Nanolaser emission under few-emitter or unity-spontaneous-emission-factor conditions

SAND2014-16518PE

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Motivation

Theoretical approach

Examples: a) Thresholdless lasing
b) Single-photon source

Co-workers: Christopher Gies and Frank Jahnke, Bremen University

Thanks to:
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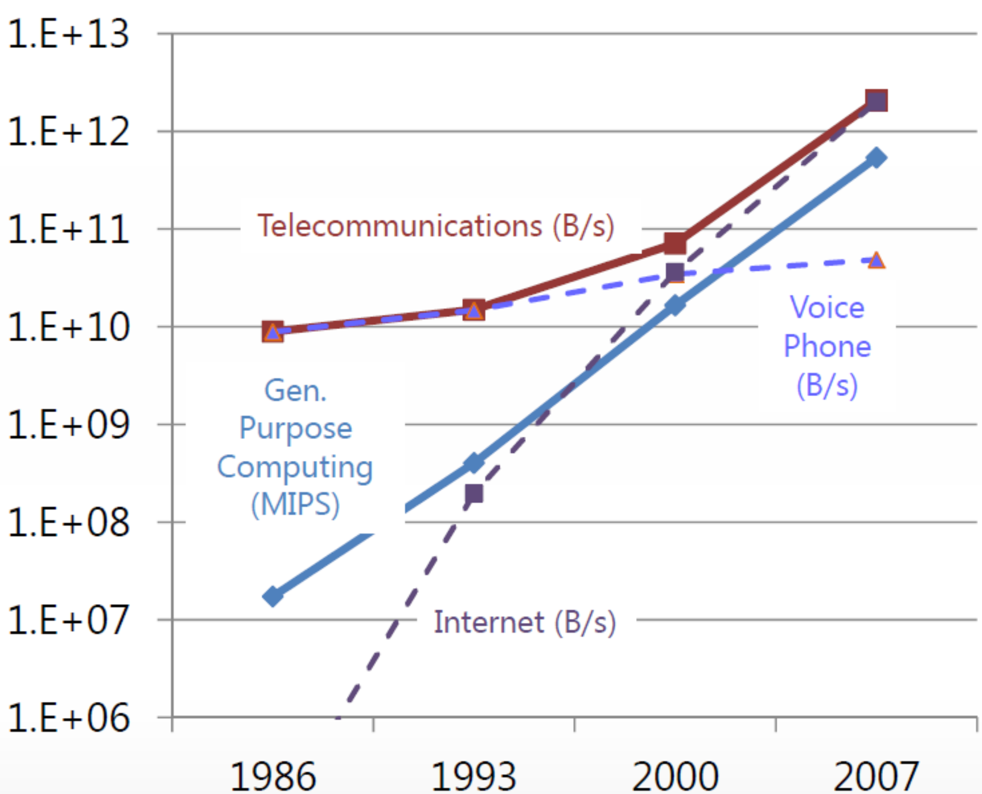
Why fund nanolaser development?

Attojoule optoelectronics – why and how

David Miller IEEE Photonics Summer Topicals, 2013



M. Hilbert and P. Lopez, "The World's Technological Capacity to Store, Communicate, and Compute Information," Science 332, 60 (2011)

MIPS – million instructions per second
~ 3 - 6 instructions = 1 floating point operation (FLOP)

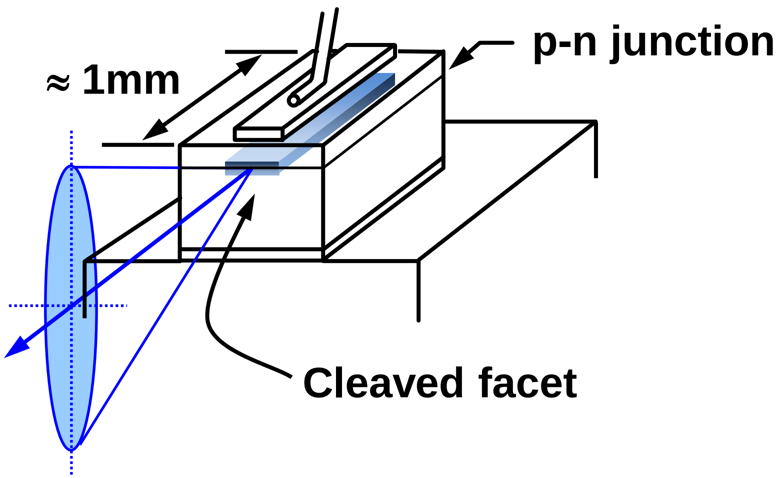
Both
- Internet traffic
- General purpose computing hardware

growing ~ 60 % per year
- ~ X 100 in 10 years

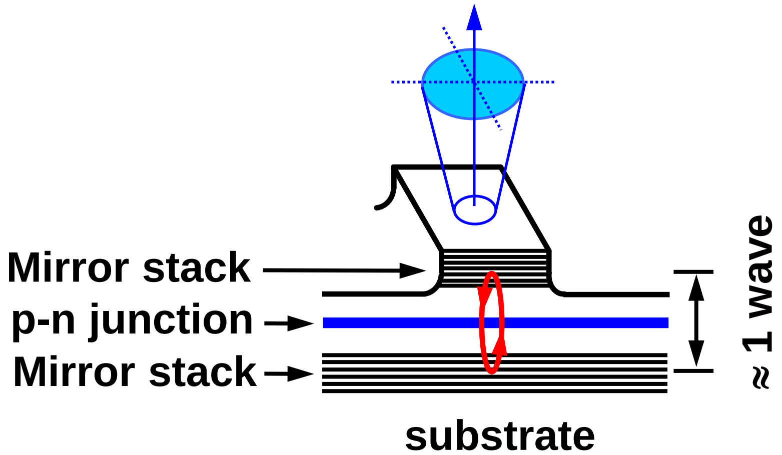
Massive challenge for hardware scaling of
- Energy
  - Energy per bit has to reduce
    - Energy scaling not environmentally sustainable
      - Information technology already (2006) consumed ~ 1.5% of US electricity
- Communication density inside systems
  - already at limits for electrical approaches

Towards smaller and smaller lasers

Edge - Emitting Laser

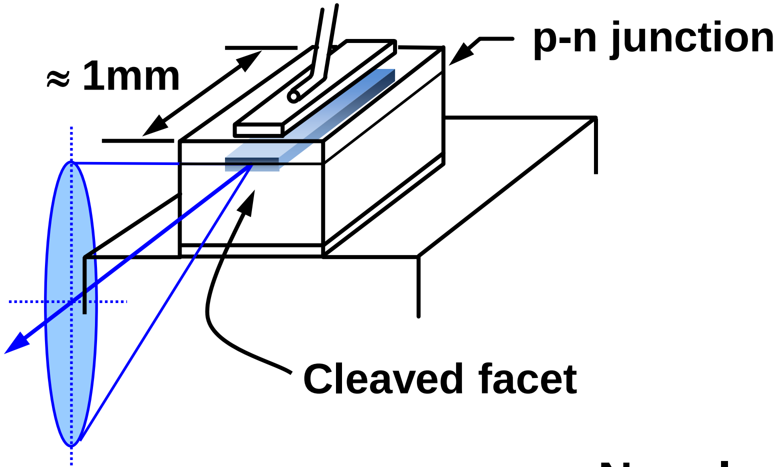


Vertical-Cavity Surface-Emitting Laser (VCSEL)



Towards smaller and smaller lasers

Edge - Emitting Laser



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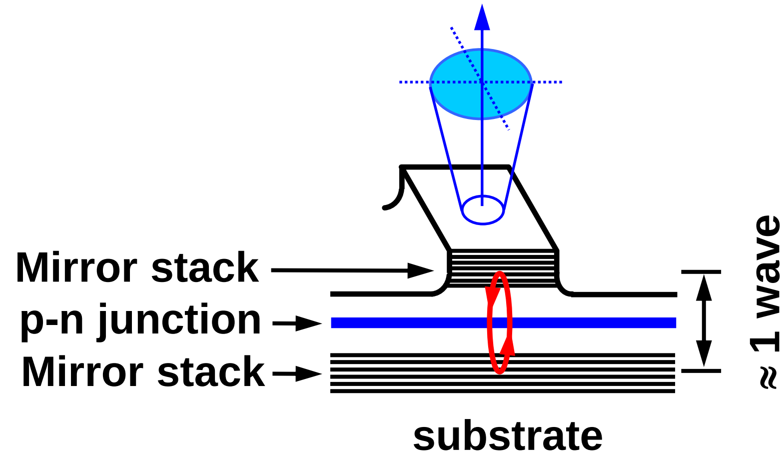
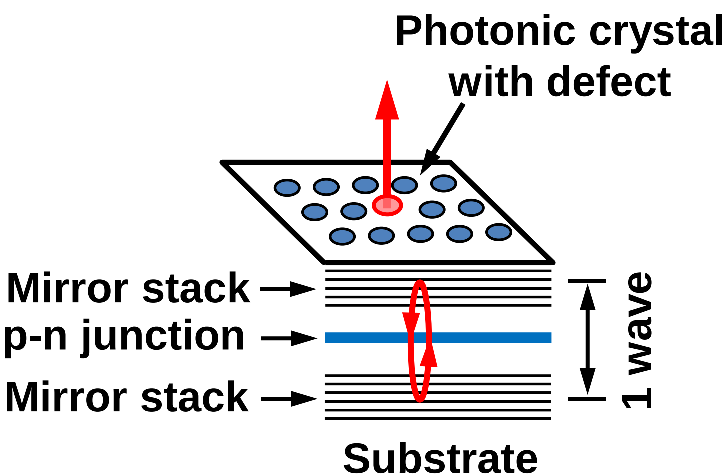
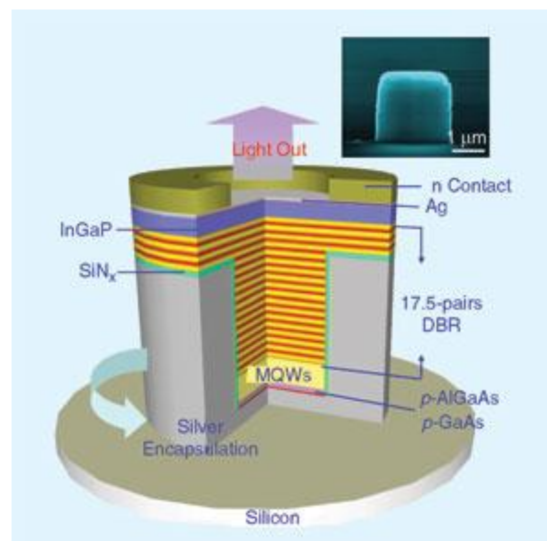


Nanolasers





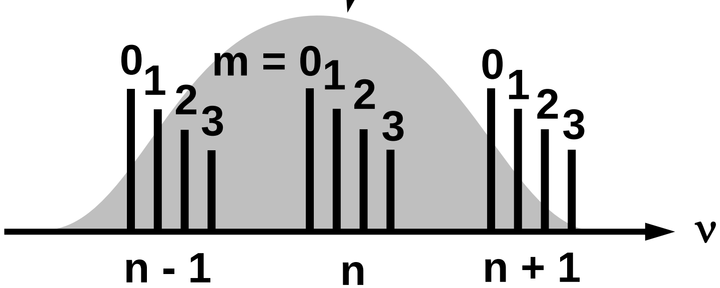
Combining semiconductors and metals enables nanolasers having a footprint roughly a factor <u>100 smaller</u> than that of a classical VCSEL. Adapted from a figure originally created by C. Y. Lu et al., UIUC.

