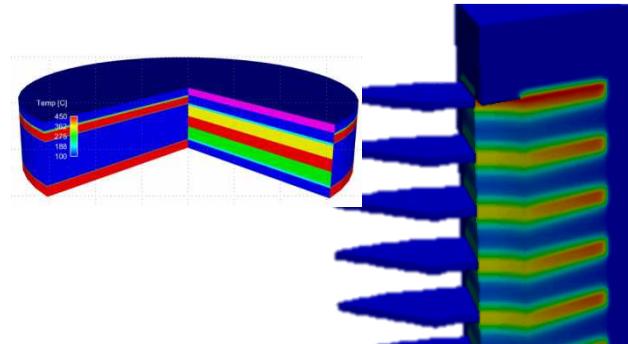
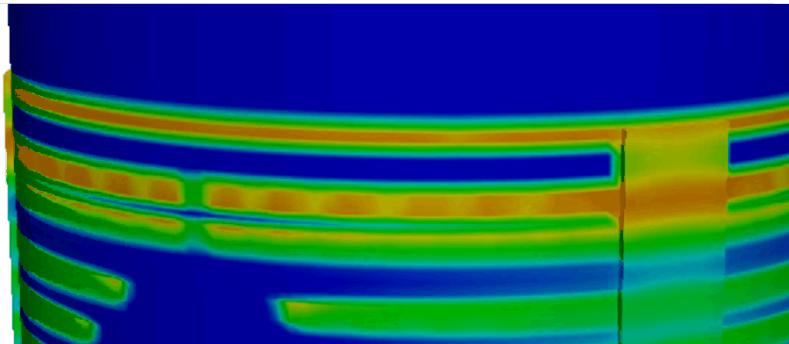


Exceptional service in the national interest



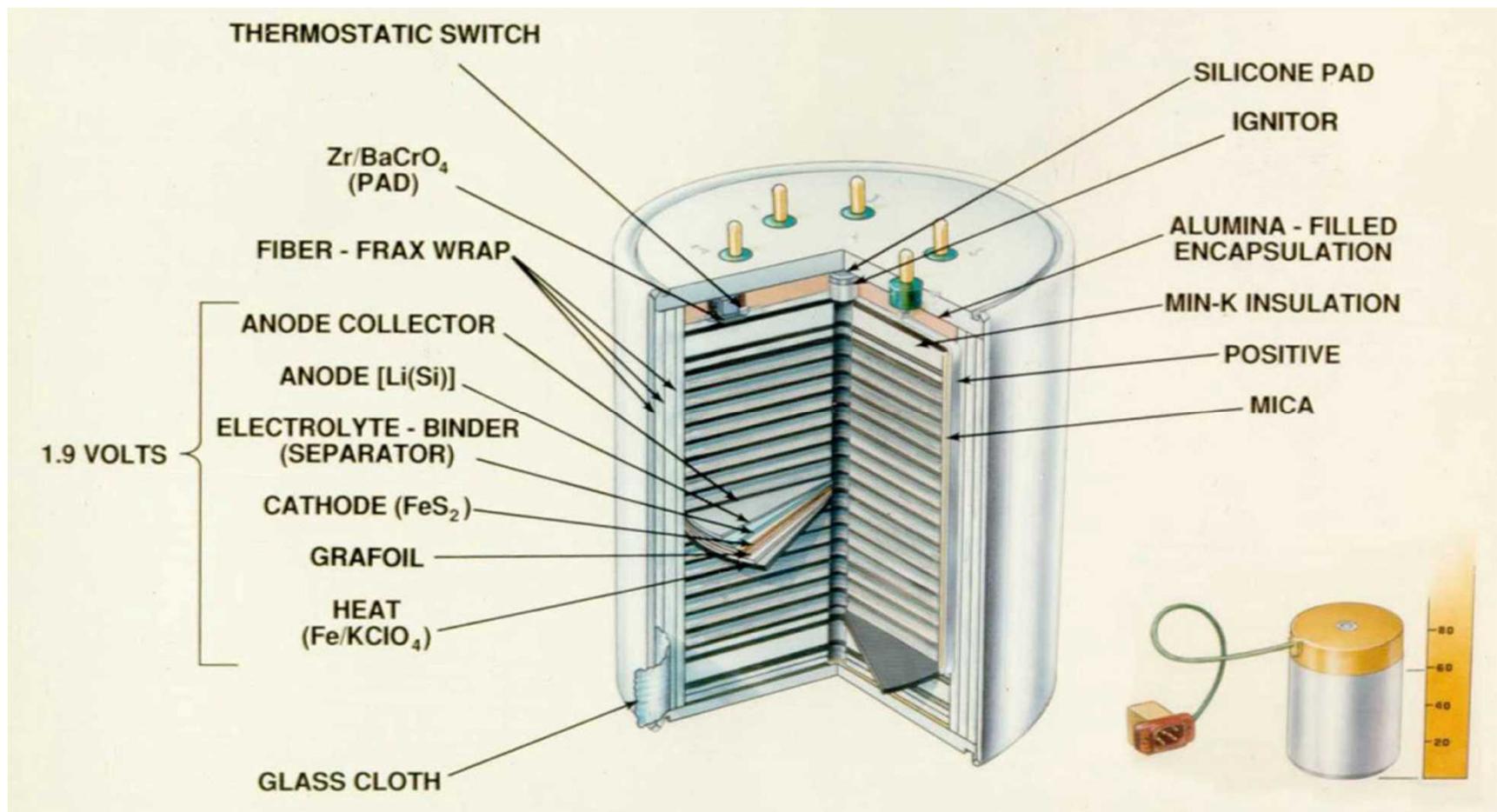
Developing a coupled thermal-mechanical-porous model for electrolyte flow in a molten salt battery

