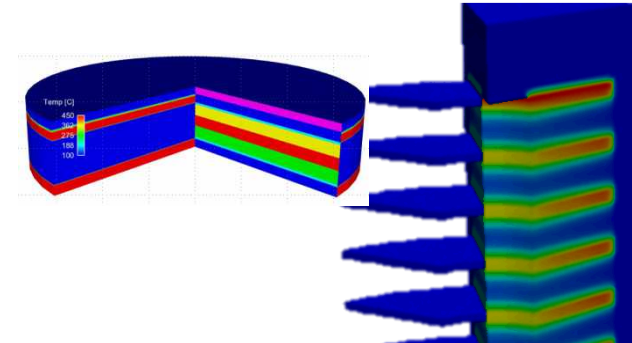
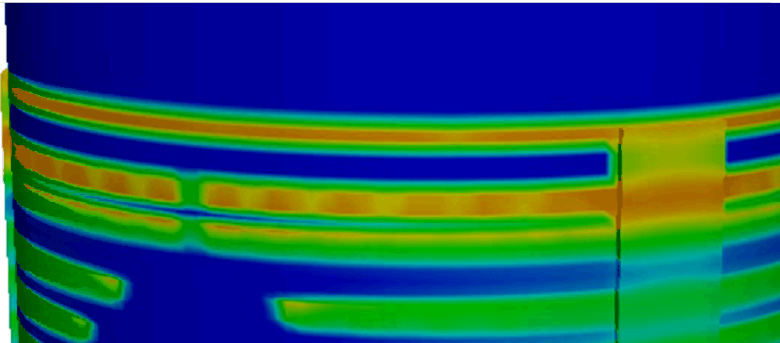


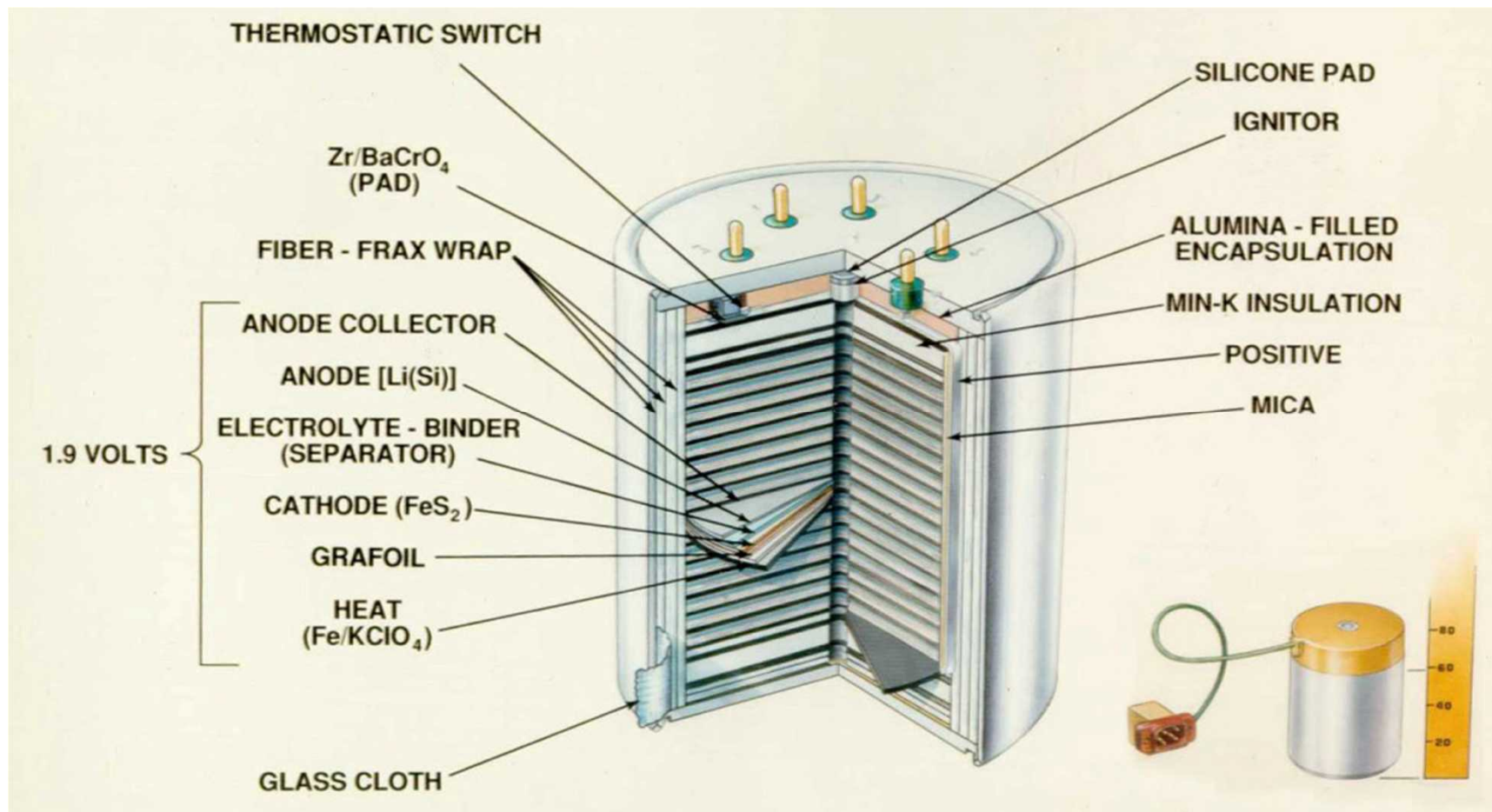
*Exceptional service in the national interest*



