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Faraday Waves Excited by Random Vibration

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Open Forum on Multiphase Flows: Work in Progress

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AUGUST 3-7, 2014
CHICAGO, ILLINOIS

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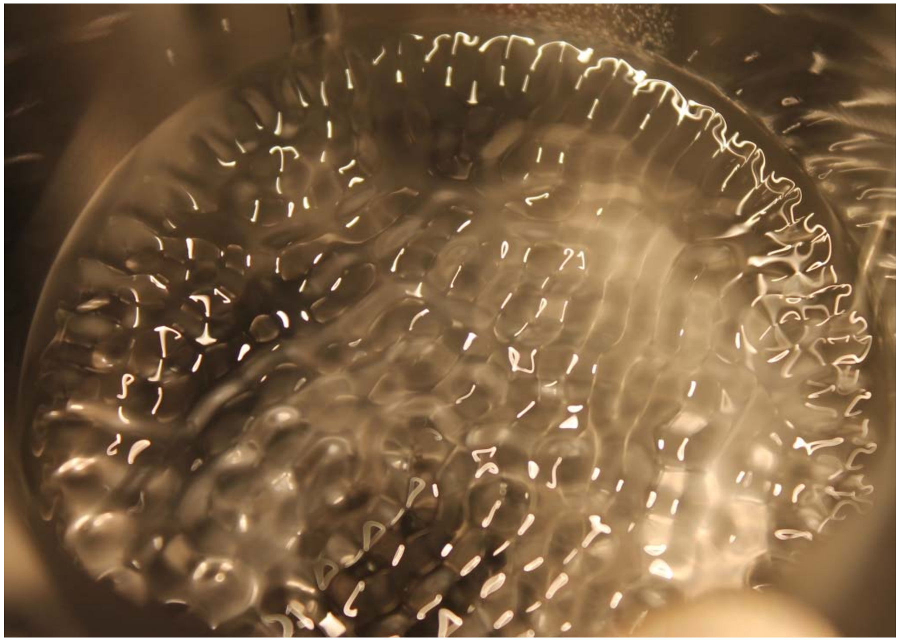
U.S. DEPARTMENT OF ENERGY

Faraday Waves



View of Faraday waves from above a vibrating free surface at: 80 Hz, 5 g, 76 mm dia, 25 mm depth, 20 cSt PDMS silicone oil

Faraday, M., (1831), "On a peculiar class of acoustical figures; and on certain forms assumed by a group of particles upon vibrating elastic surfaces," Philosophical Transactions of the Royal Society, 121, 299–318.

- Liquid-gas or liquid-liquid interfaces subjected to vertical vibration will form surface waves when the vibration amplitude exceeds a critical value.
- The initially flat free surface then becomes unstable to the formation of standing surface waves.
- These waves have a frequency half of the driving frequency (the first sub-harmonic resonance), often referred to as period-doubling
- These waves were first reported by Michael Faraday in 1831, who performed experiments using water, ink, turpentine, egg whites, alcohol, and mercury

Faraday Waves

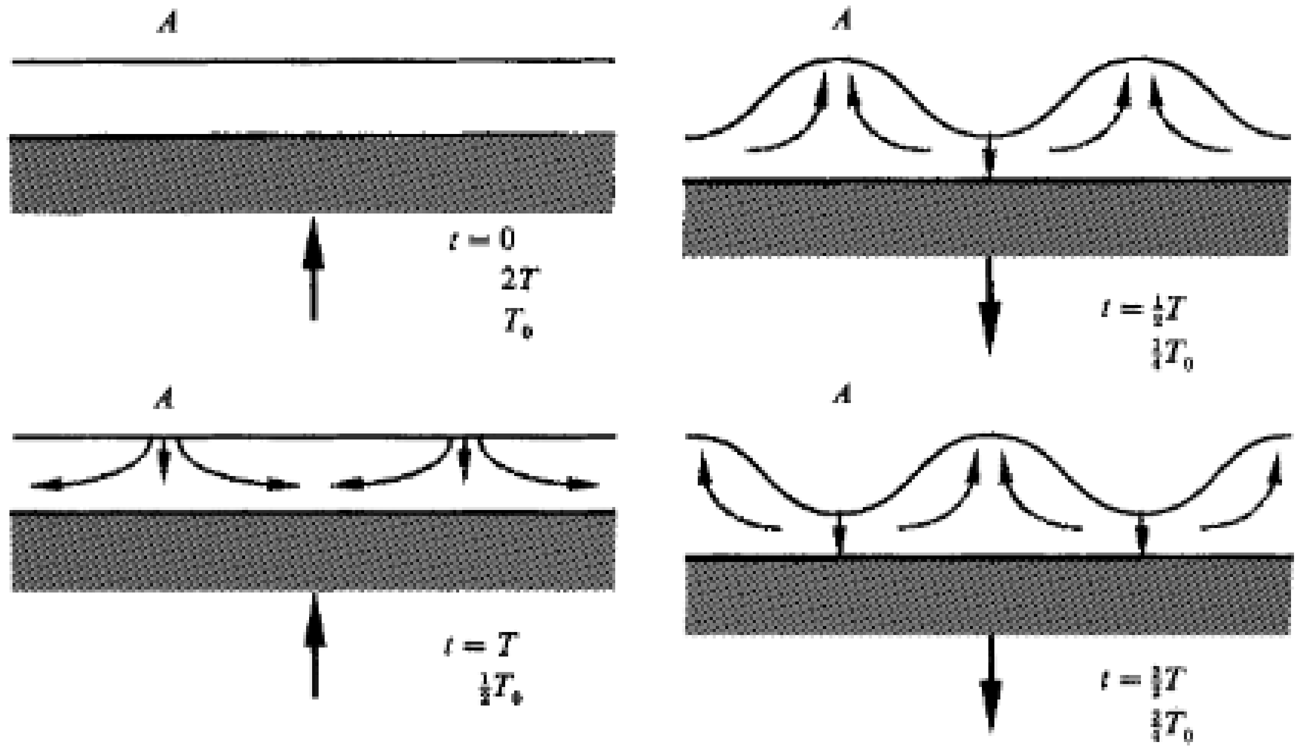
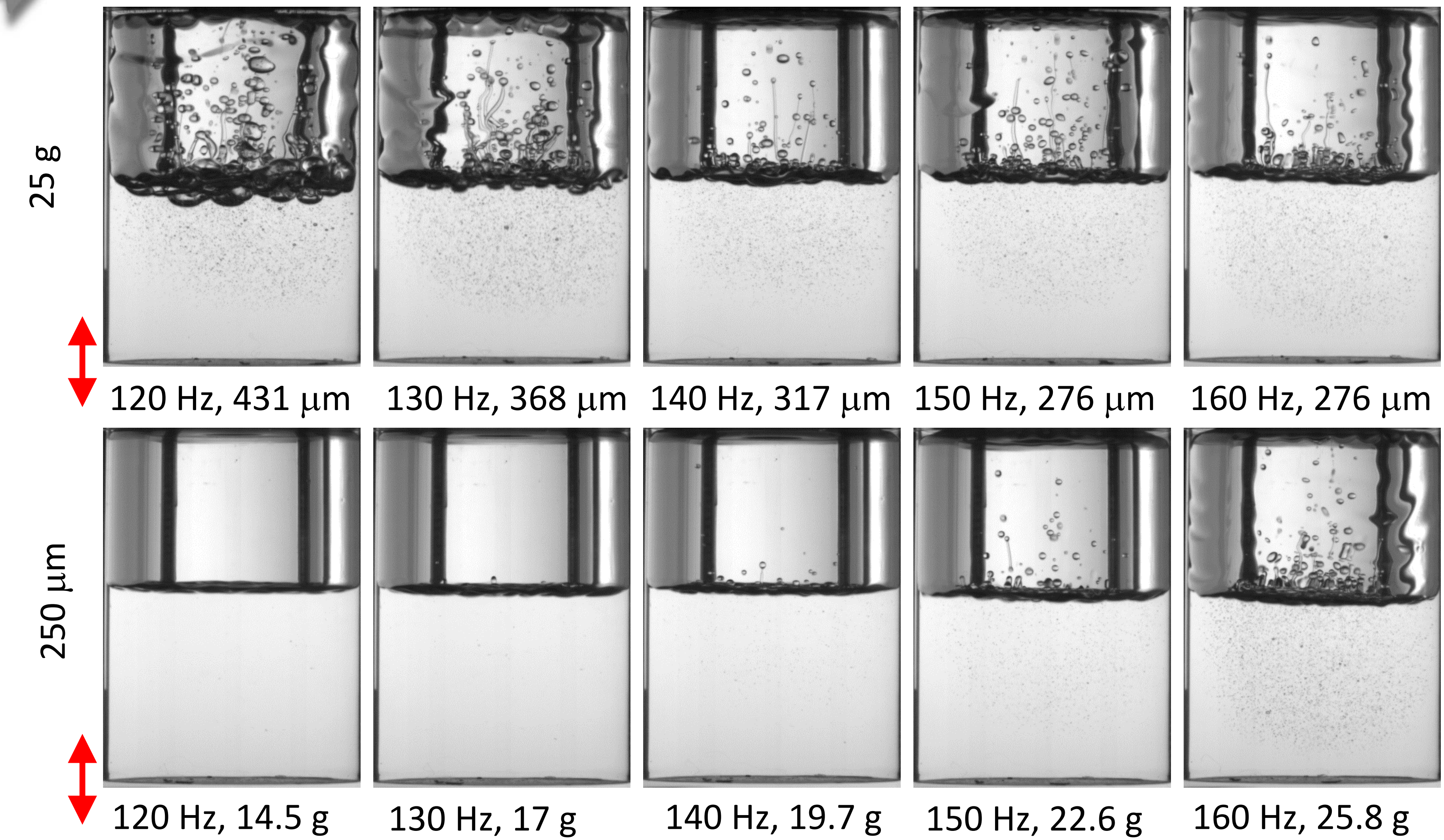