Taken from D. Bimberg, IEEE Photonics Society Research Highlights

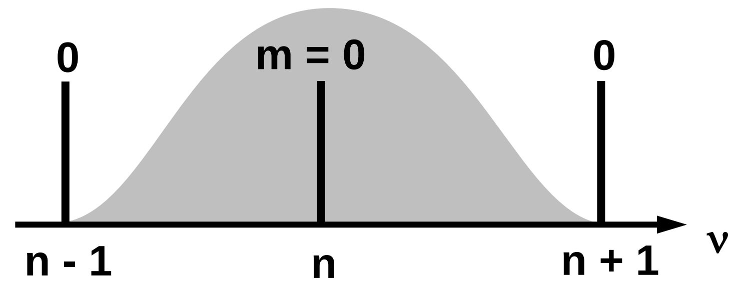
Why work on nanolasers?

<u>Most lasers</u>

Spontaneous emission spectrum

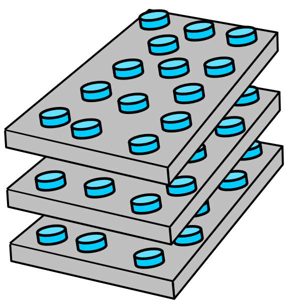


<u>Nanolasers</u>

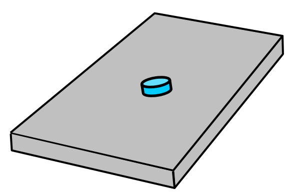


All emission into single resonator mode

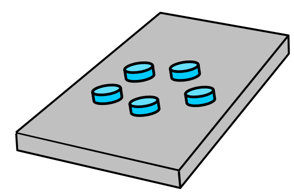
Typical QD-laser active region



Single-photon source active region

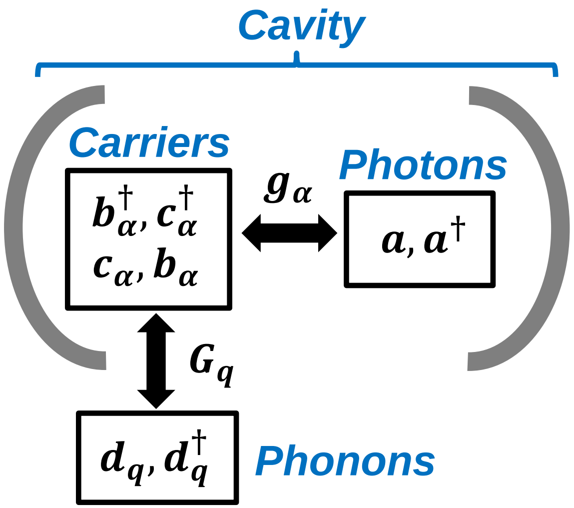


Few-QD active region



What are properties of emission from a few QDs?

How we used to study semiconductor lasers



$$H = H_0 \quad \textit{Single-particle energies}$$

$$-\hbar \sum_{\alpha} \left(g_\alpha b_\alpha^\dagger c_\alpha^\dagger a - g_\alpha^* a^\dagger c_\alpha b_\alpha \right) \quad \textit{Light-carrier}$$

$$+\hbar \sum_{\alpha\beta q} G_q \left(c_\alpha^\dagger c_\beta + b_\alpha^\dagger b_\beta \right) \left(d_q + d_q^\dagger \right) \quad \textit{Carrier-phonon}$$

Carrier-carrier

$$+\frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} c_\alpha^\dagger c_\beta^\dagger c_\eta c_\sigma + \frac{1}{2} \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_\alpha^\dagger b_\beta^\dagger b_\eta b_\sigma$$

$$- \sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta} b_\alpha^\dagger c_\beta^\dagger c_\eta b_\sigma$$

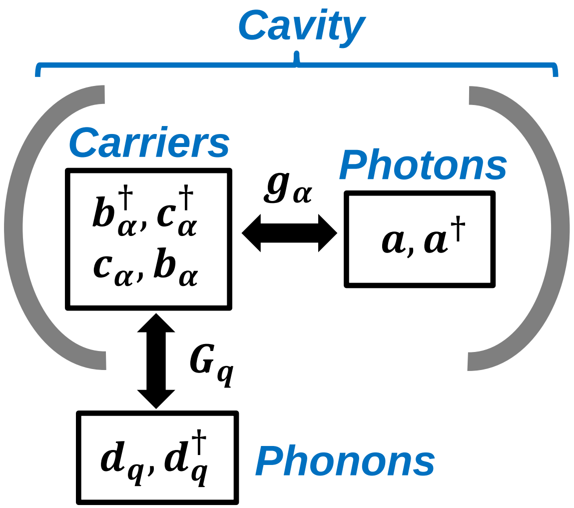
Matrix element of $\dfrac{e^2}{4\pi\varepsilon_b |r - r'|}$

Solve for:

$\langle c_\alpha^\dagger c_\alpha \rangle, \langle b_\alpha^\dagger b_\alpha \rangle, \langle a^\dagger a \rangle, \langle c_\alpha^\dagger b_\alpha^\dagger a \rangle$ *Populations and polarization*

$\left\langle c_\alpha^\dagger c_\beta^\dagger c_\eta c_\sigma \right\rangle, \left\langle c_\alpha^\dagger c_\beta^\dagger (d_q^\dagger)^n (d_q)^m \right\rangle$ *Correlations*

How we used to study semiconductor lasers



$H = H_0$ *Single-particle energies*

$$-\hbar \sum_\alpha \left(g_\alpha b_\alpha^\dagger c_\alpha^\dagger a - g_\alpha^* a^\dagger c_\alpha b_\alpha\right)$$ *Light-carrier*

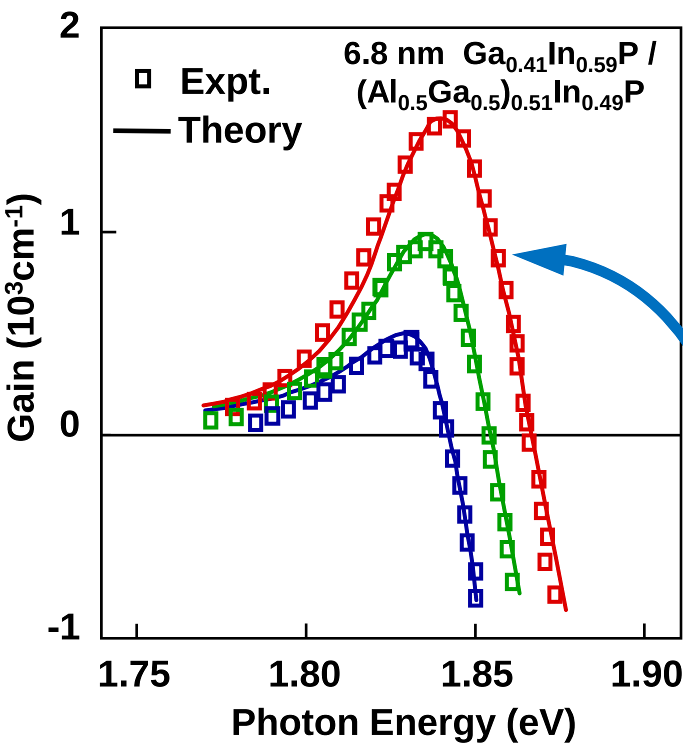
$$+\hbar \sum_{\alpha\beta q} G_q \left(c_\alpha^\dagger c_\beta + b_\alpha^\dagger b_\beta\right)\left(d_q + d_q^\dagger\right)$$ *Carrier-phonon*

Carrier-carrier

$$+\frac{1}{2}\sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta}\, c_\alpha^\dagger c_\beta^\dagger c_\eta c_\sigma + \frac{1}{2}\sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta}\, b_\alpha^\dagger b_\beta^\dagger b_\eta b_\sigma$$

$$-\sum_{\alpha\beta\sigma\eta} W_{\sigma\eta}^{\alpha\beta}\, b_\alpha^\dagger c_\beta^\dagger c_\eta b_\sigma$$

Matrix element of $\dfrac{e^2}{4\pi\varepsilon_b|r - r'|}$



Solve for:

$\langle c_\alpha^\dagger c_\alpha\rangle, \langle b_\alpha^\dagger b_\alpha\rangle, \langle a^\dagger a\rangle, \langle c_\alpha^\dagger b_\alpha^\dagger a\rangle$ *Populations and polarization*

$\left\langle c_\alpha^\dagger c_\beta^\dagger\, c_\eta\, c_\sigma\right\rangle, \left\langle c_\alpha^\dagger c_\beta^\dagger \left(d_q^\dagger\right)^n \left(d_q\right)^m\right\rangle$ *Correlations*

Information important for exploring new laser material systems

Chow, Smowton, Blood, Girdnt, Jahnke, Koch, APL.71, 157 (1997)

What we do differently with nanolasers

Injection current

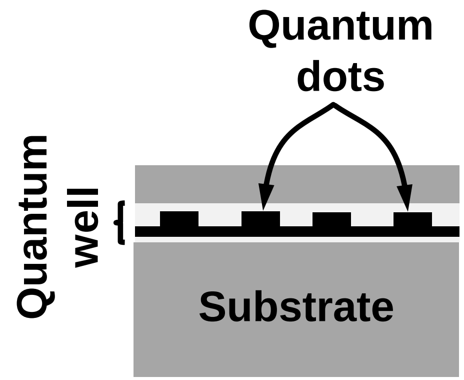
QW: $n_{e,k}^{QW}\,,\; n_{h,k}^{QW}$

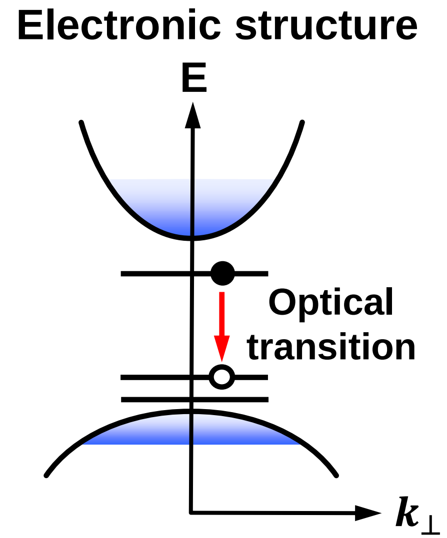
$\gamma_{c-c}\,,\;\gamma_{c-p}$

QDs: $\langle c_\alpha^\dagger c_\alpha\rangle$, $\langle b_\alpha^\dagger b_\alpha\rangle$, $\langle c_\alpha^\dagger b_\alpha^\dagger a\rangle$

