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Monte Carlo Optimization of a Hybrid Spectral / Temporal Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock*

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Abstract

The ability to achieve a given acceleration level associated with a transient shaker test (i.e., a shaker shock) is largely a function of the peak available amplifier voltage and current. For tests having spectral content within the frequency range for which the shaker behaves like a rigid body, minimizing the table acceleration should also minimize the load on the amplifier. However, for tests having spectral content in the frequency range where shaker dynamics come into play, there is no longer a direct linear relationship between the table acceleration and the amplifier performance. A test specification with frequency content out to 5 kHz, for which the required amplifier voltage was near the limit of the amplifier, provided the motivation for developing a methodology which used Monte Carlo techniques to generate an input acceleration signal that optimizes multiple user defined constraints.

Introduction

This paper discusses the Monte Carlo optimization process in the context of the need to reduce the amplifier voltage associated with this decayed sine shaker test while still matching the desired Shock Response Spectra (SRS) at the control point on the test fixture. In order to better understand the relationship between the input acceleration and the amplifier performance, two additional cases involving the minimization of the input acceleration and a joint minimization of both the input acceleration and the amplifier voltage are presented.

Data Formats

The Shock Response Spectra (SRS) is the primary metric for assessing the spectral content of a transient acceleration waveform. The SRS presented in this document were computed using a Maxi-Max Absolute Acceleration (MMAA) algorithm with a 5% critical damping ratio.

Shaker Shock Synthesis Process

The shaker shock control software used by Sandia requires an acceleration waveform to perform the test. Because SRS are a nonlinear transform, deriving an input acceleration waveform, A_{FXT}, whose SRS matches the desired reference SRS within an acceptable tolerance limit is an iterative process.

The test specification in question is defined using a set of decayed sine tones as shown in equation (1), where A_n, ζ_n, and f_n are the tonal amplitudes, damping and frequencies respectively.

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$$A(t) = \sum_{n=1}^{N} A_n e^{-2\pi\zeta_n f_n t} sin(2\pi f_n t) \quad (1)$$

A Matlab function designed to implement this process for matching a reference SRS using decayed sines was developed at Sandia by David Smallwood [1]. Smallwood's algorithm uses the following process:

1) Estimate each tonal amplitude one tone at a time starting with the lowest frequency tone.
2) Generate the composite acceleration waveform based on the sum of the currently defined tonal amplitudes.
3) Compute the SRS for that waveform.
4) Estimate the SRS error for the tonal frequency of interest.
5) Adjust the tonal amplitude for the tone of interest as needed to match the SRS at that frequency within a user defined accuracy.

Once an initial estimate for each tone is created, the entire process is repeated starting with the lowest frequency tone (typically 2-4 passes through all of the tones is sufficient to converge to an acceptable solution).

Proposed Solution

The specification in question has tones out to 5 kHz, which is well beyond the shaker's first resonant frequency. It is accepted that the shaker's ability to generate an acceleration response becomes less efficient above the first resonant frequency, and especially at the shaker's first anti-resonant frequency (≈3300 Hz). However, it is the contention of the authors that the acceleration in and of itself is not necessarily the primary measure for determining whether a test can be performed, but instead the peak amplifier voltage and current represent the true measure. Therefore, it was postulated that one or more of the higher frequency sine tones was disproportionally responsible for the high amplifier voltage.

Since the amplifier voltage was the issue, the peak current was not factored into the optimization process, but it was computed to make sure that the proposed changes to the input specification did not adversely affect the peak current.

Figure 1 shows the Transmissibility Response Function (TRF) between the amplifier voltage and the control acceleration. This TRF provides an indication as to where the shaker is more or less efficient at generating acceleration response with respect to the voltage draw on the amplifier (the shaker has to work harder at the peaks in the TRF). While the peak at 3288 Hz, which is the highest value for the entire TRF, is presumed to correspond to the shaker's first anti-resonance, there are numerous peaks in the TRF and the levels are generally high for frequencies above 2500 Hz.

The decision was made to adjust the frequencies of the decayed sine tones to avoid those frequencies where the shaker's draw on the amplifier voltage was high (i.e., the peaks in the voltage versus acceleration TRF). Because of the many peaks in the TRF, it was considered impractical to manually select the optimal tonal frequencies while still insuring that the resulting SRS was a good match for the desired SRS. Therefore, the decision was made to employ an automated Monte Carlo technique for choosing the optimal tonal frequencies.

