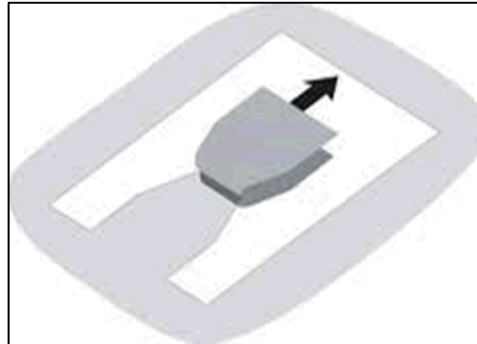


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Using Tearing as a Method to Calculate Adhesion

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Presentation Overview

- Introduction and Background
- Need Statement
- Theoretical Development
- Method of Approach
 - Preliminary Experiments
 - Test Set up

Introduction and Background

- Adhesion is a property that can be seen nearly everywhere in daily life



Introduction and Background

- Adhesion also has a large number of applications for the military

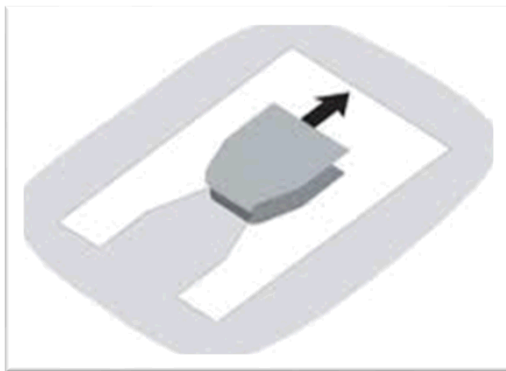
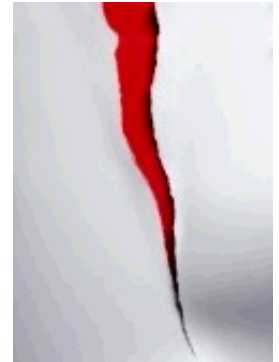


- Corrosion Inhibitors
- Radar Absorbing
- UV Protection

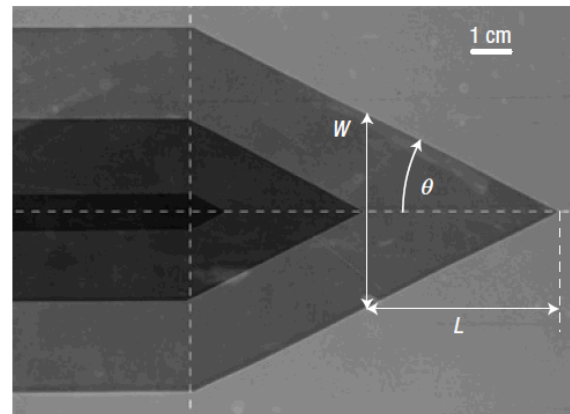
to name a few

Introduction and Background

- When a material is torn, the tear will naturally take a converging path
- This path leads to the lowest energy state
- This natural tendency can be seen in everyday life



Hamm et al., 2008



Hamm et al., 2008

Need Statement

- A method to quantify the adhesive strength between a polymer coating and a given substrate is needed.
- Additionally, a robust experimental approach that can be used to evaluate a variety of substrates is needed.

Theory

Energy of the System

Total Energy Elastic Energy Fracture Energy Adhesion Energy

$$U = U_E + 2\gamma ts + \tau A$$

Equilibrium of a half-strip requires:

$$F = \tau \frac{W}{2} + \gamma t \cos \theta$$

$$\left(\frac{\partial U_E}{\partial W}\right)_{x,t} = \gamma t \sin \theta$$

The flexural strain energy, U_E , is given as:

$$U_E = \frac{4BW}{h}, \text{ where } B = \frac{Et^3}{12(1-\nu^2)}$$

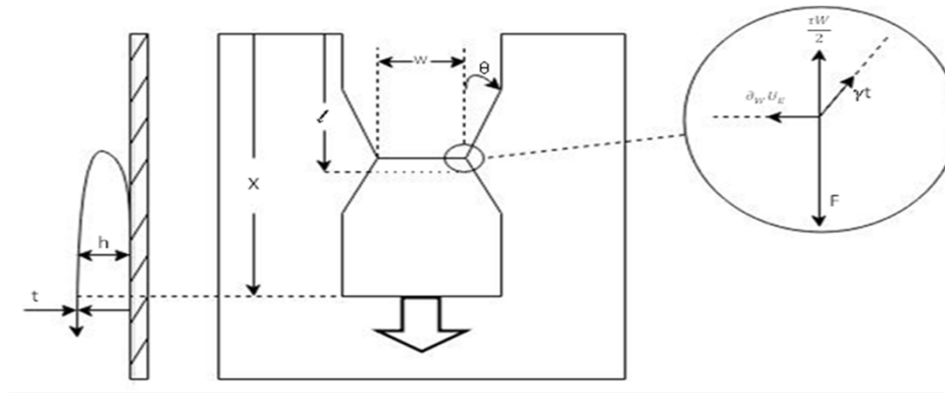
Then, $\left(\frac{\partial U_E}{\partial W}\right)_{x,t} = \frac{4B}{h}$

So,

$$\sin \theta = \frac{4B}{\gamma th}$$

The Force, F , in the force balance equation can also be described using the Work Theorem as:

$$F = (\partial_x U_E)_{W,t}$$



Therefore, taking $F = (\partial_x U_E)_{W,t}$ gives:

$$F = \frac{\partial}{\partial x} \left(\frac{4BW}{h} \right) = -\frac{4BW}{h^2} \frac{\partial h}{\partial x}$$

Letting $\eta^2 = -\partial_x h$

We can then say:

$$F = 4\eta^2 \frac{BW}{h^2}$$

For equilibrium:

$$4\eta^2 \frac{BW}{h^2} = \tau \frac{W}{2} + \gamma t \cos \theta$$

For large values of W , $\gamma t \cos \theta$ is negligible.

Solving for h then gives

$$h = 2\eta \sqrt{\frac{2B}{\tau}}$$

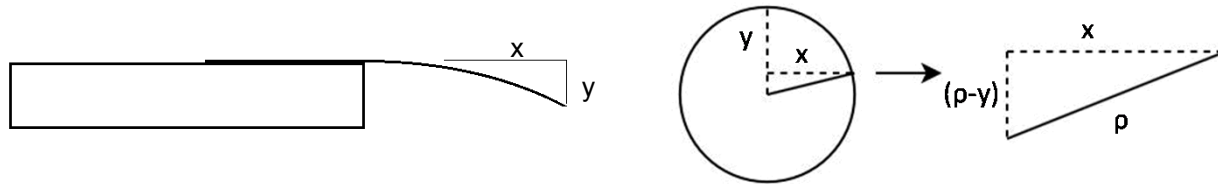
So, $\sin \theta = \frac{\sqrt{2B\tau}}{\eta \gamma t}$ or $\tau = \frac{(\eta \gamma t \sin \theta)^2}{2B}$

Method of Approach

- Before starting the tearing experiment, the bending stiffness (B), fracture force (γt), and the parameter η must be determined
- Bending stiffness, B , can be found by using a simple experiment based on elastic beam theory
- Fracture force, γt , can be found by using ASTM 1938
- η must be found graphically

$$\tau = \frac{(\eta \gamma t \sin \theta)^2}{2B}$$

Finding Bending Stiffness



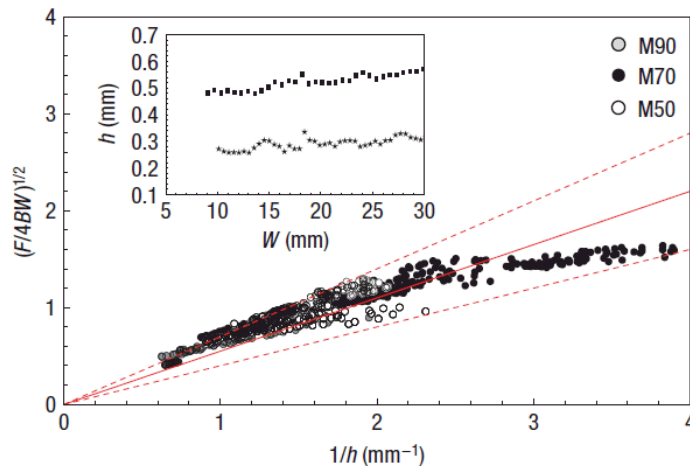
- The mass per unit length must be found and then the moment, M , calculated
- The radius of curvature, ρ , can be found using the Pythagorean theorem where x and y are known
- Utilizing the follow equation, the Bending stiffness can be found