# Developing a coupled thermal-mechanical-porous model for electrolyte flow in a molten salt battery

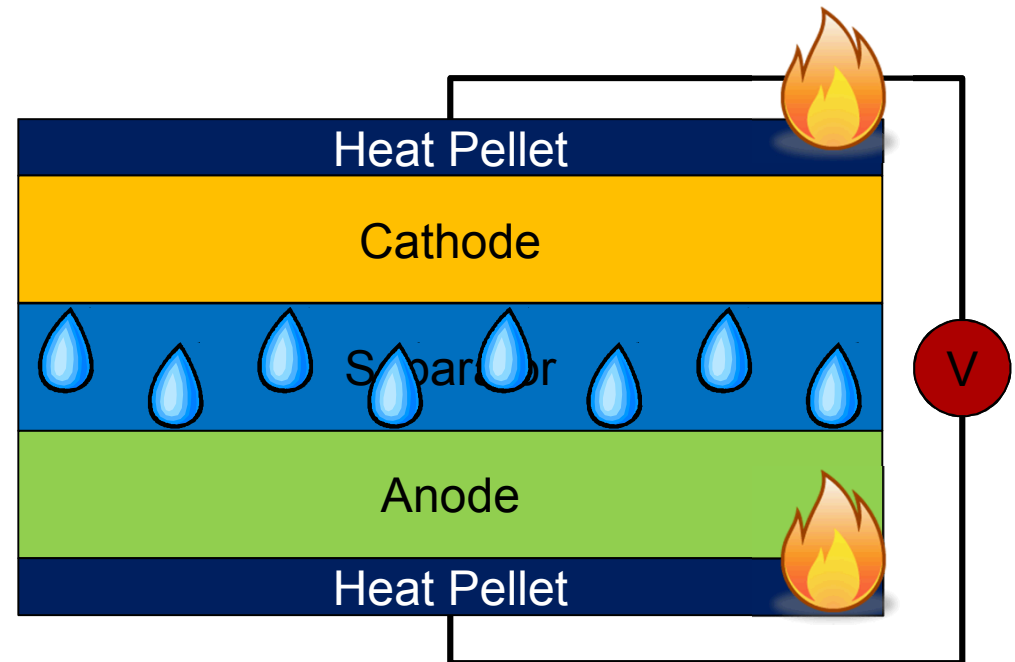
**Jonathan R. Clausen**, Scott A. Roberts,  
Mario J. Martinez, Kevin N. Long

