

SAND2014-16510PE

# Dielectric Response in Extreme Conditions Using Time-Dependent Density Functional Theory

Andrew Baczewski, Luke Shulenburger, Mike Desjarlais,  
Rudolph J. Magyar

Sandia National Laboratories, Albuquerque NM

Strongly Coupled Coulomb Systems  
July 28th, 2014

# Outline

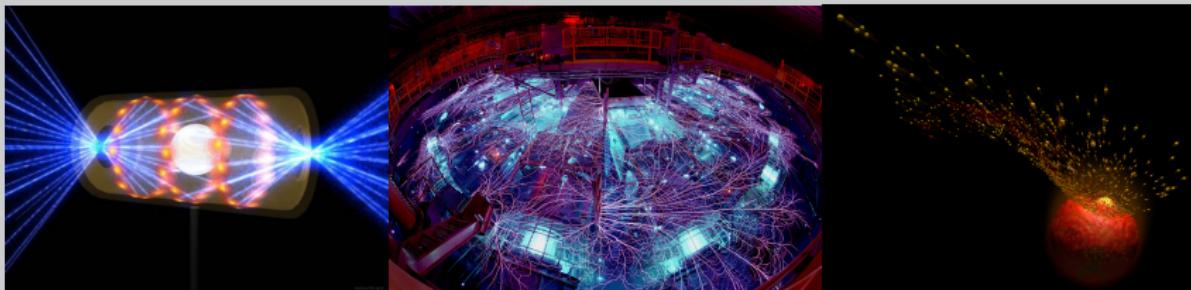
- ▶ Dielectric Response of Warm Dense Matter
- ▶ Our Implementation of Ehrenfest-TDDFT
- ▶ Stopping Power
- ▶ X-Ray Thomson Scattering
- ▶ Conclusion

- ▶ **Nomenclature:**

- ▶ BO → Born-Oppenheimer
- ▶ BZ → Brillouin Zone
- ▶ DFT → Density Functional Theory
- ▶ DSF → Dynamic Structure Factor
- ▶ KS → Kohn-Sham
- ▶ MD → Molecular Dynamics
- ▶ PAW → Projector Augmented-Wave
- ▶ TD → Time-Dependent
- ▶ WDM → Warm Dense Matter
- ▶ XRTS → X-ray Thomson Scattering

- ▶ Motivation: ICF, shock diagnostics, planetary science



- ▶ **What is it?**

- ▶ **Warm:** temperatures on the order of eVs (10kK+)
- ▶ **Dense:** electron densities  $2-4 \times$  solid
- ▶ **'Exotic':** neither standard condensed phase nor ideal plasma
- ▶ **Challenging:** both experimentally and theoretically

► **How do we study it?**

- Time-honored tradition among physicists...
- Throw something at it, watch what happens.

$$\begin{bmatrix} \delta\rho(\mathbf{r}, t) \\ \delta\mathbf{j}(\mathbf{r}, t) \end{bmatrix} = \int dt' d\mathbf{r}' \begin{bmatrix} \chi_{d,d}(\mathbf{r}, \mathbf{r}', t - t') & \chi_{d,c}(\mathbf{r}, \mathbf{r}', t - t') \\ \chi_{c,d}(\mathbf{r}, \mathbf{r}', t - t') & \chi_{c,c}(\mathbf{r}, \mathbf{r}', t - t') \end{bmatrix} \cdot \begin{bmatrix} V_{ext}(\mathbf{r}', t') \\ \mathbf{A}_{ext}(\mathbf{r}', t') \end{bmatrix}$$

► **What can we compute?**

- Stopping power
- Optical response
- XRTS spectrum
- We seek to **model** experimental process, i.e., throw and watch
- **Real-time electron dynamics → dielectric response**