As an aside, the authors have no explanation for the dip in the TRF at 500 Hz.

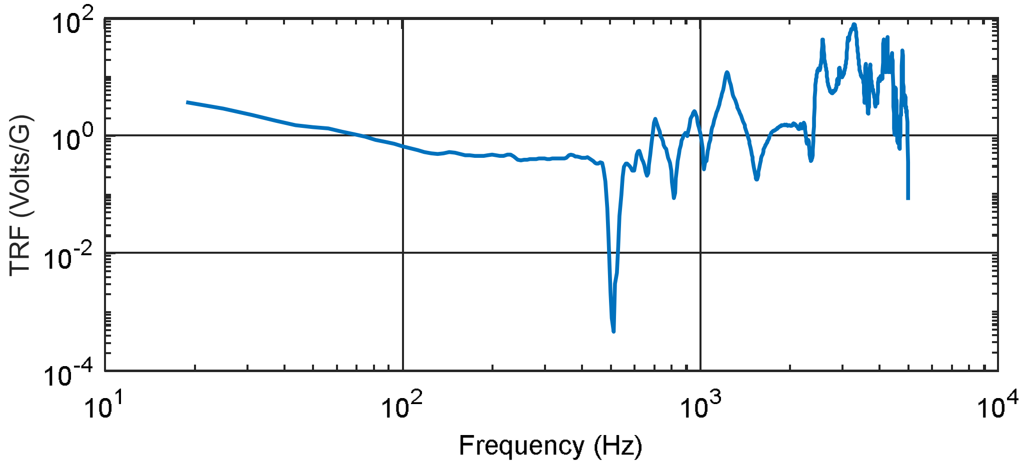

Figure 1: Amplifier Voltage vs Fixture Acceleration Transmissibility Response Function

Monte Carlo Analysis Process

The Monte Carlo approach has been used successfully by Sandia to optimize the response at locations other than the shaker table [2]. However, in that scenario the goal was to optimize the SRS at several locations in a least squares sense. That approach is not practical in this case because there is no desired SRS associated with the amplifier voltage, simply a peak temporal value.

Therefore, the following algorithm was employed.

1) Generate M sets of tonal frequencies drawing from a uniform random distribution of tones. This process, which is identical to what was done in Reference [2], is described in Appendix A.
2) For each set of tonal frequencies, compute the tonal amplitudes so that the SRS associated with the resulting acceleration waveform matches the desired SRS.
3) Compute the corresponding amplifier voltage by convolving the fixture acceleration waveform with the TRF relating the amplifier voltage to the table acceleration. The convolution is performed by multiplying the Fast Fourier Transform (FFT) of the acceleration waveform by the TRF and then taking the inverse FFT.
4) Select the set of tones for which the desired parameters have been optimized.

A valid shaker shock must not produce peak velocities and displacements in excess of the shaker capacity. Therefore, as a part of the selection process only those realizations that produced acceptable peak velocities and displacements were included in the final selection process.

Transmissibility Response Function Estimation

One can't really obtain credible TRFs from a single, short duration transient shock test. Therefore, it was necessary to use the TRF from a random vibration test. This is not an ideal situation if the system is not linear. However, it is assumed that while the resulting estimates of the parameters associated with these predictions are not necessarily correct in an absolute sense, the relative intensities of the various Monte Carlo simulations should be credible.

This TRF was also defined as magnitude only, but based on past experience SRS are only weakly sensitive to the phase of the TRF so this was not considered to be a major issue.

Optimization Metrics

In order to determine which Monte Carlo simulation is the optimal solution, it is necessary to define a suitable set of metrics. For a temporal parameter such as the amplifier voltage, the logical metric is the absolute peak value. However, no simulation can be considered valid if it does not do an adequate job of replicating the desired SRS. The measure of the best match for the SRS requires a metric that can address the entire frequency spectra using a scalar value. The choice was an "rms dB error" parameter developed at Los Alamos National Laboratory and shown in equation (2). For this equation, S_{ACH} represents the achieved SRS while S_{TAR} represents the desired or target SRS. The integral is intended to address the entire frequency range and in practical terms is simply a summation of the discrete SRS values. Δf is a weighting factor equal to the bandwidth of the SRS. Since the SRS used in this study have 1/12th octave resolution this weighting factor places more emphasis on the higher frequency responses. "N" refers to how many SRS are being optimized (for this study N=1).