**Jonathan R. Clausen, Scott A. Roberts,
Mario J. Martinez, Kevin N. Long**



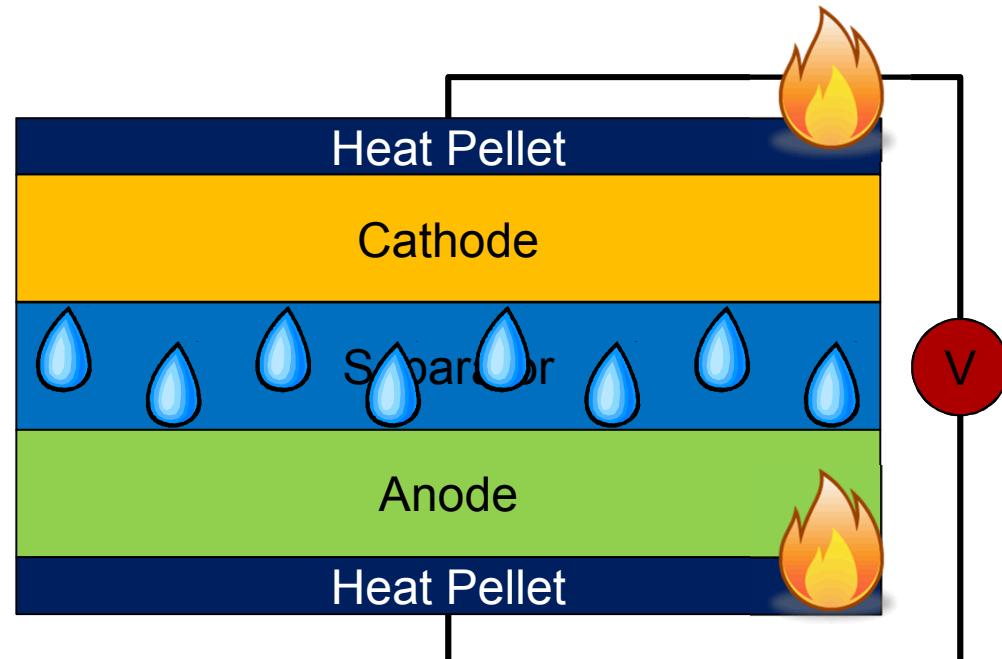
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Molten Salt Battery



Physical Mechanisms

- Multi-physics problem
 - Thermal
 - Mechanical
 - Fluid
 - Electrochemical
- Modeling goals
 - Predict activation times
 - Optimize volume, insulation, manufacturing