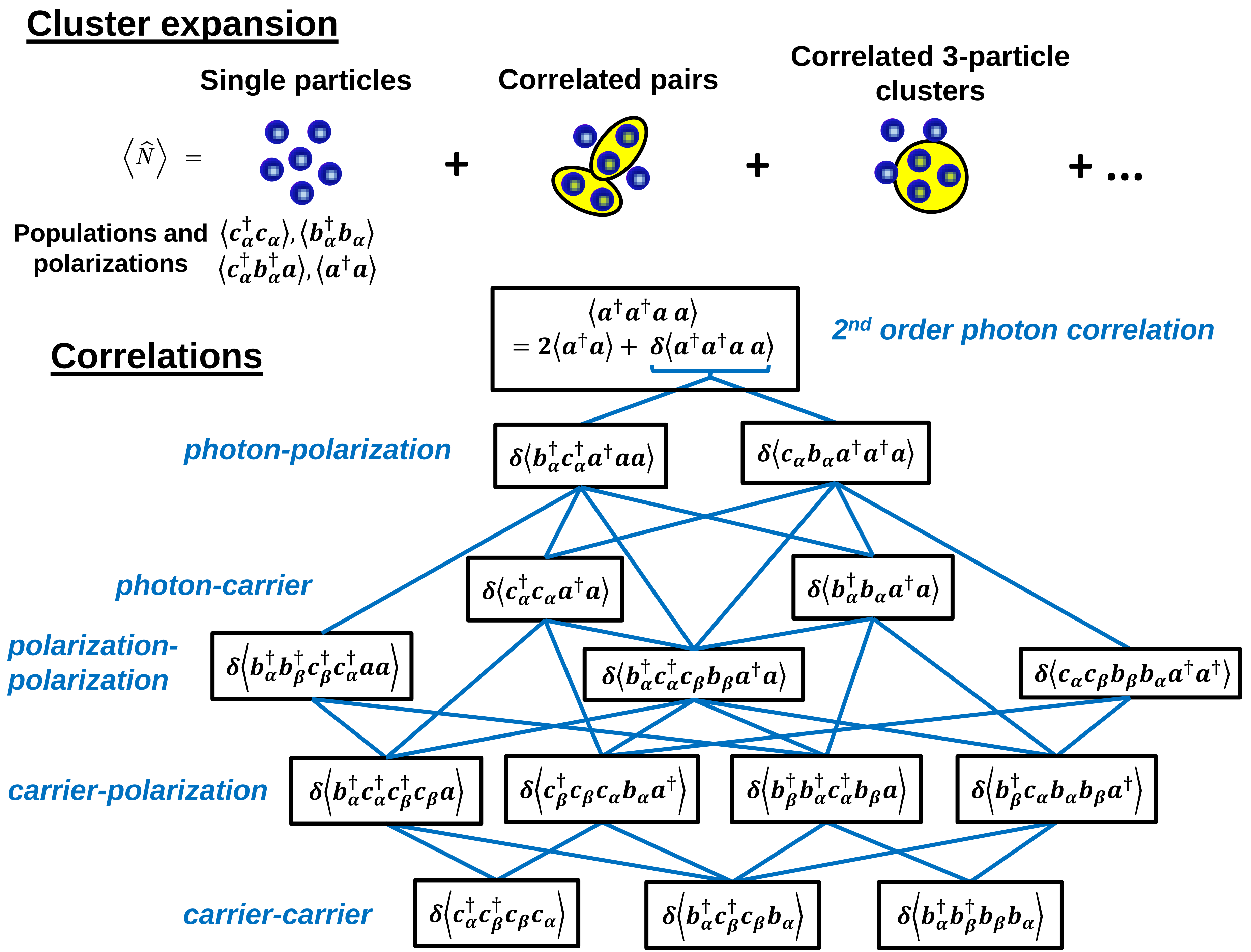
$$g_\alpha \leftarrow \wp\sqrt{\frac{\nu}{\hbar\epsilon_b V}}\,W(R_{QD})\sum_n C_\alpha(R_n)\,V_\alpha(R_n)$$

Photons: $\langle a^\dagger a\rangle$, $\langle a^\dagger a(\tau)\rangle \leftarrow$ **Coherence time**, $\langle a^\dagger a^\dagger a\, a\rangle \leftarrow$ **$g^{(2)}(0)$**





Emphasis now is on correlations involving light-matter interaction instead of Coulomb interaction


Cluster expansion
Single particles
Correlated pairs
Correlated 3-particle clusters
$\langle \hat{N} \rangle =$ + + + ...
Populations and polarizations
$\langle c_\alpha^\dagger c_\alpha \rangle, \langle b_\alpha^\dagger b_\alpha \rangle$
$\langle c_\alpha^\dagger b_\alpha^\dagger a \rangle, \langle a^\dagger a \rangle$
Correlations
$\langle a^\dagger a^\dagger a\, a \rangle = 2\langle a^\dagger a \rangle + \underbrace{\delta\langle a^\dagger a^\dagger a\, a \rangle}$
2nd order photon correlation
photon-polarization
$\delta\langle b_\alpha^\dagger c_\alpha^\dagger a^\dagger a a \rangle$
$\delta\langle c_\alpha b_\alpha a^\dagger a^\dagger a \rangle$
photon-carrier
$\delta\langle c_\alpha^\dagger c_\alpha a^\dagger a \rangle$
$\delta\langle b_\alpha^\dagger b_\alpha a^\dagger a \rangle$
polarization-polarization
$\delta\langle b_\alpha^\dagger b_\beta^\dagger c_\beta^\dagger c_\alpha^\dagger a a \rangle$
$\delta\langle b_\alpha^\dagger c_\alpha^\dagger c_\beta b_\beta a^\dagger a \rangle$
$\delta\langle c_\alpha c_\beta b_\beta b_\alpha a^\dagger a^\dagger \rangle$
carrier-polarization
$\delta\langle b_\alpha^\dagger c_\alpha^\dagger c_\beta^\dagger c_\beta a \rangle$
$\delta\langle c_\beta^\dagger c_\beta c_\alpha b_\alpha a^\dagger \rangle$
$\delta\langle b_\beta^\dagger b_\alpha^\dagger c_\alpha^\dagger b_\beta a \rangle$
$\delta\langle b_\beta^\dagger c_\alpha b_\alpha b_\beta a^\dagger \rangle$
carrier-carrier
$\delta\langle c_\alpha^\dagger c_\beta^\dagger c_\beta c_\alpha \rangle$
$\delta\langle b_\alpha^\dagger c_\beta^\dagger c_\beta b_\alpha \rangle$
$\delta\langle b_\alpha^\dagger b_\beta^\dagger b_\beta b_\alpha \rangle$

How to evaluate correlations

Similarities and difference with atomic, molecular, optical (AMO) approaches

$$c_\alpha c_\beta - c_\beta c_\alpha = 0$$

Example $\langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle = \langle c_\alpha^\dagger c_\alpha\rangle\langle a^\dagger a\rangle + \delta\langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle$

Normal ordered: $\frac{d}{dt} c_\alpha^\dagger c_\alpha a^\dagger a = -g_\alpha b_\alpha^\dagger c_\alpha^\dagger a^\dagger a a + \sum_\beta g_\beta b_\beta^\dagger c_\beta^\dagger c_\alpha^\dagger c_\alpha a + h.a.$

Factorization: $\left\langle b_\beta^\dagger c_\beta^\dagger c_\alpha^\dagger c_\alpha a\right\rangle = (1-\delta_{\alpha\beta}) p_\beta n_e^\alpha + \delta\left\langle b_\beta^\dagger c_\beta^\dagger c_\alpha^\dagger c_\alpha a\right\rangle$

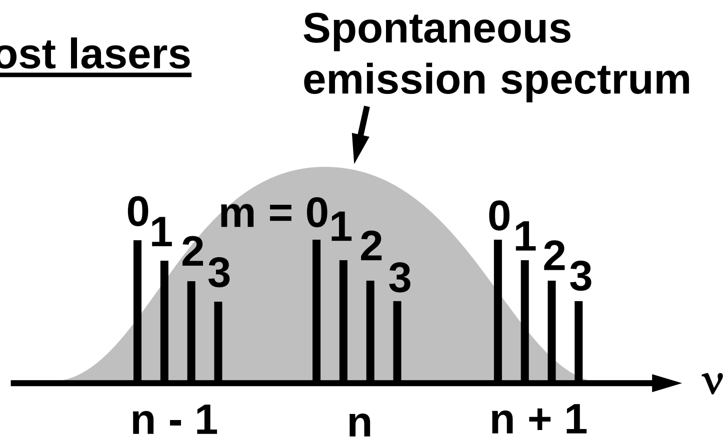
$\vdots$

$$\frac{d}{dt}\delta\langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle = \frac{d}{dt}\langle c_\alpha^\dagger c_\alpha a^\dagger a\rangle - \frac{d n_e^\alpha n_p}{dt}$$

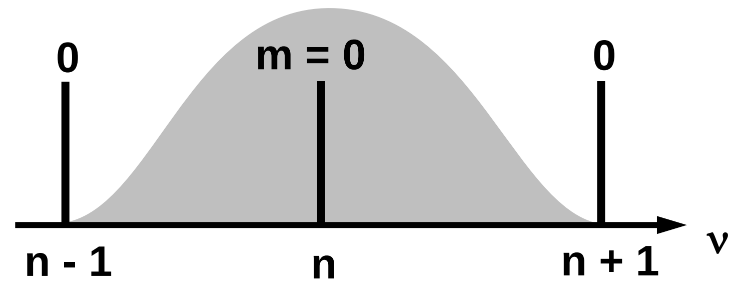
$$= -g_\alpha p_\alpha (n_e^\alpha + n_p) - g_\alpha \delta\langle b_\alpha^\dagger c_\alpha^\dagger a^\dagger a a\rangle + \sum_\beta \delta\left\langle b_\beta^\dagger c_\beta^\dagger c_\alpha^\dagger c_\alpha a\right\rangle + c.c.$$

Why work on nanolasers?

Most lasers



Nano lasers



All emission into single resonator mode

$\ll 1$

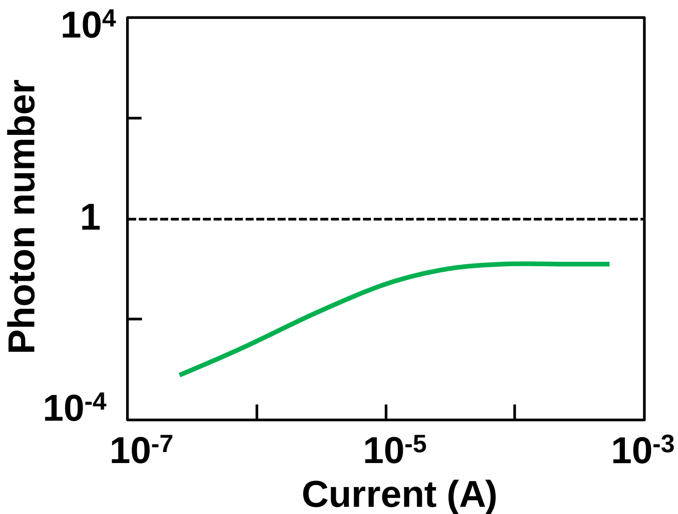
$= 1$

Spontaneous emission factor

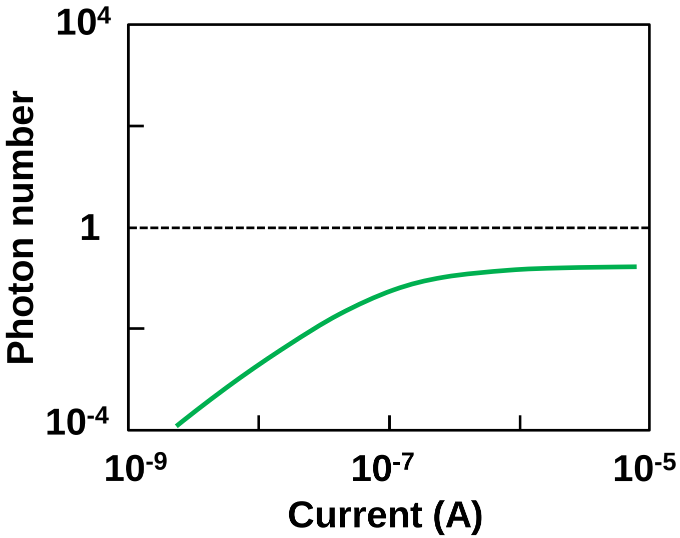
$$\beta = \frac{\gamma_l}{\gamma_{sp}}$$