$$E(dB) = \frac{1}{N}\sum_{n=1}^{N}\sqrt{\Delta f \int_{fmin}^{fmax}\left[20\log\left(\frac{S_{ACH}}{S_{TAR}}\right)\right]^2 df} \quad (2)$$

Case Study

Each Monte Carlo analysis was conducted using 200 sets of tonal frequencies based on fifty combinations of tonal frequencies for each of 4 nominal tonal densities (6, 8, 10, and 12 tones/octave). The first simulation for each tonal density used the nominal tones based on a uniform octal spacing having the desired tonal density.

The case study considered the baseline deterministic set of tonal frequencies for the current test specification (denoted as "DET") along with three different cases based on different optimization metrics:

1) The initial Monte Carlo simulation minimized the fixture input acceleration with respect to the rms dB error. This scenario is referred to as the "Fixture Acceleration" or 'FA" case.
2) The second Monte Carlo simulation minimized the amplifier voltage with respect to the rms dB error. This scenario is referred to as the "Amplifier Voltage" or "AV" case.
3) The third Monte Carlo simulation looked at jointly minimizing the amplifier voltage and the fixture acceleration. This scenario is referred to as the "Joint Voltage / Acceleration" or "AV/FA" case.

Figure 2 presents the fixture acceleration and amplifier voltage for the deterministic case. This gives the reader a sense of the waveform shape.

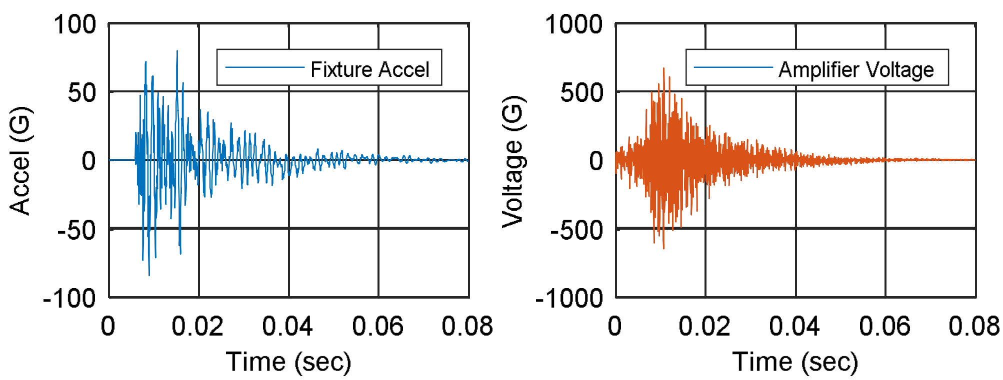

Figure 2: Deterministic Fixture Acceleration and Amplifier Voltage

Figure 3 presents the Monte Carlo scatter plots used to select the optimal realization. The upper plot in this figure presents the peak amplifier voltage versus the rms dB error while the lower plot presents the fixture acceleration versus the rms dB error. The resulting values for all four cases are included on each plot. Table 1 summarizes the peak acceleration, voltage, and current values.