# Molten Salt Battery

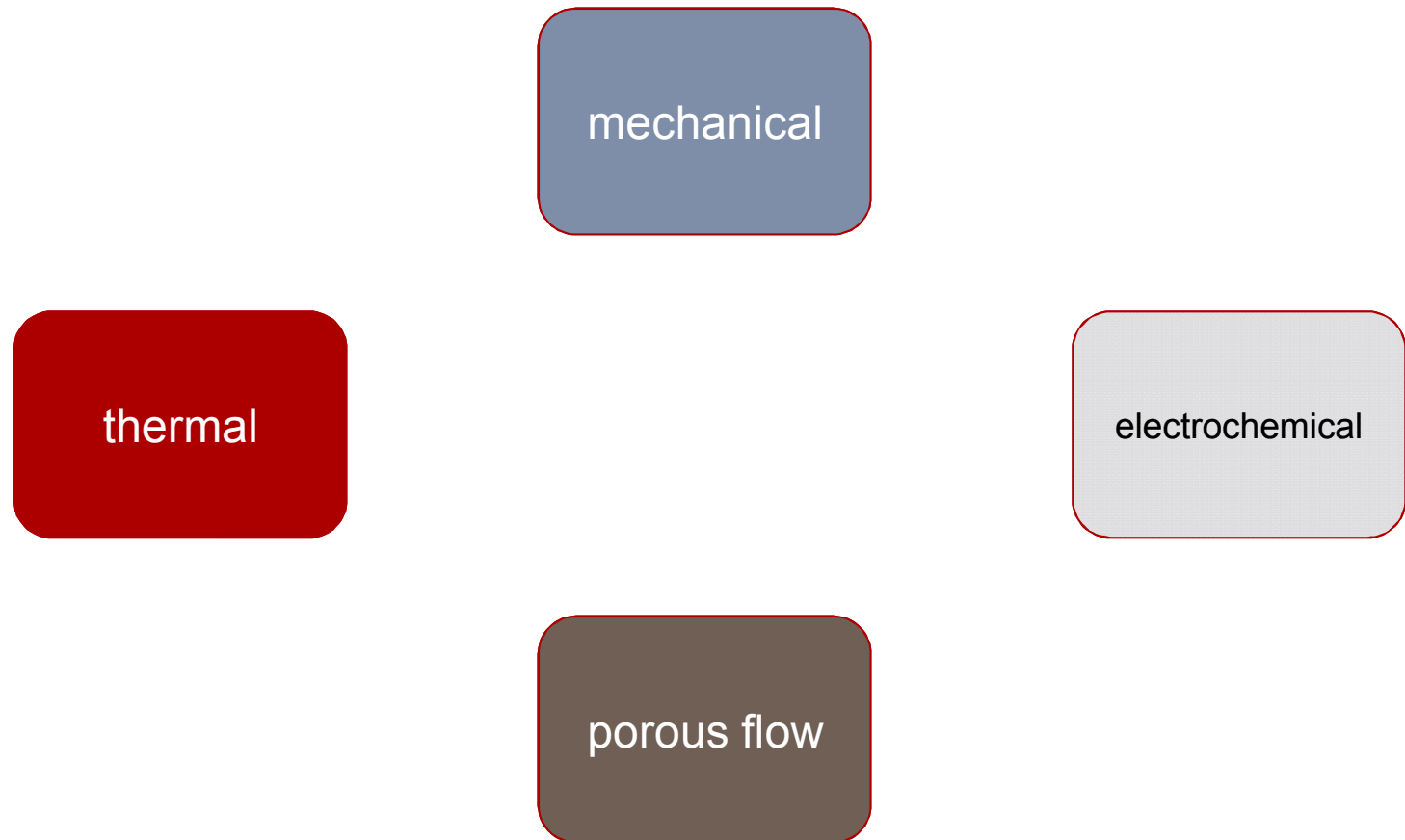


# Physical Mechanisms

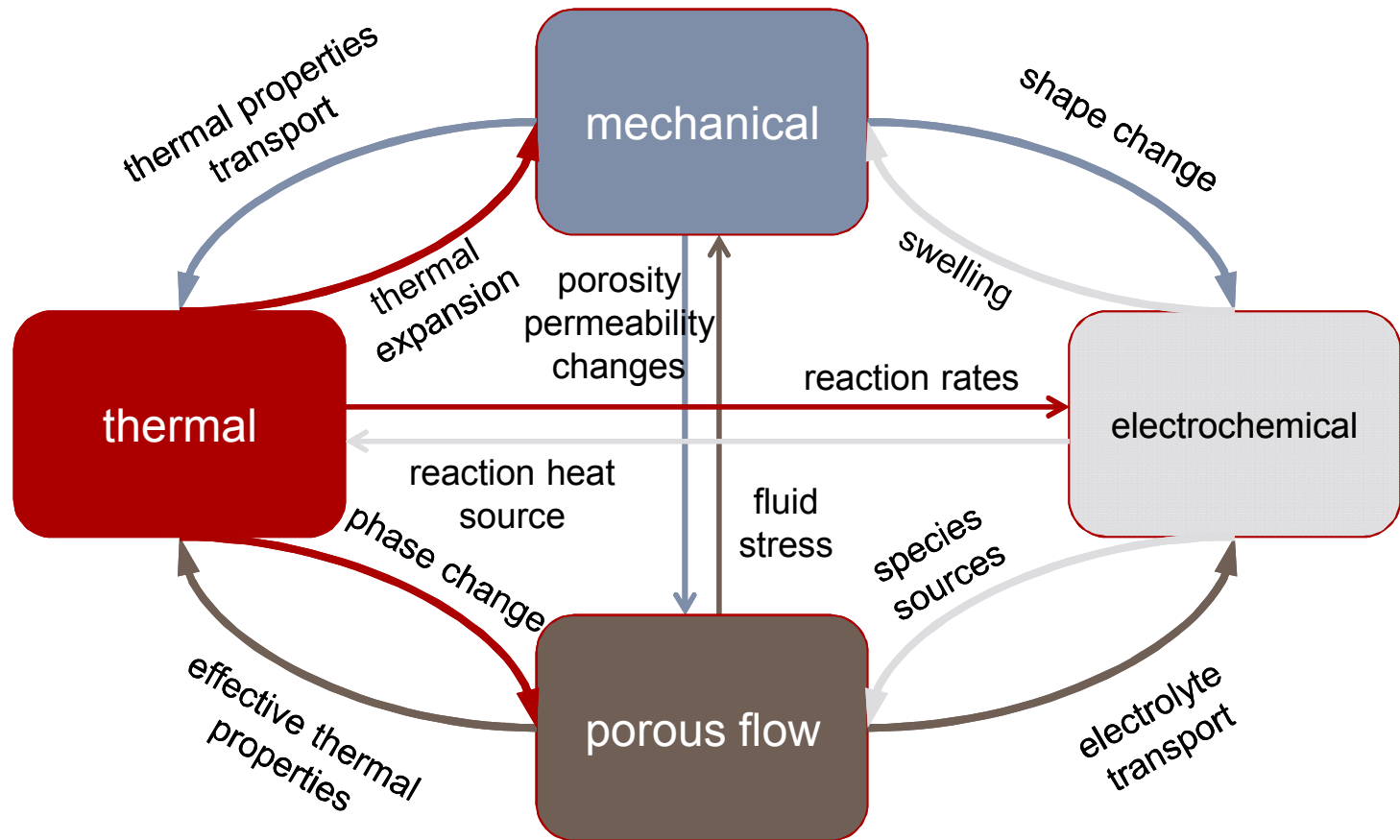
- Multi-physics problem
  - Thermal
  - Mechanical
  - Fluid
  - Electrochemical
- Modeling goals
  - Predict activation times
  - Optimize volume, insulation, manufacturing



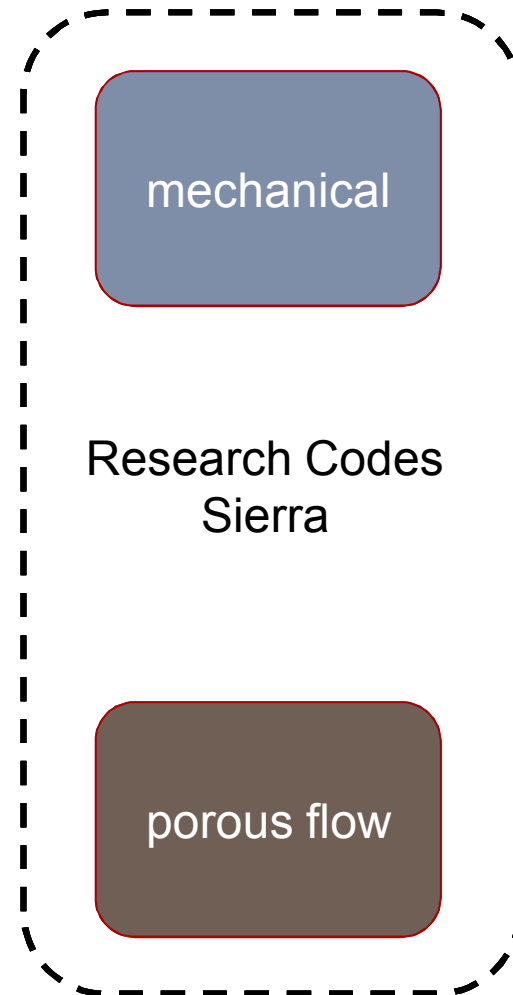
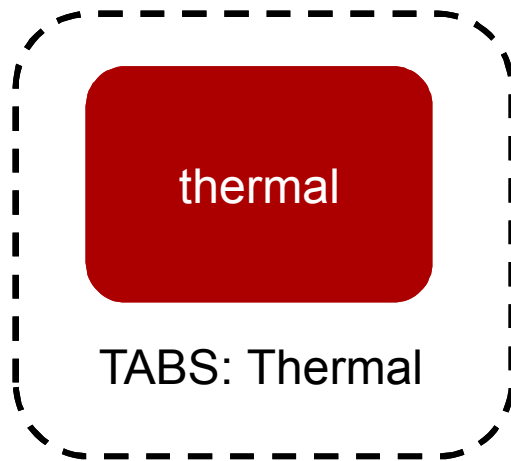
# Multiphysics Coupling



