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Local Smoothers for CDFEM with Sub-Element Discontinuities

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Thanks to: David Noble (SNL) and Geoffrey Dillon (TTU)

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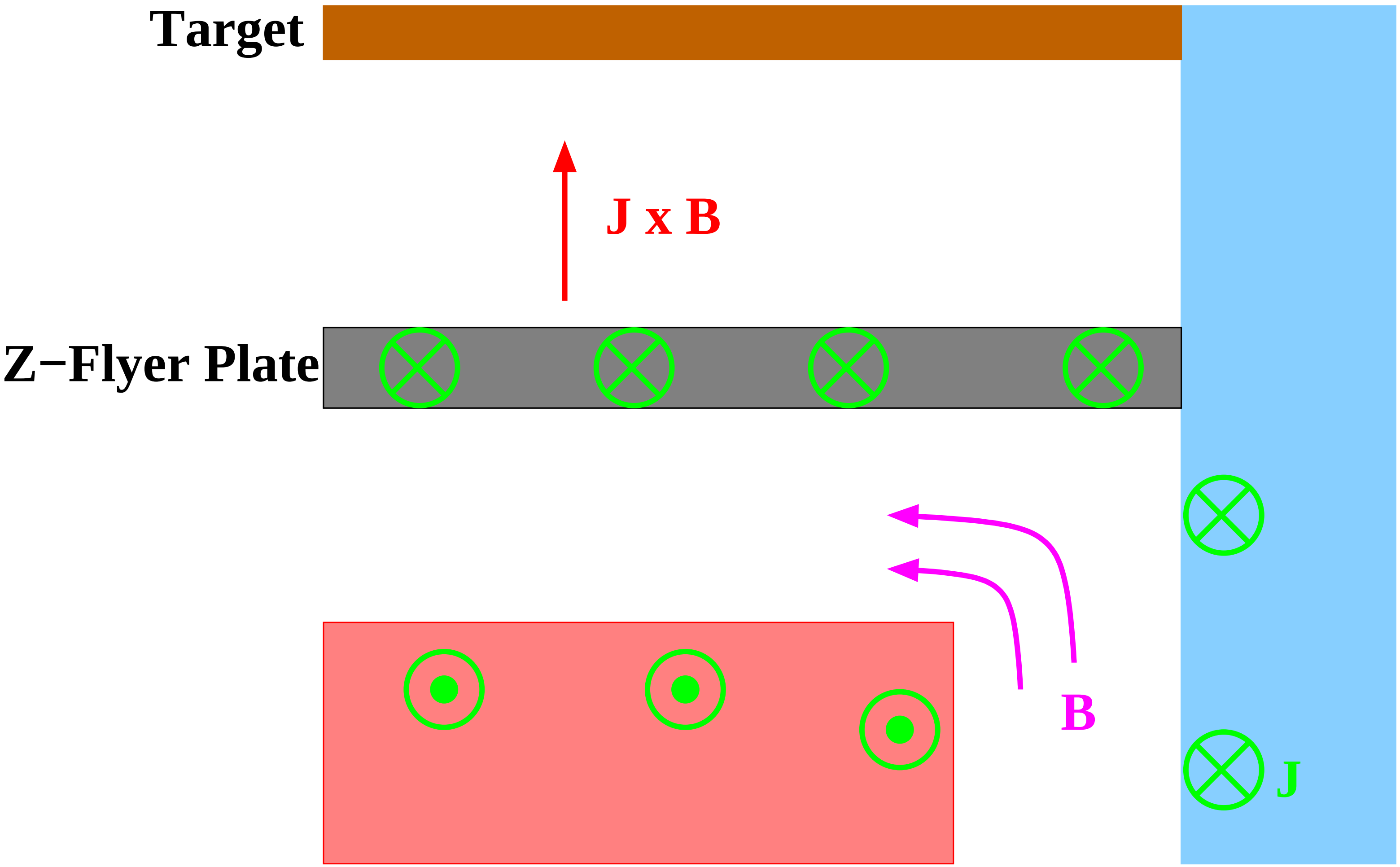
Outline

- Introduction
- What is CDFEM?
- Solvers for CDFEM
- Results
- Final Thoughts

Sample Application



How Can We Handle This?

