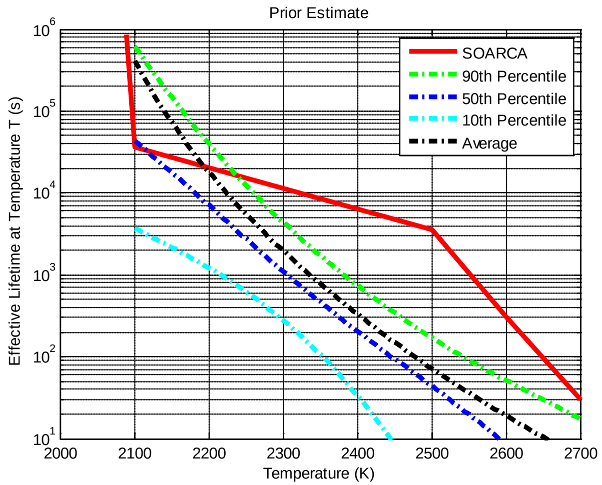
Define the Likelihood

- Does the model support the data?
- What type of variance from ideal is acceptable?

Compute Posterior

- Multiply the likelihood by the prior to develop your new understanding of the system



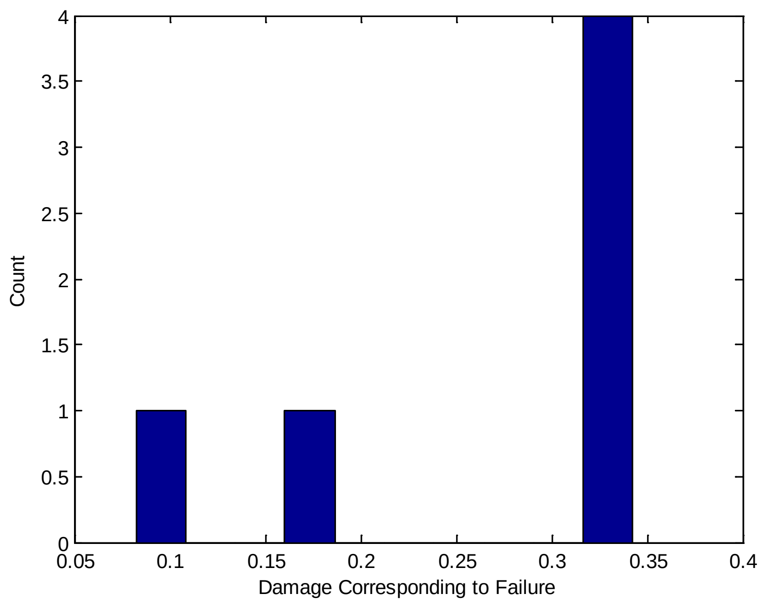


1. What is our current model?
2. What parameters (a, b, σ) are uncertain?
3. Are there relationships between the parameters?

DEFINE THE PRIOR

What is our current model?

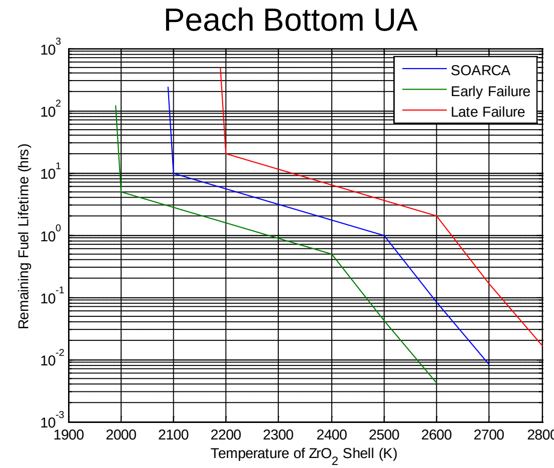
- Assume simple damage model – Arrhenius
  - $\frac{1}{t(T)} = A * \exp(BT)$,D(t) $= \sum\left(\frac{1}{t(T)} * \Delta t\right)$
- How well does it describe the data?
  - Low damage corresponding to failure
  - Does not appear lognormal
    - Small data-set?
  - Lognormal fit
    - μ
      - MLE = -1.42
      - 95%CI = [-2.04, -0.80]
    - σ
      - MLE = 0.59
      - 95%CI = [0.37, 1.45]

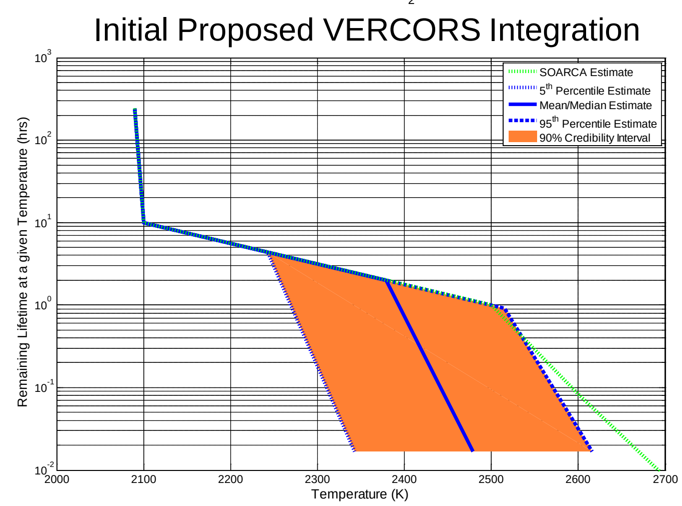


Defining prior understanding of A and B

Previous Uncertainty Characterization

- Two uncertainty treatments were leveraged:
  - Peach Bottom UA (Top Right)
  - Surry UA (Bottom Right)
    - Since removed as a UA parameter
- Proposed Surry Treatment
  - Lower temperature data point was assumed at 2200K
    - Beginning of the VERCORS temp. ramp
  - The median lifetime at 2200K was assumed to be 2 hours, with an error factor $(\frac{\lambda_{95}}{\lambda_{05}})$ of 10.
    - SOARCA curve predicts 5 hours at 2200K
    - The ratio of early failure to late failure lifetimes is 13.