$$M = B \frac{\partial^2 y}{\partial x^2}$$

- In this case $\frac{\partial^2 y}{\partial x^2} = \frac{1}{\rho}$

Finding η

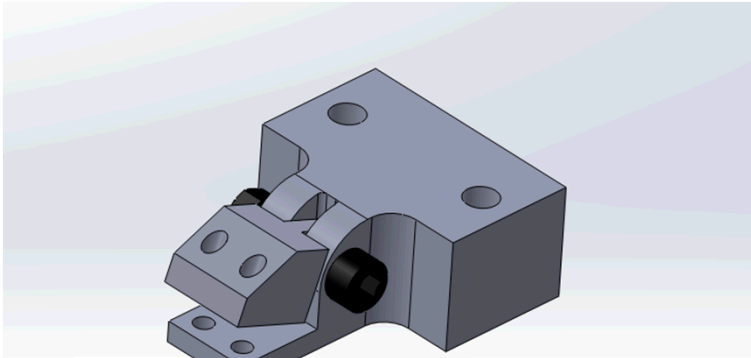
- Under the assumption that η is a constant and h is a variable, $F = 4\eta^2 \frac{BW}{h^2}$ can be rearranged $\sqrt{\frac{F}{4BW}} = \frac{1}{h} \eta$ and plotted
- Once plotted, the slope of the line can be extracted and will give the value of η (show typical plot)



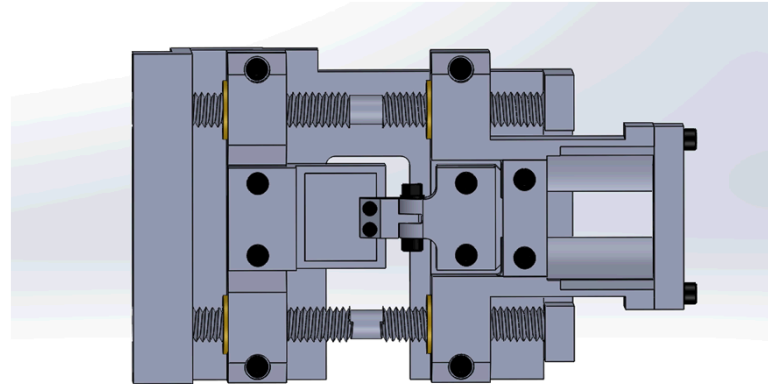
Hamm et al., 2008

- The equation $\sin \theta = \frac{4B}{\gamma t h}$ implies that h , is a constant. However, in reality, there are small variations in h which allow us to assume this relation is correct, but these variations are measured and used to help find η

