Probabilistic Methods for Power Grid Networks

Cosmin Safta, Habib N. Najm, Ali Pinar, Richard Chen, and Jean-Paul Watson
Sandia National Laboratories

Objective

Design improved algorithms for handling uncertainties in forecast demand for stochastic unit commitment models

- Adopt advanced modeling and sampling techniques from the uncertainty quantification (UQ) community, and leverage them to impact power systems operations problems such as stochastic unit commitment (UC) and economic dispatch (ED)
- Most studies analyzing uncertain power system operations problems generate forecast scenarios by drawing random samples from a stochastic process using standard Monte Carlo (MC) techniques.
- We consider an alternative approach based on using Polynomial Chaos expansions, built using sparse quadrature methods, valid over the range of the forecast uncertainty

Stochastic Commitment and Dispatch

- Stochastic UC includes start-up, $\boldsymbol{c^u}$, and shutdown, $\boldsymbol{c^d}$, costs, and includes the expected generation cost $\overline{Q}(x)$ for a set $\boldsymbol{x}$ of UC decisions

$$\min_{\boldsymbol{x}} \quad c^u(\boldsymbol{x}) + c^d(\boldsymbol{x}) + \overline{Q}(\boldsymbol{x}) \qquad \text{s.t.} \quad \boldsymbol{x} \in \mathcal{X}, \quad \boldsymbol{x} \in \{0,1\}^{|G|\times|T|}$$

- The uncertain loads $\boldsymbol{D}$ are treated as Random Variables (RVs), expressed in terms of a set of standard RVs $\boldsymbol{\xi} = \{\xi_1, \xi_2, \ldots, \xi_{|T|}\}$. Hence the expected cost is defined as

$$\overline{Q}(\boldsymbol{x}) = \mathrm{E}_{\boldsymbol{\xi}} Q(\boldsymbol{x}, \boldsymbol{\xi}(\omega))$$

- The economic dispatch (ED) problem under fixed unit commitment $\boldsymbol{x}$ is given by:
- Decision variables $\boldsymbol{p}$ and $\boldsymbol{q}$ correspond to generator output levels and load shedding.
- The generation cost $\boldsymbol{c^P}$ is typically a quadratic function, while the loss of load penalty $\boldsymbol{M}$ is tipically a large value.
- ***RU*** and ***RD*** are ramp up/down constraints

$$Q(\boldsymbol{x}, \boldsymbol{\xi}(\omega)) = \min_{\boldsymbol{p},\boldsymbol{q}} \sum_{t\in T}\sum_{g\in G} c_g^P(p_g^t) + \sum_{t\in T} M q^t$$

$$\begin{aligned} \text{s.t.} \quad \sum_{g\in G} p_g^t - q^t &= D^t(\xi_t(\omega)), \quad \forall t \in T \\ \underline{P}_g x_g^t &\le p_g^t \le \overline{P}_g x_g^t, \quad \forall g \in G, t \in T \\ p_g^t - p_g^{t-1} &\le RU(x_g^{t-1}, x_g^t), \\ p_g^{t-1} - p_g^t &\le RD(x_g^{t-1}, x_g^t). \end{aligned}$$

Estimation of expected cost by Monte Carlo sampling requires a large number of solutions for the ED problem to obtain converged estimates for the expected generation cost

Representation of Uncertainty Using Polynomial Chaos

- We employ a truncated Legendre-Uniform PC expansion to represent the dependenc of the cost on the uncertain demand, ***ξ=ξ(D)***

$$Q_{\mathrm{PC}}(x, \boldsymbol{\xi}) = \sum_{k=0}^{P} c_k(x)\Psi_k(\boldsymbol{\xi})$$

- The PC coefficients ck are computed by Galerkin projection, thus the L2 error is controlled by using an appropriate number of terms

$$c_k(x) = \frac{\langle Q\Psi_k\rangle}{\langle \Psi_k^2\rangle} = \frac{1}{\langle \Psi_k^2\rangle}\int_{[-1,1]^n} Q(x,\boldsymbol{\xi})\Psi_k(\boldsymbol{\xi})p(\boldsymbol{\xi})d\boldsymbol{\xi}$$

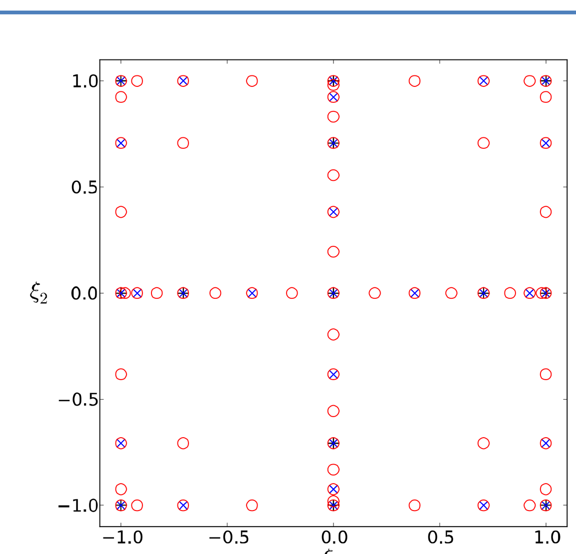
- Evaluation of expectation and/or higher order moments incurs no additional cost

$$\overline{Q}(x) = \mathrm{E}_{\boldsymbol{\xi}}[Q(x,\boldsymbol{\xi})] = \langle Q(x,\boldsymbol{\xi})\rangle = c_0$$