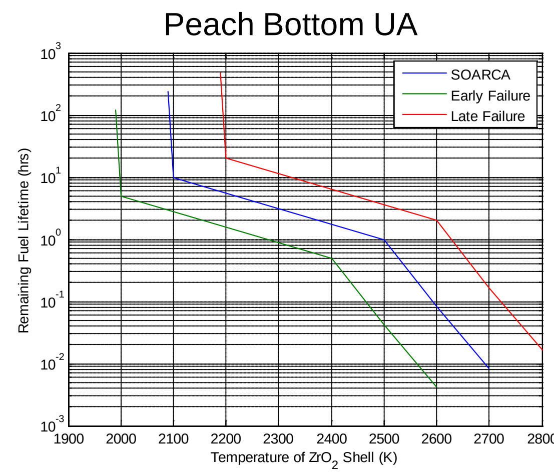


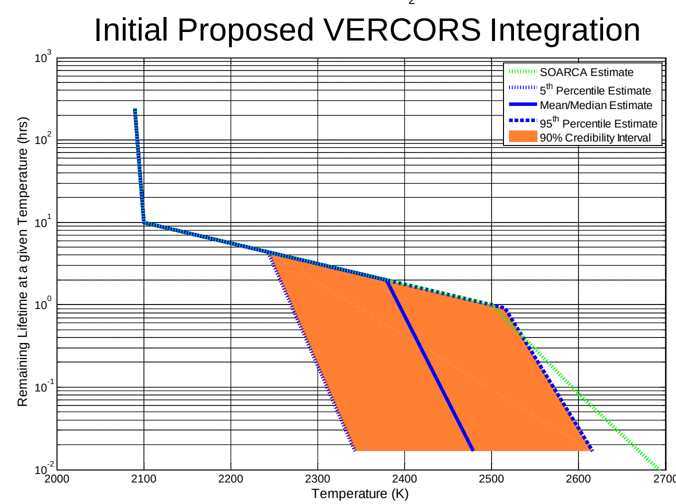


Defining prior understanding of A and B

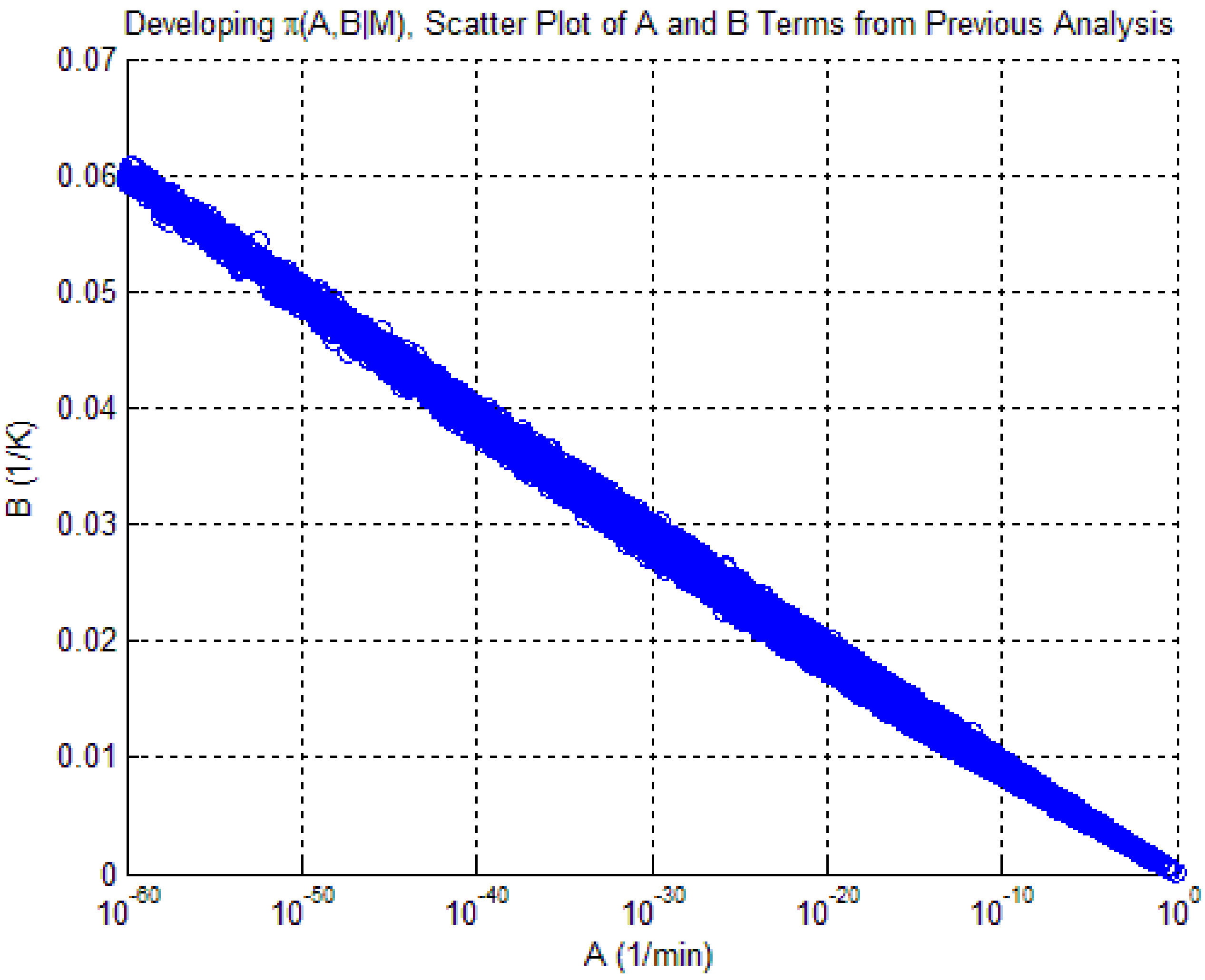
Previous Uncertainty Characterization

- Proposed Surry Treatment
  - Used to allow for high temperature / low lifetime variability
  - Small remaining lifetime (assumed to be one minute) should occur at:
    - The sampled effective fuel slumping temperature from the high burn VERCOR Tests
    - $N(\mu = 2479K, \sigma = 89K)$
- Combine high lifetime and low lifetime samples to determine range of Arrhenius functions.
  - $\frac{1}{t(T)} = A * \exp(BT)$
  - $B = \ln\left(\frac{t(T_2)}{t(T_1)}\right) * \frac{1}{T_1 - T_2}, A = \frac{1}{t(T_1) * \exp(B * T_1)}$
  - This structure is abbreviated as the failure model *M*





Can the uncertainty in A and B be treated as independent? No!



How can this scatter be used to make a prior distribution?