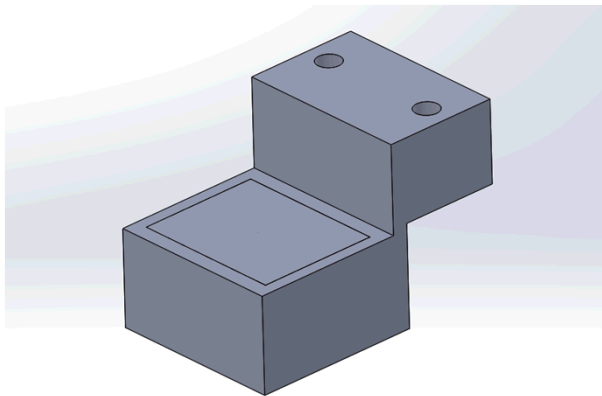
Test Set up



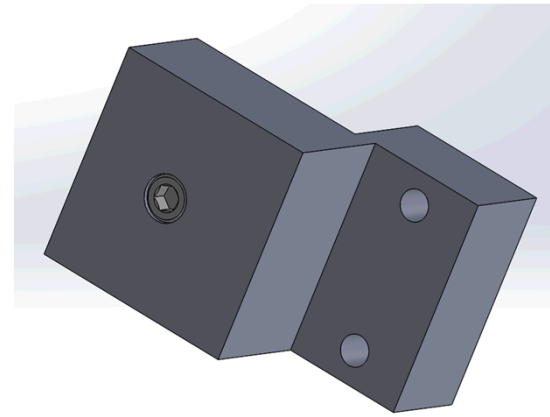
Initial Clamp Design



Top View of Test Set up

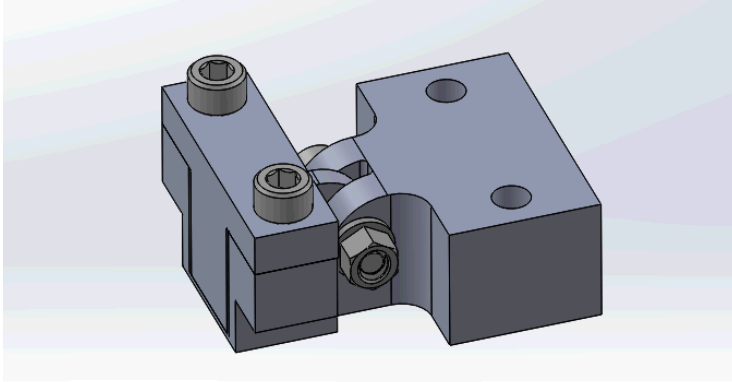


Substrate frame with Substrate Plate

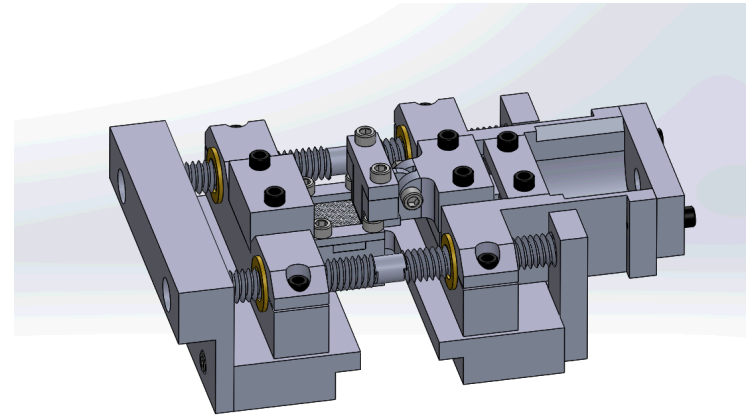


Bottom of Substrate frame

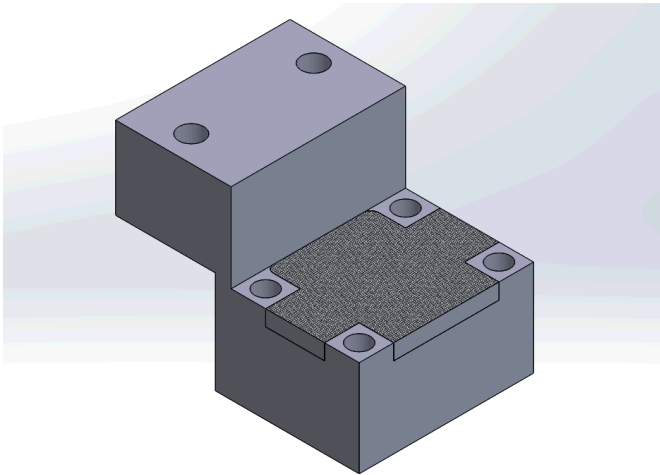
Test Set up



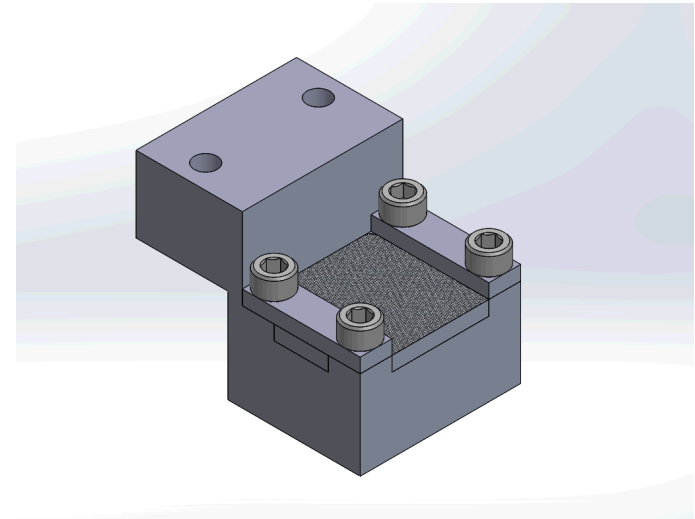
Updated Clamp design



Updated Test Set up



Modified Substrate Frame



Modified Substrate Frame with top plate

Summary

- Adhesion was shown to be a characteristic needed for a variety of critical applications
- The adhesion strength was shown to be governed by elastic and fracture properties
- A delineation between the material properties and geometric parameters was made
- A conceptual design was created in order to quantify the adhesion strength for a variety of different substrates