Multiphysics Coupling

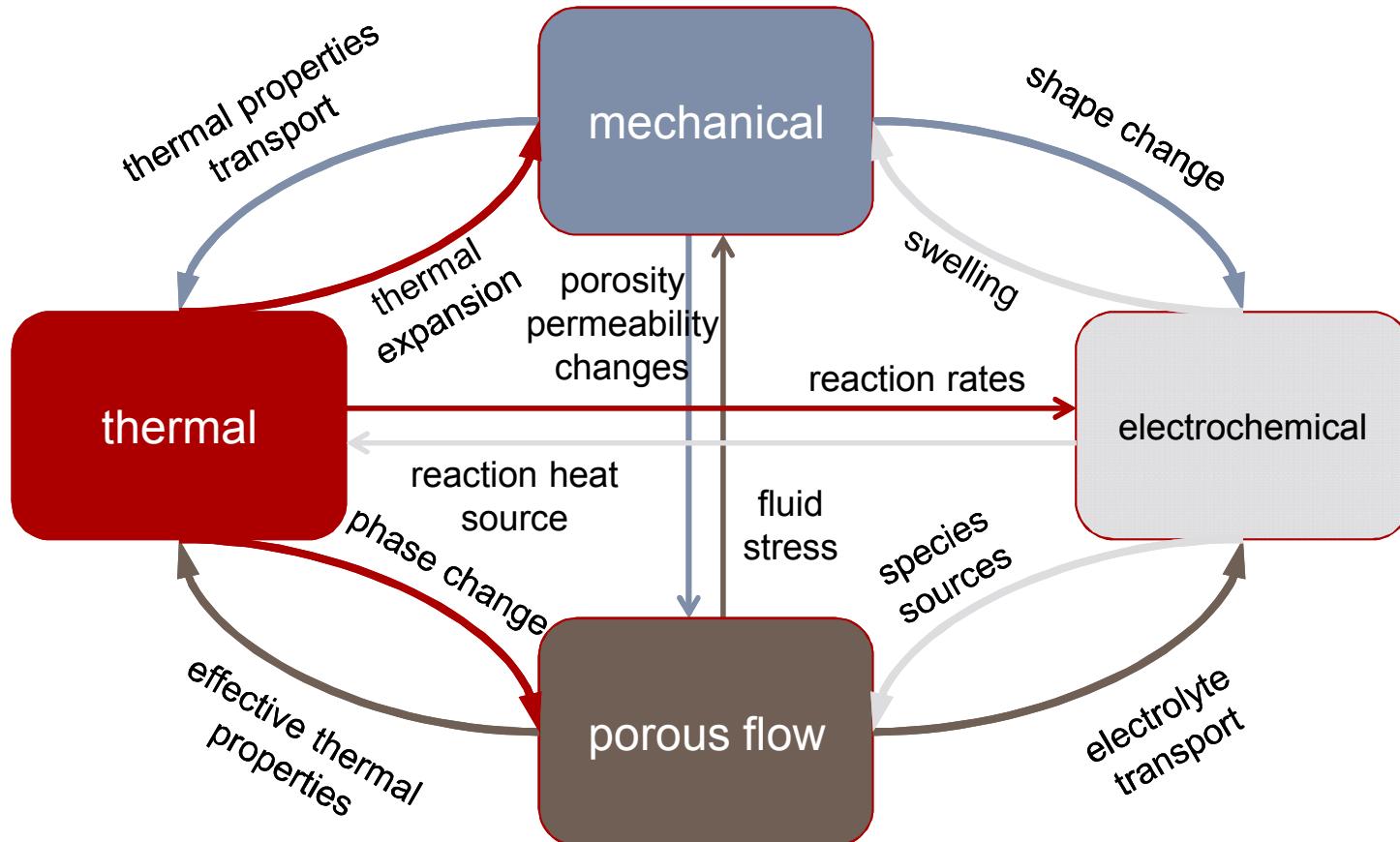
mechanical

thermal

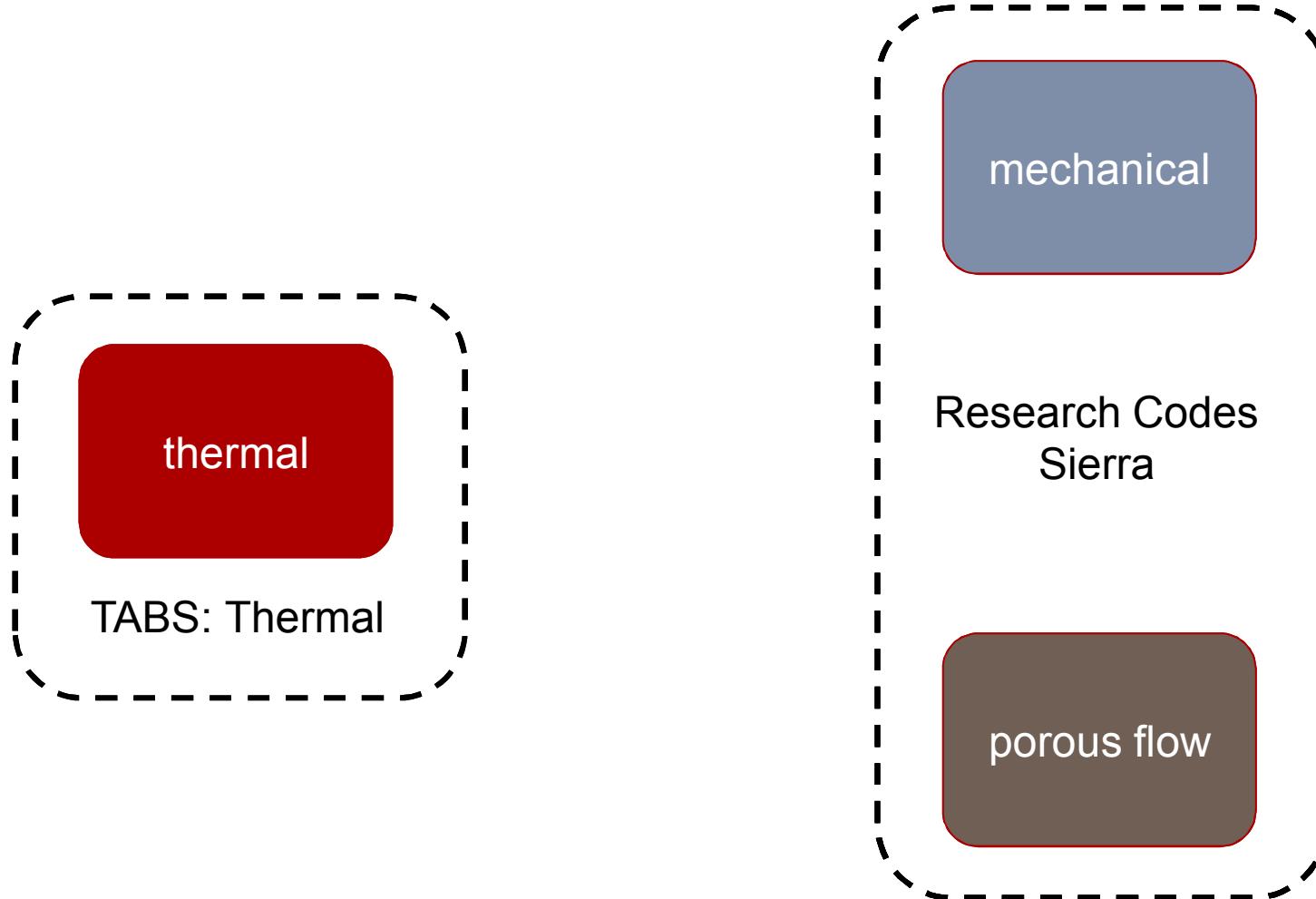
electrochemical

porous flow

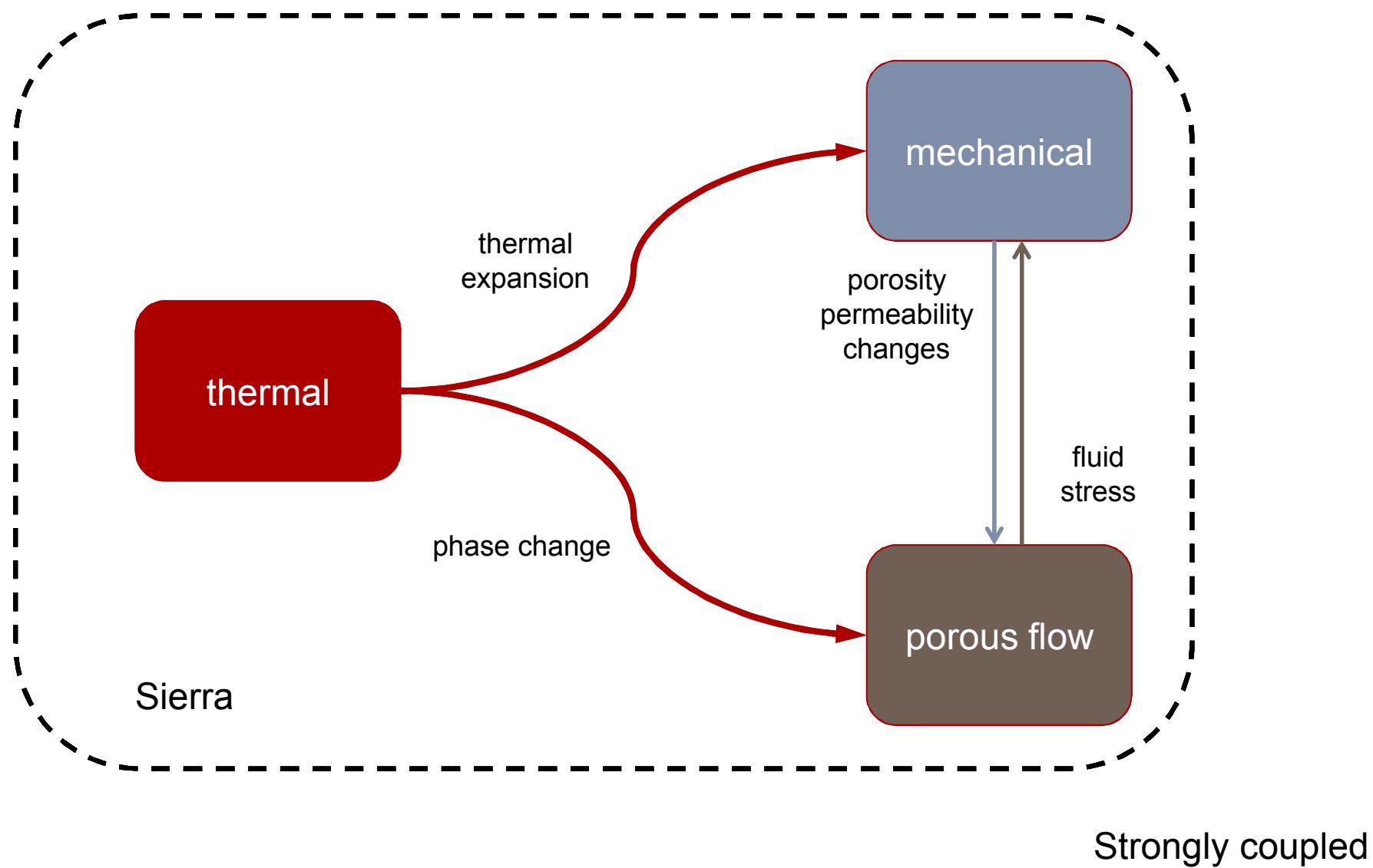
Multiphysics Coupling



Multiphysics Coupling



Multiphysics Coupling

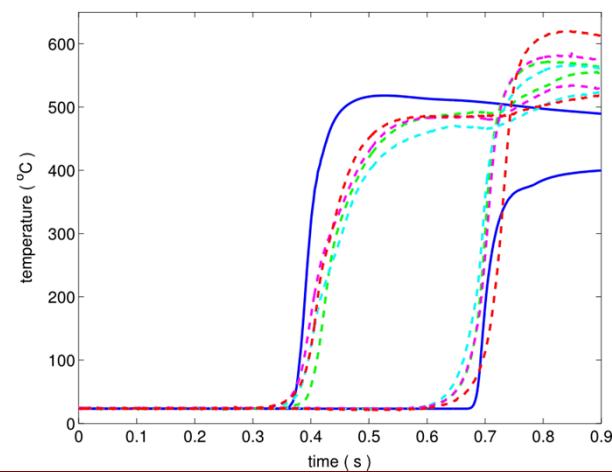
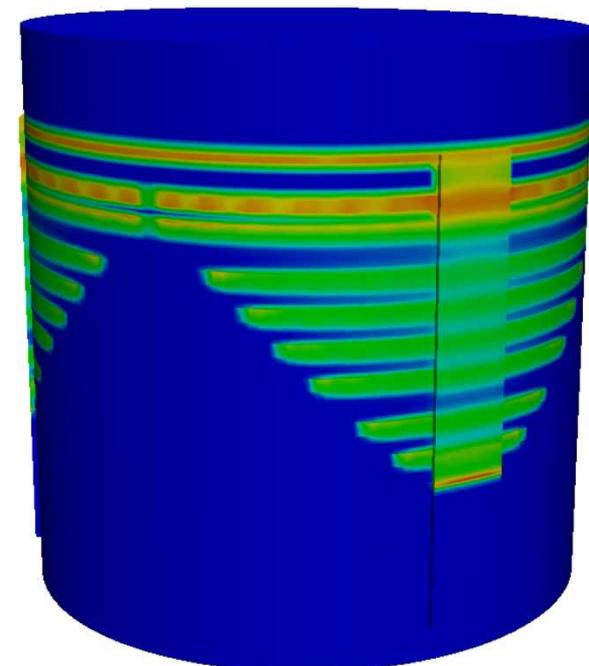


Models: Thermal

- Heat equation

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + Q$$

- Source term Q applies to heat pellet, paper
- Level set tracking of burn fronts
 - Constant propagation speed
 - Heat released over a narrow region near burn-front position



Model: Mechanical Deformation

- Solid constitutive model
 - inelastic volumetric and isochoric deformation of the *MgO skeleton* before, during, and after activation
 - Isotropic, thermal-elastic-plasticity
 - **Plasticity governs activation deformation**
 - Kinematic split of deformations

$$\underline{\mathbf{F}} = \underline{\mathbf{F}}^e \underline{\mathbf{F}}^p \underline{\mathbf{F}}^T$$

- Rule of mixtures for phase decomposition

$$\chi = \frac{\underline{T} - (\underline{T} - T_w/2)}{T_w}$$

- Kirchoff stress: $\underline{\underline{\tau}} = \mu_x(\underline{T}) \text{dev}(\underline{b}^3) + \frac{\kappa_x(\underline{T})}{2} (J_e^2 - 1) \underline{\underline{\delta}}$

- Conservation of momentum: $\underline{\nabla} \cdot \underline{\underline{\sigma}} = \underline{0}$

- Coupled to porous-flow through effective stress: $\underline{\underline{\sigma}} = \hat{\underline{\underline{\sigma}}} + \underline{p} \underline{\underline{\delta}}$