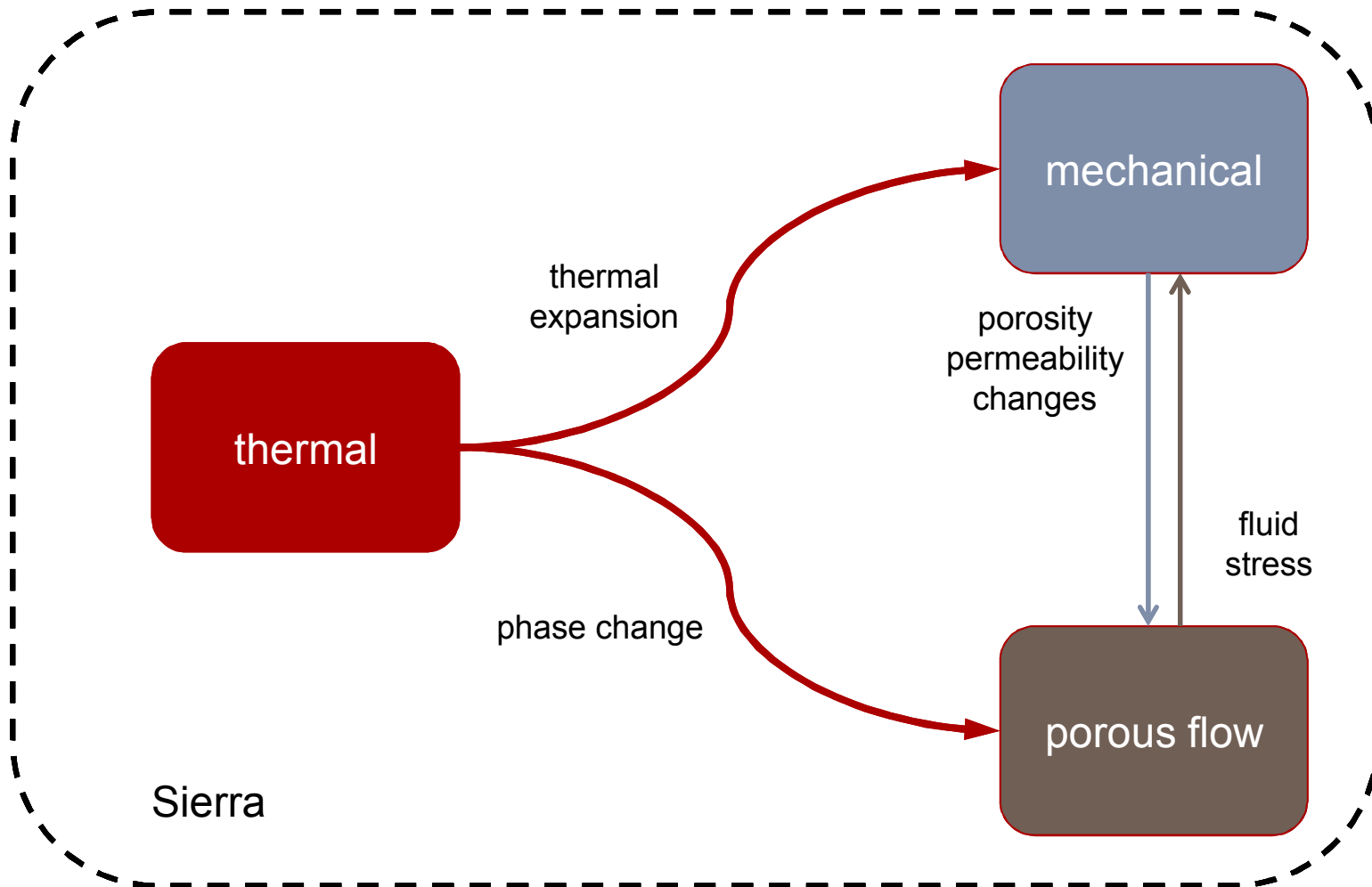
# Multiphysics Coupling



# Multiphysics Coupling



# Multiphysics Coupling



Strongly coupled

# Models: Thermal

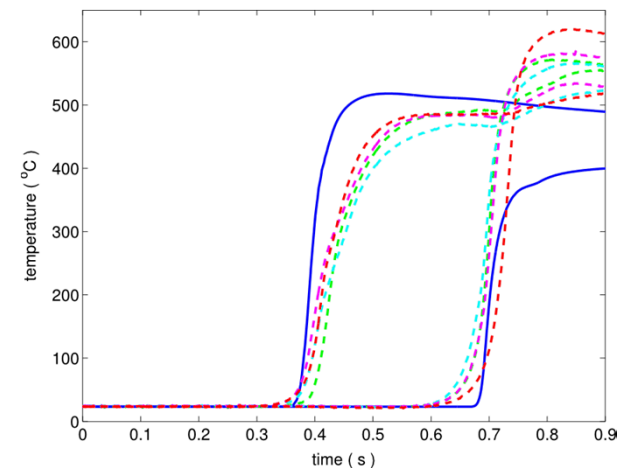
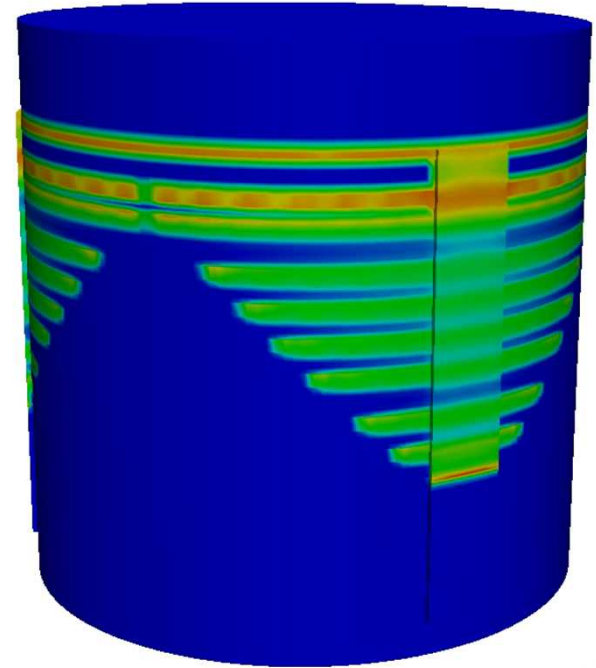
- Heat equation

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + Q$$

- Source term  $Q$  applies to heat pellet, paper

- Level set tracking of burn fronts

- Constant propagation speed
  - Heat released over a narrow region near burn-front position





