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Toward estimating the extinction frequency in turbulent non-premixed flames with a simple stochastic model

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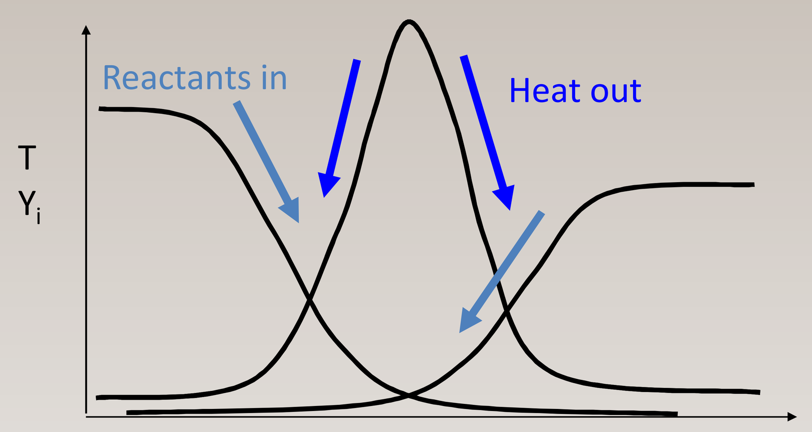
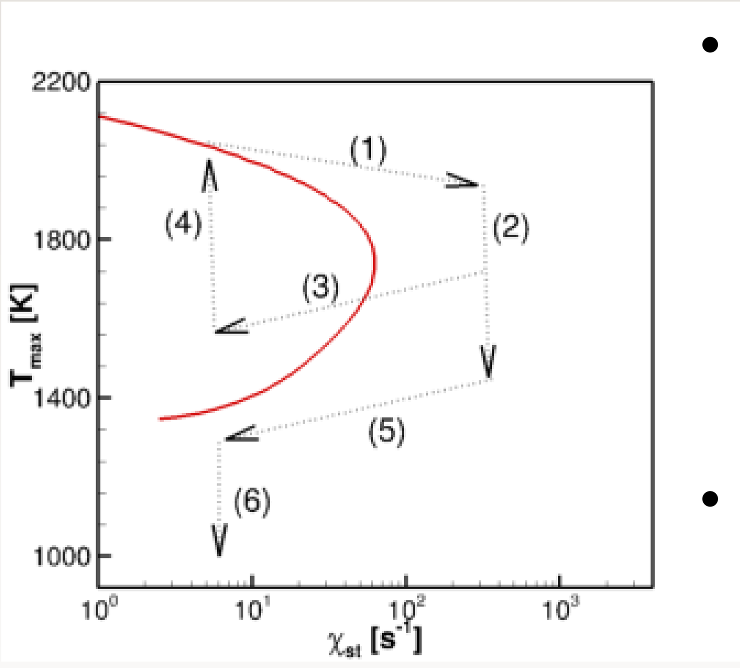
Summary

- Unsteady extinction characterized by extinction impulse: time integrated dissipation rate exceeding steady extinction value.
- Critical dissipation impulse related to S-curve characteristics.
- Frequency of a given extinction impulse magnitude has been estimated using a simple Ornstein-Uhlenbeck stochastic process model. Results suggest power law scalings in impulse magnitude and Prob($\chi=\chi_q$) when results are appropriately normalized.

Unsteady Extinction by Dissipation Impulse

- Extinction occurs if chemistry is slower than mixing, if heat losses exceed heat release.
- **Critical dissipation rate** above which *steady* solutions are not possible: χ_q
- Turning point of S-curve at χ_q.





- In unsteady turbulent flows, dissipation rates can exceed χ_q briefly without extinguishing flame – depends on whether flame state is above or below unstable middle branch.
- Path 1,2,3,4 versus 1,2,5,6.

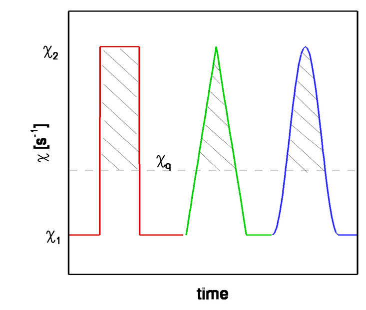
- **Unsteady extinction criterion:**
- Motivated by trajectories in T-χ phase space (S-curve).
- Consider time when $\chi>\chi_q$
- Set reaction rate at χ_q (max) rate.

$$\frac{dT}{dt} = \frac{\chi}{2}\frac{d^2T}{dZ^2} - \left(\sum_i \frac{\omega_i h_i}{\rho c_p}\right)_{max}$$
$$\approx \frac{(\chi-\chi_q)}{2}\frac{d^2T}{dZ^2}$$
$$\approx \frac{(\chi_q-\chi)(T-T_\infty)}{Z_{st}(1-Z_{st})\,\varepsilon}$$

- Estimate heat loss from **dissipation impulse** to get temperature decrement:

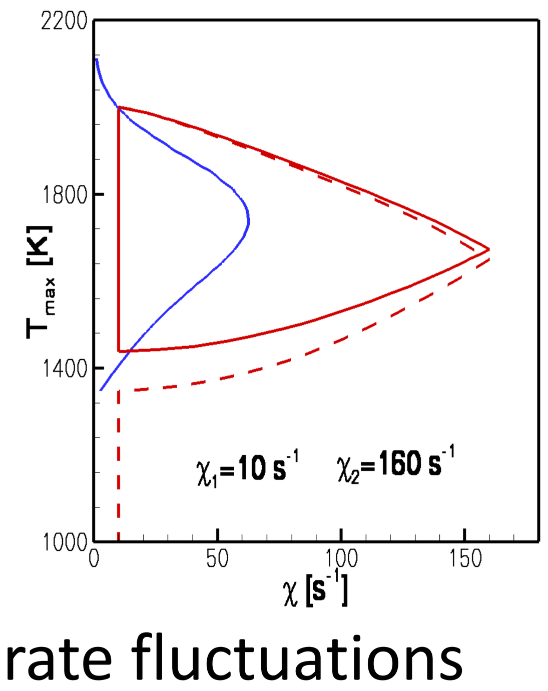
$$\frac{T_2-T_\infty}{T_1-T_\infty} = \exp(-\Xi)$$

with $$\Xi \approx \frac{A\int_{\chi>\chi_q}(\chi-\chi_q)\,dt}{Z_{st}(1-Z_{st})\,\varepsilon}$$



- Critical value corresponds to T_2 on middle branch.

$$\Xi_q = \ln\left(\frac{T_1-T_\infty}{T_m-T_\infty}\right)$$



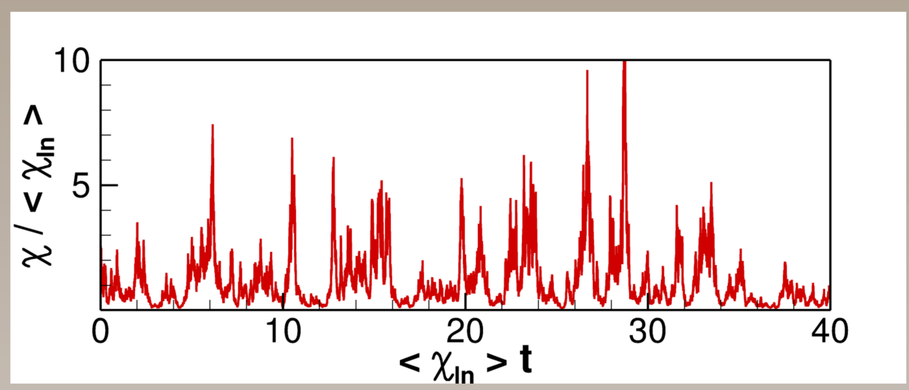
- Works well for large dissipation rate fluctuations characteristic of high Reynolds number turbulence.

Simple Stochastic Model for the Dissipation Rate

Ornstein-Uhlenbeck process can be used to simulate lognormal dissipation rate fluctuations.

$$d\ln\chi^* = -\left(\ln\chi^* + \frac{\sigma^2}{2}\right)dt^* + \sqrt{2}\sigma\, dW$$

$$\chi^* = \chi/\langle\chi\rangle, \quad t^* = t/\tau_\chi$$

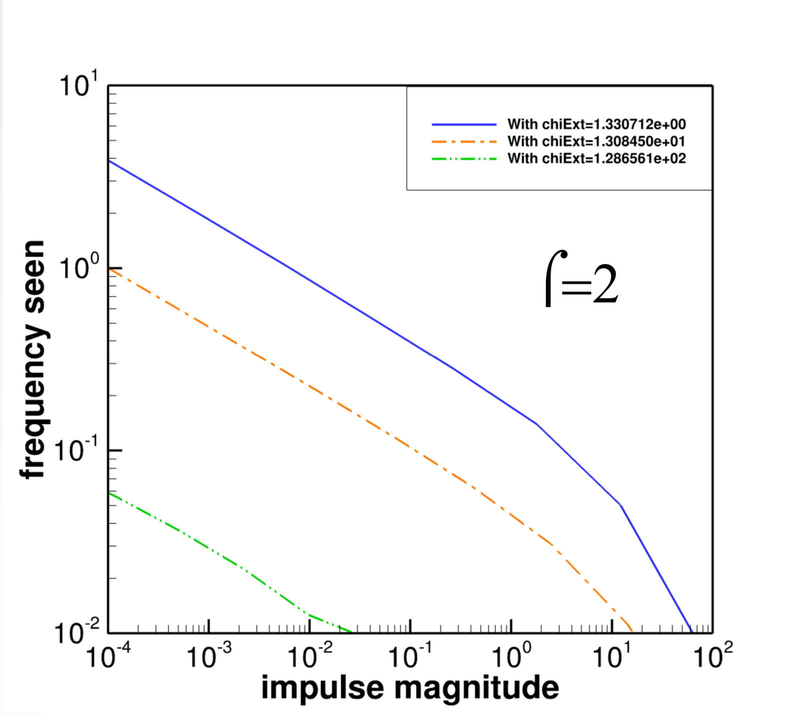


- Normalized dissipation impulse: $\Xi^* = \int_{\chi>\chi_q} (\chi^* - \chi_q^*)\,dt^* = \left[\frac{2Z_{st}(1-Z_{st})}{A\langle\chi\rangle\tau_\chi}\right]\Xi$