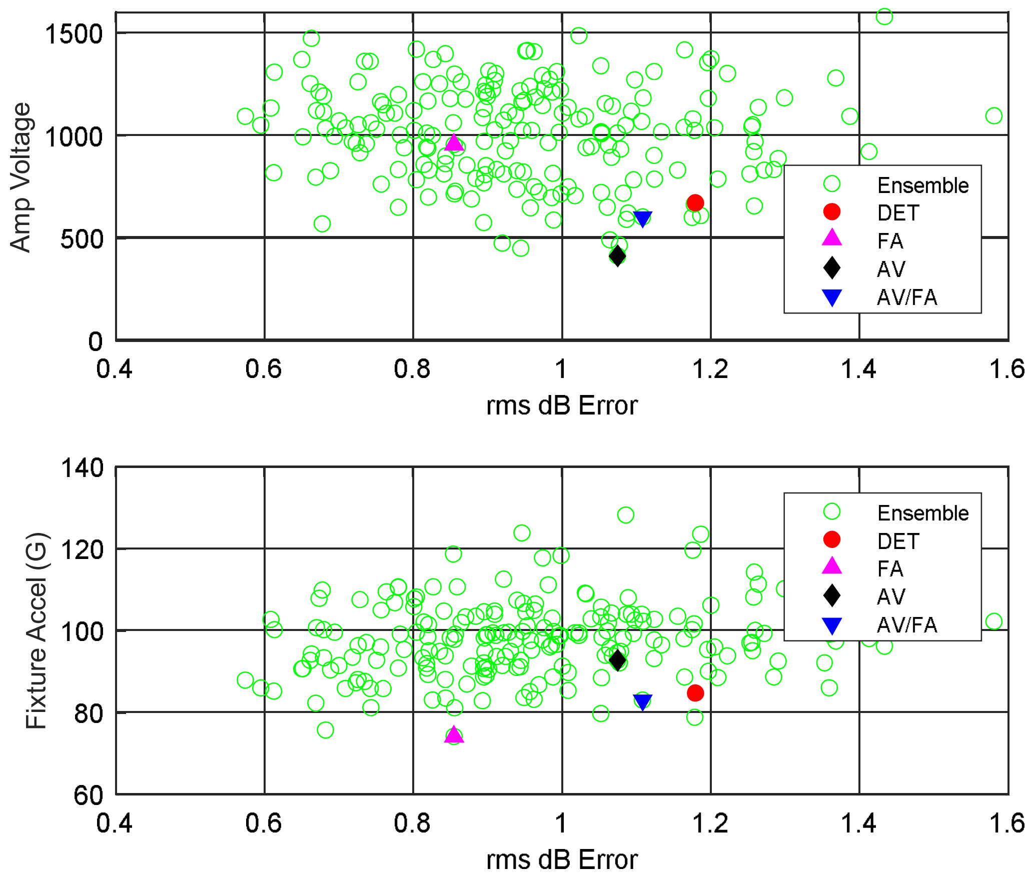

Figure 3: Comparison of Monte Carlo Simulations

Table 1: Summary of Monte Carlo Results

Method	Rms dB Error	Fixture Acceleration	Amplifier Voltage	Amplifier Current
Deterministic	1.18	84.7g	670.5V	366.9A
Fixture Acceleration / rms dB Error	0.86	74.2g	954.5V	440.8A
Amplifier Voltage / rms dB Error	1.08	92.7g	412.0V	242.0A
Joint Amplifier Voltage / Fixture Acceleration	1.11	83.0g	602.0V	326.1A

The reader should take note of several trends in these results.

1) While minimizing the fixture acceleration did reduce the peak acceleration by ≈12% with respect to the deterministic case, it actually caused the amplifier voltage to rise by ≈40%.
2) The amplifier voltage case lowered the voltage by ≈40% with respect to the deterministic case, but the resulting fixture acceleration increased by 10%.
3) The joint case did achieve a 10% improvement in voltage with respect to the deterministic case while keeping the acceleration nearly constant.

The upper left plot in Figure 4 overlays the SRS corresponding to the deterministic case against the desired SRS. The remaining plots in Figure 4 overlay the SRS for the optimal simulation for the three Monte Carlo cases against the desired SRS. These plots show that each case produced an acceptable match for the desired SRS.