# Model: Mechanical Deformation

- Solid constitutive model
  - inelastic volumetric and isochoric deformation of the *MgO skeleton* before, during, and after activation
  - Isotropic, thermal-elastic-plasticity
  - **Plasticity governs activation deformation**
  - Kinematic split of deformations

$$\underline{\underline{F}} = \underline{\underline{F}}^e \underline{\underline{F}}^p \underline{\underline{F}}^T$$

- Rule of mixtures for phase decomposition

$$\chi = \frac{T - (T - T_w/2)}{T_w}$$

- Kirchoff stress:  $\underline{\underline{\tau}} = \mu_x(T) \text{dev}(\underline{\underline{b}}^3) + \frac{\kappa_x(T)}{2} (J_e^2 - 1) \underline{\underline{\delta}}$

- Conservation of momentum:  $\underline{\underline{\nabla}} \cdot \underline{\underline{\sigma}} = \underline{\underline{0}}$
- Coupled to porous-flow through effective stress:  $\underline{\underline{\sigma}} = \underline{\underline{\hat{\sigma}}} + p \underline{\underline{\delta}}$

# Models: Porous flow

- Electrolyte and gas form two immiscible phases upon mel

$$\frac{\partial(\rho_w \phi S_w)}{\partial t} = \underline{\nabla} \cdot \left( \rho_w \frac{k_{rw}}{\mu_w} \underline{\underline{K}} \cdot (\underline{\nabla} p_w - \rho_w \underline{g}) \right) + Q_w$$

$$\frac{\partial(\rho_n \phi S_n)}{\partial t} = \underline{\nabla} \cdot \left( \rho_n \frac{k_{rn}}{\mu_n} \underline{\underline{K}} \cdot (\underline{\nabla} p_n - \rho_n \underline{g}) \right) + Q_n$$

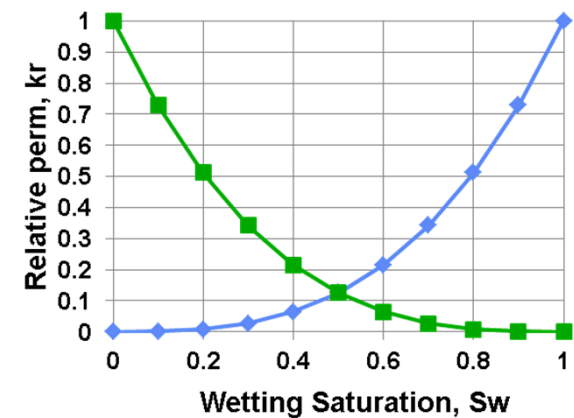
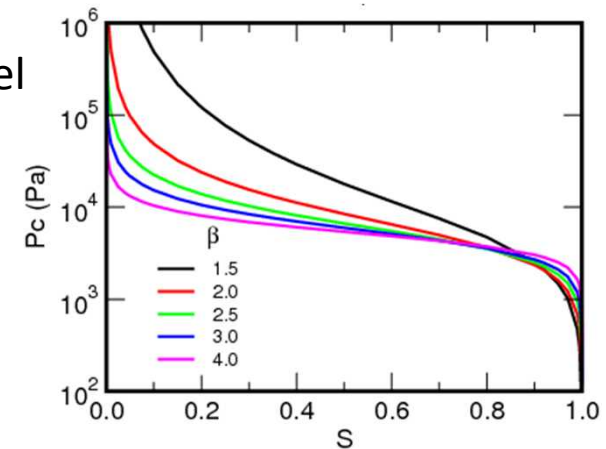
- Saturation and capillary pressure related to DOFs (wetting and non-wetting pressures) through model relations

$$S = S(p_c); \quad p_c = p_n - p_w$$

- Coupling to other physics important!

$$\phi = \phi(\underline{d}); \quad \mu_i = \mu_i(T)$$

$$S_i = S_i(p_c, \underline{d}); \quad \underline{\underline{K}} = \underline{\underline{K}}(\underline{d})$$



Capillary pressure (top) and relative permeability (bottom) depend on wetting phase saturation and electrode pore structure

- Thermal and Mechanical

- GFEM method

- Porous Flow

- Upwinded version of Darcy flux (Forsyth)

$$R_I^{flux} = - \sum_{J \neq I} \lambda_{(I,J)}^u K_{IJ} (P_J - P_I)$$

$$K_{IJ} = - \int_{\Omega} \nabla N_I \cdot \mathbf{k} \cdot \nabla N_J \, d\Omega$$