Statistics of the Dissipation Impulse

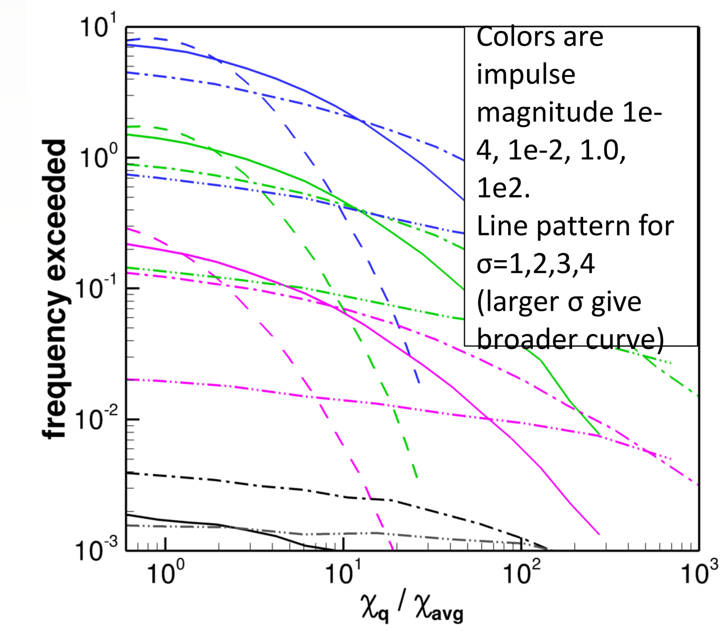
- Based on assumption of OU evolution, compute statistics for various $\sigma_{\ln\chi}$, $\chi_q/\langle\chi\rangle$, $\in^*$
  - Select values for $\in^*$ based on S-curve (10^{-2} to 1 are reasonable values).
  - Integrate dissipation impulse: $\chi-\chi_q$ when $\chi>\chi_q$.
  - Determine statistics as function of $\sigma_{\ln\chi}$, $\chi_q/\langle\chi\rangle$.

- Reporting **cumulative frequency** $\in^*$ **exceeds given value** in t^* units as function of $\chi_q/\langle\chi\rangle$.

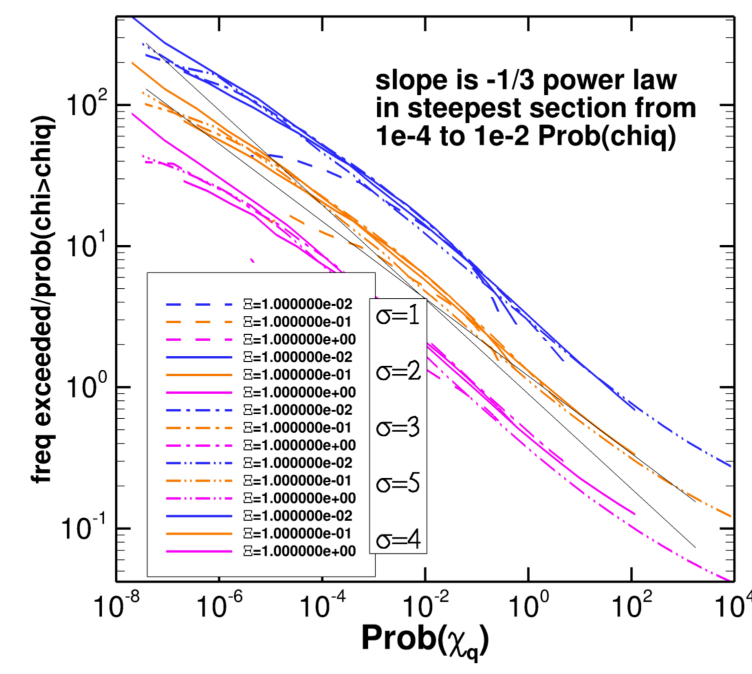


- Frequency $\in^*$ exceeds given value decreases with increasing magnitude (larger gap in S-curve temperatures).
  - Power law scaling for $\in^*<1$.

- As $\chi_q/\langle\chi\rangle$ increases (harder to extinguish) the frequency exceeded decreases.
- Reduction in frequency strongest for smaller σ (when large dissipation rates less frequent).



- Normalize rate by Prob($\chi>\chi_q$) (cumulative distribution) as measure of time and magnitude of $\chi-\chi_q$ when $\chi>\chi_q$.
- Normalized frequency then decreases approximately as power law in Prob($\chi=\chi_q$).



Reference for dissipation impulse:

Hewson, *Combust. Flame,* 160: 887-897, 2013.