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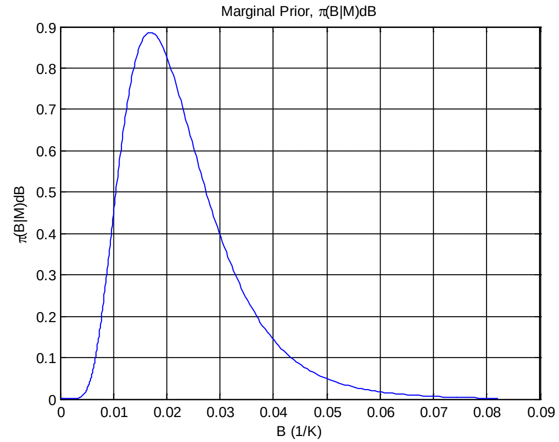
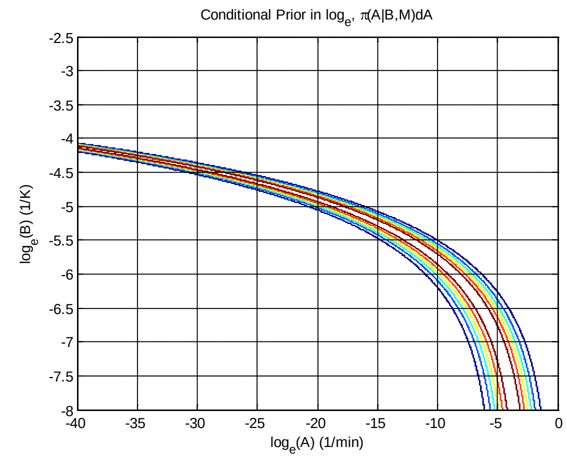
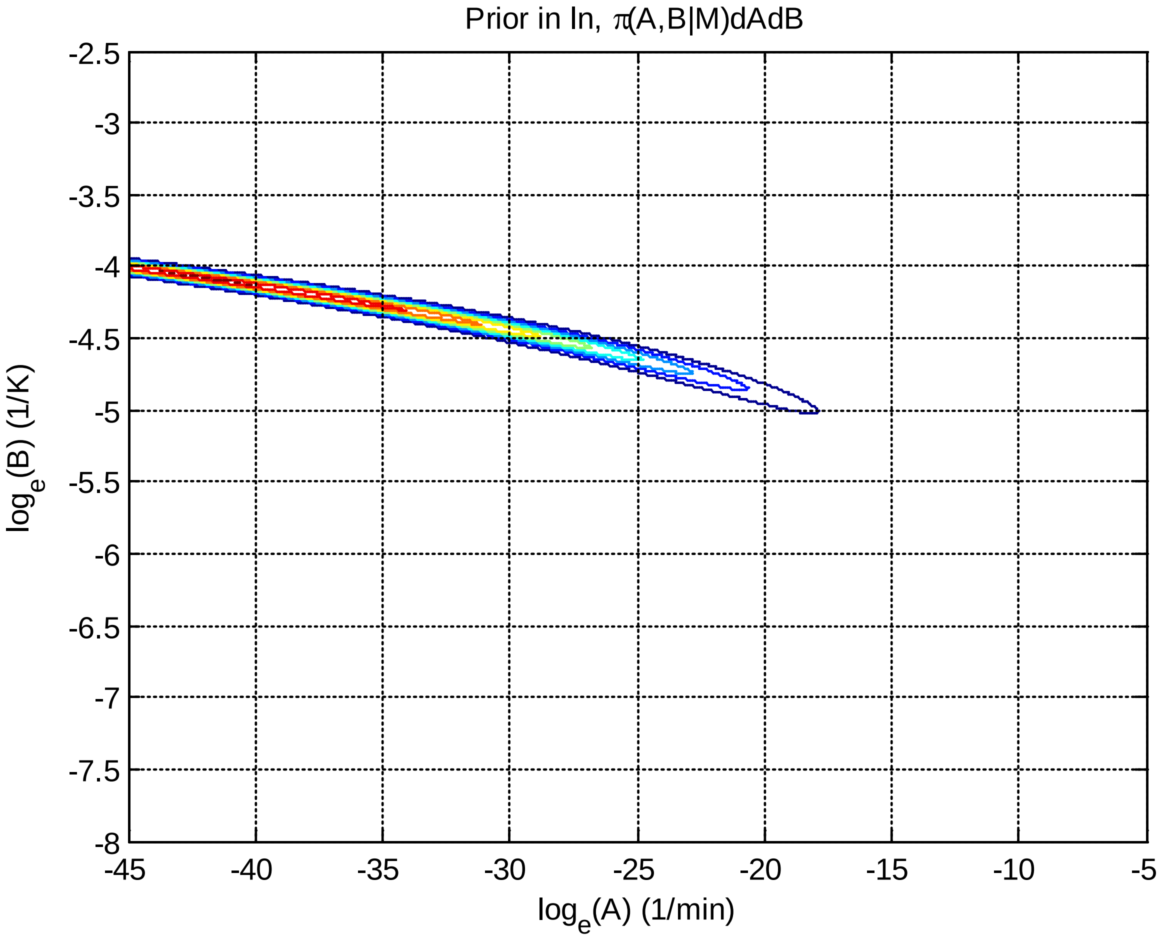
1. Create a marginal distribution for the independent variable (B-Lognormal)
2. Define the relationship between A and B.
   - The ln(A) and B are linear
   - $\mu_{\ln(A)}(B) = -3 * B - 2295 + \epsilon$
   - $\pi(\ln(A)) = N\left(\ln(A) \,\middle|\, \mu_{\ln(A)}(B), (\sigma|\epsilon), M\right)$
3. Multiply the marginal distribution of B to the conditional distribution of A|B to create the joint distribution.
   - $\pi(A, B|M) = \pi(A|B, M) * \pi(B|M)$





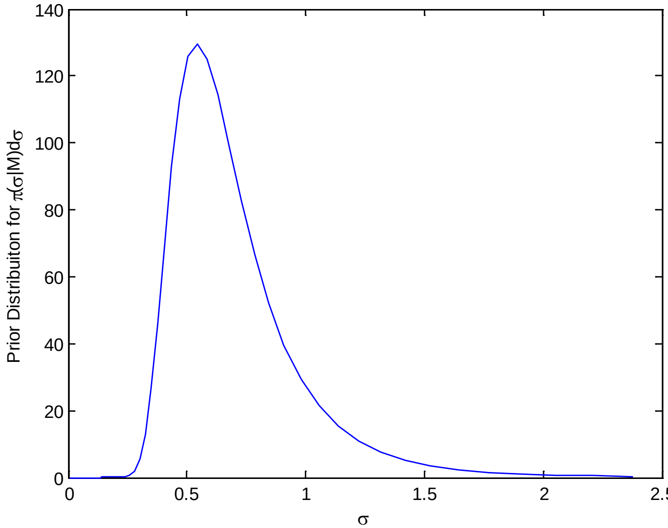
Create a joint prior

$$\pi(A, B|M)dAdB$$



$\pi(\sigma|E, M^*)\mathrm{d}\sigma$ - Prior Distribution for Uncertainty in Damage Estimates Corresponding to Failure

- Failure is characterized as Arrhenius: D=1.0 is idealized failure
  - $\frac{1}{t(T)} = A * \exp(BT)$,$\mathrm{D(t)} = \sum\left(\frac{1}{t(T)} * \Delta t\right)$
- Failure can occur when D≠1.0 due to:
  - Inherent variability
  - Model inaccuracy
- It is assumed that:
  - Experimental variability from D=1.0 is log normally distributed
  - Expected variability in final model should be similar (≈) to that of the current failure model
  - The likelihood of the variability in the historical TatT model (M^*) is:
    - $\pi(\sigma|E, M^*)\mathrm{d}\sigma = \frac{\ln(\sigma|E,\mu=0,M^*)}{\int \ln(\sigma|E,\mu=0,M^*)d\sigma}$