# Our Approach

- ▶ **Ehrenfest-TDDFT:** MD simulation with concurrent electron-ion motion
  - ▶ Ehrenfest: ionic forces on average potential energy surface
  - ▶ TDDFT: theory in which real-time electron dynamics evolve
- ▶ **Method:**
  - ▶ Start in **Mermin** state:  $n_0(\mathbf{r}) = \sum_{n,\mathbf{k}} f_{n,\mathbf{k}} |\psi_{n,\mathbf{k}}(\mathbf{r})|^2$
  - ▶ Integrate TD-KS equations in the presence of **perturbing potential**:

$$i \frac{\partial}{\partial t} \psi_{n,\mathbf{k}}(\mathbf{r}, t) = \left[ -\frac{\nabla^2}{2} + V_{KS}(\mathbf{r}, t) + V_{ext}(\mathbf{r}, t) \right] \psi_{n,\mathbf{k}}(\mathbf{r}, t)$$

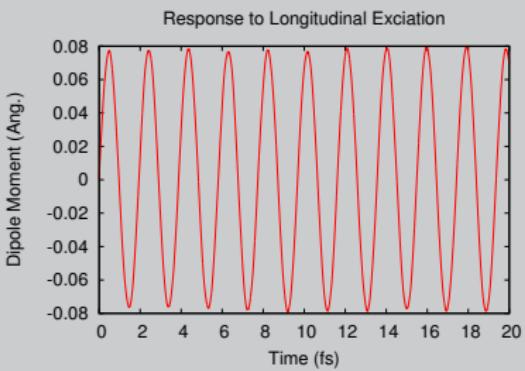
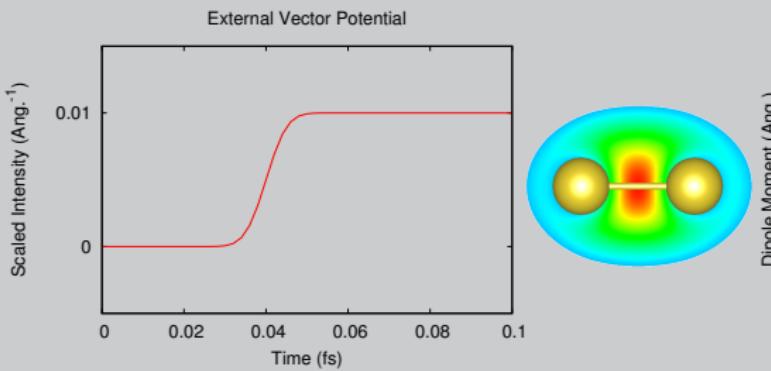
- ▶ Compute **ionic forces** along the way, update (x,p) if needed
- ▶ Record response in terms of observables (**density functionals**):

$$\langle \hat{O}(t) \rangle = O[n(\mathbf{r}, t)], \quad n(\mathbf{r}, t) = \sum_{n,\mathbf{k}} f_{n,\mathbf{k}} |\psi_{n,\mathbf{k}}(\mathbf{r}, t)|^2$$