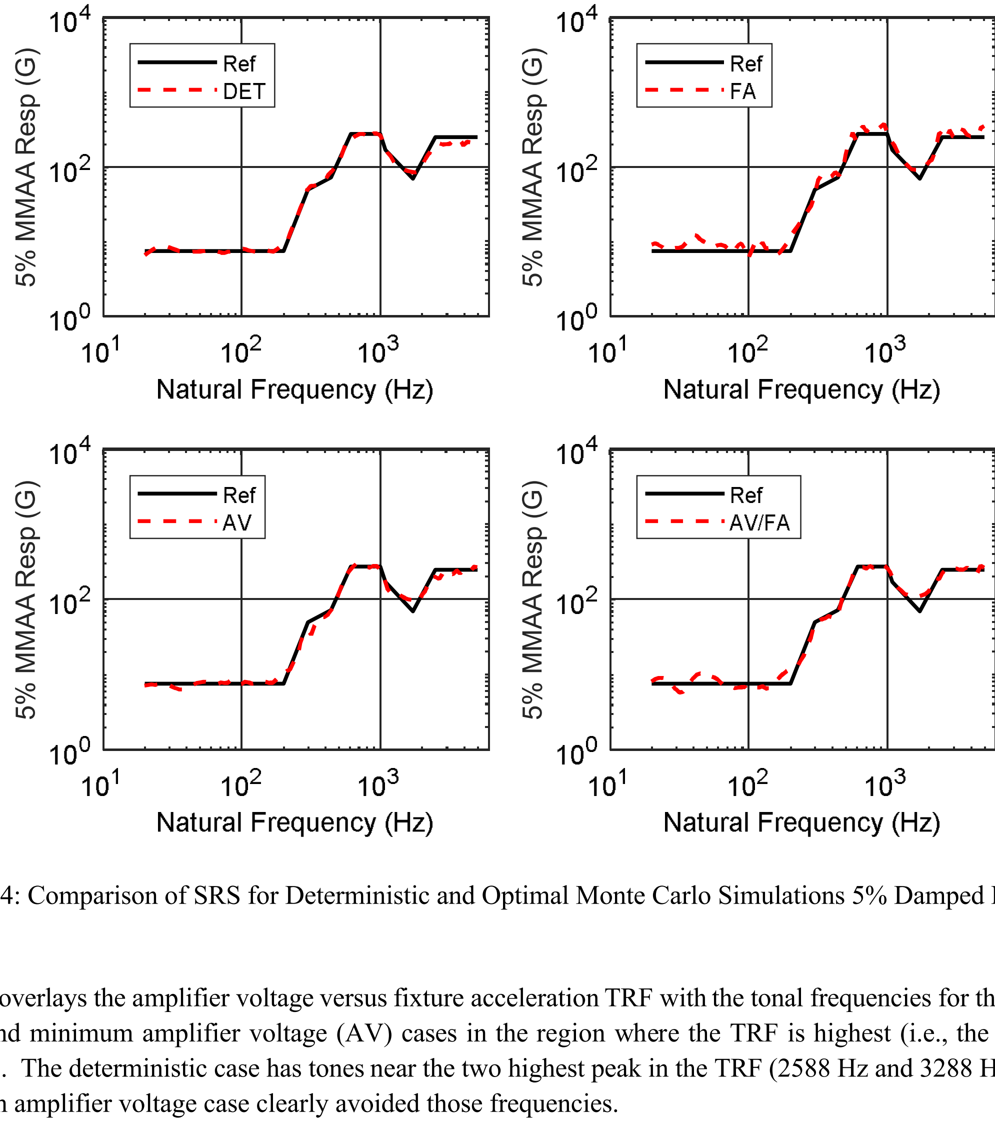

Figure 4: Comparison of SRS for Deterministic and Optimal Monte Carlo Simulations 5% Damped MMAA SRS

Figure 5 overlays the amplifier voltage versus fixture acceleration TRF with the tonal frequencies for the deterministic (DET) and minimum amplifier voltage (AV) cases in the region where the TRF is highest (i.e., the shaker is least efficient). The deterministic case has tones near the two highest peak in the TRF (2588 Hz and 3288 Hz) whereas the minimum amplifier voltage case clearly avoided those frequencies.

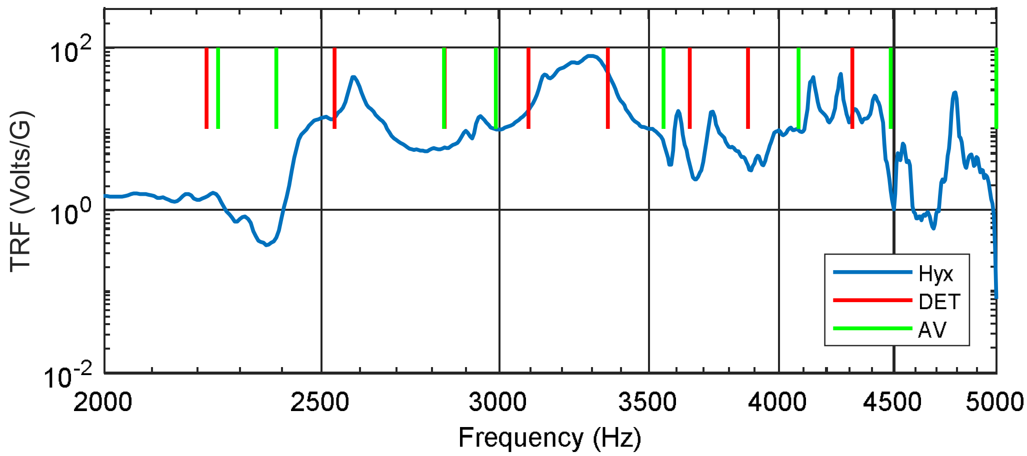

Figure 5: Evaluation of Tonal Frequency Selection versus Shaker TRF Anti-Resonances