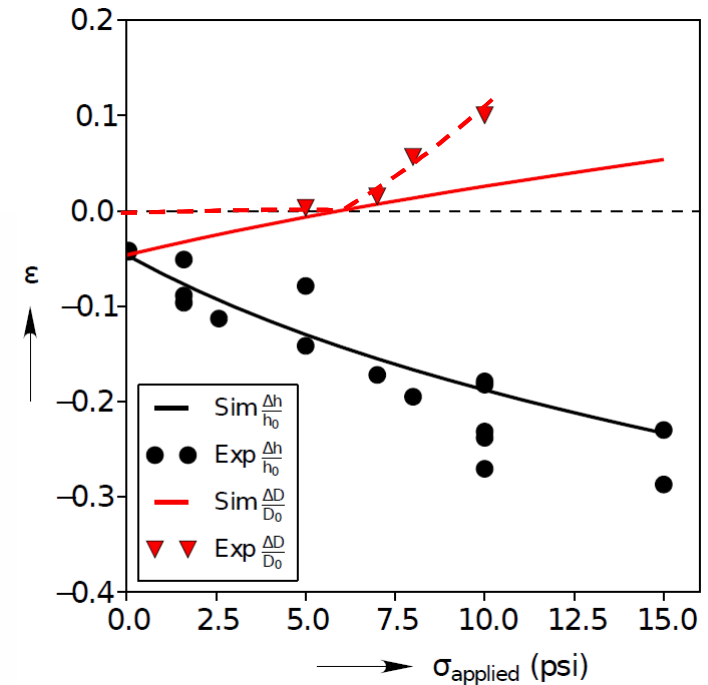
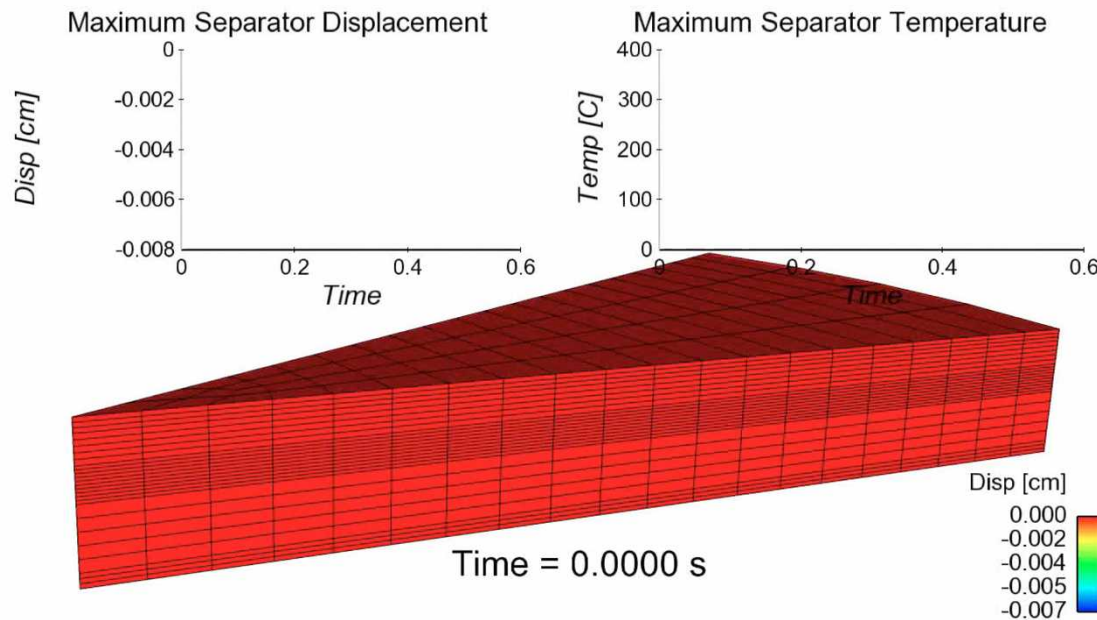
$$\lambda_{(I,J)}^u = \begin{cases} \lambda_J & \text{if } K_{IJ}(P_J - P_I) > 0 \\ \lambda_I & \text{if } K_{IJ}(P_J - P_I) < 0 \end{cases}$$

$$\lambda = \rho Y k_r / \mu$$

- Vertex Quadrature
  - Discontinuous Saturations
    - pressure—pressure formulation

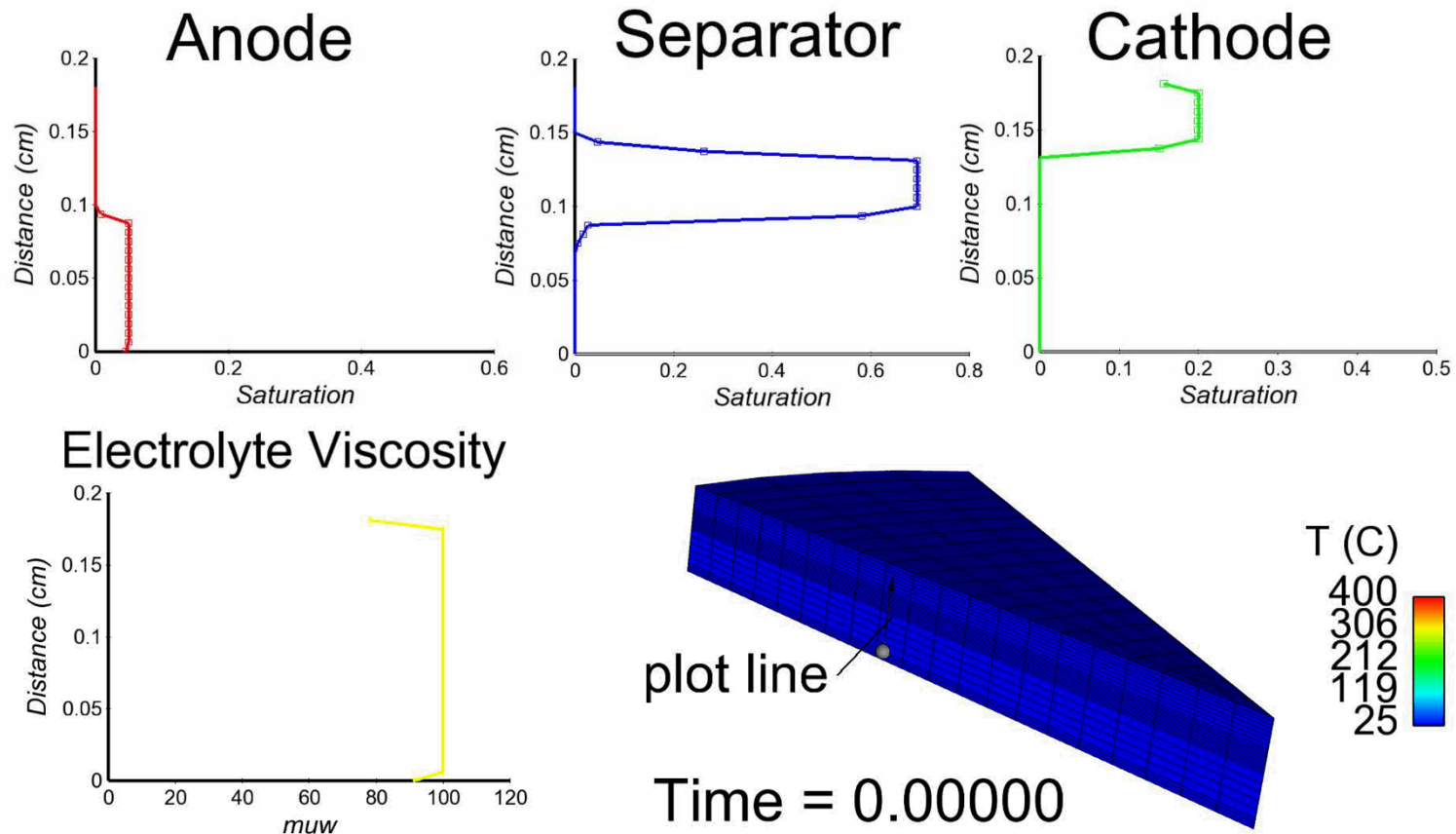
# Demonstration: Themo-mechanical deformation