- ▶ **Goal of Talk:** describe capabilities developed over the past 16 months

# A Simple Example

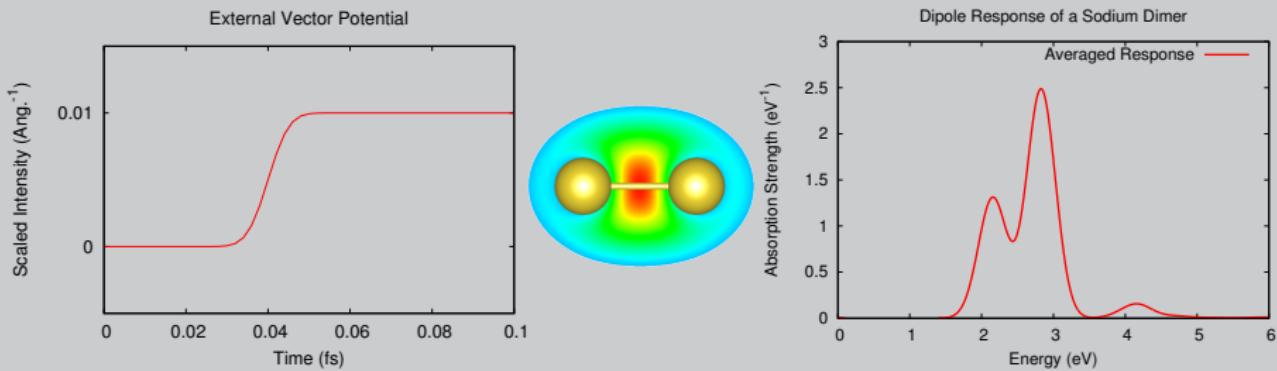
- ▶ Ignoring ionic motion for a moment...
- ▶ Consider subjecting a Na dimer to a homogeneous vector potential



- ▶ Time evolved dipole moment → polarizability/optical absorption

# A Simple Example

- ▶ Ignoring ionic motion for a moment...
- ▶ Consider subjecting a Na dimer to a homogeneous vector potential



- ▶ Time evolved dipole moment → polarizability/optical absorption

# Implementation Details

## ► Ehrenfest-TDDFT in VASP:

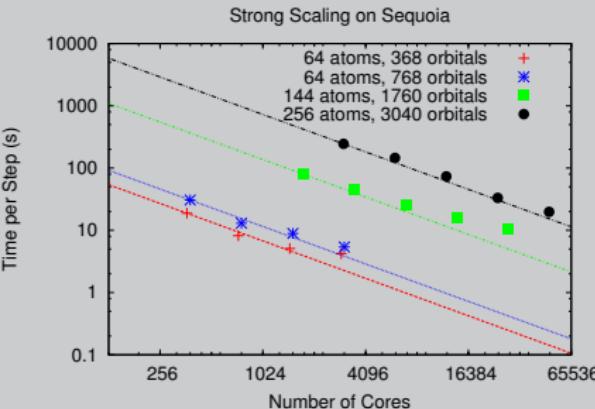
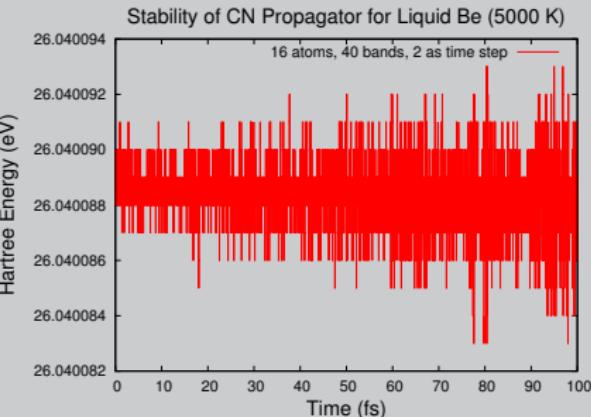
- ▶ Uses PAW method
- ▶ Crank-Nicolson time integration w/gauge correction
- ▶ Demonstrated scalability on BGQ
- ▶ 1 of 3 implementations for extended systems

## ► Advantages:

- ▶ Easier to scale than BOMD
- ▶ Lower cost complexity
- ▶ “Real” information about excitations