- Important physics happens at the interface!
  - Current concentrates on interfaces.
  - Conditions of contact may be critical.
- Treatment Options
  - Lagrangian Mesh: Not for large-scale deformation.
  - Eulerian Mixture Models: Mixed bag.
  - XFEM/CDFEM: Interface tracking + local “mesh” refinement.
- For this talk we consider the heat advection-diffusion

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi - \nabla \cdot \alpha \nabla \phi = f$$

Outline

- Introduction
- What is CDFEM?
  - Basic Theory
  - Sliver Cuts & Contact Resistance
- Solvers for CDFEM
- Results
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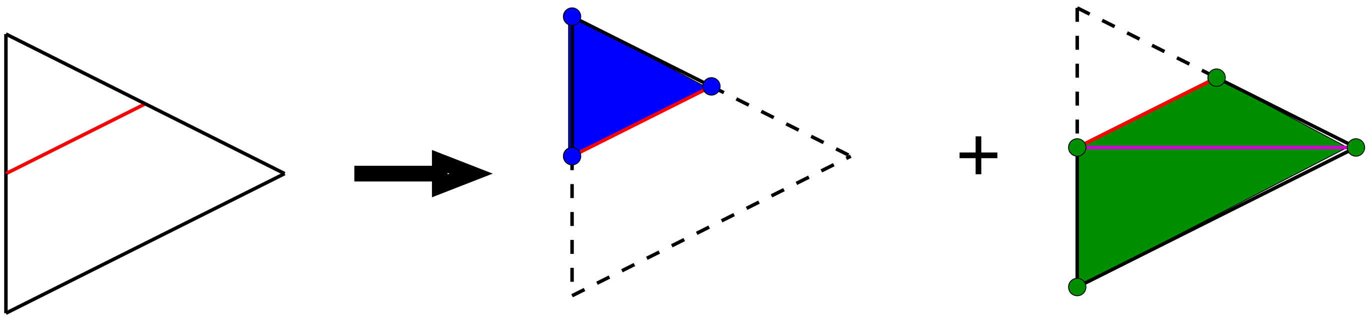
CDFEM 101

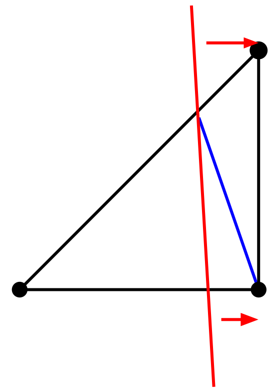
- Conformal Decomposition Finite Element Method (CDFEM) [1]
- Adds intra-element discontinuities
  - Weak (e.g. bonded materials)
  - Strong (e.g. imperfect contact)
- Tracks interface location via a level set.
- Adds local mesh adaptivity only around interfaces.



[1] *Noble, Newren and Lechman, 2009.*

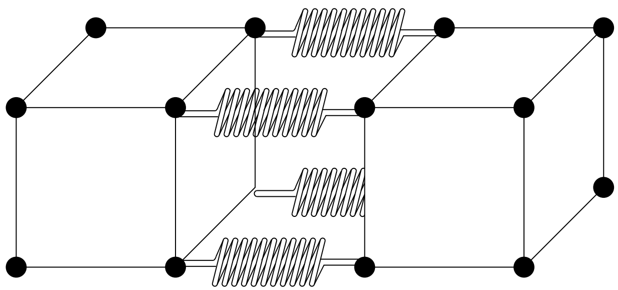
Slivers & Contact

Sliver cut:



- If it is really small, it is now the solver's problem.