The full joint prior distribution $\pi(A, B, \sigma|M)dAdBd\sigma$

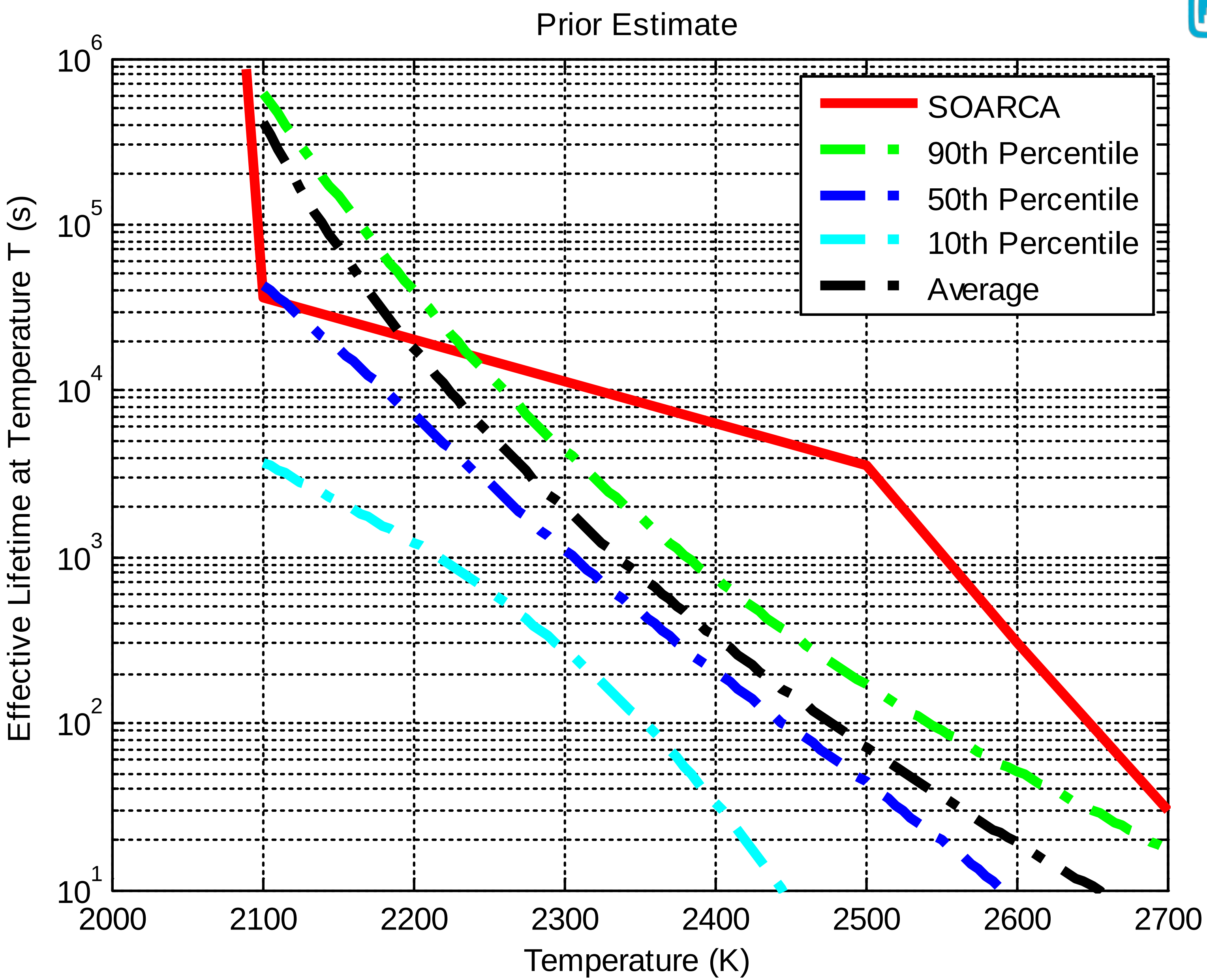
- Now that all of the basic uncertainty relationships are characterized, they can be multiplied together to create a joint prior.

- $$\pi(A, B, \sigma|M)dAdBd\sigma = \pi(A, B|M)dAdB * \pi(\sigma|M^*, E)d\sigma = \pi(A|B, M)dA * \pi(B|M)dB * \pi(\sigma|M^*, E)d\sigma$$

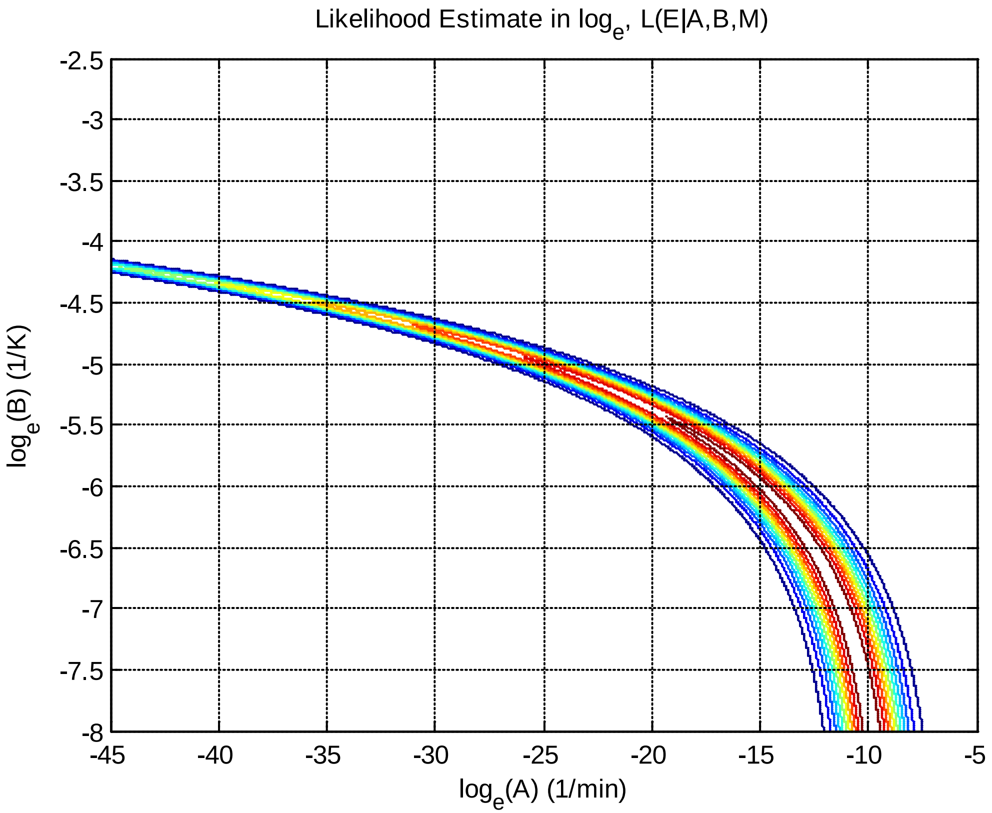
- In this analysis, $\pi(A, B, \sigma|M)dAdBd\sigma$ is calculated numerically by discretizing A, B, and σ over likely values and then calculating the likelihood for each point in the set of A, B, and σ