FIGURE 1. Excitation at half the excitation frequency of a fluid layer undergoing a vertical oscillation. When the vessel goes down, the fluid inertia tends to create a surface deformation, as in the Rayleigh–Taylor instability. This deformation disappears when the vessel comes back up, in a time equal to a quarter-period of the corresponding wave (T_0). The decay of this deformation creates a flow which induces, for the following excitation period T, the exchange of the maxima and the minima. Thus one obtains $T_0 = 2T$.

Douady, S., (1990), "Experimental study of the Faraday instability," J. Fluid Mech., 221, 383-409.

Why do we care about Faraday Waves?



As part of a larger study on the effect of vibration on bubbles we need to understand when free surface breakup occurs and when breakup acts as a bubble source (20 cSt PDMS silicone oil with air above)

Why do we care about Faraday Waves?

175 μm p-p displacement, 4.3 g

250 μm p-p displacement, 6.1 g

10 cSt PDMS, 110 Hz

Single Sine Frequency Vibration

Labworks ET-140 shaker

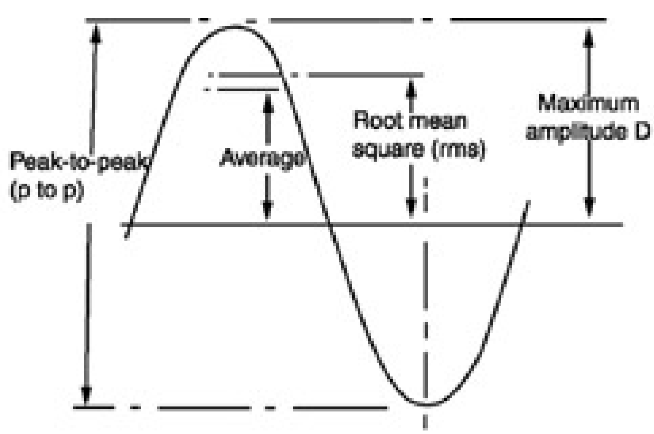
Single frequency vibrations

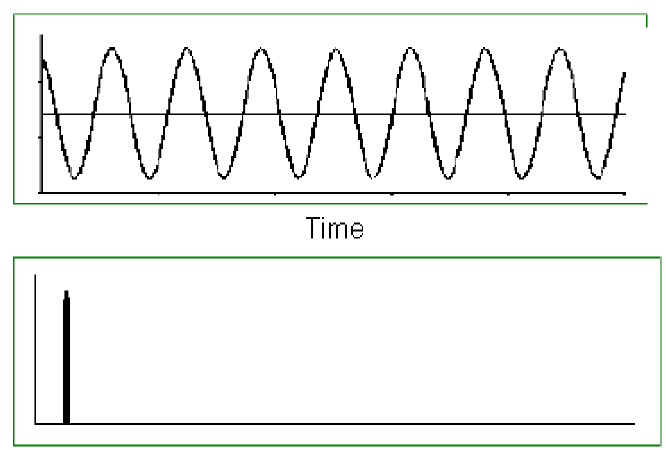
$$z = z_0 \sin\omega t$$

$$v = \frac{dz}{dt} = \omega z_0 \cos\omega t$$

$$a = \frac{d^2z}{dt^2} = -\omega^2 z_0 \sin\omega t$$

Vibration conditions completely defined by $\omega = 2\pi f$, z_0, and a (pick 2, third is determined)





The Spectrum of a Sine Wave

Single Sine Frequency Faraday Waves

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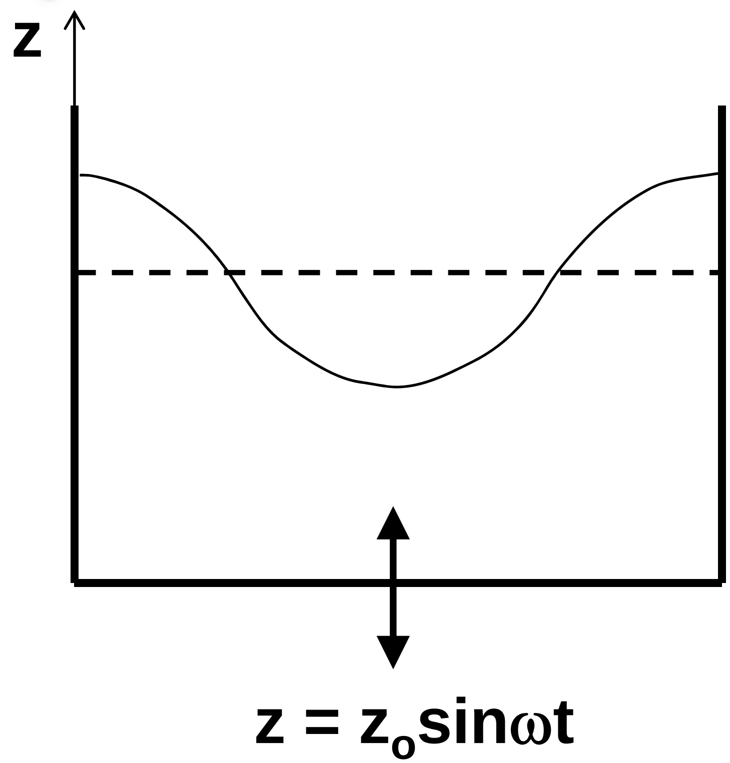


- Vibration leads to a modulated gravity

  $g' = g - \omega^2 z_0 \sin\omega t$

- Small amplitude waves in a vessel undergoing sinusoidal vertical oscillations can be described by the Mathieu equation (when damping is small):

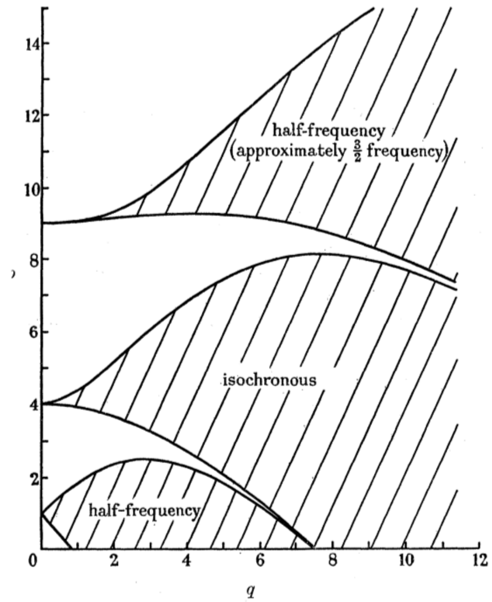
$$\frac{d^2z}{dt^2} + \gamma\frac{dz}{dt} + \left[\omega_0^2 + z_0 sin(\omega t)\right]z = 0$$

  where ω is the excitation frequency and ε is the amplitude (proportional to z_0). This describes a simple harmonic oscillator with a periodically time-varying spring constant.