## ► Disadvantages:

- ▶ Small time step



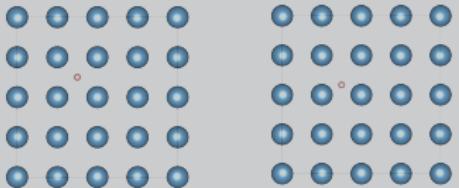
# Stopping Power

- ▶ **Electronic contribution:** dielectric response to charged particle

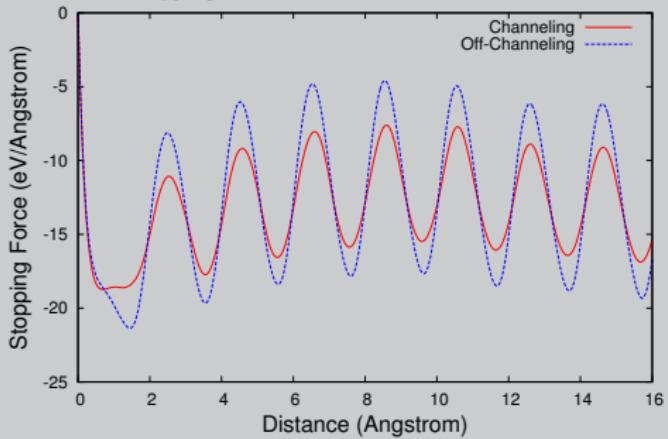
$$\delta\rho(\mathbf{r}, t) = \int dt' \int d\mathbf{r}' \chi_{d,d}(\mathbf{r}, \mathbf{r}', t - t') V_{ion}(\mathbf{r}' - \mathbf{v}t')$$

- ▶ Average force of medium on ion as a function of velocity and Z
- ▶ **BOMD:** stopping power zero
- ▶ **Ehrenfest-TDDFT:** agrees well with experiment
- ▶ **Note:** Movies are 2D projection of 3D simulation

# Stopping Validation

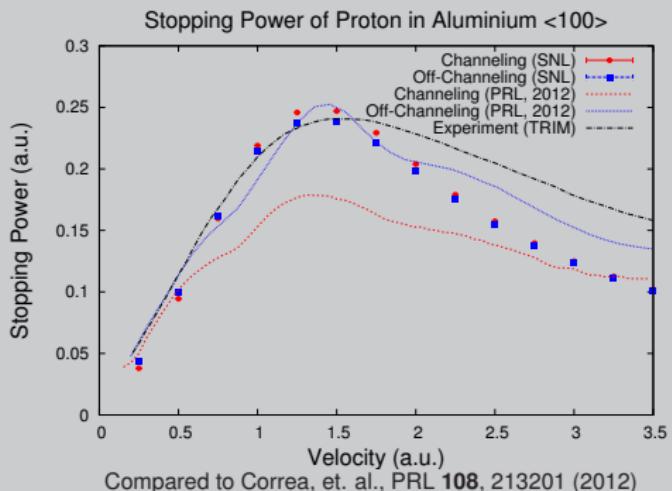


Stopping Forces on Proton in Aluminium <100>



- ▶ Validation against Correa, et. al., PRL **108**, 213201 (2012)
- ▶ **Proton stopping in fcc Al**
  - ▶ 193 electrons
  - ▶ 20 k-points in BZ
  - ▶ Fixed resolution of path
  - ▶  $\Delta t \leq 2$  as
- ▶ Experimental error  $\sim 10\%$
- ▶ Disagreement beyond peak
  - ▶ Core excitations?
  - ▶ Trajectory sampling?

# Stopping Validation



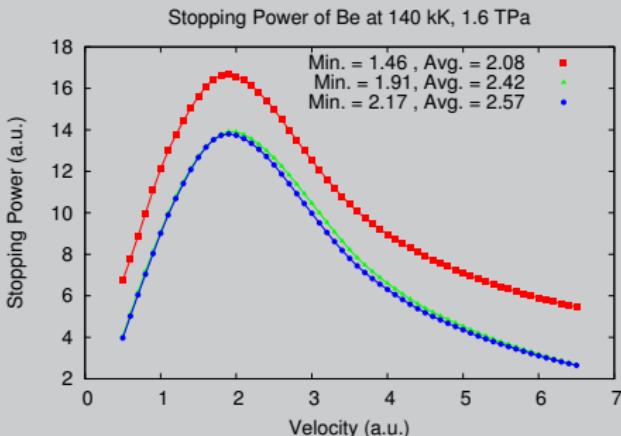
- ▶ Validation against Correa, et. al., PRL 108, 213201 (2012)
- ▶ **Proton stopping in fcc Al**
  - ▶ 193 electrons
  - ▶ 20 k-points in BZ
  - ▶ Fixed resolution of path
  - ▶  $\Delta t \leq 2$  as
- ▶ Experimental error  $\sim 10\%$
- ▶ Disagreement beyond peak
  - ▶ Core excitations?
  - ▶ Trajectory sampling?