- Electrolyte melting causes separator deformation
- Height vs. diameter change
  - Missing effective stress



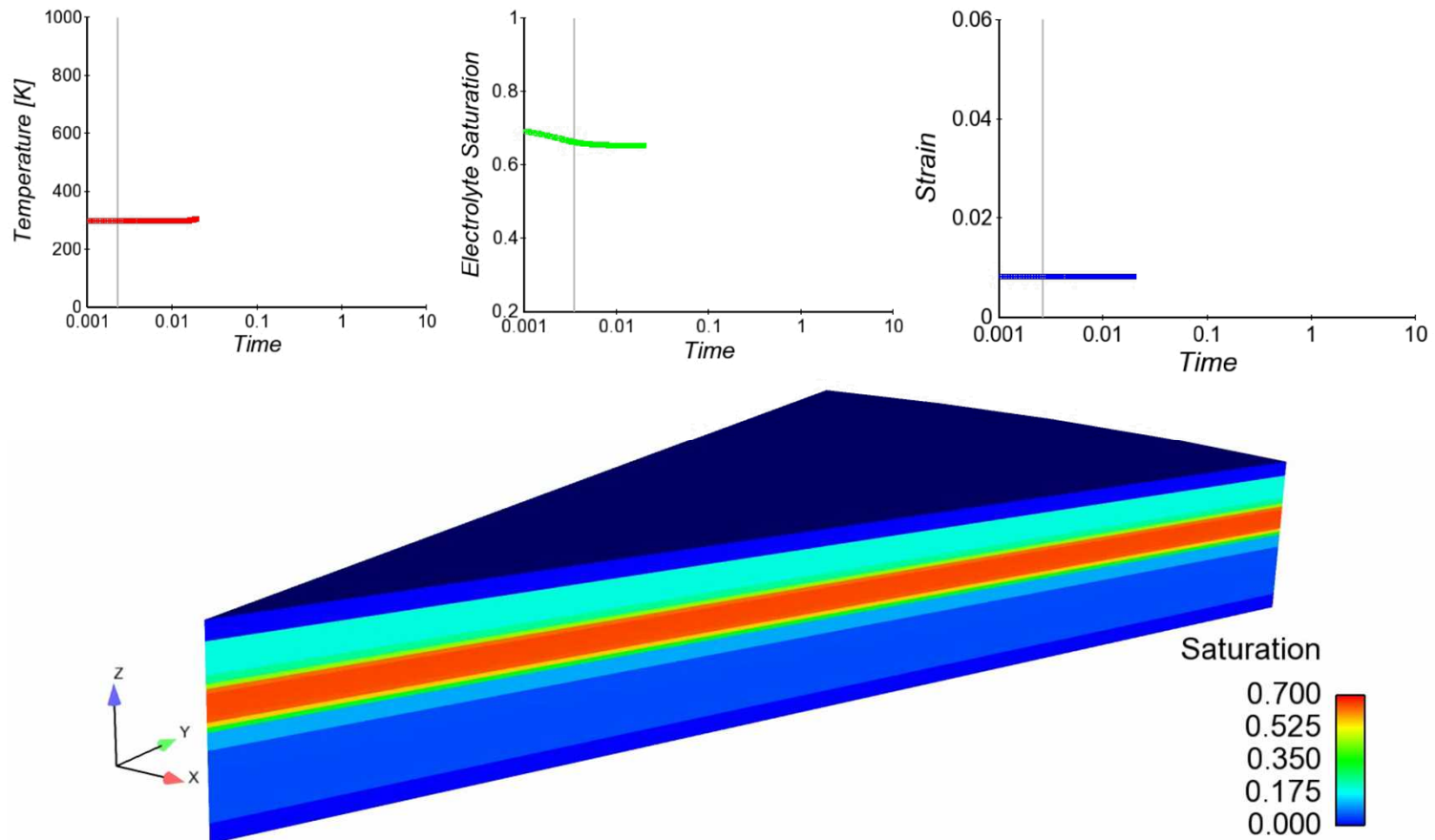
# Demonstration: Thermo-porous flow

- Two-pressure porous-flow formulation enables stable solution of flow from the separator to the cathode and



# Demonstration: Thermo-poro-mechanical coupling

- Thermo-poro-mechanical single-cell simulation with full coupling:



# Conclusions & Future Work

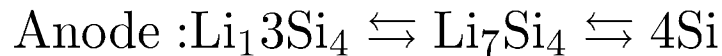
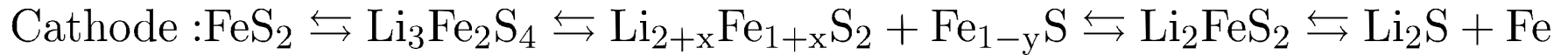
- Proof of concept simulation
- Experimental determination of material properties
- Calibration of solid model via experiments
- Electrochemical modeling
  
- Acknowledgements:
  - DoD/DoE Joint Munitions Program, TCG V
  - ASC/IC, ASC/P&EM, WSEAT
  - Victor Brunini, Lindsay Erickson, Adrian Kopacz





# Models: Electrochemistry

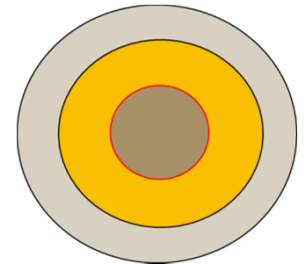
- Reactions, especially for the cathode, are stoichiometrically complicated



- Cantera's "Electrode Object" deploys multiple sub-grid models

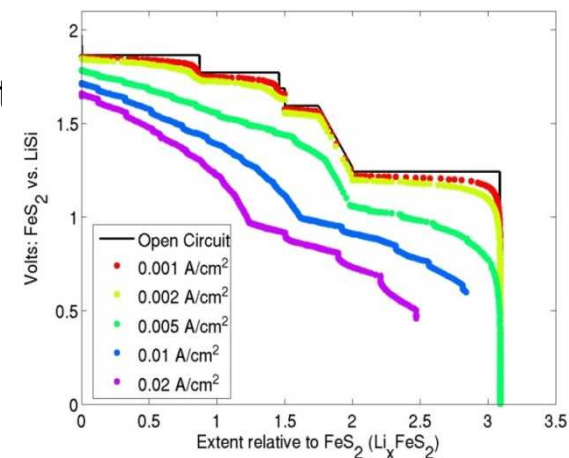
- Infinite capacity
- Multi-plateau
- Newman reaction extend
- Finite capacity