- After a spatial Fourier transform, each wavenumber *k* also satisfies the Mathieu equation

$$\frac{d^2z}{dt^2} + 4\nu k^2\frac{dz}{dt} + k\left[g - \omega^2 z_0 sin(\omega t)\right]z = 0$$

Single Sine Frequency Faraday Waves



IGURE 2. Stability chart for the solutions of Mathieu's equation

$$\frac{d^2a}{dT^2} + (p - 2q\cos 2T)\, a = 0.$$

There are instabilities for <u>certain ranges</u> of the unperturbed frequency ω_0, the damping γ, the forcing frequency ω, and the forcing amplitude z_0. The easiest instability to excite gives free surface oscillation at half the forcing frequency ω.

•**Benjamin and Ursell, The Stability of the Plane Free Surface of a Liquid in Vertical Periodic Motion, *Proc. R. Soc. Lond. A,* 1954, 225**

Single Sine Frequency Faraday Waves

Wave patterns depend on excitation frequency, the shape of the container, fluid properties (esp. viscosity), and the depth.

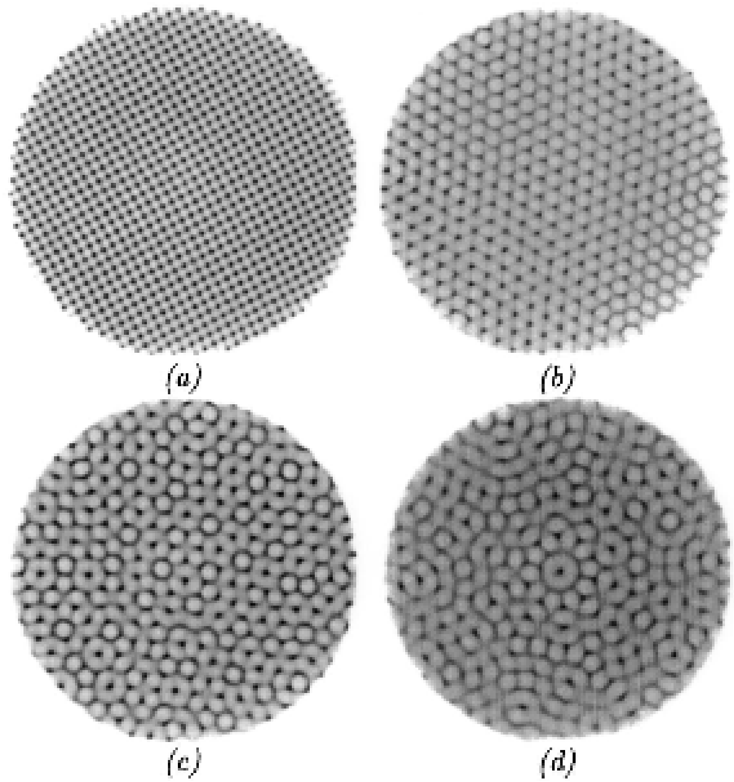
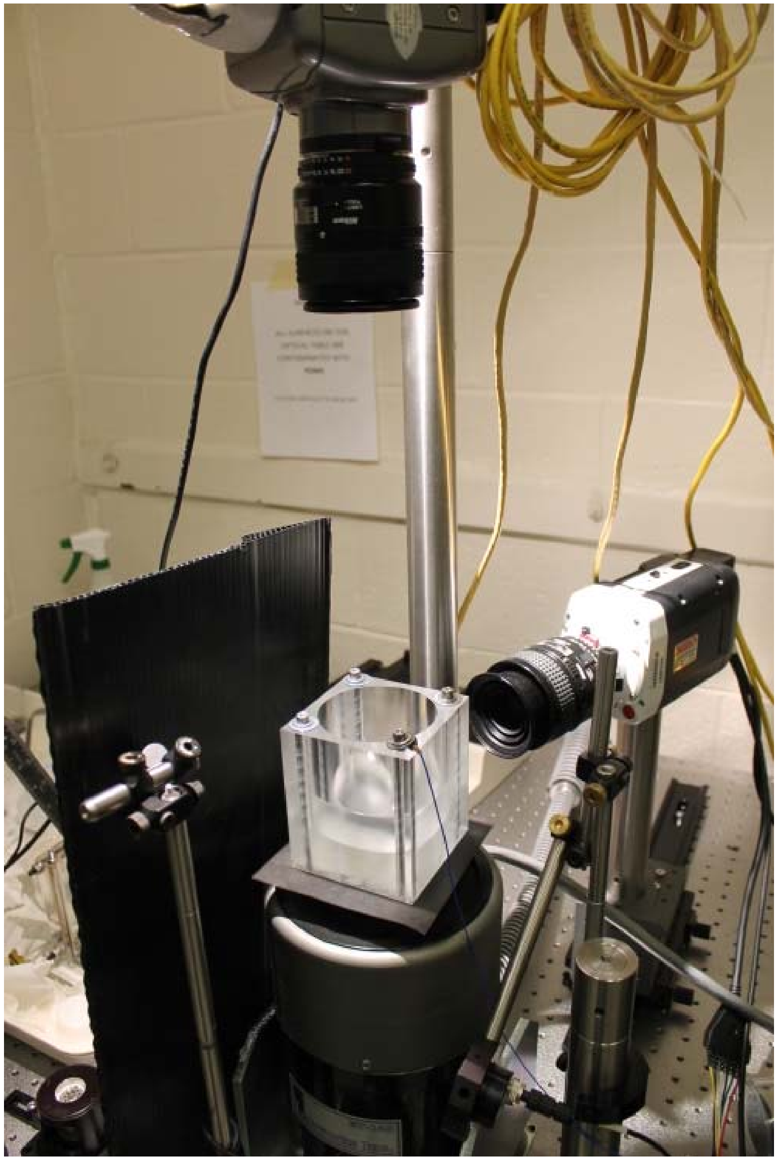


FIG. 1. Images of the fluid surface at frequencies f where patterns of square symmetry are observed: (a) $f = 45.0$ Hz, hexagonal symmetry; (b) $f = 30.0$ Hz, 8-fold quasiperiodic; (c) $f = 29.0$ Hz, and 10-fold quasiperiodic; (d) $f = 27.0$ Hz. The visualized region is of diameter 26 cm, approximately 1/2 the diameter of the container.

Binks and van de Water, Nonlinear Pattern Formation of Faraday Waves, Phys. Rev. Letters, 1997

Experimental Setup



Labworks ET-140 electrodynamic shaker

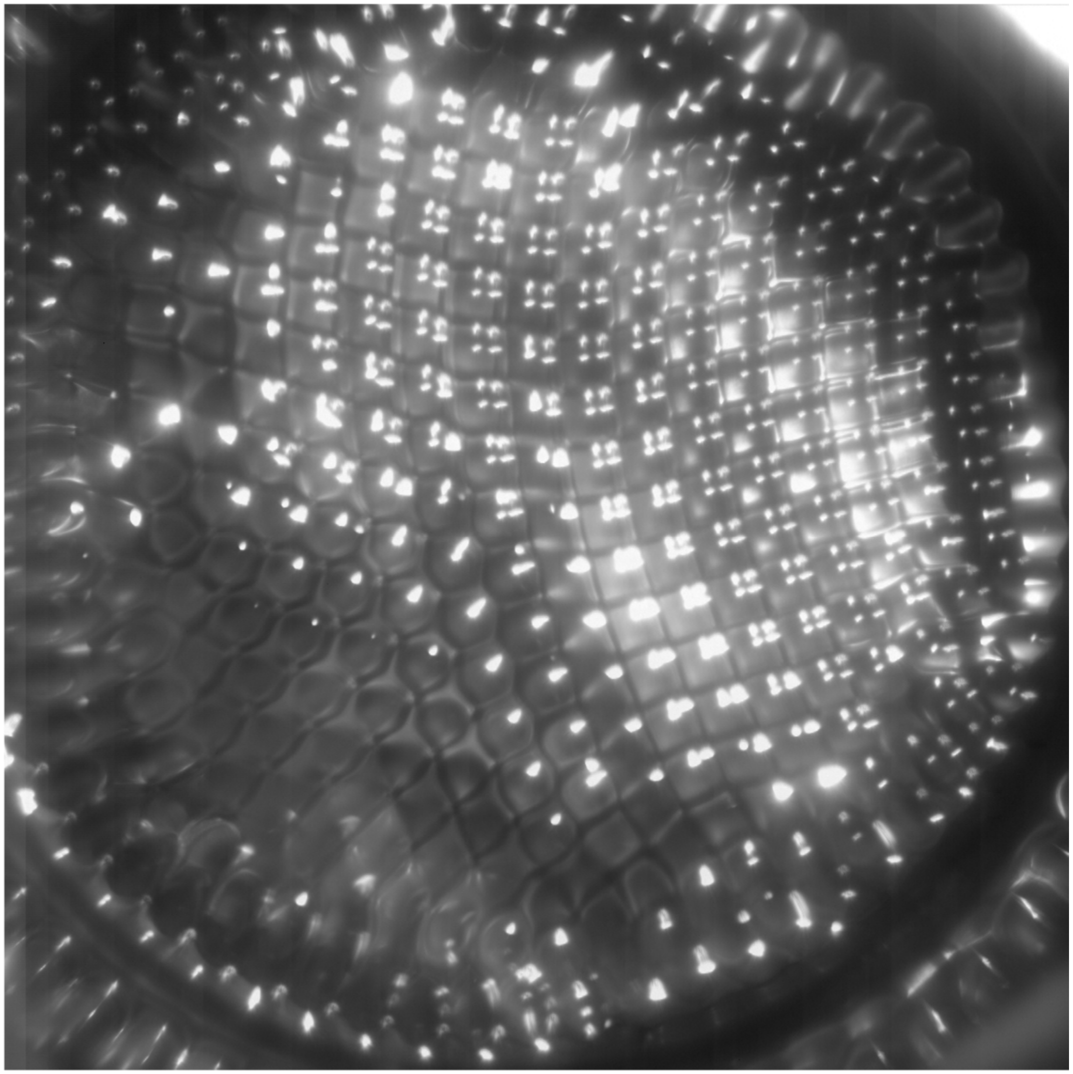
Acrylic boxes of several shapes and sizes mounted on top of shaker