Contact resistance:



- We add a 1D line elements across the cut:

$$\beta \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \phi_L \\ \phi_R \end{bmatrix} = \begin{bmatrix} \Delta/h \\ -\Delta/h \end{bmatrix}$$

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- Introduction
- What is CDFEM?
- Solvers for CDFEM
  - Standard SA Approaches
  - CD2VA Method
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Smoothed Aggregation AMG

- Want to use multigrid-preconditioned CG/GMRES.
- How to handle ill-conditioning?
  - Aggregation threshold (aka strength of connection):

$$|a_{ij}| < \epsilon \sqrt{|a_{ii}a_{jj}|}$$

  - Distance Laplacian — Do thresholding on a matrix where:

$$d_{ij} = \begin{cases} -(x_i - x_j)^{-2}, & i \neq j, a_{ij} \neq 0 \\ -\sum_{k \neq i} d_{ik}, & i = j \end{cases}$$

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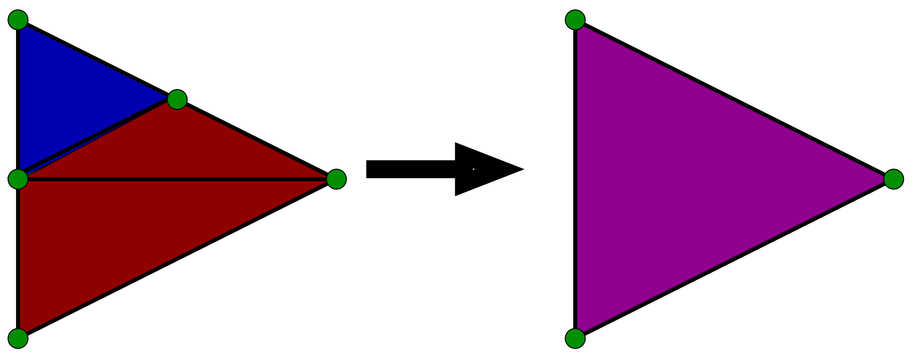
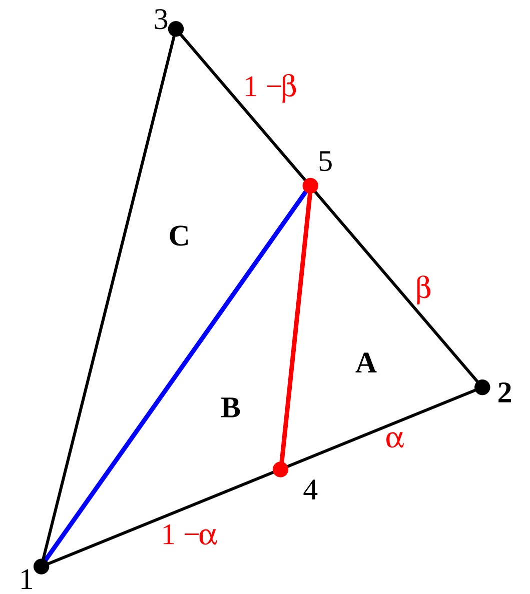
Alternatives

- A geometry-aware semi-coarsening might do better.
- Idea: Background FEM mesh is well conditioned. . . just remove the CD part of CDFEM.
- Process: Use a CDFEM-to-VA (volume-averaged) prolongator.



- Standard SA from VA level down.

CD2VA Prolongator



- We can write a CDFEM-to-VA prolongator, P, as follows:

$$
\begin{array}{c} \\ CD_1 \\ CD_2 \\ CD_3 \\ CD_4 \\ CD_5 \end{array}
\begin{array}{c} \begin{array}{ccc} VA_1 & VA_2 & VA_3 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-\alpha & \alpha & 0 \\ 0 & \beta & 1-\beta \end{pmatrix} \end{array}
$$

- Idea: Transfer unknowns using weights derived from the basis functions.