Models: Porous flow

- Electrolyte and gas form two immiscible phases upon melt

$$\frac{\partial(\rho_w \phi S_w)}{\partial t} = \nabla \cdot \left(\rho_w \frac{k_{rw}}{\mu_w} \underline{\underline{K}} \cdot (\nabla p_w - \rho_w \underline{g}) \right) + Q_w$$

$$\frac{\partial(\rho_n \phi S_n)}{\partial t} = \nabla \cdot \left(\rho_n \frac{k_{rn}}{\mu_n} \underline{\underline{K}} \cdot (\nabla p_n - \rho_n \underline{g}) \right) + Q_n$$

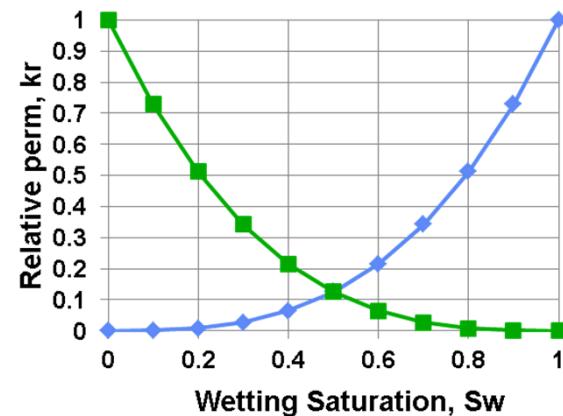
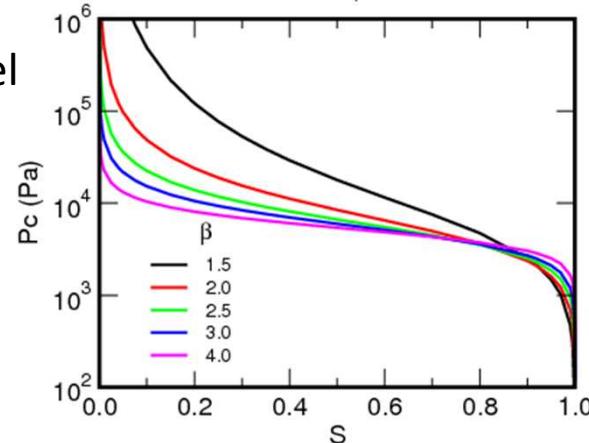
- Saturation and capillary pressure related to DOFs (wetting and non-wetting pressures) through model relations

$$S = S(p_c); \quad p_c = p_n - p_w$$

- Coupling to other physics important!

$$\phi = \phi(\underline{d}); \quad \mu_i = \mu_i(T)$$

$$S_i = S_i(p_c, \underline{d}); \quad \underline{\underline{K}} = \underline{\underline{K}}(\underline{d})$$



Capillary pressure (top) and relative permeability (bottom) depend on wetting phase saturation and electrode pore structure

Numerics

- Thermal and Mechanical
 - GFEM method
- Porous Flow
 - Upwinded version of Darcy flux (Forsyth)

$$R_I^{flux} = - \sum_{J \neq I} \lambda_{(I,J)}^u K_{IJ} (P_J - P_I)$$

$$K_{IJ} = - \int_{\Omega} \nabla N_I \cdot \mathbf{k} \cdot \nabla N_J \, d\Omega$$

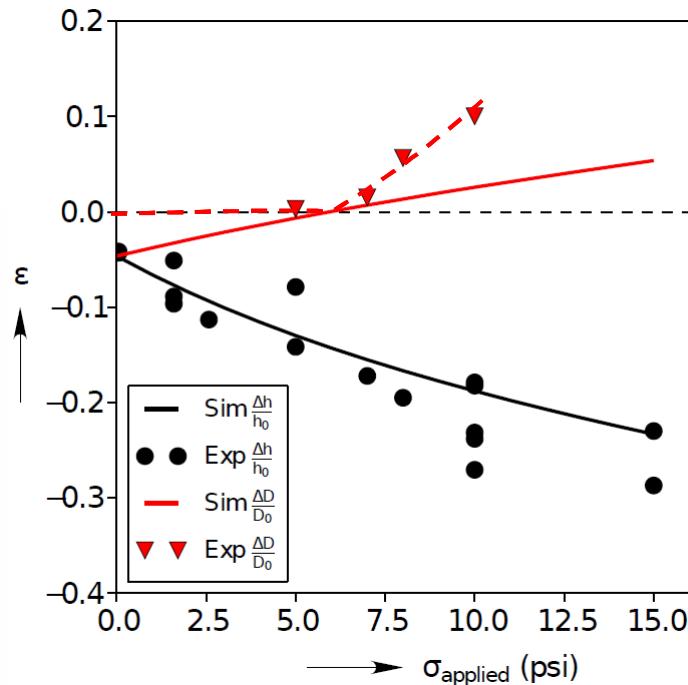
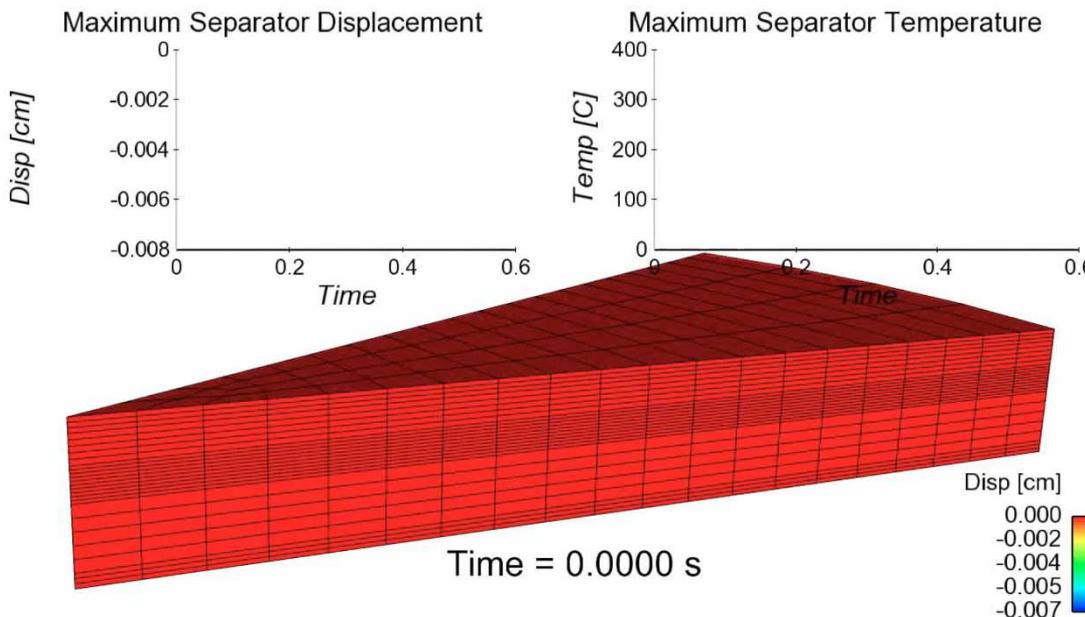
$$\lambda_{(I,J)}^u = \begin{cases} \lambda_J & \text{if } K_{IJ}(P_J - P_I) > 0 \\ \lambda_I & \text{if } K_{IJ}(P_J - P_I) < 0 \end{cases}$$