PDMS silicone oil in box (ν = 20 cSt for most experiments to date)

Size and depth considerations

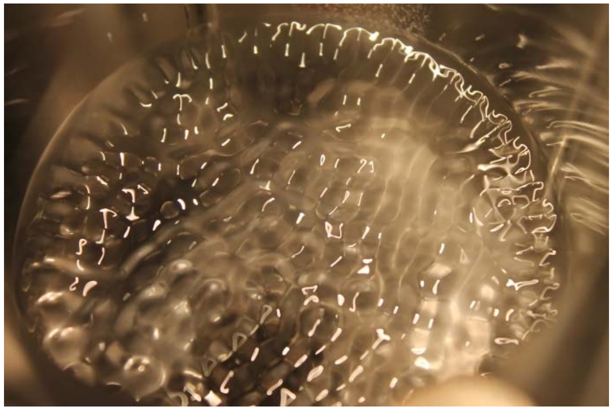
Aspect ratio: Bechhoefer et al. (JFM, 1995) discuss sidewall meniscus wave influence at different width:depth ratios. Ours range from 3:1 to 10:1.

Results – Sine excitation



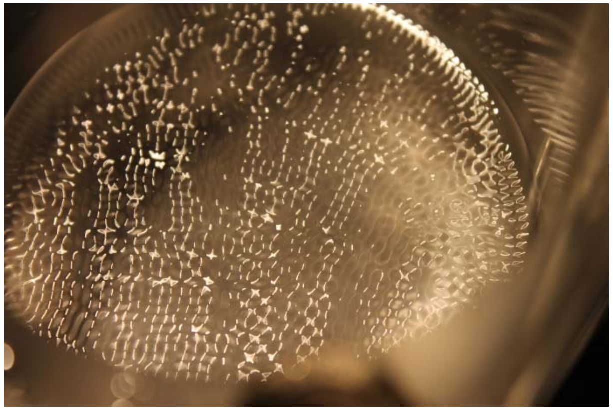
View of Faraday waves from directly above a vibrating free surface at: 130 Hz, 9.4 g, 76 mm dia, 25 mm depth, 20 cSt PDMS silicone oil

Results – Sine excitation



80 Hz, 5 g
388 μm displacement

130 Hz, 9.4 g
276 μm displacement



200 Hz, 20 g
248 μm displacement

Faraday waves on vibrating free surface

76 mm dia, 25 mm depth

20 cSt PDMS silicone oil

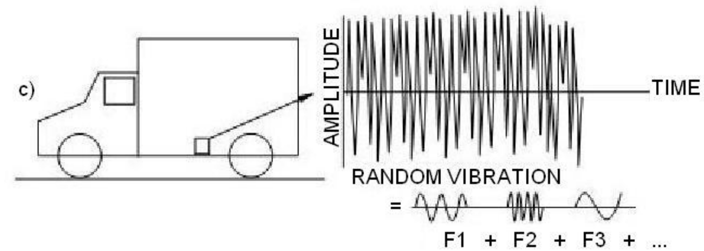
Random Vibrations

Most real world vibrations don't occur at a single frequency, but are a combination or distribution of frequencies

Random vibration is motion at many frequencies simultaneously, with the amplitude of each frequency varying randomly with time.

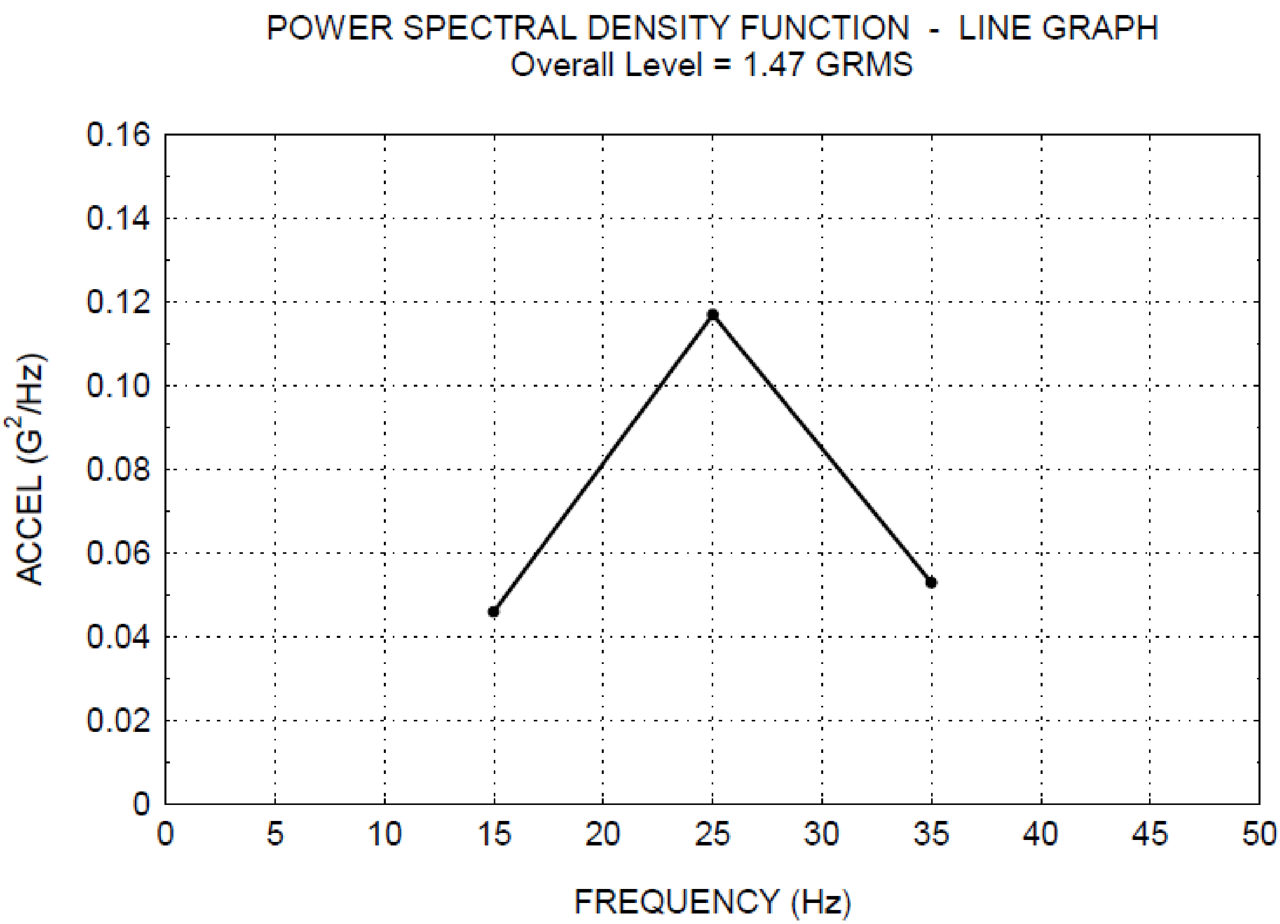
Random vibration is usually described through its Power Spectral Density (PSD) with units of g^2/Hz

Acceleration, velocity, displacement no longer directly related to frequency as in sine excitation. The RMS amplitude vs. time is statistically defined.