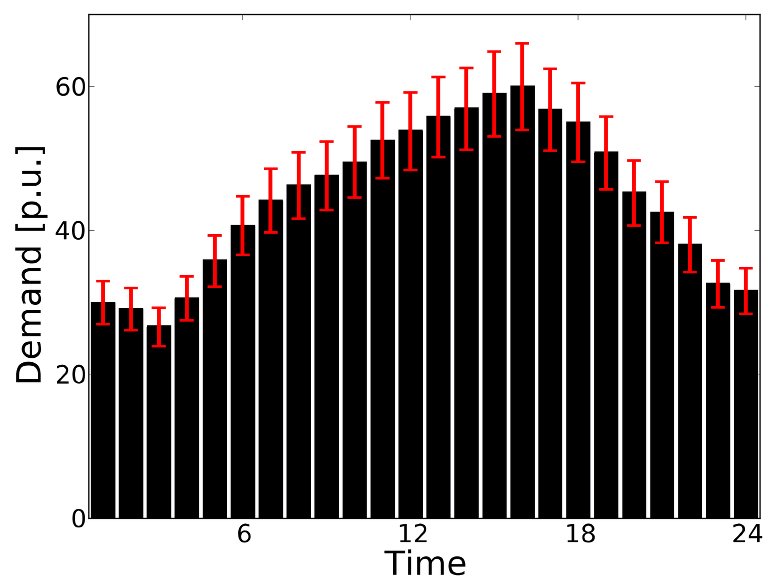
- The PC representation can be used as a surrogate for the ED problem if the uncertainty model for the demand changes

$$\xi_t = \frac{2D^t - (D^t_{\max} + D^t_{\min})}{D^t_{\max} - D^t_{\min}}, \quad \forall t \in T$$

- The projection integrals for the evaluation of the PC coefficients are evaluated via sparse quadrature methods.
- For low-order PC representations, e.g. 2nd order polynomial, this approach works well for moderate to high-dimensional cases, in particular when using nested quadrature rules
- The sparse quadrature methodology requires smooth integrands