Note on the Treatment of σ

- What is useful to a MELCOR analysis
  - Useable information - Epistemic uncertainty of the shape parameters
    - $\pi(A, B|M)d$AdB
  - Unusable information – Aleatory variability
    - $\pi(D^*|A, B, \sigma, M)dD = \ln(D^*|A, B, \sigma, M) = \ln(D^*|\sigma, M)$
- Because the choice of σ effects the model fitting, its effects are averaged out by integrating $\pi(A, B, \sigma|M)dAdBd\sigma$ over σ, producing $\pi(A, B|M)dAdB$.
  - $\pi(A, B|M)dAdB = \int \pi(A, B, \sigma|M)dAdBd\sigma$



Prior Estimate
SOARCA
90th Percentile
50th Percentile
10th Percentile
Average
Effective Lifetime at Temperature T (s)
Temperature (K)
10^1
10^2
10^3
10^4
10^5
10^6
2000
2100
2200
2300
2400
2500
2600
2700



How do we judge the proposed model parameters given the data?

DEFINE THE LIKELIHOOD

Evaluating the Damage Model given the Failure Data

- Every set of (A,B) will produce a different damage estimate.

$$\frac{1}{t(T)} = A * \exp(BT)\ ,\mathrm{D(t)} = \sum\left(\frac{1}{t(T)} * \Delta t\right)$$

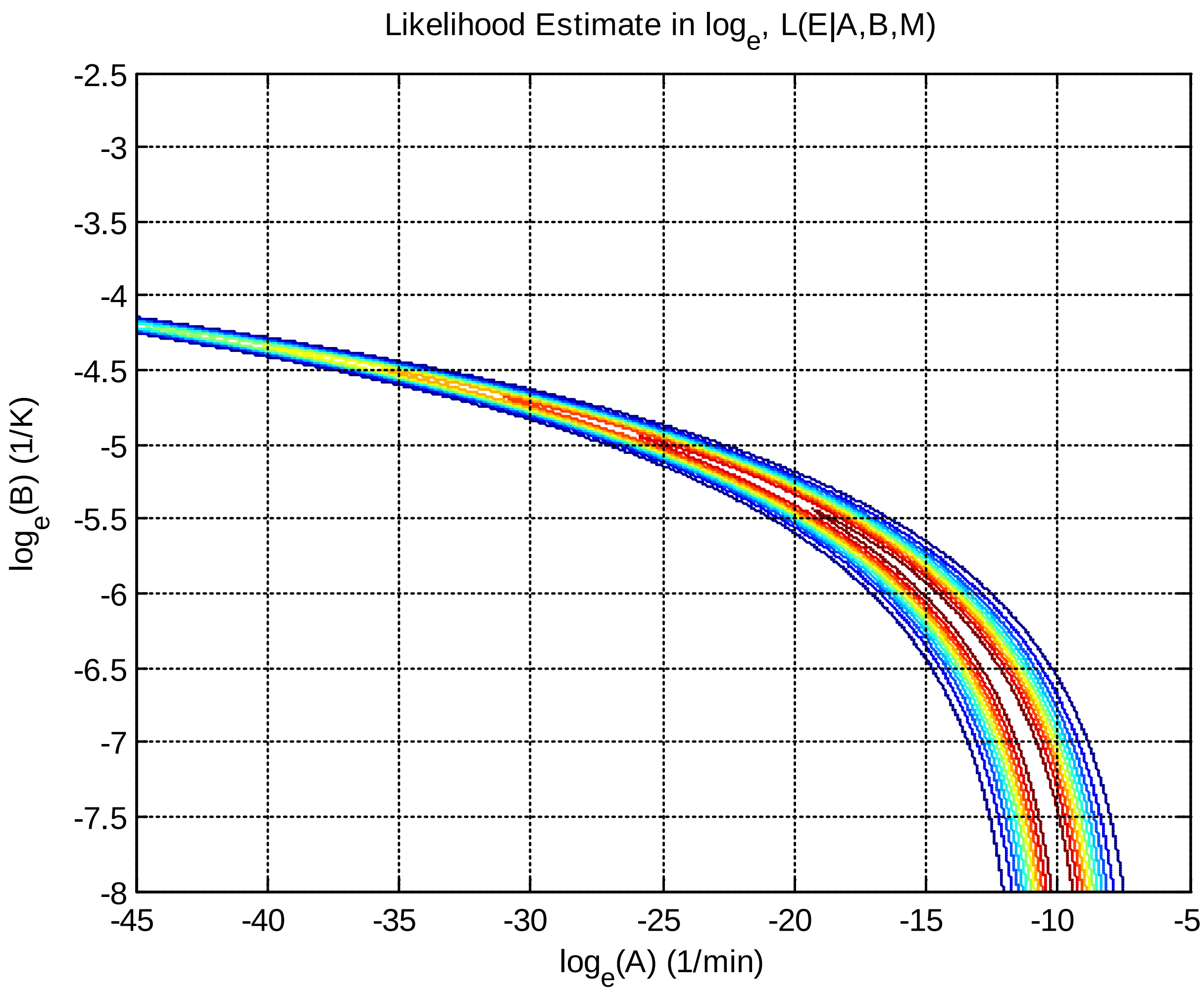
- No combination of A,B will produce a model estimated D=1.0 at all experimental failure temperatures ->

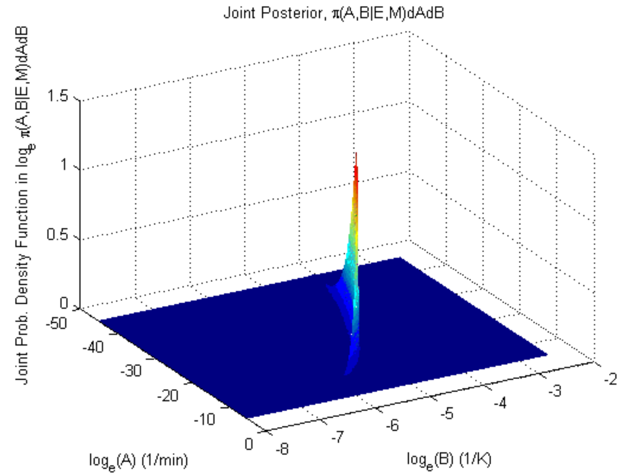
Test	*Collapse Temperature (K)*	*Driving Phenomena*
(R?)T1	2525	H_2O oxidizing atmosphere
HT1	2550	H_2 reducing atmosphere
HT2	2400	H_2O oxidizing atmosphere U–Zr–O–FP interaction
HT3	2525	H_2 reducing atmosphere
V_6 (RT4?)	2525	ZrO_2-“fuel”-FP Interaction
RT6	2350	H_2O oxidizing atmosphere
Mean	2479	
Standard Deviation	83	