N_{QD} = 50, Δ_{inh} = 20meV, γ_c = 10^{10}, 5 x 10^{10}, 10^{11}, 5 x $10^{11}$$s^{-1}$

β = 0.01



β = 1



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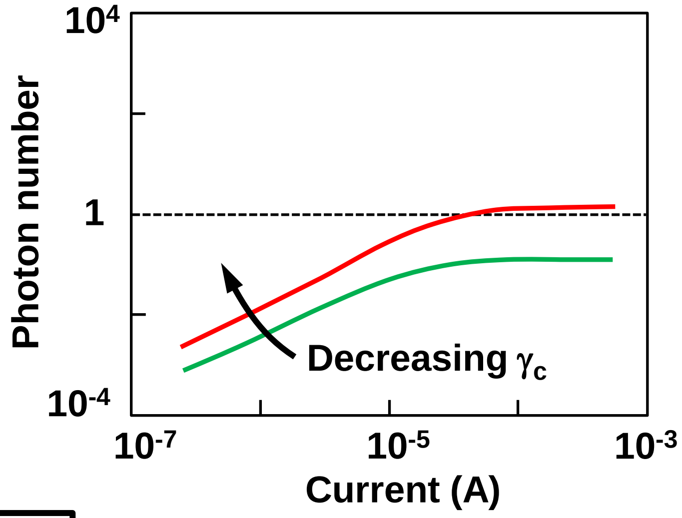

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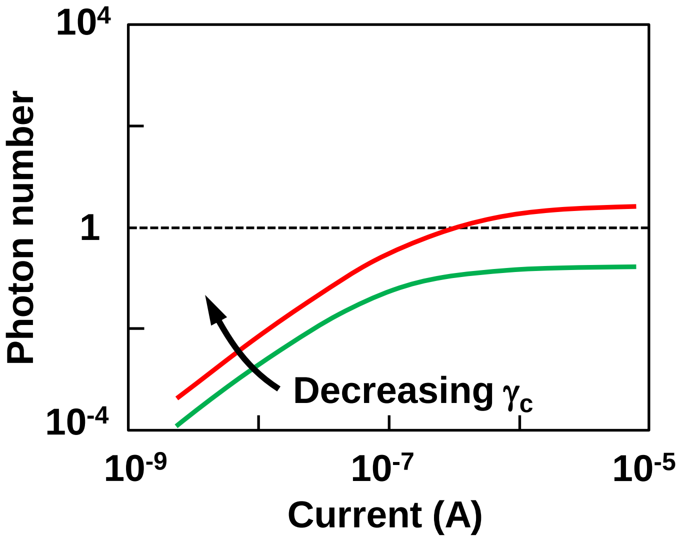

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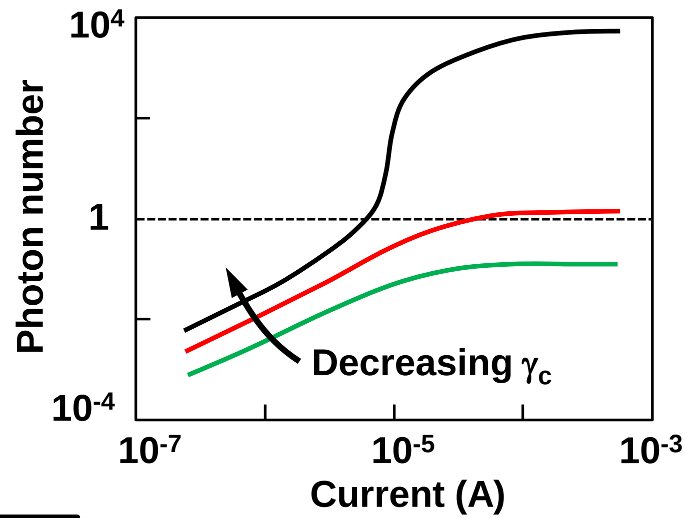
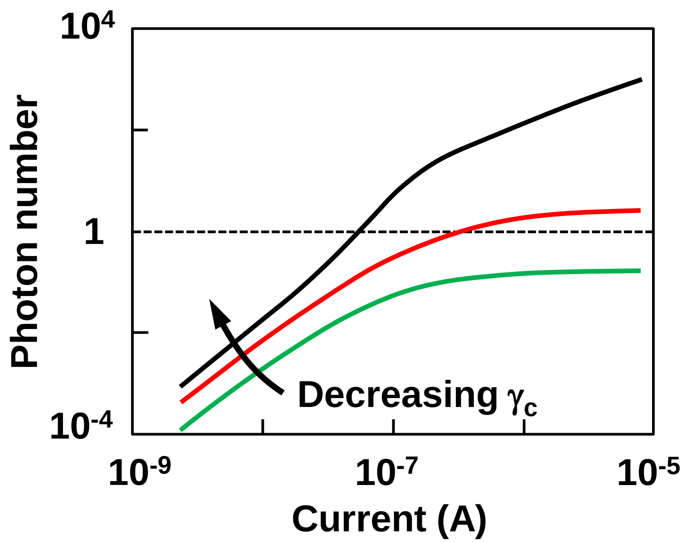


β = 1



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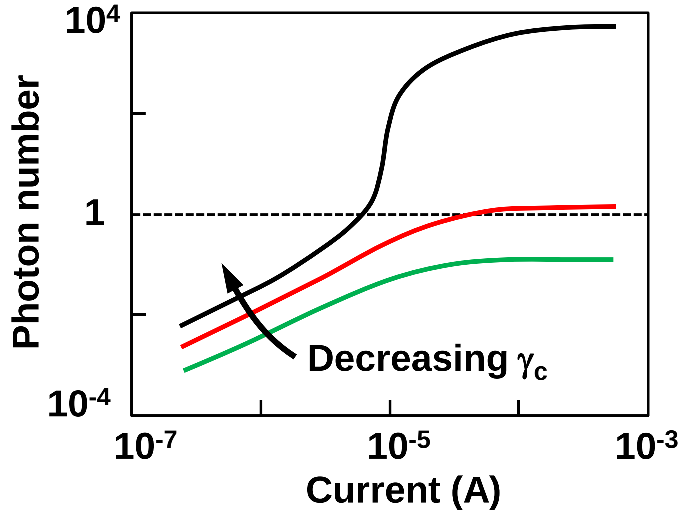
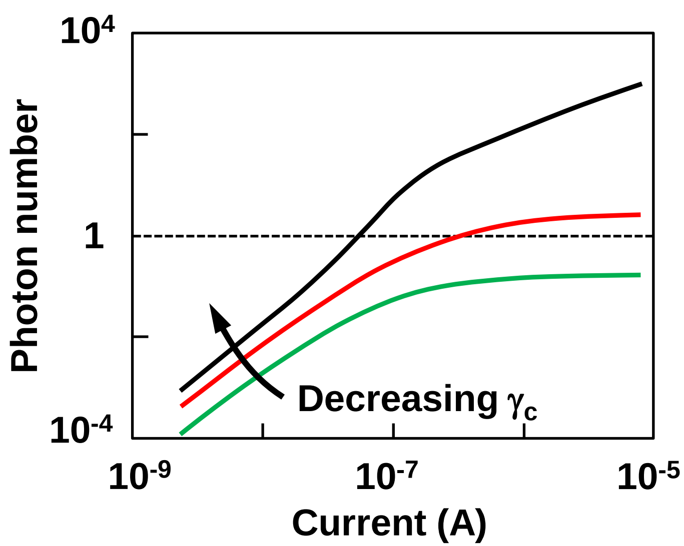
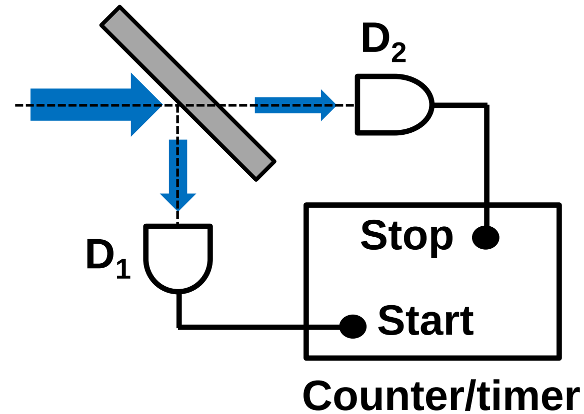
$\beta = 0.01$



$\beta = 1$

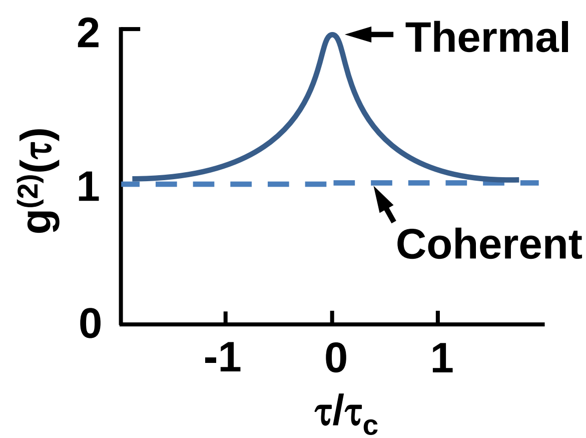


Hanbury-Brown-Twiss experiment



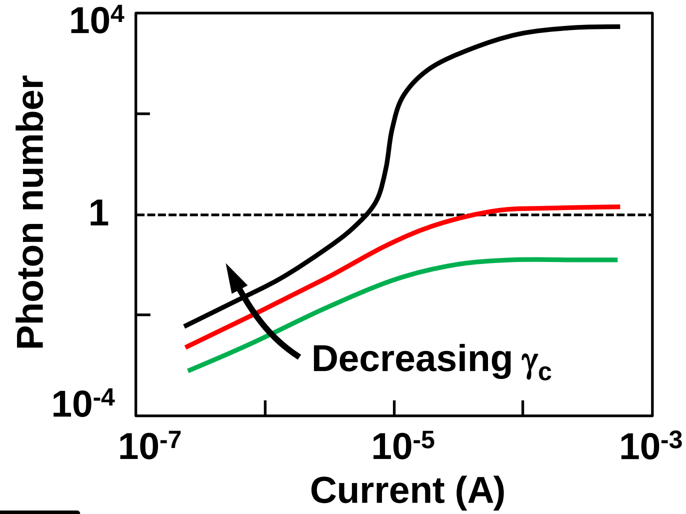
Second-order correlation function

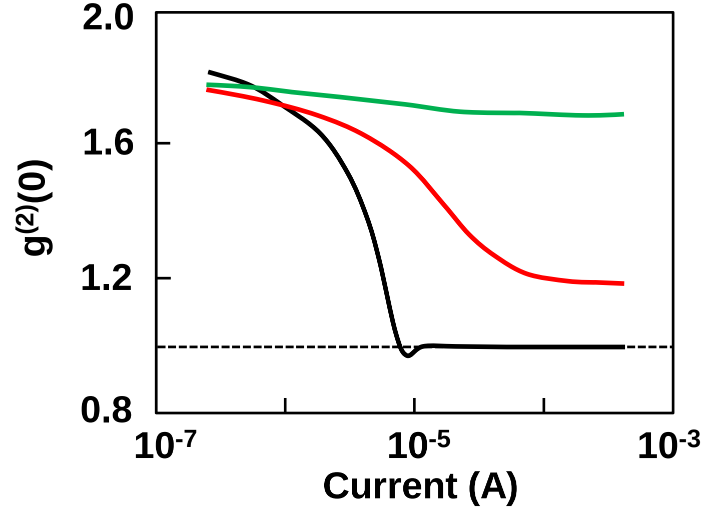
$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$