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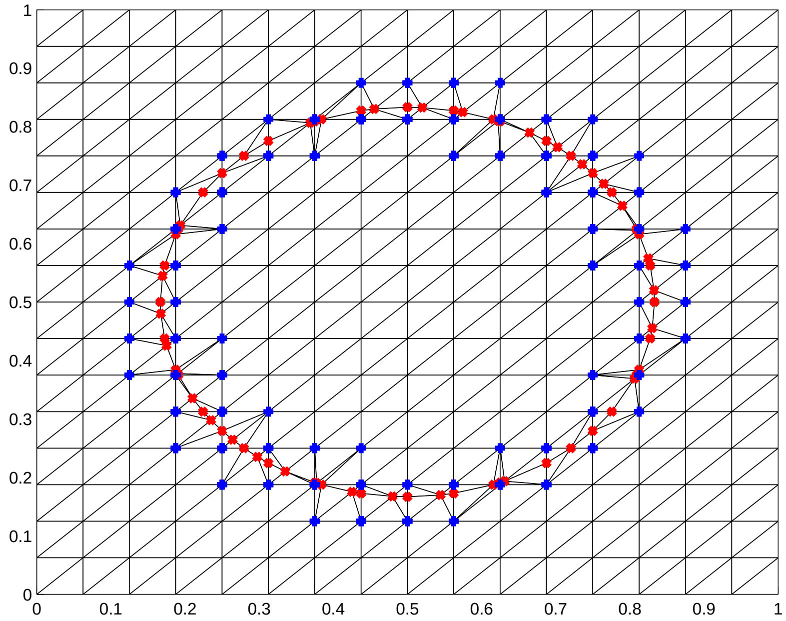
Local Smoothing

- Smoothing rule: If my unknown would touch a cut on the CDFEM mesh, smooth me.

CDFEM Unknowns: 363
Local Unknowns:74
Eff. Op. Complexity: 1.20



- Conclusion: Cost is reasonable if the smoothing is sufficient.

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  - CDFEM w/o contact resistance
  - CDFEM w/ contact resistance
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Test Problems (1)