$$\lambda = \rho Y k_r / \mu$$

- Vertex Quadrature
- Discontinuous Saturations
 - pressure—pressure formulation

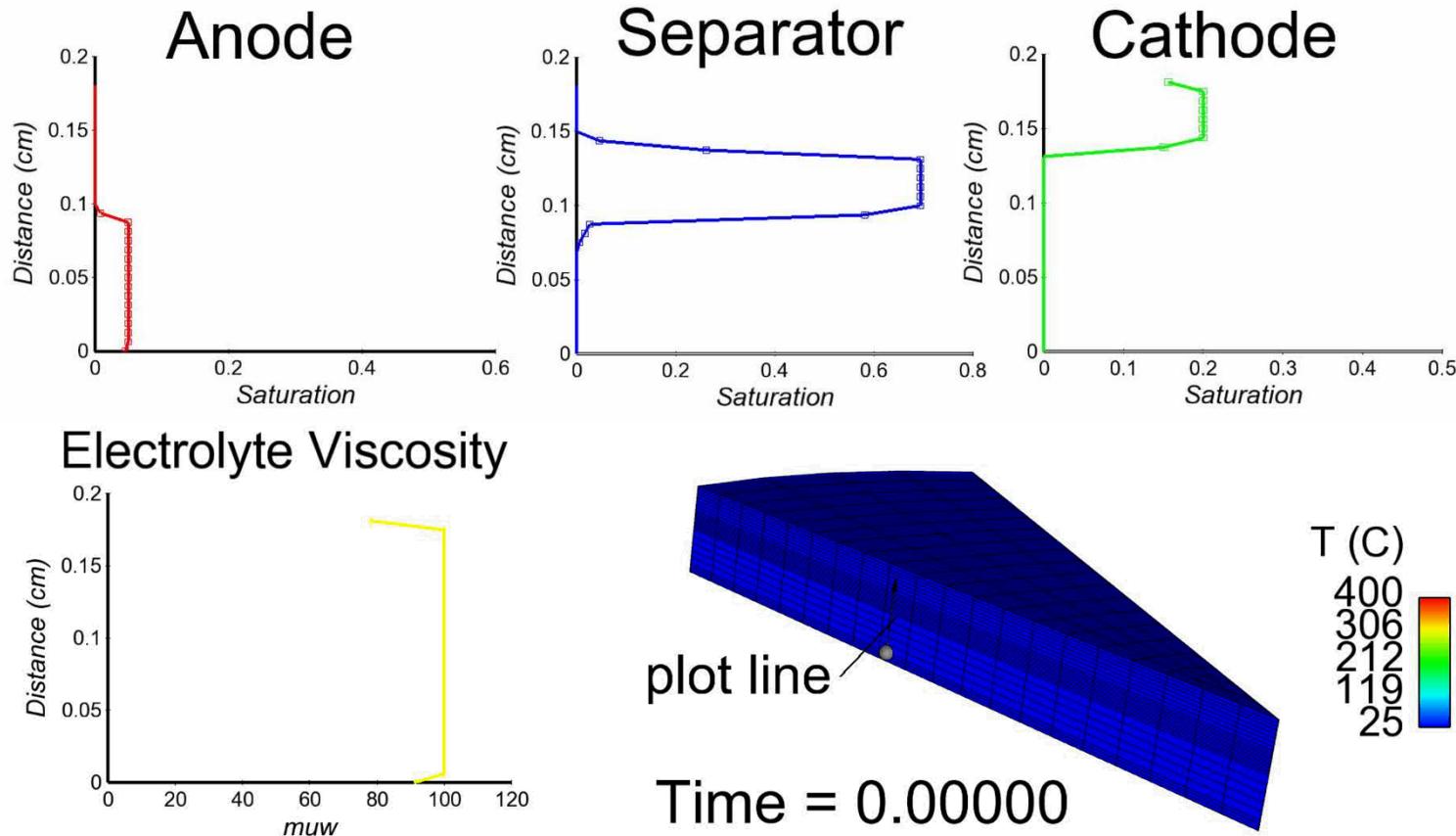
Demonstration: Themo-mechanical deformation

- Electrolyte melting causes separator deformation
- Height vs. diameter change
 - Missing effective stress