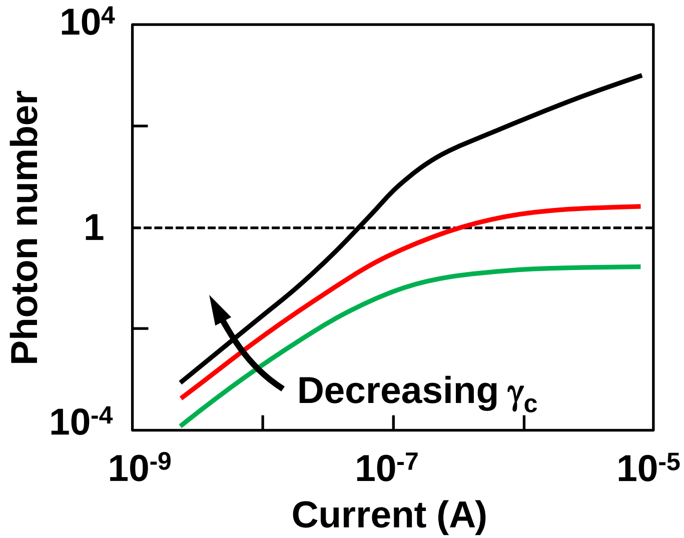
N_{QD} = 50, Δ_{inh} = 20meV, γ_c = 10^{10}, 5 x 10^{10}, 10^{11}, 5 x 10^{11} s^{-1}

β = 0.01



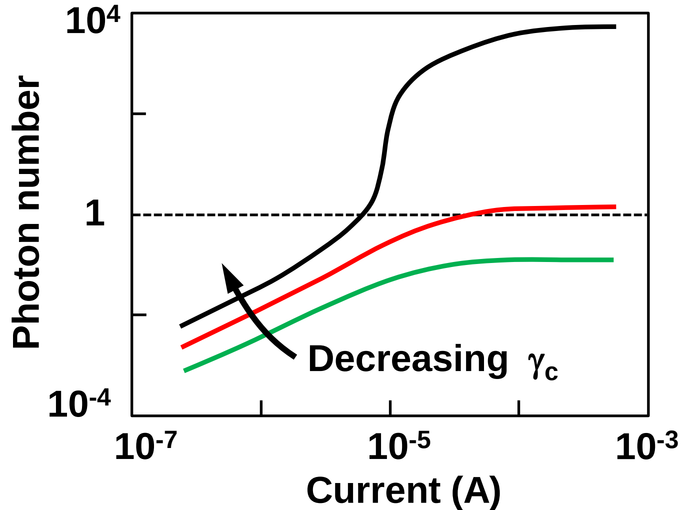


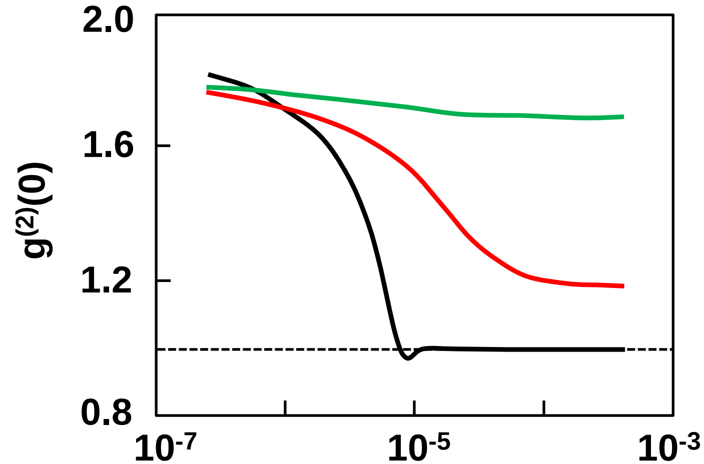
β = 1



$N_{QD} = 50$, $\Delta_{inh} = 20$meV, $\gamma_c = 10^{10}, 5 \times 10^{10}, 10^{11}, 5 \times 10^{11} s^{-1}$

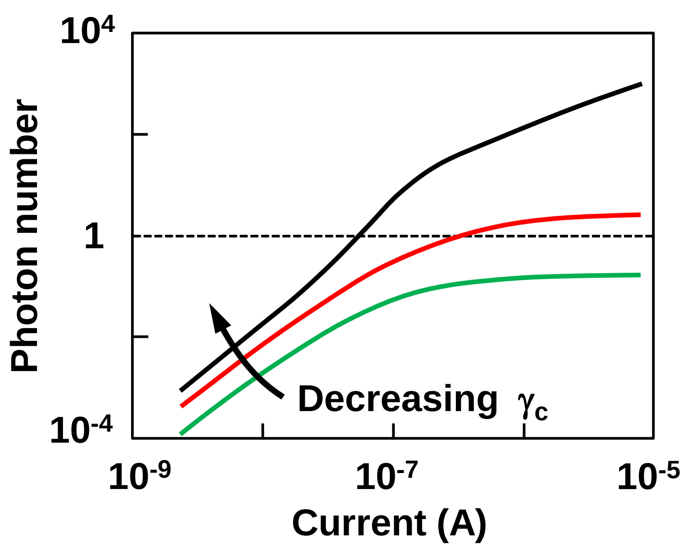
$\beta = 0.01$

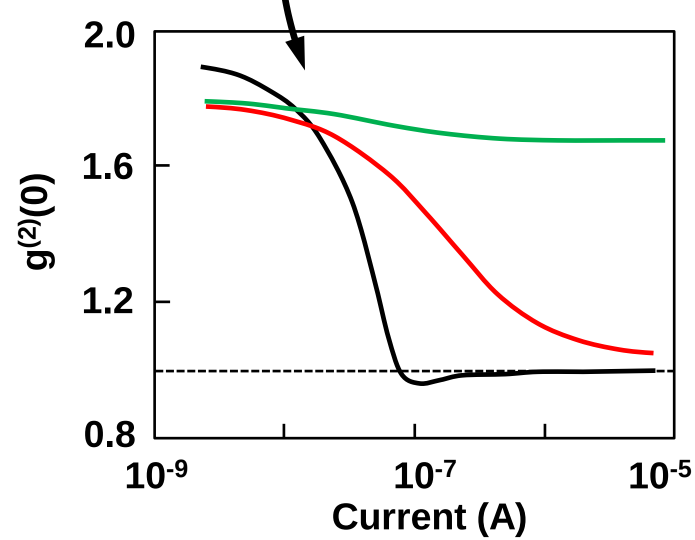




$\beta = 1$

No such thing as thresholdless lasing?

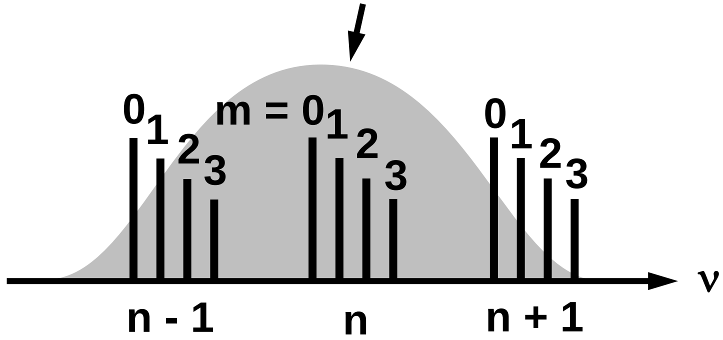




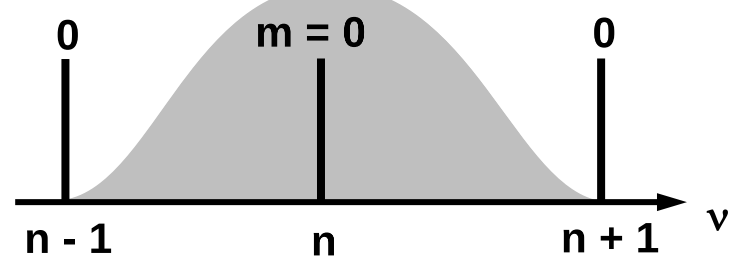
Why work on nanolasers?

Most lasers

Spontaneous emission spectrum

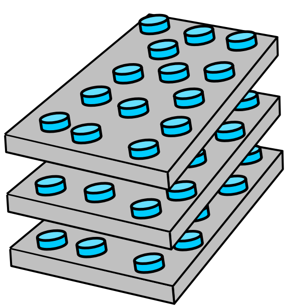


Nanolasers

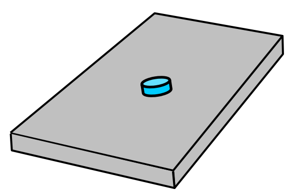


All emission into single resonator mode

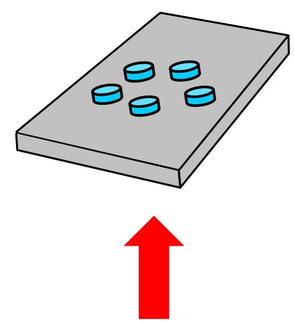
Typical QD-laser active region



Single-photon source active region



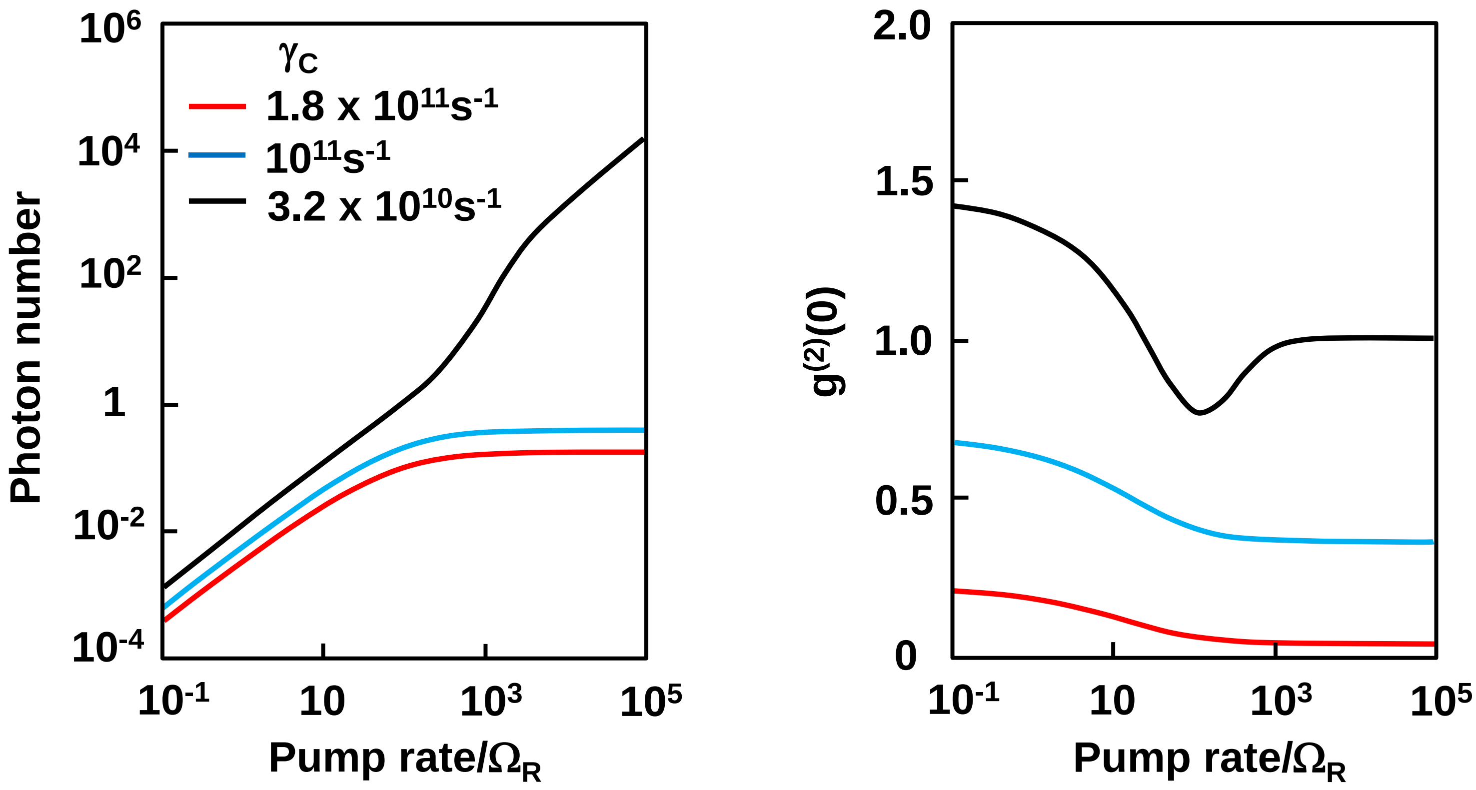
Few-QD active region



What are properties of emission from a few QDs?