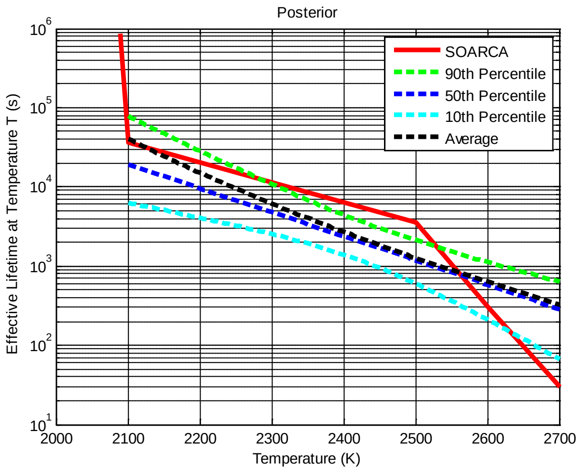
Evaluating the Damage Model given the Failure Data

- The likelihood of a given set of (A,B)'s damage estimate is assumed to be lognormal because of its range [0,∞) and small number of shape parameters (μ,σ).
  - Failures should be distributed around D=1.0, thus μ=ln(1.0)=0.0 was fixed in the analysis.
    - $\pi(\sigma|E,M) = Numerical, Defined\ by\ VERCORS\ data\ and\ SOARCA\ Model$
    - $\pi(A,B|E,M)dAdB = \int_{\sigma\_min}^{\sigma\_max} \pi(A,B,\sigma|E,M)\, dAdBd\sigma$
  - Choice of likelihood function can be explored as a sensitivity study
- $L(E|A,B,\sigma,E,M) = \prod_{i=1}^{N}\left[\frac{1}{D_i * \sigma * \sqrt{2\pi}} * \exp\left(-\frac{\ln(D_i)-\mu}{2\sigma^2}\right)\right]$
  - Where D_i is the i^{th} damage calculated from A, B, the E is the set evidence (VERCORS failure temperatures), and M (the Arrhenius damage accrual model)

Calculation of the Likelihood
Integrating (Averaging) over σ







$$\pi(A,B,\sigma|M,E) = \frac{L(E|A,B,\sigma,M) * \pi(A,B|M)dAdB * \pi(\sigma|E,M^*)d\sigma}{\{\int\int\int L(E|A,B,\sigma,M) * \pi(A,B|M)dAdB * \pi(\sigma|E,M^*)d\sigma\}}$$

Once the posterior is known, it can be sampled to create a distribution of TatT curves.

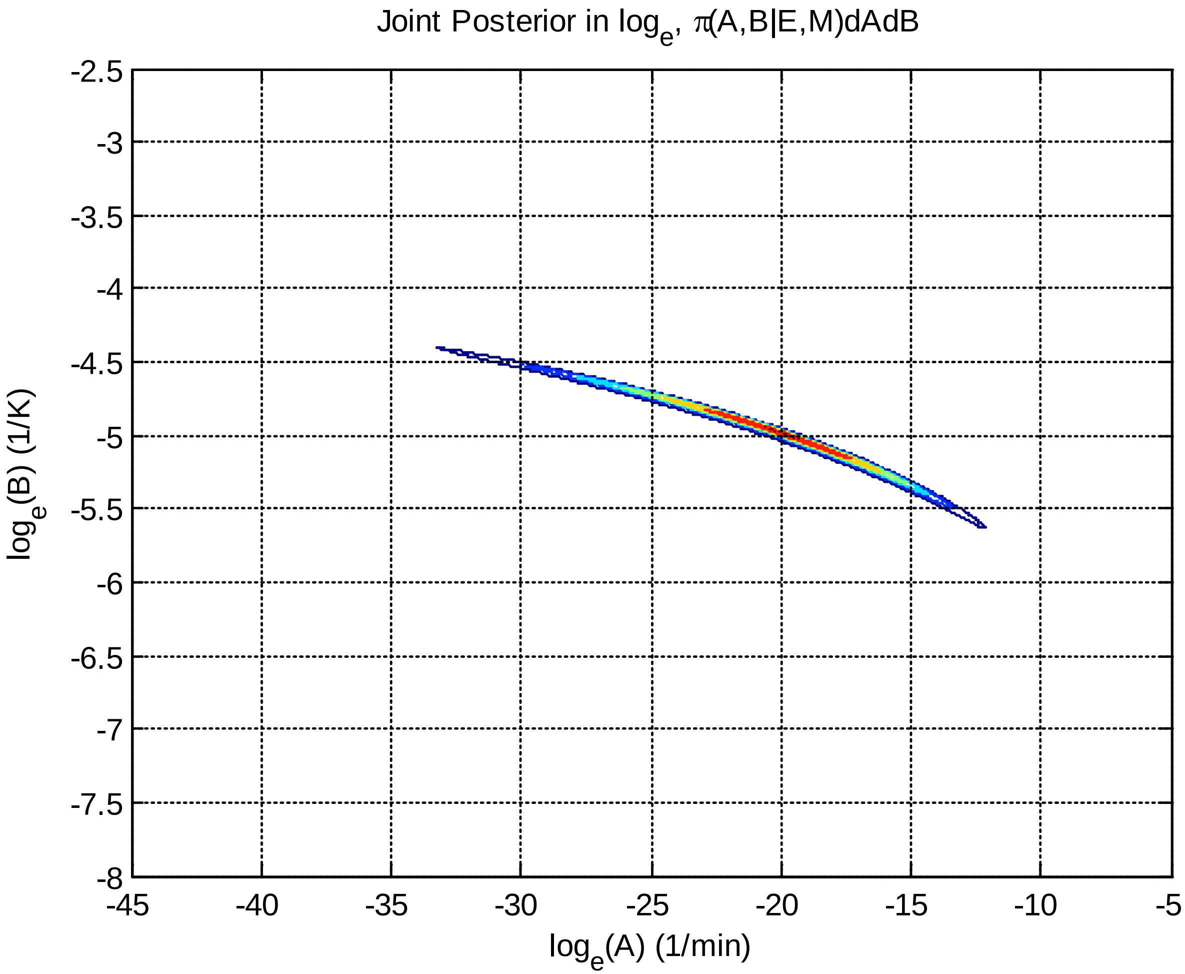
CREATE THE POSTERIOR

Note on the Treatment of σ

Same as before, only with the evidence variable E

- What is useful to a MELCOR analysis
  - Usable information - Epistemic uncertainty of the shape parameters
    - $\pi(A, B|M, E)d$AdB
  - Unusable information – Aleatory variability
    - $\pi(D^*|A, B, \sigma, M, E)dD = \ln(D^*|A, B, \sigma, M, E) = \ln(D^*|\sigma, M, E)$
- Because the choice of σ affects the model fitting, its effects are averaged out by integrating $\pi(A, B, \sigma|M, E)dAdBd\sigma$ over σ, producing $\pi(A, B|M, E)dAdB$.
  - $\pi(A, B|M, E)dAdB = \int \pi(A, B, \sigma|M, E)dAdBd\sigma$