# Stopping in WDM

- ▶ Cold metallic systems are not our primary interest:
  - ▶ Warm dense systems
  - ▶ Heterogeneous systems (e.g., GDP, Xe+D/T, etc.)
- ▶ Stopping in **warm dense Be**
- ▶ Different paths show **stronger variation** in stopping
- ▶ Characteristic of **condensed phase** rather than plasma
- ▶ Gathering data for model
  - ▶ Average nearest neighbor distance to projectile
  - ▶ Other dependencies?

# Stopping in WDM

- ▶ Cold metallic systems are not our primary interest:
  - ▶ Warm dense systems
  - ▶ Heterogeneous systems (e.g., GDP, Xe+D/T, etc.)
  
  
  
  
  
  
  
  
  
- ▶ Stopping in warm dense Be
- ▶ Different paths show **stronger variation** in stopping
- ▶ Characteristic of **condensed phase** rather than plasma
- ▶ Gathering data for model
  - ▶ Average nearest neighbor distance to projectile
  - ▶ Other dependencies?

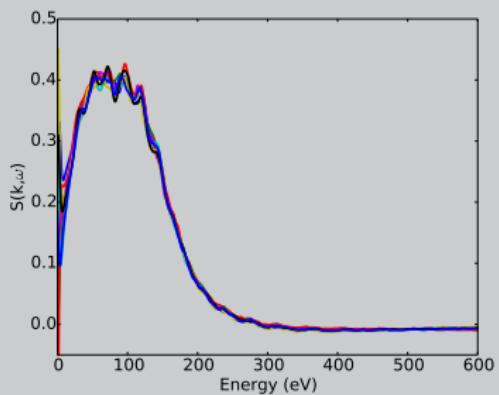


- ▶ Compute response to  $V_{ext}(\mathbf{r}, t) = V_0 e^{i\mathbf{k} \cdot \mathbf{r}} f(t)$
- ▶ Studying conditions in Lee, et. al., PRL **102**, 213201 (2009)
- ▶ 3x compressed Be at T=13 eV, measured in collective regime

## ▶ Method:

- ▶ Ion configuration from DFT-MD
- ▶ Compute Mermin state
- ▶ Integrate TD-KS equations
- ▶ Ramp up x-ray pulse
- ▶ Record  $\Delta\rho(\mathbf{k}, t)$
- ▶  $\chi(\mathbf{k}, \omega) = \mathcal{F}\{\Delta\rho(\mathbf{k}, \cdot)\}(\omega) / \mathcal{F}\{V_0 f(\cdot)\}(\omega)$
- ▶  $S(\mathbf{k}, \omega) = -\frac{f(\beta_e \omega)}{\pi} \text{Im} [\chi(\mathbf{k}, \omega)]$

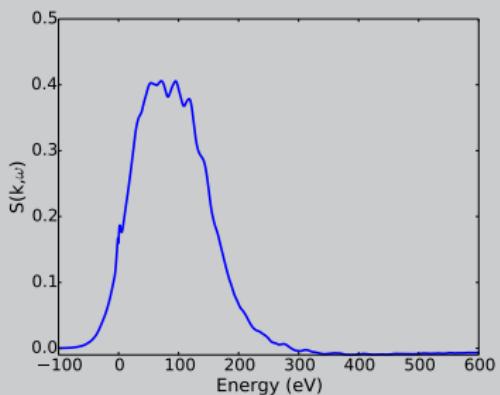
- ▶ Compute response to  $V_{ext}(\mathbf{r}, t) = V_0 e^{i\mathbf{k} \cdot \mathbf{r}} f(t)$
- ▶ Studying conditions in Lee, et. al., PRL **102**, 213201 (2009)
- ▶ 3x compressed Be at T=13 eV, measured in collective regime



DSF for 10 independent ionic configurations.