Random Vibration PSD

Example



•http://www.vibrationdata.com/tutorials2/psd.pdf

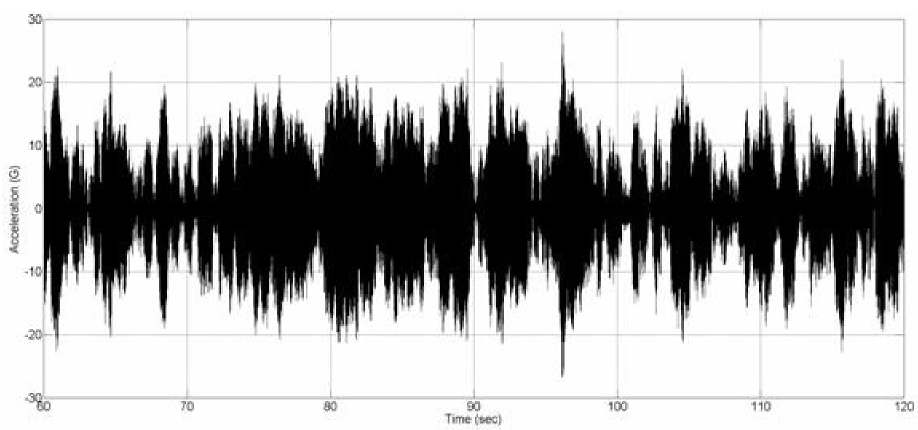
Random Vibration PSD

Strange Units (g^2/Hz)

- PSD defined by mean squared acceleration (g^2) divided by the bandwidth (Hz)

PSD (Power Spectral Density) really should be ASD (Acceleration Spectral Density).

- However, commonly used PSD indicates the "power" from the output of accelerometers during structural test.



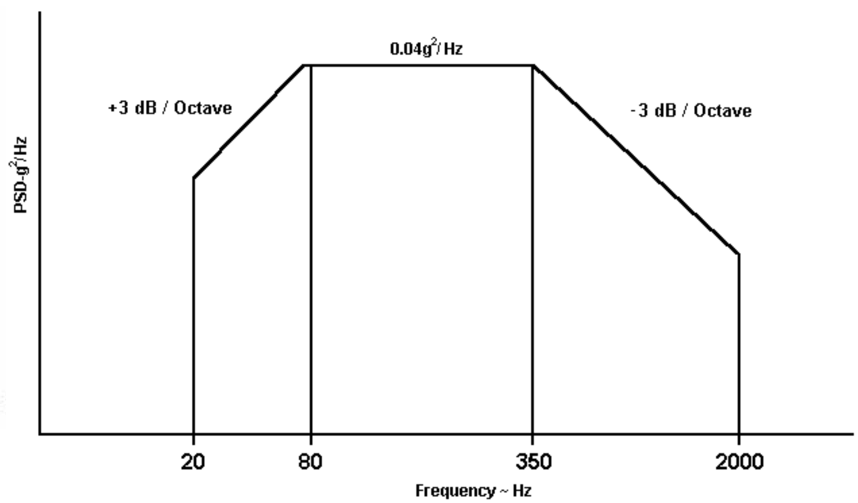
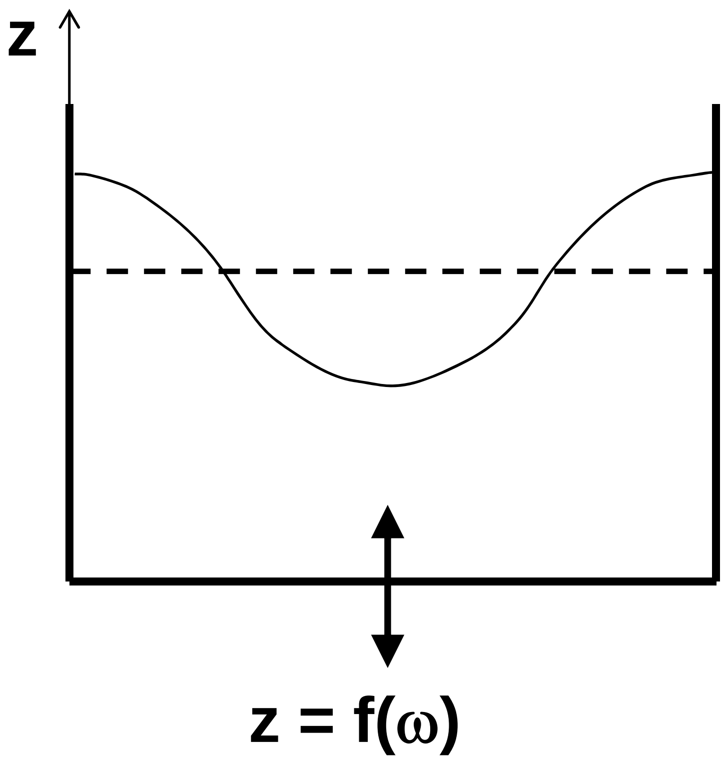


Figure 5. Random Vibration Spectrum

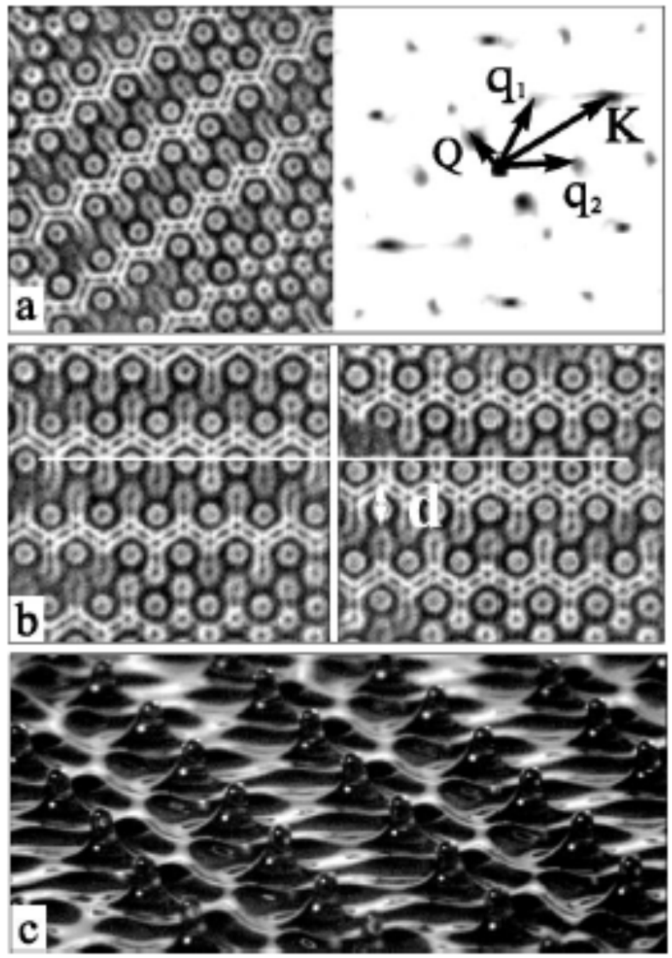
Random Frequency Faraday Waves



- Most literature on Faraday waves considers single sine frequencies
- A few groups have examined multiple discrete frequency excitation (Zhang and Viñals, JFM, 1997)
- For most or all applications, the forcing is via a continuous spectrum.
- Zhang et al. (Phys. Fluids, 1993) showed that random forcing broadens the unstable frequency range and that the threshold acceleration needed to form waves increases as the spectrum broadens.
- Repetto and Galletta (JFM, 2002) performed theoretical study showing that the range of unstable frequencies broadens under narrow-band spectrum random forcing compared to single frequency forcing