- Basic Problem

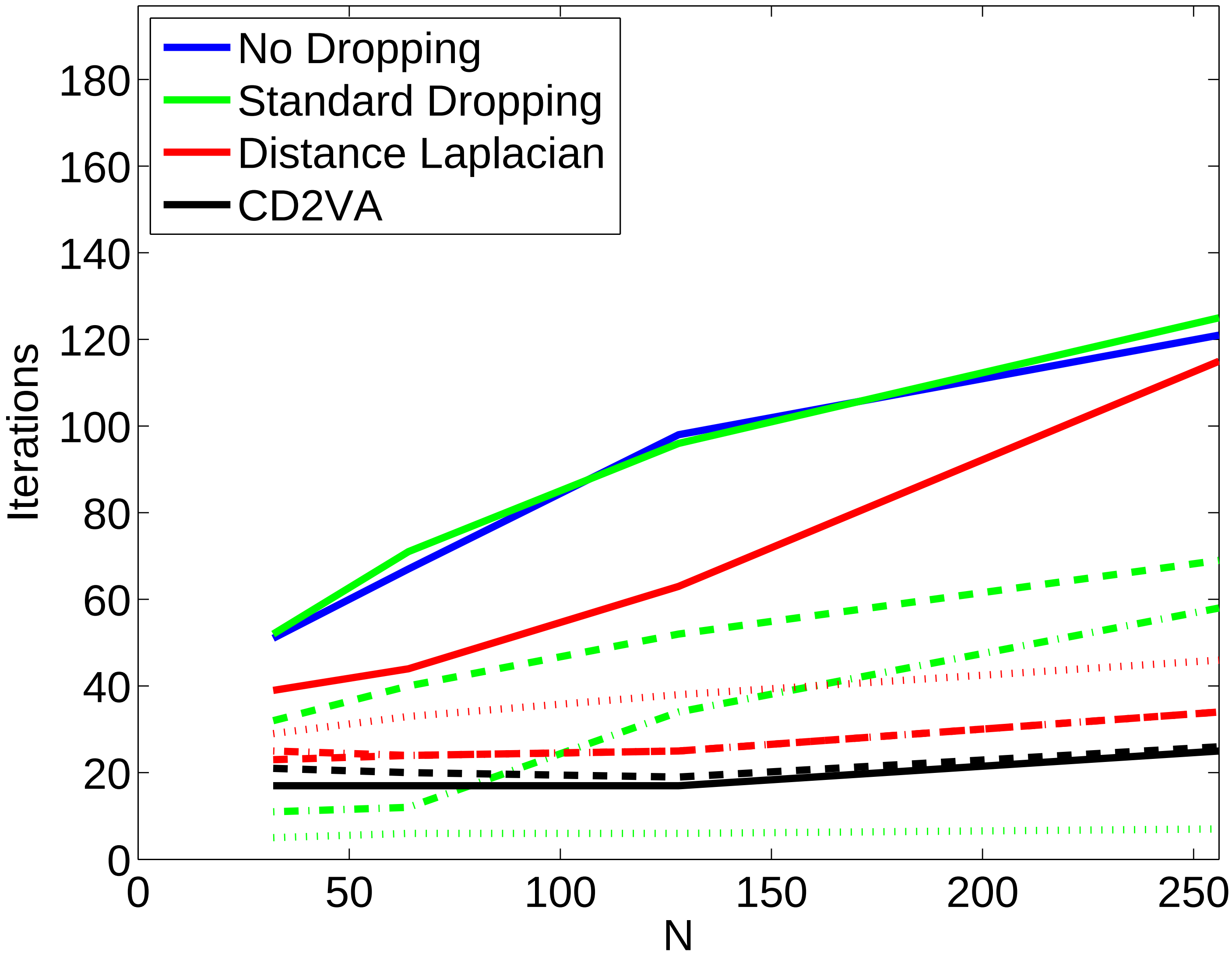
$$\phi(x,t) = \begin{cases} \kappa \sin\left(c_x(x - x_0)\right)\exp(-t/c_t), & x \le x_0 \\ \sin\left(c_x(x - x_0)\right)\exp(-t/c_t) + \Delta, & x > x_0 \end{cases}$$

- Problem 1: $\Delta = 1$ (fixed jump; $Pe = 1.0$)
- Problem 2: $\Delta = \Delta(\beta)$ (contact resistance; $Pe = 1.0$). Contact resistance ($1/\kappa$) varies from $1e - 6$ to $1e6$.