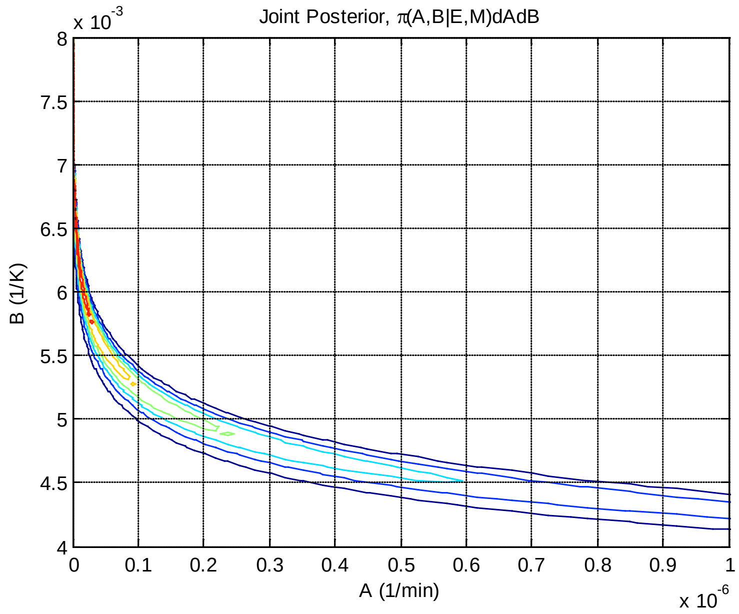
The Updating Process



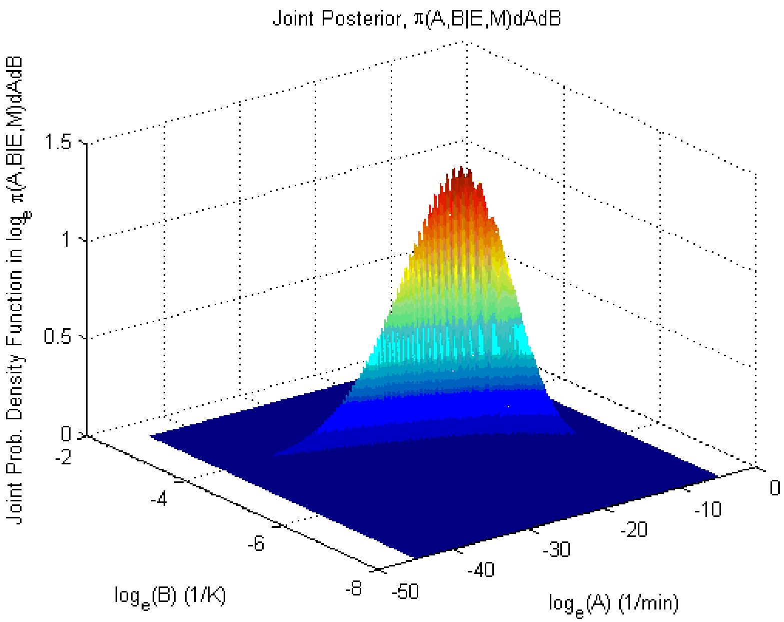
$$\pi(A,B|E,M)dAdB = \frac{L(E|A,B,M) * \pi(A,B|M)dAdB}{\int L(E|A,B,M) * \pi(A,B|M)dAdB}$$

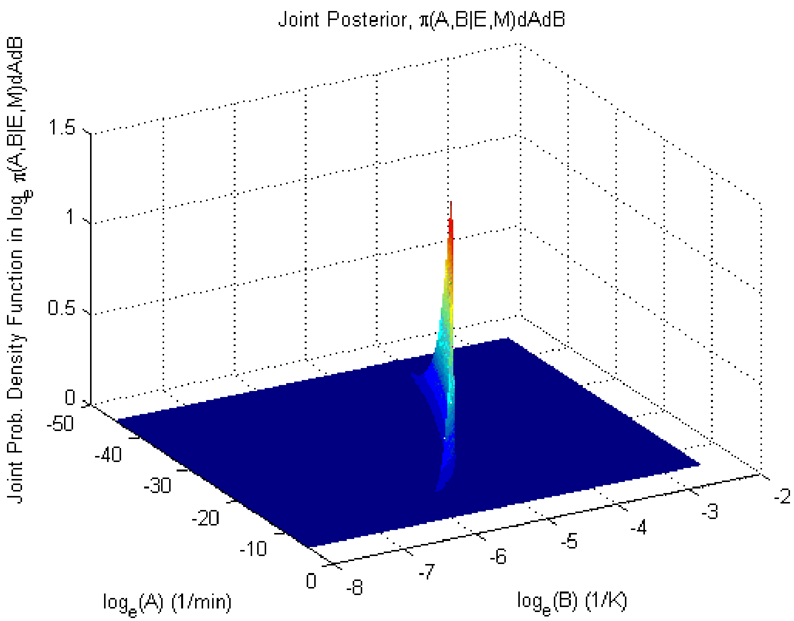
Combine the Prior and the Likelihood

- This joint posterior distribution can be sampled to produce (A,B) pairs which are informed by:
  - Prior analysis
  - VERCORS test data