- The design is ready to move forward to the manufacturing stage for proof of concept

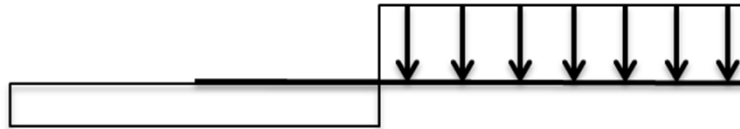
Acknowledgements

- Tim Briggs- A big thank you to Tim for helping me break down and understand the math and theory behind the nature of tearing
- Staci Dorsey- Thank you to Staci for setting up the internship and giving me this opportunity

Questions?

References

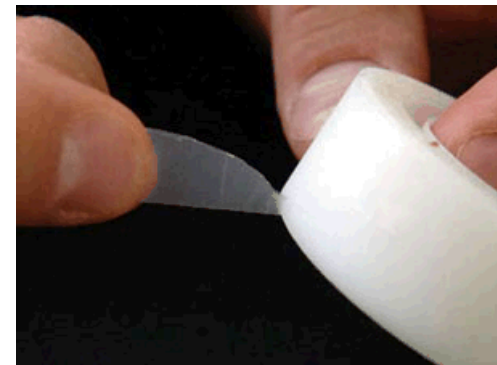
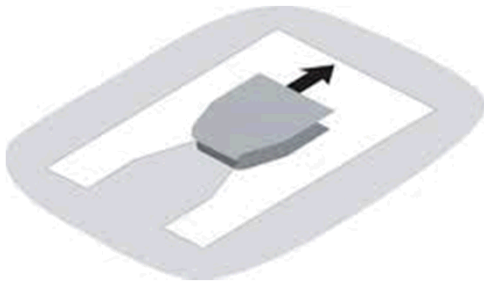
1. Hamm, E., Reis, P., LeBlanc, M., Roman, B. & Cerda, E. Tearing as a test for mechanical characterization of thin adhesive films. *Nature Materials* **7**, 386-390 (2008).
2. ASTM standards
3. Books and other papers



Theory

- When a film is peeled from a substrate, the tear will naturally take a converging path
- The first step in finding adhesion from this natural path is to mathematically describe it
- To start, an equation to characterize the energy of the system is created