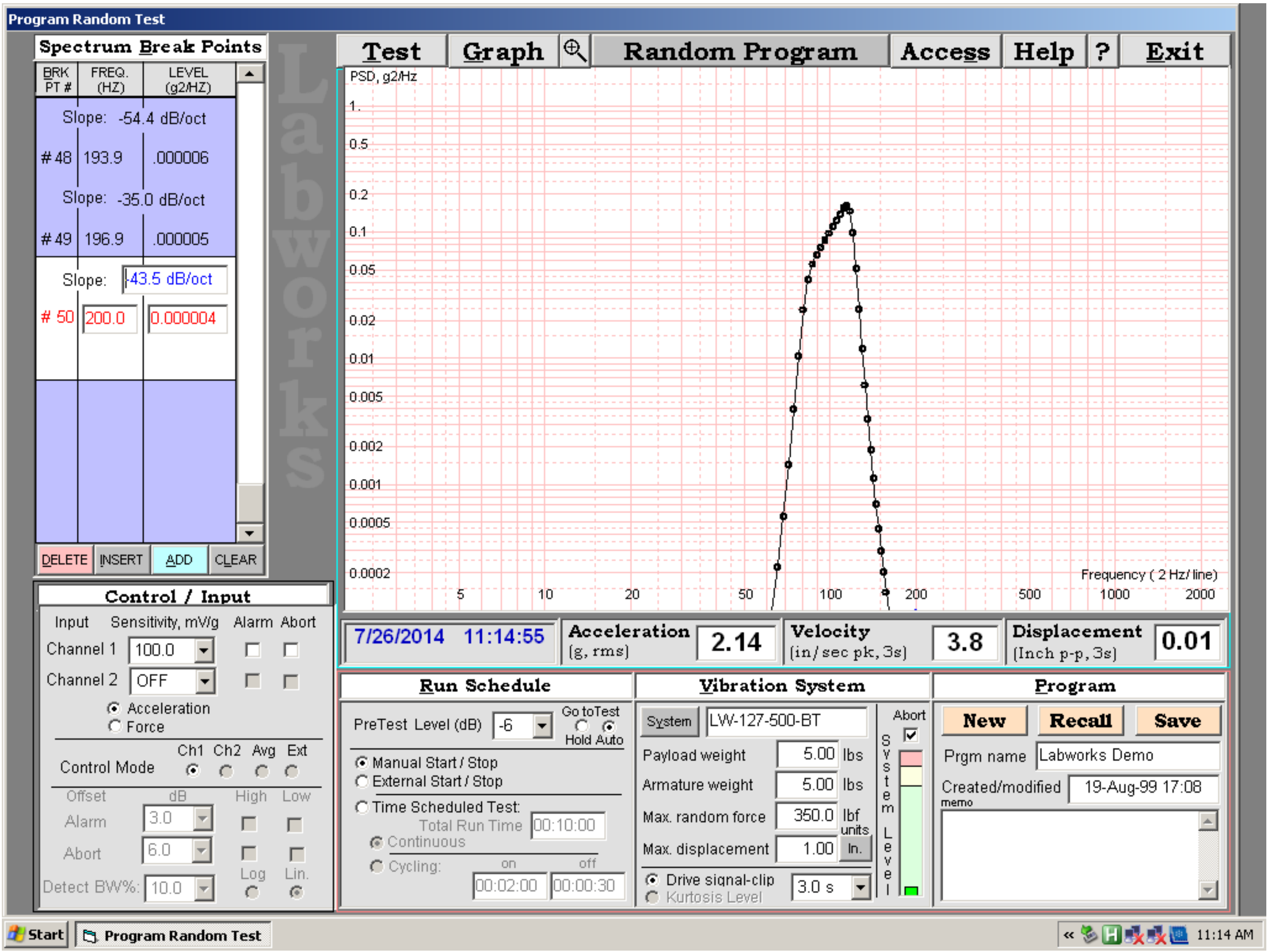
Random Frequency Faraday Waves

Wave patterns caused by each forcing frequency and interactions between waves



Arbell and Fineberg, Pattern Formation in two-frequency forced parametric waves, Phys. Rev. E, 2002

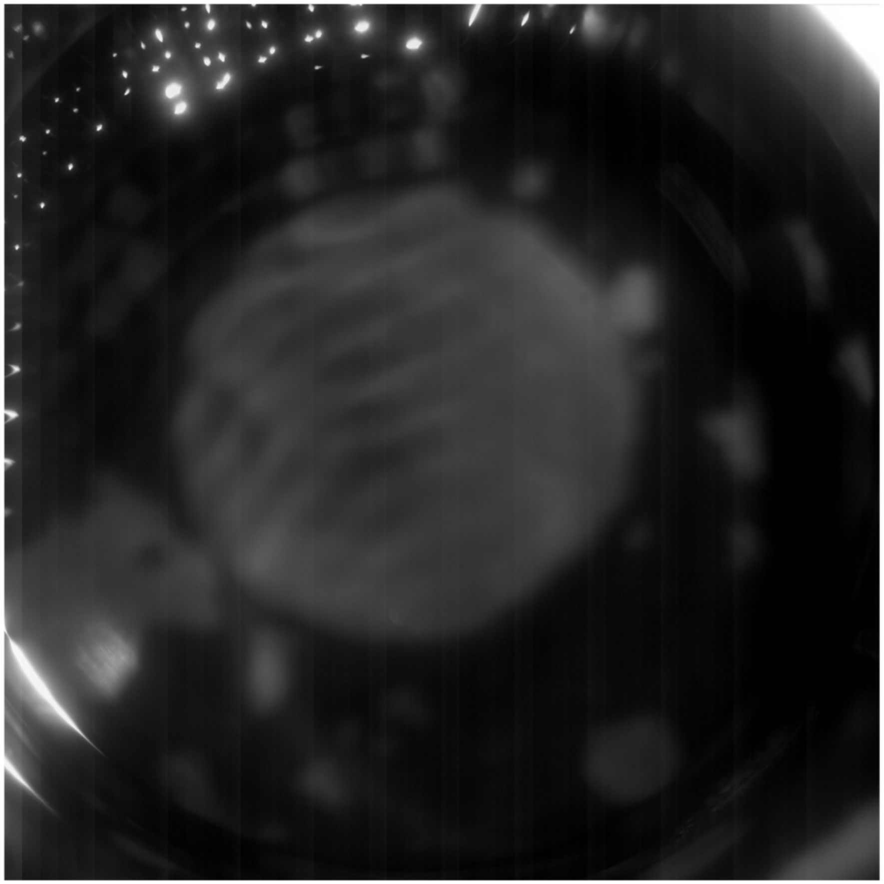
Results – Random excitation



PSD defined as shown here

– **1x scaling or 2.14 g rms amplitude**

Results – Random excitation



20 cSt PDMS, 4.23 g rms

Critical amplitude

75 mm dia, 25 mm depth

2 cSt PDMS: 0.96 g rms

10 cSt PDMS: 2.03 g rms

20 cSt PDMS: 4.23 g rms

50 cSt PDMS: 4.49 g rms

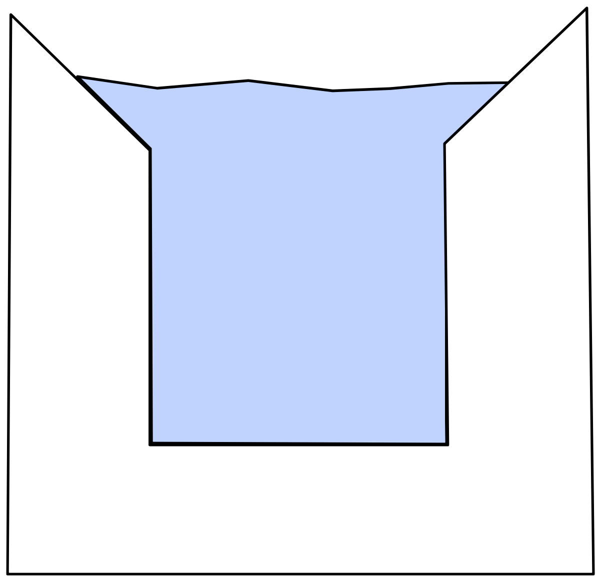
Defining transition difficult. For single frequency excitation Faraday waves can be distinguished from meniscus wave sloshing by their frequency. Not the case for random excitation.