Single-quantum-dot active region

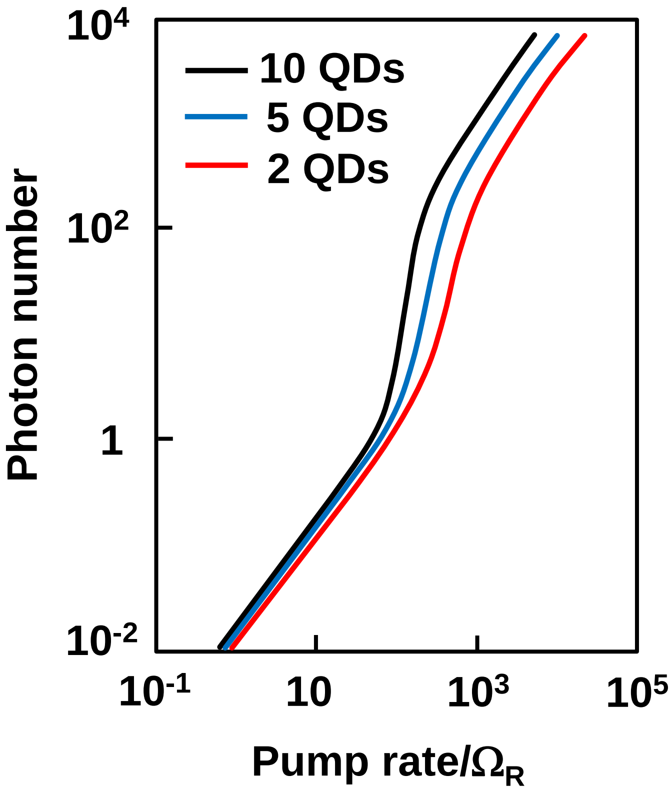
Single-photon source ➡ Nonclassical-light source ➡ Laser

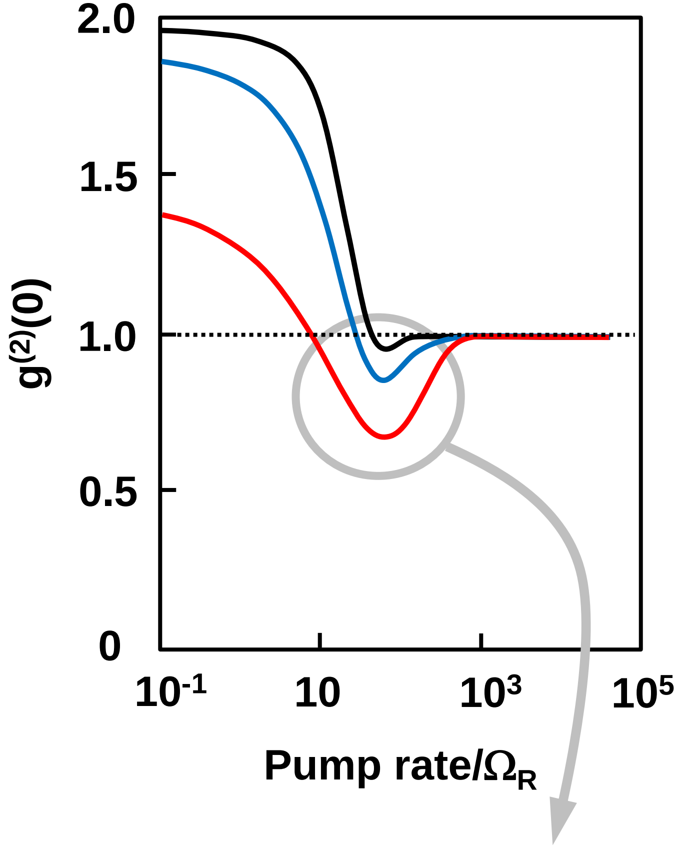


Vacuum Rabi frequency: $\Omega_R = 2.7 \times 10^{11} s^{-1}$

Few-quantum-dot active region

Question: Can we increase single-photon production rate?

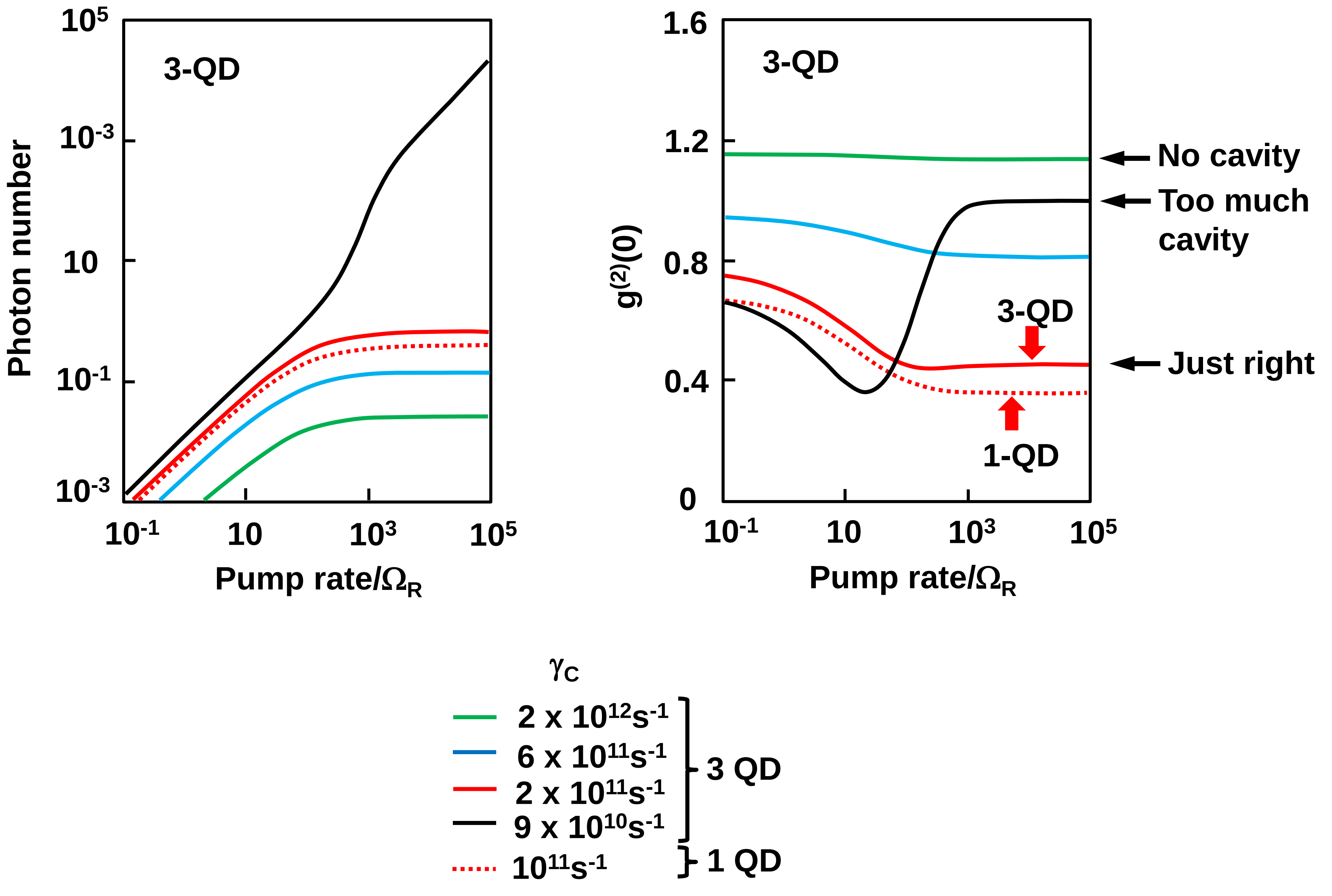




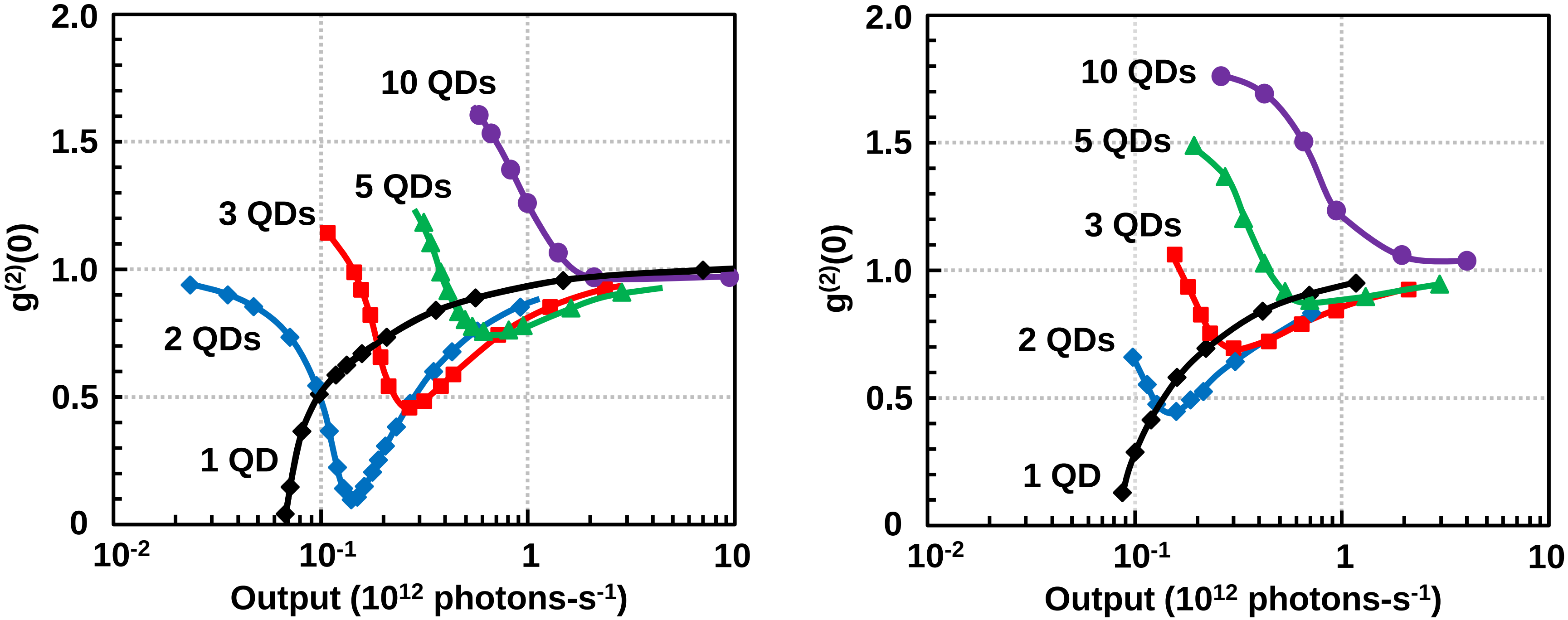
QD	γ_C
10	$2 \times 10^{11} s^{-1}$
5	$10^{11} s^{-1}$
2	$6 \times 10^{10} s^{-1}$

Light-carrier correlation
Leading terms:
$\delta\langle c^\dagger c a^\dagger a\rangle, \delta\langle b^\dagger b a^\dagger a\rangle,$
$\delta\langle b^\dagger c^\dagger a^\dagger a a\rangle$

Single-photon source with few-emitter active region?