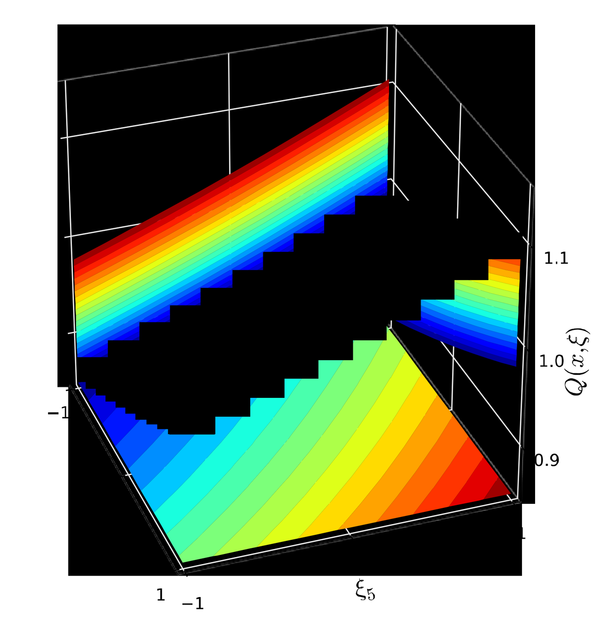


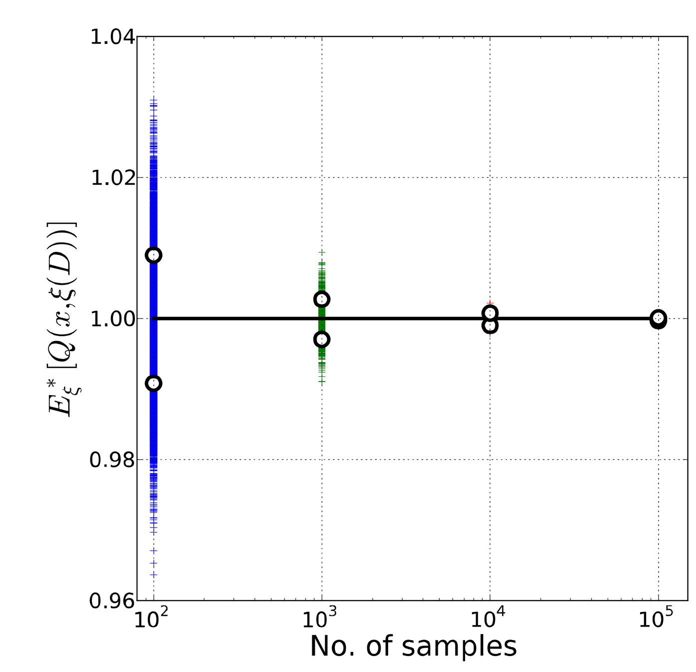
Numerical Results

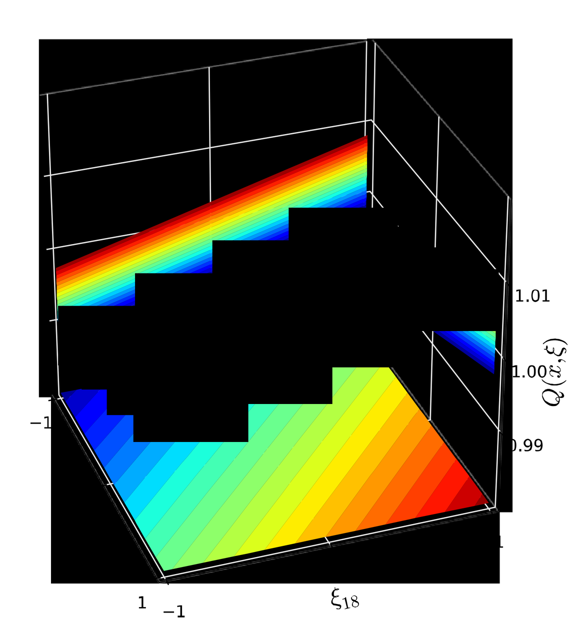


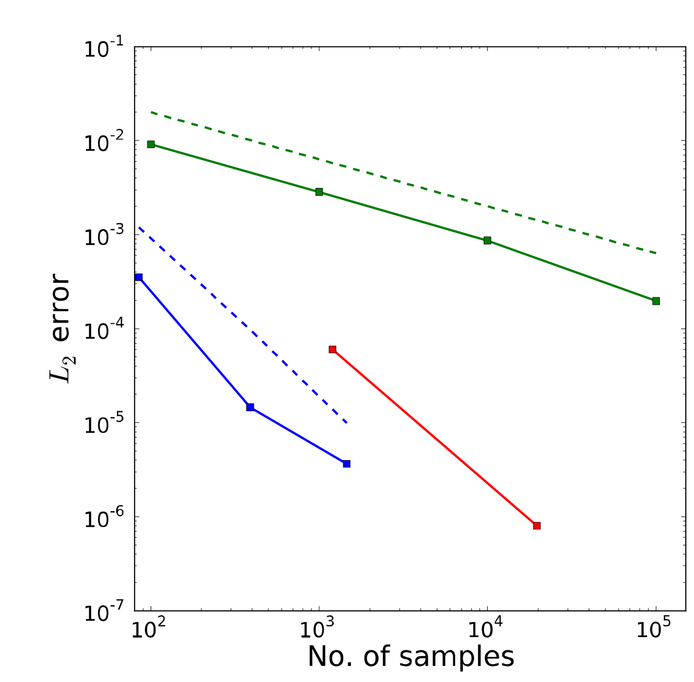
- Two cases: a 9-bus, and 118-bus test system.
- Number of time periods ***|T|*** varied from 6 to 24.
- Figure to the left shows typical load series for the IEEE 118 bus test system over a 24 h period.
- The red bars depict a **20%** uncertainty range around the nominal values.
- The table shows relative $\boldsymbol{L_2}$ errors at training points for PC representations of degree up to 4, using sparse quadrature methods based on nested rules with 2 to 5 levels.
- Additional test indicate the PC representation errors are very small across the entire range of input paremeters

Order	Sparse Quadrature			
	L2, 85p	L3, 389p	L4, 1457p	L5, 4865p
1	1.62e-05	2.90e-05	2.15e-05	2.18e-05
2	-	7.48e-07	2.17e-07	7.83e-08
3	-	-	1.92e-07	5.36e-08
4	-	-	-	2.10e-08









- Dependence of cost on the load in specific time period for the 9-bus (top frame) and 118-bus example; Second order PC representations are sufficient to capture the model behavior

Comparison of convergence rates for the Monte Carlo sampling and PC expansion approach. The PC representation is several orders of magnitude cheaper.

Conclusions

We present an approach to reduce the computational cost associated with stochastic unit commitment and economic dispatch, by reducing the number of required forecast samples.

- The approach is based on Polynomial Chaos representation for the dependency of cost on the uncertain demands; the accuracy of the representation is controlled in L2 sense
- Results for 9-bus and 118-bus power grid models show that PC approach typically requires one to two order of magnitude less samples compared to Monte Carlo sampling for given requisite accuracy of the expected cost.

Acknowledgements

This work was funded by the Laboratory Directed Research & Development (LDRD) program at Sandia National Laboratories