Conclusions and Future Work

Early results show that random excitation Faraday waves ... Theory

Continue experiments and comparison with model

Vary aspect ratio



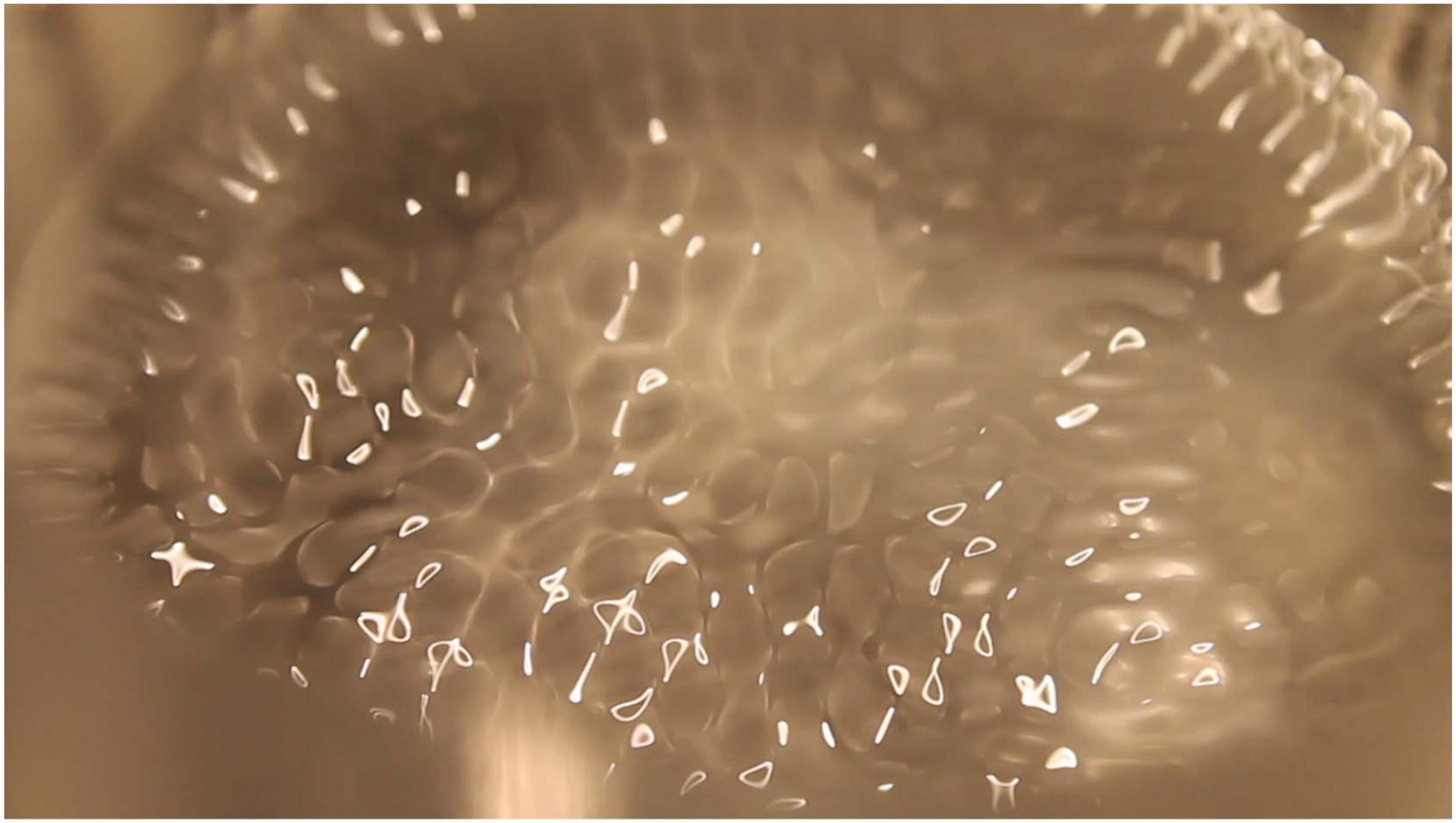
Determine quantitative test for instability

- Distinguish from sloshing (period doubling works for single frequency sine excitation, not clear for random vibe)

Minimize sloshing and wave reflection

- Angled walls (Arbell and Fineberg, 2002)
- Fill to brim (Douady, 1990)

Questions?

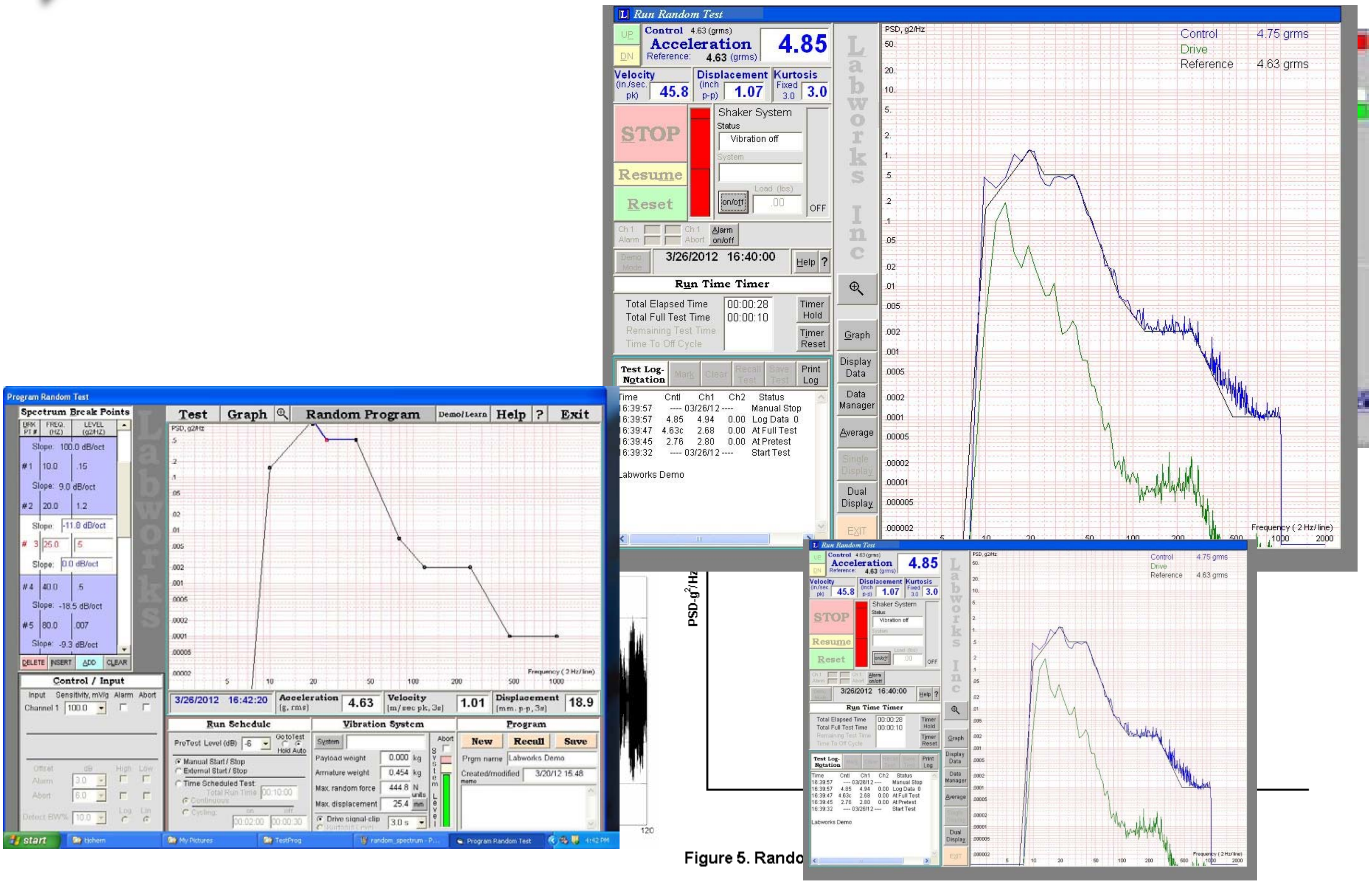


View of Faraday waves on a vibrating free surface.
80 Hz, 5 g, 20 cSt PDMS

- **Faraday waves are generated at the interface between two fluids when parametrically excited by vertical vibration. This is most commonly seen at liquid-gas interfaces where Faraday waves can form a variety of patterns on the free surface, with wave period twice the excitation period (Faraday, 1831). Many studies have examined Faraday waves in a wide range of liquids and containers under a wide range of single frequency vibration excitation conditions, both experimentally and analytically (Benjamin and Ursell (1954); Wright et al., 1996; Wernet et al., 2001). Most vibrations encountered in the real world are composed of multiple simultaneous frequencies, with the vibration amplitude varying randomly with time. However, very few studies have examined the effect of multiple frequencies on the generation of Faraday waves (Zhang, 1993; Repetto and Galletta, 2002). In the present work we review the literature on random vibration experiments and Faraday waves excited by such vibrations. Experimental results will be presented and discussed, and compared with analytical predictions. These phenomena have been observed in several PDMS oils over a broad range of viscosities, as well as in water. Random vibration conditions are varied, and include fairly broad power spectra.**
- **6 talks in 2 hours – shoot for 15-17 minutes**