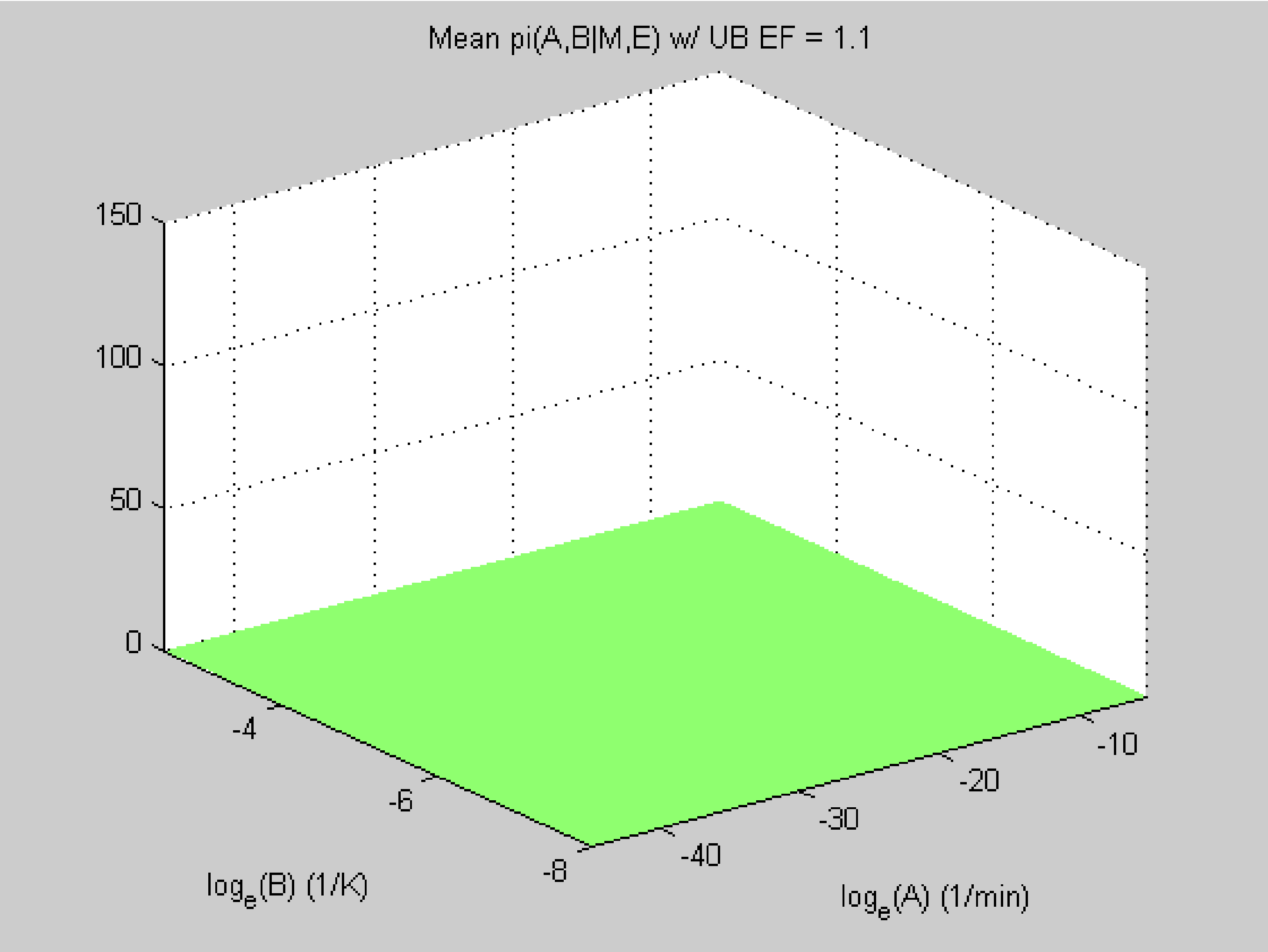


Surface Plots of Posterior Distribution

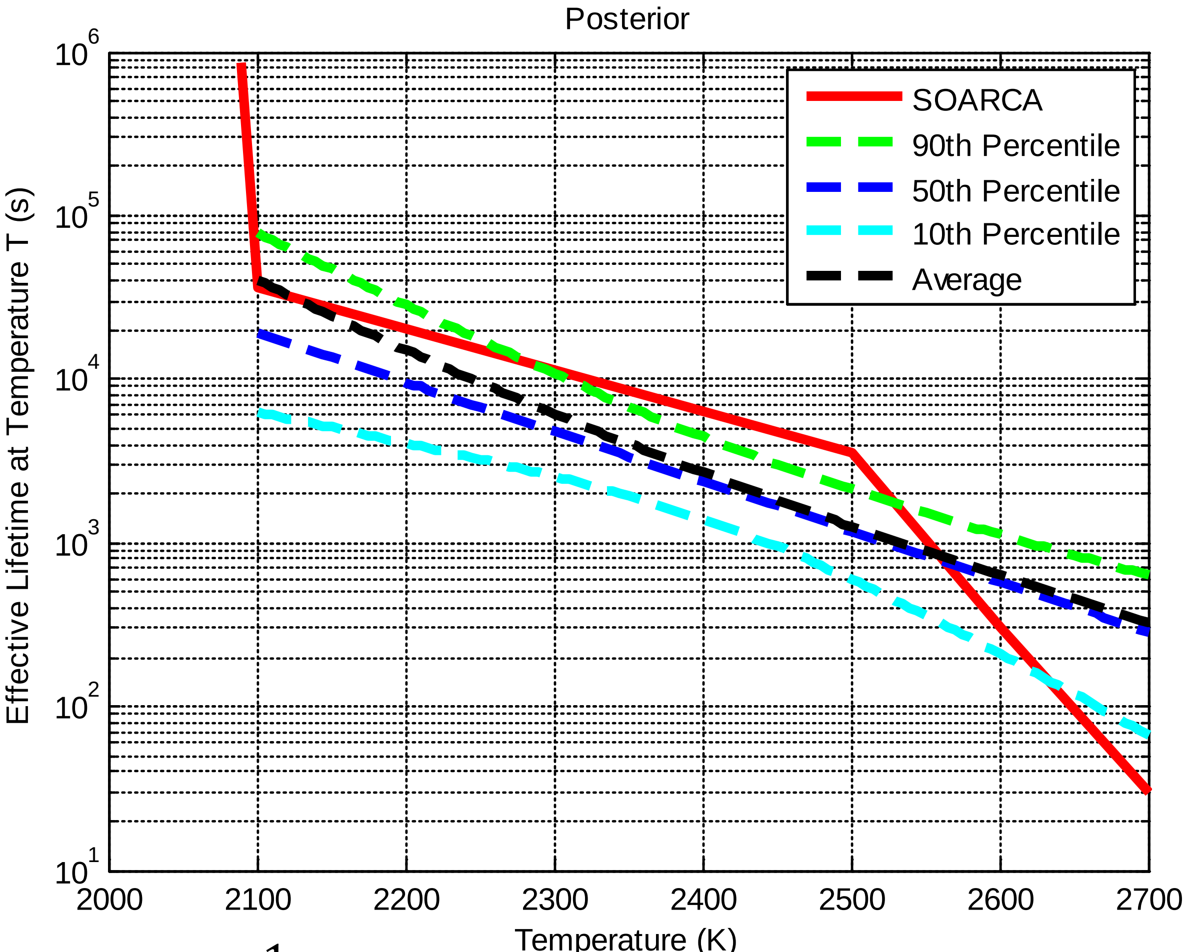




Shark-Fin Movie
The Effect of Integration Over Sigma



Effect on TatT curves



$$\frac{1}{t_{50^{th}}(T)\{sec\}} = 2.16x10^{-11} * exp(7x10^{-3} * T)$$

Conclusions

- The Time at Temperature relationship has been transformed from an expert judgment relationship to a data informed relationship through Bayesian Regression analysis
  - A new point estimate curve has been developed to replace the old SOARCA TatT curve
  - A numerical uncertainty distribution of A and B $\{\pi(A, B)dAdB\}$ has been created which can be sampled from to support subsequent uncertainty analysis
- Not only is the new TatT curve more rigorous and defensible than the old curve, the expected uncertainty in the output is built into the model.