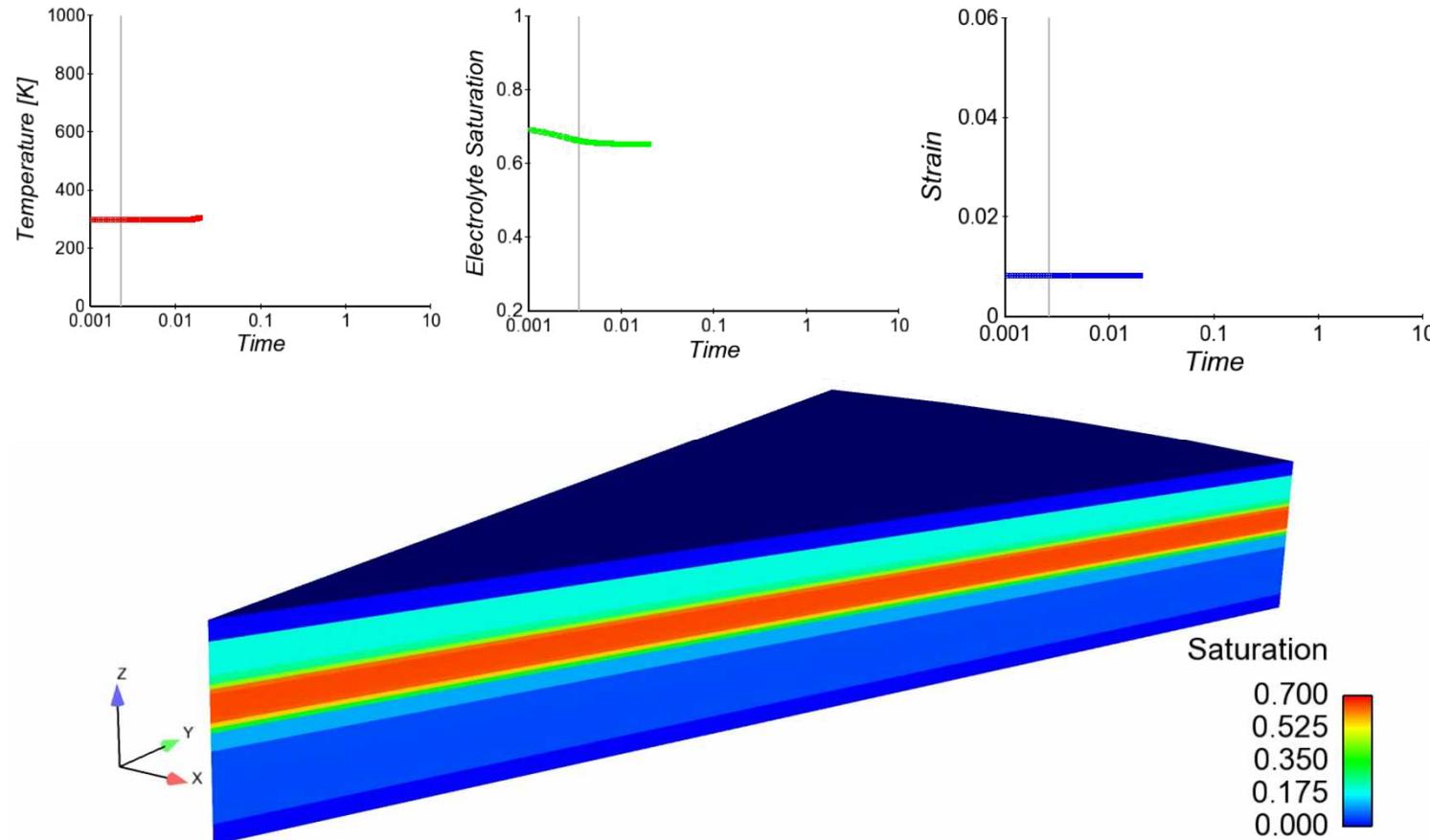
Demonstration: Thermo-porous flow

- Two-pressure porous-flow formulation enables stable solution of flow from the separator to the cathode and



Demonstration: Thermo-poro-mechanical coupling

- Thermo-poro-mechanical single-cell simulation with full coupling:

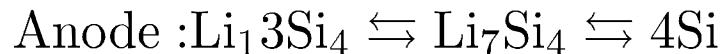
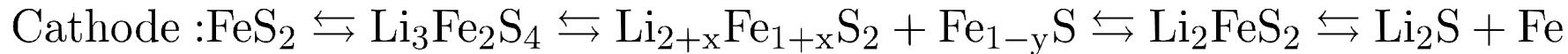


Conclusions & Future Work

- Proof of concept simulation
- Experimental determination of material properties
- Calibration of solid model via experiments
- Electrochemical modeling
- Acknowledgements:
 - DoD/DoE Joint Munitions Program, TCG V
 - ASC/IC, ASC/P&EM, WSEAT
 - Victor Brunini, Lindsay Erickson, Adrian Kopacz

Models: Electrochemistry

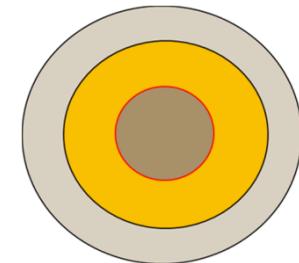
- Reactions, especially for the cathode, are stoichiometrically complicated



- Cantera's "Electrode Object" deploys multiple sub-grid models

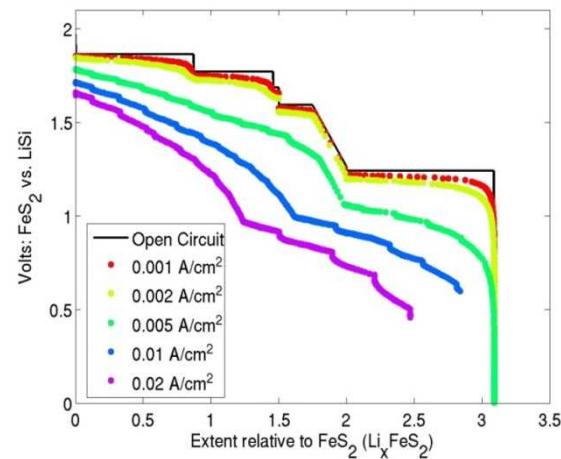
- Infinite capacity
- Multi-plateau
- Newman reaction extend
- Finite capacity

- Primary electrochemical coupling is the temperature
 - Cantera's thermodynamics all temperature-dependent
- Future: Use deformed geometry to affect porosity in electrochemical calculations