Increasing single-photon production rate with few-emitter active region



Concerns and next refinements

<u>Present model</u>

Injection current

More rigorous treatment of excitation necessary for few-QD situations

QW

$n^{QW}_{e,k}$, $n^{QW}_{h,k}$

γ_{c-c} , γ_{c-p}

QDs

$\langle c^\dagger_n c_n \rangle$

$\langle b^\dagger_n b_n \rangle$

$\langle c^\dagger_n b^\dagger_n a \rangle$

ω_n

g

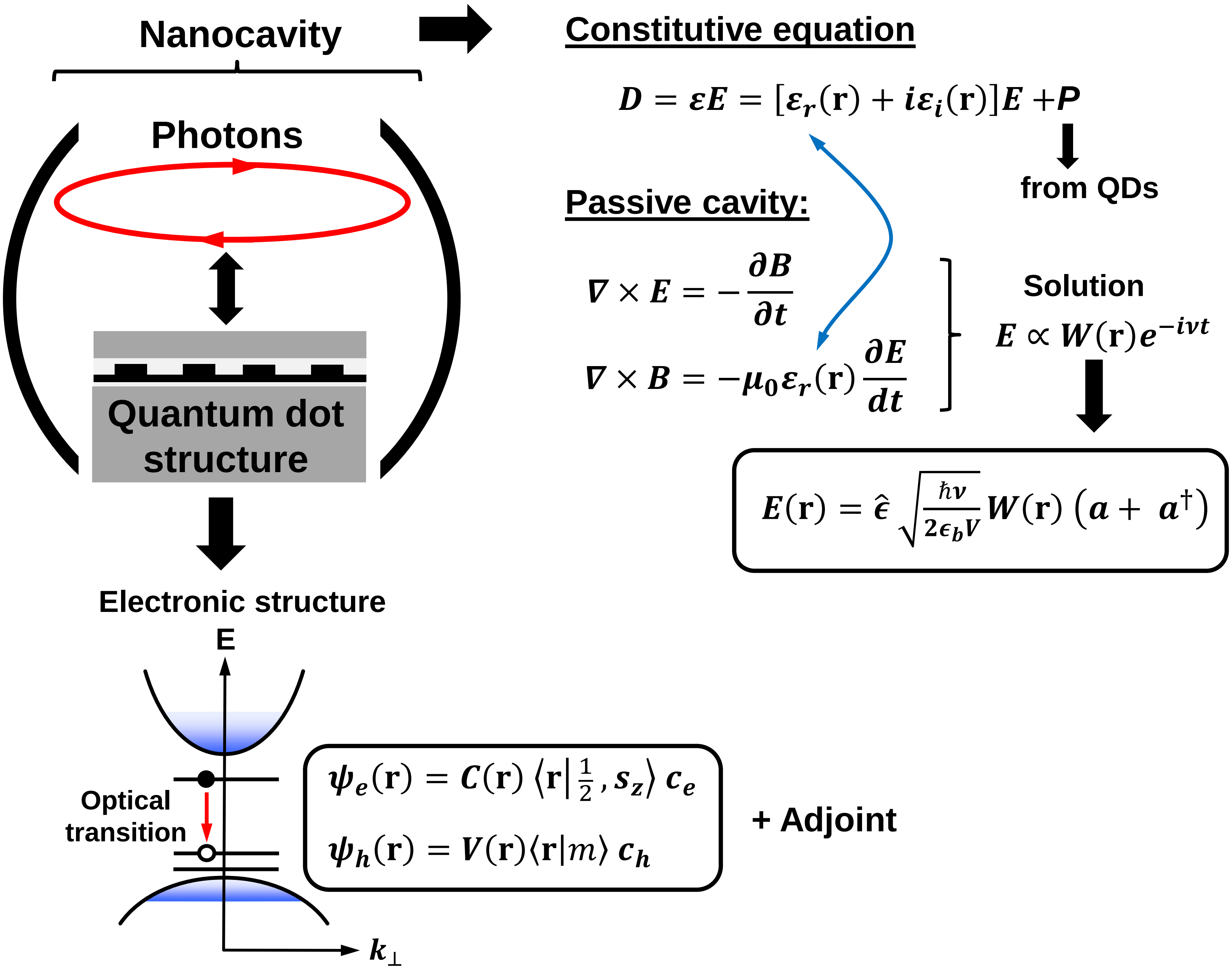
Photons

$\langle a^\dagger a \rangle$ $\langle a^\dagger a(\tau) \rangle$

$\langle a^\dagger a^\dagger a\, a \rangle$ $\langle a^\dagger a^\dagger a\, b_n c_n \rangle$

$\langle a^\dagger a\, c^\dagger_n c_n \rangle$ $\langle a^\dagger a\, b^\dagger_n b_n \rangle$

Beyond doublet



Nanocavity
Photons
Quantum dot structure
Electronic structure
E
$k_\perp$
Optical transition
$\psi_e(\mathbf{r}) = C(\mathbf{r}) \langle \mathbf{r} | \frac{1}{2}, s_z \rangle c_e$
$\psi_h(\mathbf{r}) = V(\mathbf{r}) \langle \mathbf{r} | m \rangle c_h$
+ Adjoint
Constitutive equation
$D = \varepsilon E = [\varepsilon_r(\mathbf{r}) + i\varepsilon_i(\mathbf{r})]E + P$
from QDs
Passive cavity:
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times B = -\mu_0 \varepsilon_r(\mathbf{r}) \frac{\partial E}{dt}$
Solution
$E \propto W(\mathbf{r}) e^{-i\nu t}$
$E(\mathbf{r}) = \hat{\epsilon} \sqrt{\frac{\hbar\nu}{2\epsilon_b V}} W(\mathbf{r}) \left(a + a^\dagger\right)$

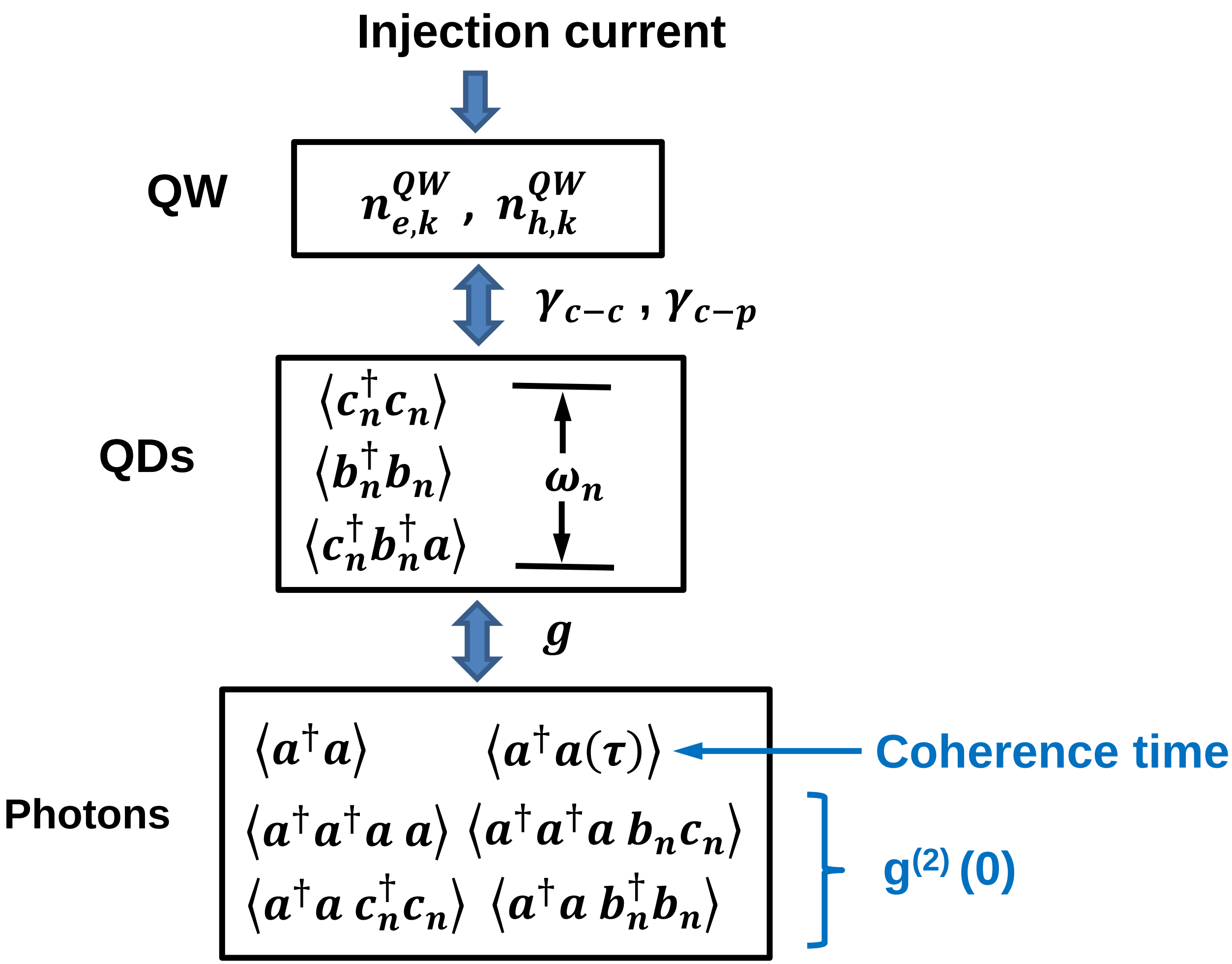
Hamiltonian

$$\mathbf{H} = \hbar\nu\left(a^{\dagger}a + \frac{1}{2}\right) + \sum_{\alpha}\left[\varepsilon_{e,\alpha}^{QD}\, c_{\alpha}^{\dagger}c_{\alpha} + \varepsilon_{h,\alpha}^{QD}\, b_{\alpha}^{\dagger}b_{\alpha} - \hbar g\left(c_{\alpha}^{\dagger}b_{\alpha}^{\dagger}a - a^{\dagger}b_{\alpha}c_{\alpha}\right)\right]$$