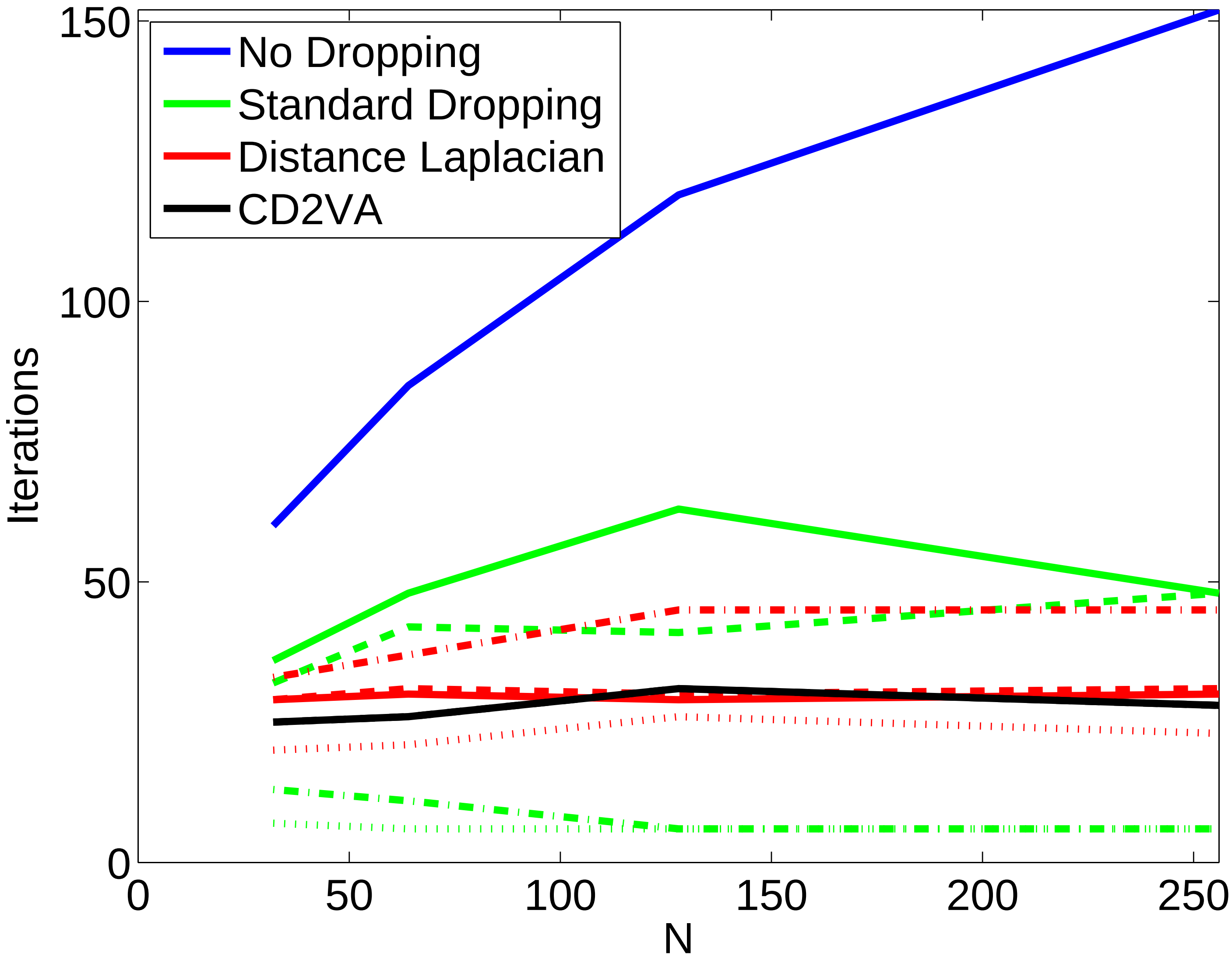
Test Problems (2)

- Preconditioning Methods
  - SA w/ no thresholding.
  - SA w/ standard thresholding, $\epsilon = 1e-4$, $1e-3$, $1e-2$, $1e-1$.
  - SA w/ Distance Laplacian, $\epsilon = 1e-4$, $1e-3$, $1e-2$, $1e-1$.
  - CD2VA w/ global/local smoothing on VA level (SA below).
- Solver Parameters
  - One pre/post sweep of SGS; LU on coarse level.
  - Preconditioned full GMRES, $1e-10$ tolerance.