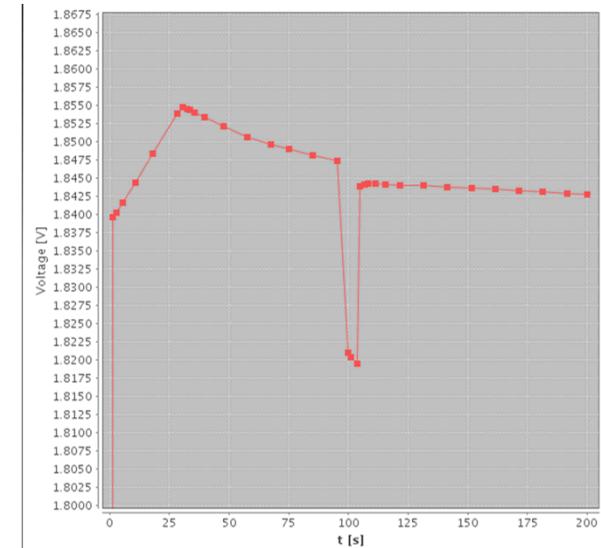
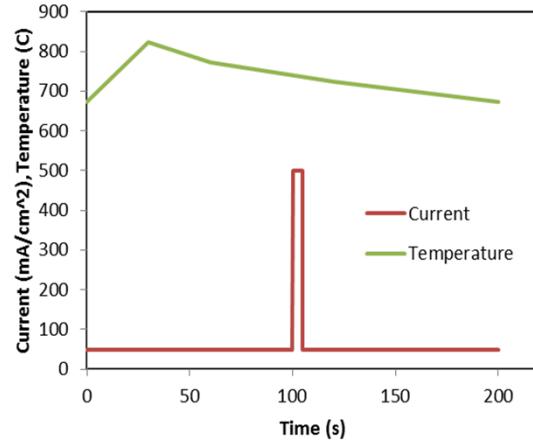
Shrinking Core Model

- Multiple plateaus can react simultaneously
- Diffusional losses with transport

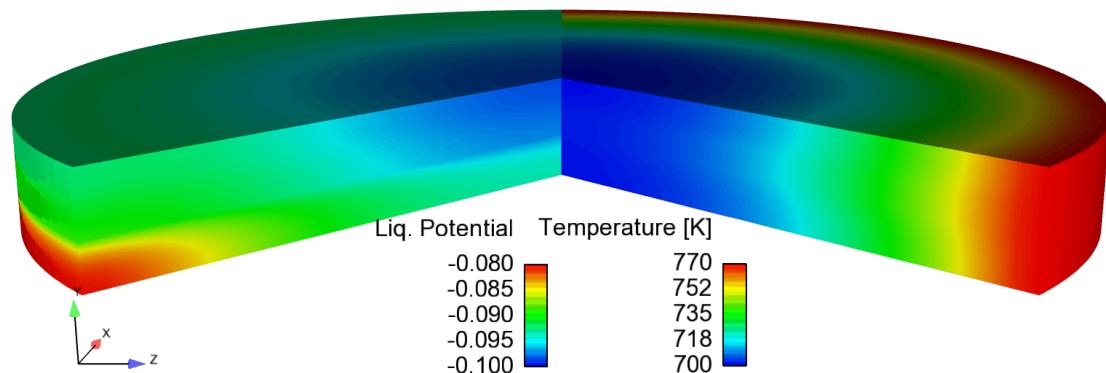


Demonstration: Thermo-electrochemical coupling

- Voltage responds to temperature and current



- Spatial temperature variations affect local potentials and current densities



Thermal-Mechanical Behavior of the Solid Skeleton

Free Energy Density

$$\rho_0 \psi = \mu_x[T] (\bar{I}_{1e} - 3) + \frac{\kappa_x[T]}{4} (J_e^2 - 1 - 2 \log J_e)$$

Kirchoff Stress

$$\boldsymbol{\tau} = 2\rho_0 \mathbf{b} \frac{\partial \psi}{\partial \mathbf{b}} = \mu_x[T] \text{dev}[\mathbf{b}^e] + \frac{\kappa_x[T]}{2} (J_e^2 - 1) \mathbf{1}$$

Isochoric (Radial) Yield

$$\phi_\mu = \sqrt{J_2} - A_x \frac{I_1}{3} - B_x \left(\frac{I_1}{3} \right)^2 - Y_{ps\,x} - H_{\mu\,x} \epsilon_\mu^{m_x}$$

Plastic Flow Rules

$$\mathcal{L}[\mathbf{b}^e] = \mathbf{F} \dot{\mathbf{C}}^{p-1} \mathbf{F}^T = -2 \left(\dot{\lambda}_{iso} \mathbf{n}_{iso} + \dot{\lambda}_{vol} \mathbf{n}_{vol} \right) \mathbf{b}^e$$

$$\mathbf{n}_{iso} = \frac{\text{dev} \boldsymbol{\tau}}{\|\text{dev} \boldsymbol{\tau}\|}, \quad \mathbf{n}_{vol} = \frac{1}{3} \mathbf{1}$$

Kinematic Split of the Deformation Gradient: Thermal, Elastic, and Plastic Parts

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p \mathbf{F}^\theta$$

Cold/Hot State Phase Decomposition

$$\begin{aligned} & \text{if} & T < T_m - T_w/2, & \chi = 0, \\ & \text{elseif} & T > T_m + T_w/2, & \chi = 1, \\ & \text{else} & \chi = \frac{T - (T - T_w/2)}{T_w}, \end{aligned}$$

Volumetric Yield

$$\phi_\kappa = \frac{I_1}{3} - Y_{P\,x} - H_{\kappa\,x} \epsilon_\kappa^{n_x}$$

Net Yield Surface is the Phase Volume Fraction Weighted Sum

$$\phi = (1 - \chi) (\phi_\mu^C + \phi_\kappa^C) + \chi (\phi_\mu^H + \phi_\kappa^H)$$

Model: Mechanical Deformation