- **Benjamin, T. B. and Ursell, F., (1954), "The Stability of the Plane Free Surface of a Liquid in Vertical Periodic Motion," Proc. Royal Soc. A, 225, 505-515.**
- **Faraday, M., (1831), "On a peculiar class of acoustical figures; and on certain forms assumed by a group of particles upon vibrating elastic surfaces," Philosophical Transactions of the Royal Society, 121, 299–318.**
- **Repetto, R. and Galletta, V. (2002) "Finite amplitude Faraday waves induced by a random forcing," Phys. Fluids, 14(12), 4284-4289.**
- **Wernet, A., Wagner, C., Papathanassiou, D., Muller, H. W., and Knorr, K., (2001), "Amplitude Measurements of Faraday Waves," Phys. Rev. E, 63, 06305:1-9.**
- **Wright, W. B., Budakian, R., and Putterman, S. J., (1996), "Diffusing Light Photography of Fully Developed Isotropic Ripple Turbulence," Phys. Rev. Let.. 76(24), 4528-45313.**
- **Zhang, W., Casademunt, J., and Vinals, J., (1993), "Study of the Parametric Oscillator Driven by Narrow-Band Noise to Model the Response of a Fluid Surface to Time-Dependent Accelerations," Phys. Fluids A, 5(12), 3147-3161.**

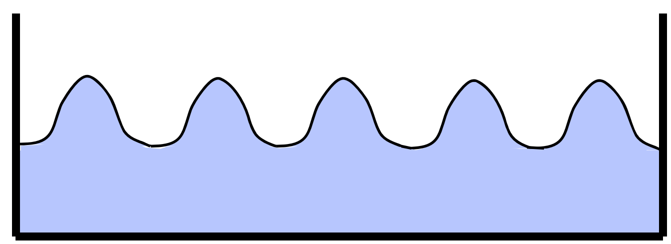
Random Vibration PSD



Measuring Wave Pattern and Height

Difficult since the interface is not very reflective (Wernet et al., 2001). Techniques include:

- Shadowgraphy: Pass parallel beam of diameter equal to container size through interface and record pattern caused by peaks and valleys acting as array of lenses. Limited to small surface deflections.
- Laser reflection: Track focused laser beam reflection onto position-sensitive detector
- Diffusive scattering: Seed liquid with dye or particles to scatter light. Wave crests contain more liquid so absorb or scatter more light than thinner valleys do.
- Contacting permittivity
- Interferometry
- Radar back scatter
- X-ray absorption



Measuring Wave Height

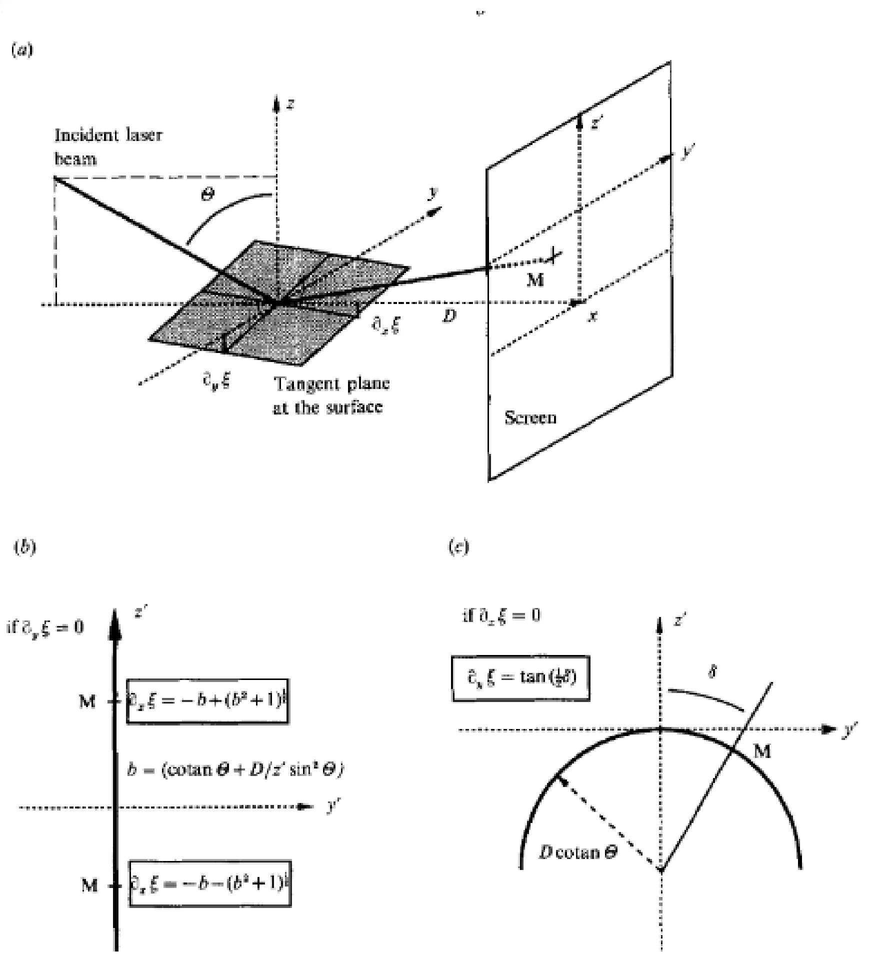


FIGURE 16. (a) The principle of the measurement method of the surface wave amplitude: a laser beam is reflected at the free surface, and one looks at the image point M on a vertical screen far away. The origin is chosen as the reflection with a horizontal surface. In the geometrical approximation, M only depends on the slope of the surface at the reflection point. (b) and (c) The image points on the screen in two cases: the relation can be easily inverted to give the local slopes, and thus to measure the amplitude.

Position Sensitive Detector

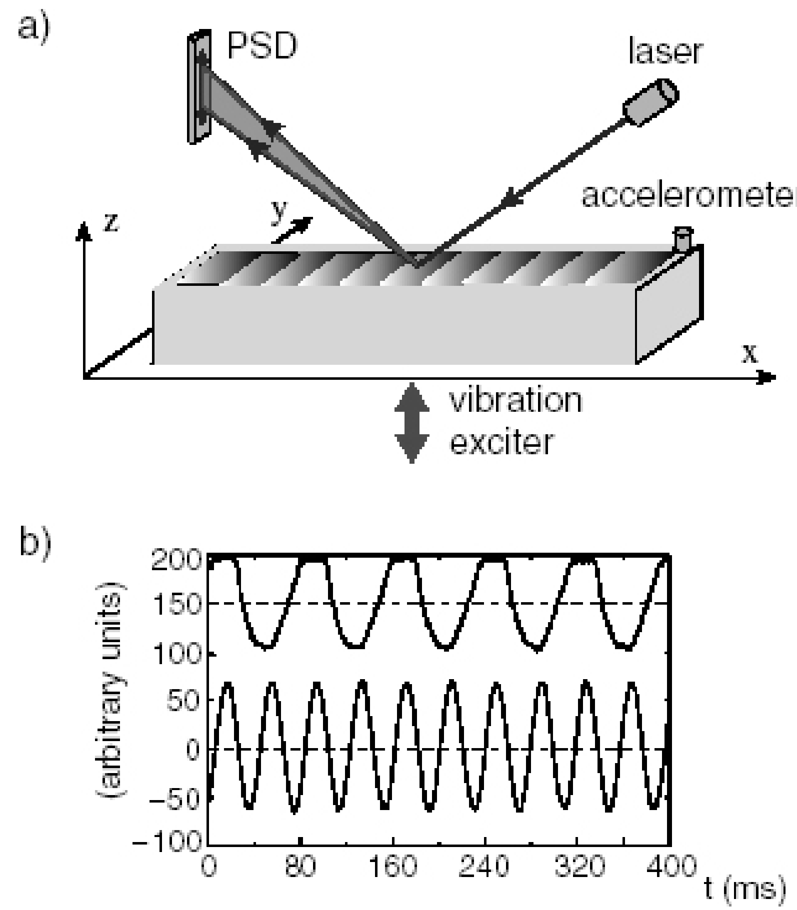


Figure 1. a) Experimental setup; b) typical signal recordings for the PSD (upper trace) and the accelerometer (lower trace), showing the parametric resonance of the surface oscillations.

Douady, S., (1990), "Experimental study of the Faraday instability," J. Fluid Mech., 221, 383-409.

Residori et al. 2007), "Two-mode competition in Faraday instability," Europhysics Letters, 77 44003 (5pp).

Defining Transition

- Photos or movies: Left clear wave pattern, right unclear

Quantitative definition of transition from undisturbed to Faraday waves is difficult

- Sloshing, or meniscus waves, formed at the walls have no stability threshold so are always present. These waves have a frequency equal to the driving frequency