- Primary electrochemical coupling is the temperature
  - Cantera's thermodynamics all temperature-dependent
- Future: Use deformed geometry to affect porosity in electrochemical calculations



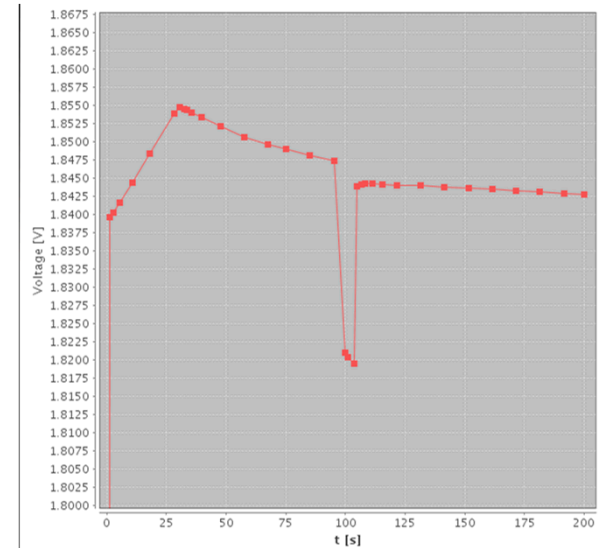
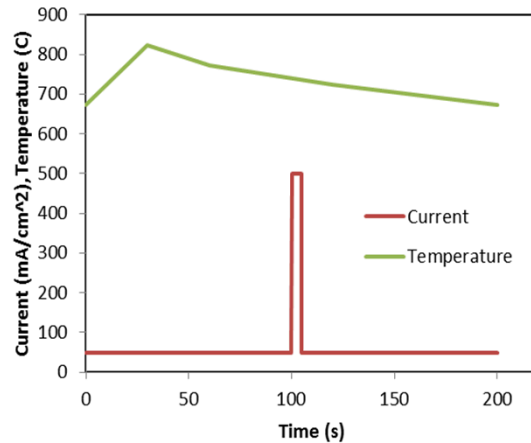
Shrinking Core Model

- Multiple plateaus can react simultaneously
- Diffusional losses with transport

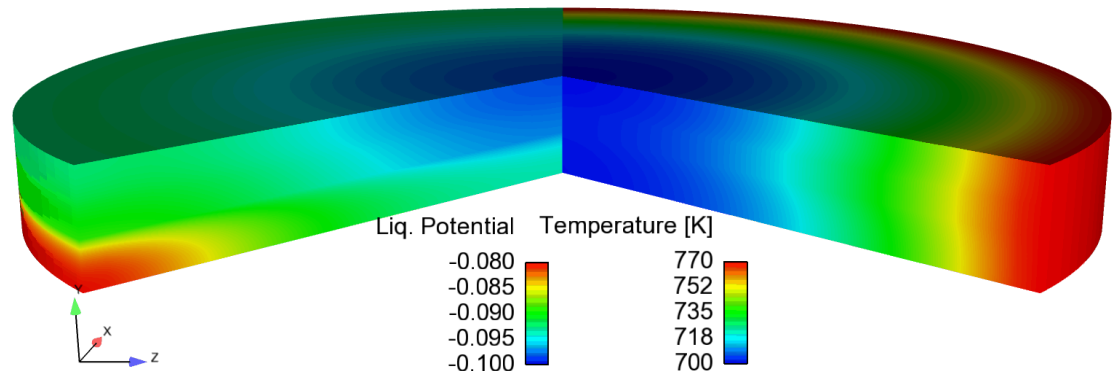


# Demonstration: Thermo-electrochemical coupling

- Voltage responds to temperature and current



- Spatial temperature variations affect local potentials and current densities



# Thermal-Mechanical Behavior of the Solid Skeleton

## Free Energy Density

$$\rho_0 \psi = \mu_x [T] (\bar{I}_{1e} - 3) + \frac{\kappa_x [T]}{4} (J_e^2 - 1 - 2 \log J_e)$$

## Kirchoff Stress

$$\boldsymbol{\tau} = 2\rho_0 \mathbf{b} \frac{\partial \psi}{\partial \mathbf{b}} = \mu_x [T] \text{dev}[\mathbf{b}^e] + \frac{\kappa_x [T]}{2} (J_e^2 - 1) \mathbf{1}$$

## Isochoric (Radial) Yield

$$\phi_\mu = \sqrt{J_2} - A_x \frac{I_1}{3} - B_x \left( \frac{I_1}{3} \right)^2 - Y_{ps\ x} - H_{\mu\ x} \epsilon_\mu^{m_x}$$

## Plastic Flow Rules

$$\mathcal{L}[\mathbf{b}^e] = \mathbf{F} \dot{\mathbf{C}}^{p-1} \mathbf{F}^T = -2 \left( \dot{\lambda}_{iso} \mathbf{n}_{iso} + \dot{\lambda}_{vol} \mathbf{n}_{vol} \right) \mathbf{b}^e$$

$$\mathbf{n}_{iso} = \frac{\text{dev} \boldsymbol{\tau}}{\|\text{dev} \boldsymbol{\tau}\|}, \quad \mathbf{n}_{vol} = \frac{1}{3} \mathbf{1}$$

## Kinematic Split of the Deformation Gradient: Thermal, Elastic, and Plastic Parts

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p \mathbf{F}^\theta$$

## Cold/Hot State Phase

### Decomposition

$$\begin{aligned} &\text{if} \quad T < T_m - T_w/2, \quad \chi = 0, \\ &\text{elseif} \quad T > T_m + T_w/2, \quad \chi = 1, \\ &\text{else} \quad \chi = \frac{T - (T_m - T_w/2)}{T_w}, \end{aligned}$$

## Volumetric Yield

$$\phi_\kappa = \frac{I_1}{3} - Y_P\ x - H_{\kappa\ x} \epsilon_\kappa^{n_x}$$

## Net Yield Surface is the Phase Volume Fraction Weighted Sum

$$\phi = (1 - \chi) (\phi_\mu^C + \phi_\kappa^C) + \chi (\phi_\mu^H + \phi_\kappa^H)$$

# Model: Mechanical Deformation