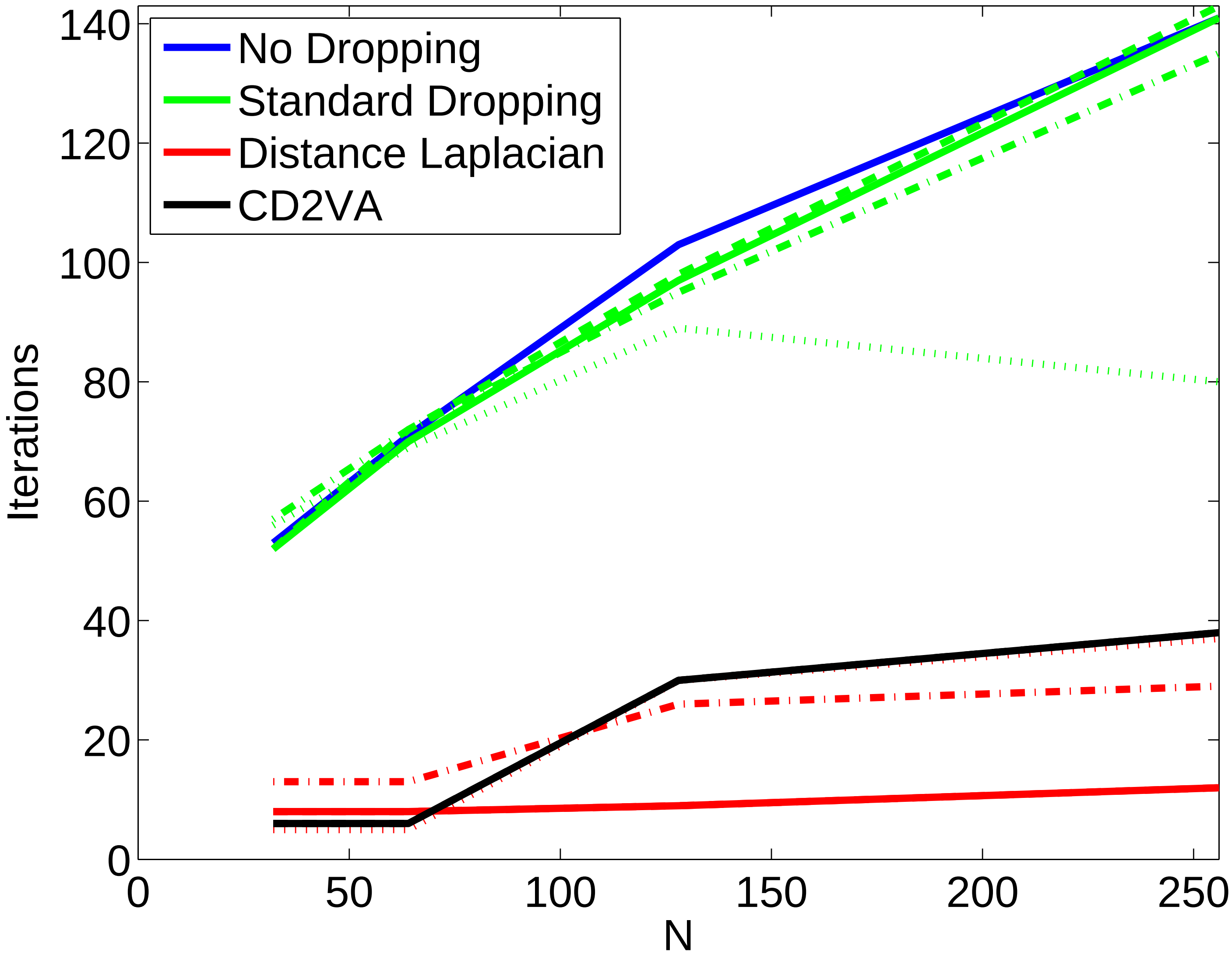
Problem 1: $\Delta = 1$ ($1e - 3$ cut)



Problem 2: Good Contact



Problem 2: Poor Contact



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- XFEM with Algebraic Constraints
- Multigrid for XFEM-AC
- Results
- Final Thoughts

Final Thoughts

- Conclusions
  - Thresholding works great. . . if you get the right threshold.
  - CD2VA local smoother approach is pretty robust.
  - Contact resistance doesn't seem to cause too big a problem for CD2VA.
- Future work
  - Implementation in production fluids code.
  - Navier-Stokes w/ contact resistance.
  - Cohesive elements for solid mechanics (maybe).

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