- ▶ Our calculation:
  - ▶ Adiabatic AM05 xc-functional
  - ▶ Excitation has FWHM of  $\approx 780$  eV
  - ▶ Integrated for 2.5 fs at  $\Delta t = 1$  as
  - ▶  $|\mathbf{k}| = 2.25a_B^{-1}$
  - ▶ 4 electron Be PAW
- ▶ Independent of Chihara model

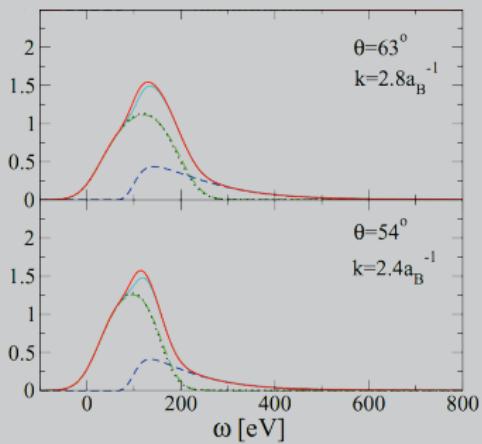
- ▶ Compute response to  $V_{ext}(\mathbf{r}, t) = V_0 e^{i\mathbf{k} \cdot \mathbf{r}} f(t)$
- ▶ Studying conditions in Lee, et. al., PRL **102**, 213201 (2009)
- ▶ 3x compressed Be at T=13 eV, measured in collective regime



Averaged DSF including negative part of spectrum.

- ▶ Our calculation:
  - ▶ Adiabatic AM05 xc-functional
  - ▶ Excitation has FWHM of  $\approx 780$  eV
  - ▶ Integrated for 2.5 fs at  $\Delta t = 1$  as
  - ▶  $|\mathbf{k}| = 2.25a_B^{-1}$
  - ▶ 4 electron Be PAW
- ▶ Independent of Chihara model

- ▶ Compute response to  $V_{ext}(\mathbf{r}, t) = V_0 e^{i\mathbf{k} \cdot \mathbf{r}} f(t)$
- ▶ Studying conditions in Lee, et. al., PRL **102**, 213201 (2009)
- ▶ 3x compressed Be at T=13 eV, measured in collective regime



Souza, et. al., PRE **89**, 023108 (2014)

▶ Our calculation:

- ▶ Adiabatic AM05 xc-functional
- ▶ Excitation has FWHM of  $\approx 780$  eV
- ▶ Integrated for 2.5 fs at  $\Delta t = 1$  as
- ▶  $|\mathbf{k}| = 2.25a_B^{-1}$
- ▶ 4 electron Be PAW

▶ Independent of Chihara model

# Summary and Future Work

- ▶ **New tool at SNL:**

- ▶ Ehrenfest-TDDFT for coupled electron-ion dynamics in bulk
- ▶ Stable, accurate, and scalable PAW implementation
- ▶ Real-time electron dynamics → dielectric response

- ▶ **Ongoing work:** statistics for stopping and XRTS in WDM

- ▶ **Future work:** investigation of time-dependent current DFT

- ▶ **Long-term challenge:** electron-ion energy transfer in TDDFT?

- ▶ **Acknowledgements:**

- ▶ Rudolph J. Magyar, Luke Shulenburger, and Mike Desjarlais
- ▶ Stephen Bond and Phil Van Every
- ▶ LDRD funding

- ▶ Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.