(the vertical lines denote the sine tones)

Test Results

Only by trying out each of these cases experimentally is it possible to determine which approach, if any, is better than the current specifications. Therefore, all four cases were used to conduct shaker shock tests. Data were recorded for several different test levels. The absolute magnitude of the results did not match the predicted levels, but that is attributed to bias introduced by the use of the random vibration TRF and the convolution process. Unfortunately, the only test that the engineer took to full level was the amplifier voltage case, and the SRS for the reduced level tests did not correlate well with the stated "dB" levels so comparison of results was difficult. However, for the -3dB level the minimum amplifier voltage case did result in lower amplifier voltages relative to the deterministic case (207 amps versus 247 amps).

Conclusions

The application of the Monte Carlo techniques represents a simple approach for optimizing multiple parameters associated with a shaker shock. The results of the Amplifier Voltage case indicate that minimizing the amplifier voltage is a viable means for improving the shaker performance when the shaker dynamics are important. However, the results of the Fixture Acceleration case indicate that it is possible to minimize the fixture acceleration on the order of 10-15%. Therefore, applying this technique when generating the initial estimate of the input acceleration would appear to make sense when the upper frequency of the test specification is low enough that the shaker dynamics do not come into play.

Appendix A

The first step in the Monte Carlo technique is the selection of the baseline nominal set of tonal frequencies. The nominal tonal frequencies were octal spaced with a tonal density of N tones/octave. This decision was based on the fact that the SRS is the response of a series of Single Degree of Freedom (SDOF) Oscillators and SDOF oscillators tend to respond in an octal manner.

The second step is the randomization of the tones. Each tone is varied about the nominal frequency, fc, using a uniform random distribution over the frequency range [fmin fmax] where fmin and fmax are defined in equations (3) and (4). The 1.01 and 0.99 terms are intended to prevent the randomization process from choosing redundant tones.

$$\log_2(fmin) = 1.01*[\log_2(fc) - 1/N] \tag{3}$$

$$\log_2(fmax) = 0.99*[\log_2(fc) + 1/N] \tag{4}$$

Figure 6 presents a notional example for 50 Monte Carlo tones for a 4 tone/octave case. "fband" represents the boundaries between 1/N octave frequency bands associated with fmin and fmax.

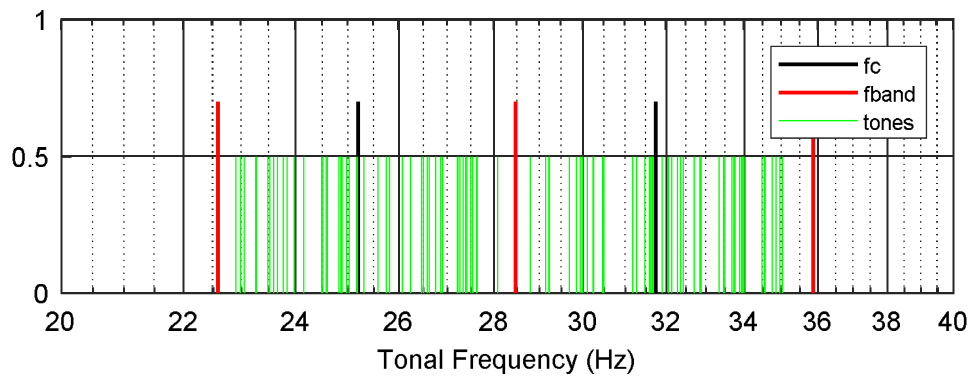

Figure 6: Example of Monte Carlo Tones

References

[1] Smallwood DS. An Improved Recursive Formula for Calculating Shock Response Spectra. The Shock and Vibe Bulletin. May 1981.

[2] Heitman, Chad, et.al; "Case Study of a Single Input Multiple Output (SIMO) Input Derivation for an Oscillatory Decaying Shock"; Proceedings of the 87th Shock and Vibration Symposium, 10/17-20/2016; New Orleans, LA.