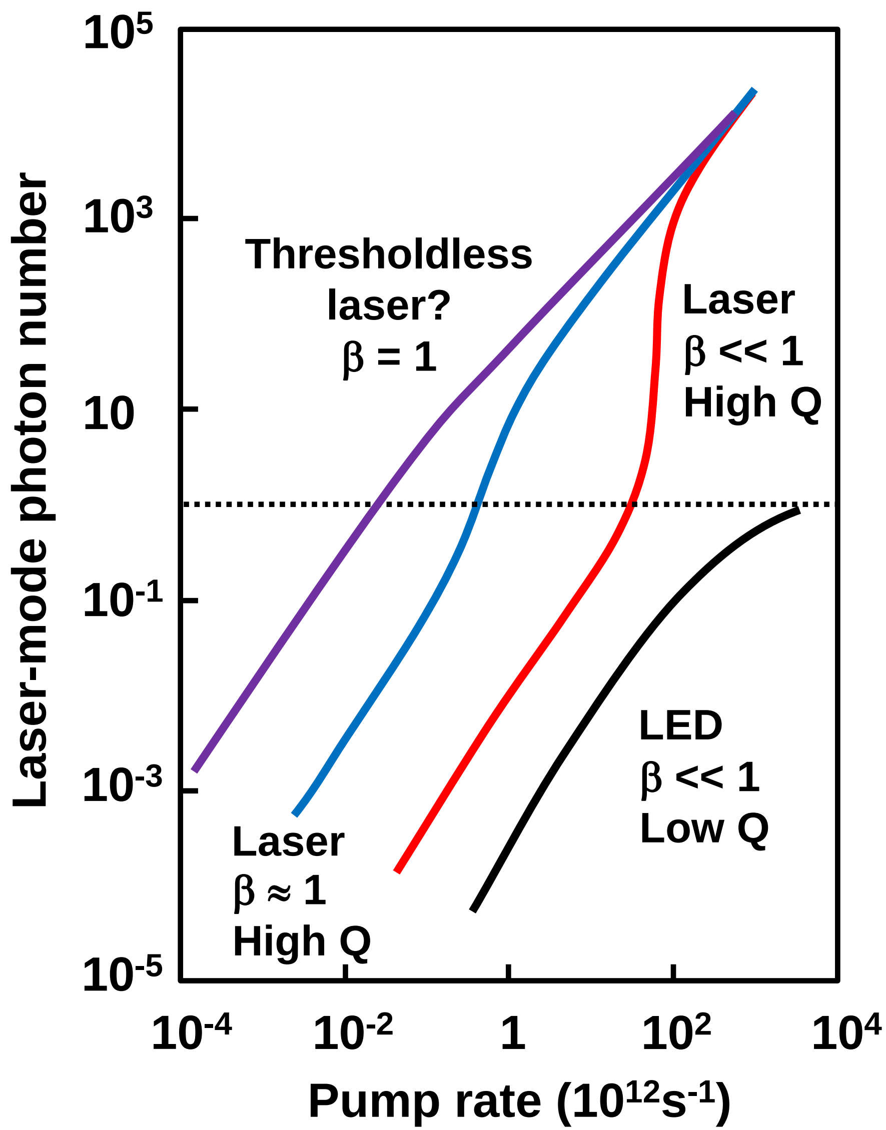
Inhomogeneous broadening

$$\wp\sqrt{\frac{\nu}{\hbar\epsilon_b V}}\, W\left(R_{QD}\right)\sum_{n} C(R_n)\, V(R_n)$$

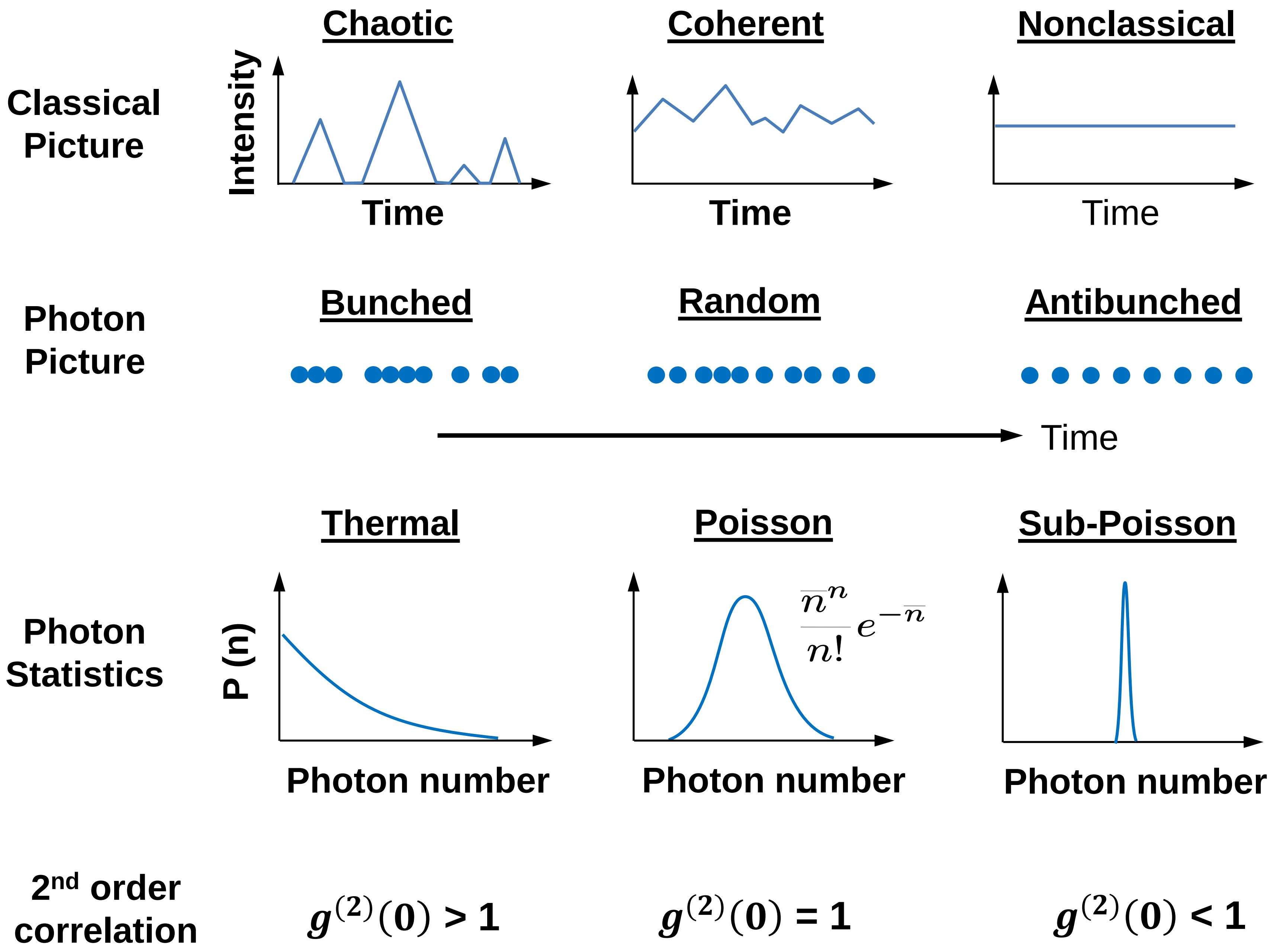
Present model



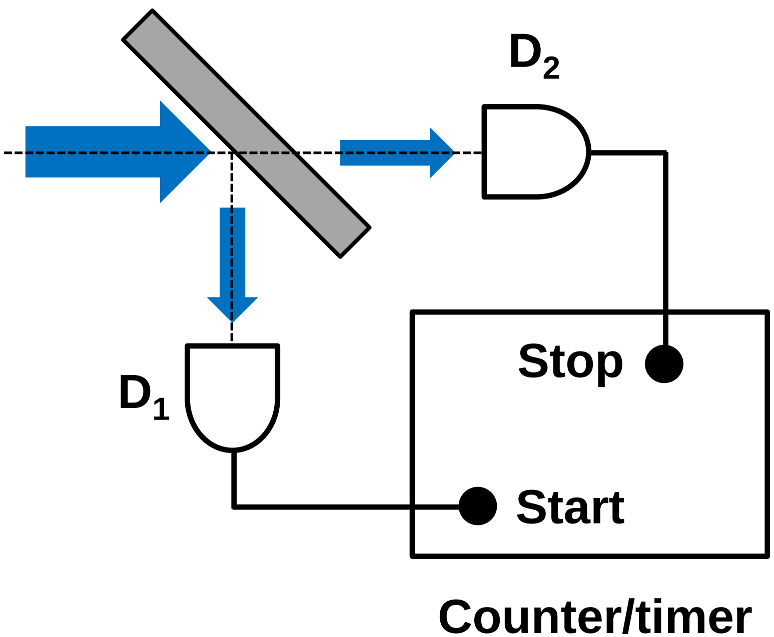
LED ⟶ Laser ⟶ Thresholdless laser



Types of light



Hanbury-Brown-Twiss experiment

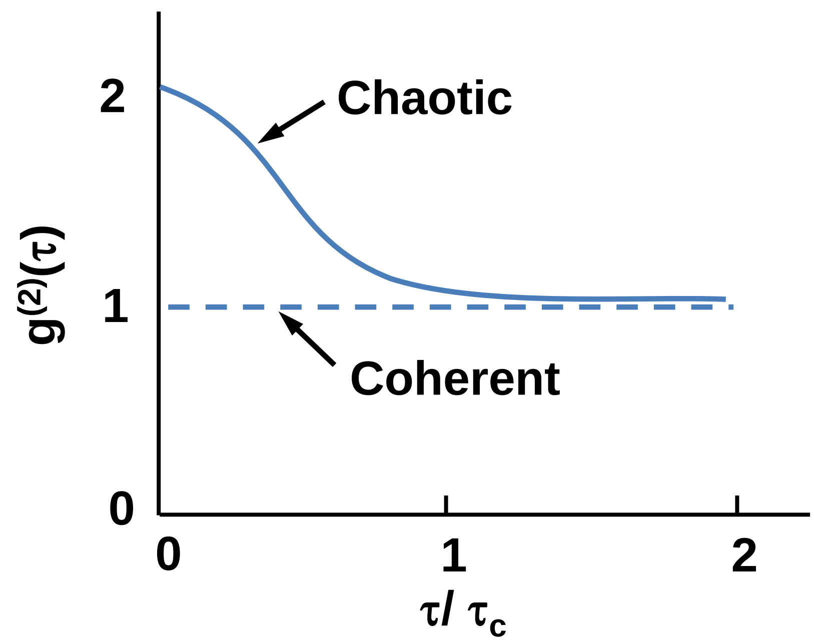


Second-order correlation function

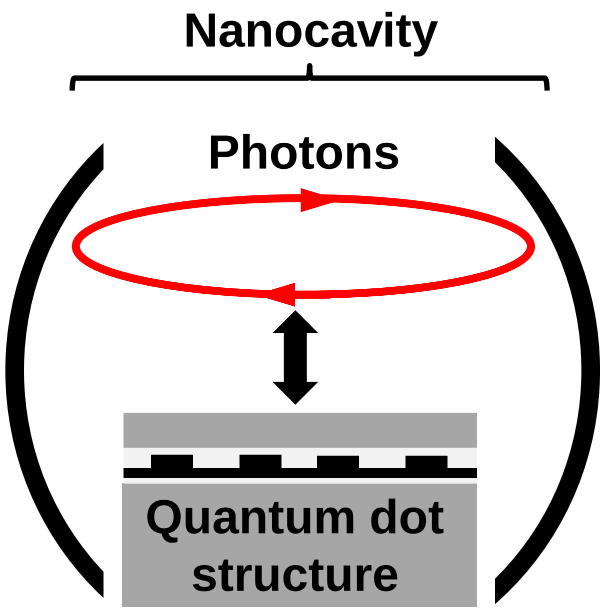
$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

$$= \frac{\langle n_1(t)n_2(t+\tau)\rangle}{\langle n_1(t)\rangle\langle n_2(t+\tau)\rangle}$$

$$= \frac{\left\langle a^\dagger(t)a^\dagger(t+\tau)\,a(t+\tau)a(t)\right\rangle}{\langle a^\dagger(t)\,a(t)\rangle\langle a^\dagger(t+\tau)\,a(t+\tau)\rangle}$$



Theoretical model



Example of Nanolaser