$$U = U_E + 2\gamma ts + \tau A \quad (1)$$

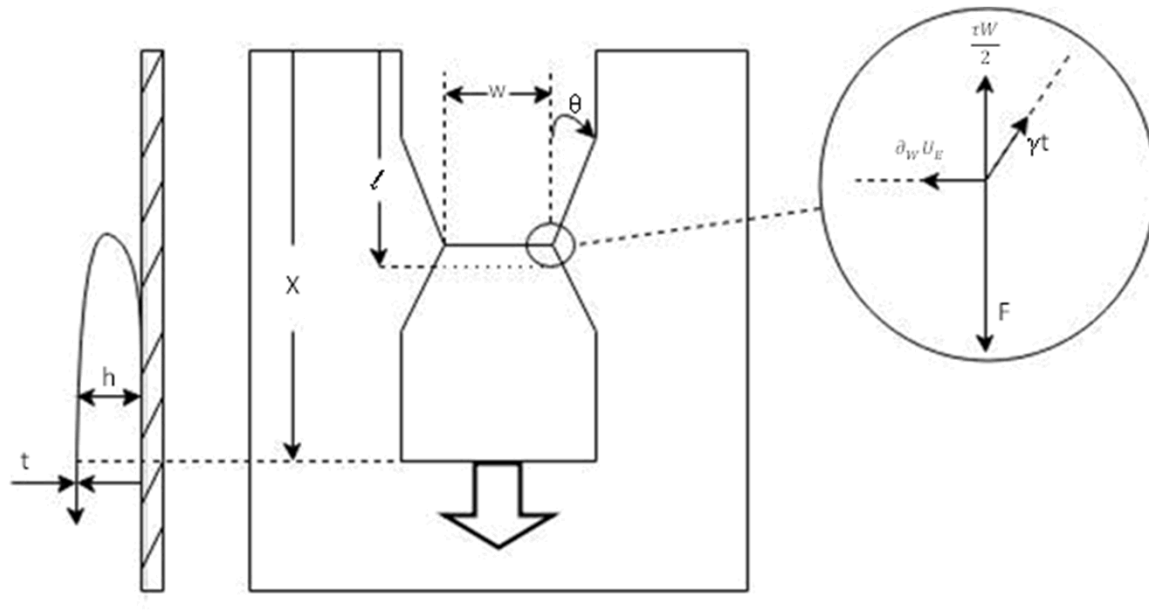


Theory

- From the energy equation, a balance of forces can be derived for $\frac{1}{2}$ of the strip and yields:

$$F = \tau \frac{W}{2} + \gamma t \cos \theta \quad (2)$$

$$\left(\frac{\partial U_E}{\partial W}\right)_{x,\ell} = \gamma t \sin \theta \quad (3)$$



Theory

- The classical elastica of Euler yields that the elastic energy available is $U_E = \frac{4BW}{h}$, where B is the Bending Stiffness
- Plugging the equation for elastic energy into $(\frac{\partial U_E}{\partial W})_{x,\ell} = \gamma t \sin \theta$, equation (3) now becomes

$$\sin \theta = \frac{4B}{\gamma t h} \quad (4)$$

$$\begin{aligned} U &= U_E + 2\gamma t s + \tau A \quad (1) \\ F &= \tau \frac{W}{2} + \gamma t \cos \theta \quad (2) \\ (\frac{\partial U_E}{\partial W})_{x,\ell} &= \gamma t \sin \theta \quad (3) \end{aligned}$$

- Because $\sin \theta$ is a constant, this new equation implies that the height, h , is constant.

- The force in equation (2) can also be described as

$$F = (\partial_x U_E)_{W,t}$$
- To make the partial derivative of the elastic energy work in the above equation, a new parameter, η^2 , is defined.

$$\eta^2 = -\partial_x h$$
- By using η^2 and the equation for elastic energy, the force in equation (2) can now be written as $F = 4\eta^2 \frac{BW}{h^2}$
- Substituting in the above equation for the force in equation (2) yields

$$4\eta^2 \frac{BW}{h^2} = \tau \frac{W}{2} + \gamma t \cos \theta \quad (5)$$

$$U = U_E + 2\gamma ts + \tau A \quad (1)$$

$$F = \tau \frac{W}{2} + \gamma t \cos \theta \quad (2)$$

$$\left(\frac{\partial U_E}{\partial W}\right)_{x,t} = \gamma t \sin \theta \quad (3)$$

$$\sin \theta = \frac{4B}{\gamma th} \quad (4)$$

Theory

- It was found that for large values of W that the last term in equation (5) is negligible
- This allows us to solve for the height, h , through the elimination of the last term in equation (5) and gives us that $h = 2\eta\sqrt{\frac{2B}{\tau}}$
- By plugging the above equation for height into equation (4), we get the final equation needed to find adhesion and yields

$$\sin \theta = \frac{\sqrt{2B\tau}}{\eta\gamma t} \quad (6)$$

- A quick rearrangement of equation (6) gives the more usable form of

$$\tau = \frac{(\eta\gamma t \sin \theta)^2}{2B}$$

$$\begin{aligned} U &= U_E + 2\gamma ts + \tau A \quad (1) \\ \sin \theta &= \frac{4B}{\gamma th} \quad (4) \\ 4\eta^2 \frac{BW}{h^2} &= \tau \frac{W}{2} + \gamma t \cos \theta \quad (5) \end{aligned}$$