Greenland Ice-sheet Initialization: Optimal Control and Bayesian Calibration Approaches

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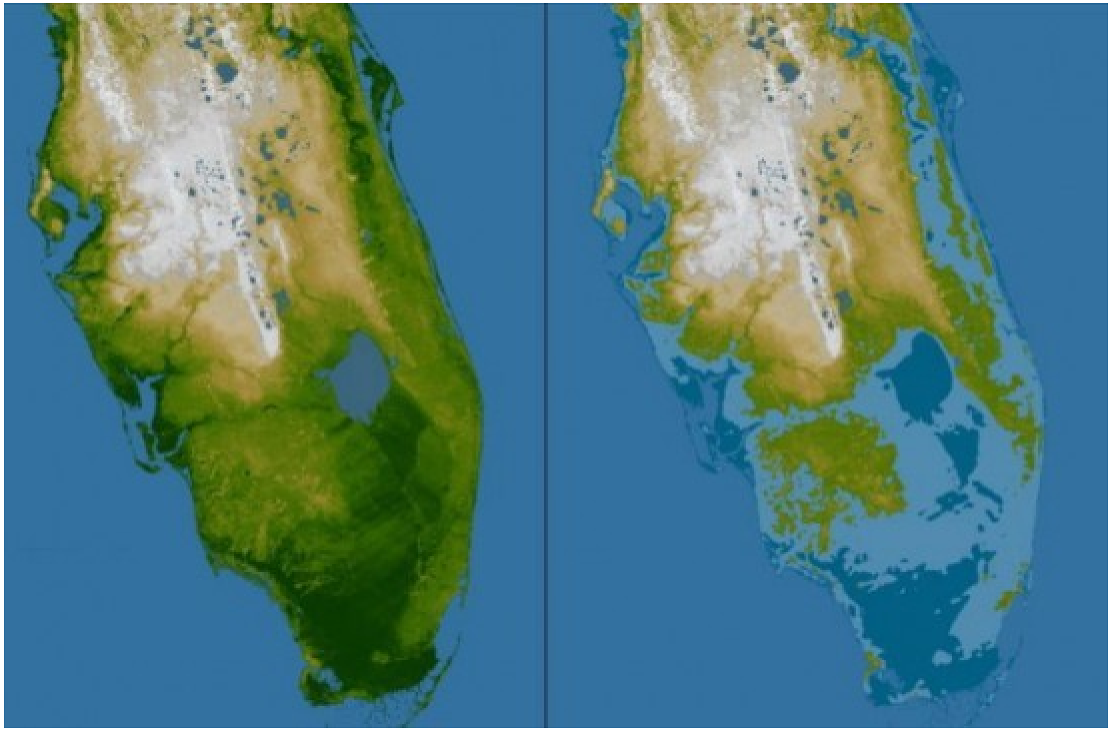
Motivations

- Glaciers and ice sheets influence the global climate, and vice-versa
- Melting of land ice determines the **sea level rise**
  melting of the Greenland ice sheet: 7 m
  melting of the Antarctic ice sheet: 61 m



South Florida projection for a sea levels rise
of 5m (dark blue) and 10m (light blue)

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- Glaciers and ice sheets influence the global climate, and vice-versa
- Melting of land ice determines the **sea level rise**
  melting of the Greenland ice sheet: 7 m
  melting of the Antarctic ice sheet: 61 m
- The Fourth Report of the Intergovernmental Panel on Climate Change (**IPCC 2007**) declared that the current models and programs for ice sheets did not provide credible predictions

Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} \rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) - \nabla\cdot\sigma = \rho\mathbf{g} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

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with:

$$\sigma = 2\mu - \Phi I, \qquad \mathbf{D}_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Non linear viscosity:

$$\mu = \frac{1}{2}\alpha(T)\,|\mathbf{D}(\mathbf{u})|^{(p-2)}, \quad p \in (1,2] \quad (\text{tipically } p \simeq \frac{4}{3})$$

Viscosity is singular when ice is not deforming

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Ice Sheet Modeling

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$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

- **Model for the evolution of the boundaries**
(thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot \int_z \mathbf{u} \, dz$$

- **Temperature equation**

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) - \rho c \mathbf{u} \cdot \nabla T + 2 \dot{\varepsilon} \sigma$$

- **Coupling with other climate components (e.g. ocean, atmosphere)**

Stokes Approximations

"Reference" model: **STOKES**[1]

$O(\delta^2)$ ***FO,*** Blatter-Pattyn first order model[2] (3D PDE, in horizontal velocities)

$O(\delta)$ Zeroth order, depth integrated models:
SIA, Shallow Ice Approximation (slow sliding regimes) ,
SSA Shallow Shelf Approximation (2D PDE) (fast sliding regimes)

$\simeq O(\delta^2)$ Higher order, depth integrated (2D) models: ***L1L2***[3], (L1L1)...

[1] *Gagliardini and Zwinger, 2008. The Cryosphere.*

[2] *Dukowicz, Price and Lipscomb, 2010. J. Glaciol.*

[3] *Schoof and Hindmarsh, 2010. Q. J. Mech. Appl. Math.*

First order equation.

FO is a nonlinear system of elliptic equations in the horizontal velocities:

$$\begin{cases} -\nabla \cdot (2\mu\, \dot{\boldsymbol{\varepsilon}}_1) = -\rho g \dfrac{\partial s}{\partial x} \\[2ex] -\nabla \cdot (2\mu\, \dot{\boldsymbol{\varepsilon}}_2) = -\rho g \dfrac{\partial s}{\partial y}, \end{cases} \qquad \begin{aligned} \mu &= \frac{1}{2} A^{-\frac{1}{n}} \dot{\varepsilon}_e^{\left(\frac{1}{n}-1\right)} \\ \dot{\varepsilon}_e &= \sqrt{\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{xx}\dot{\varepsilon}_{yy} + \dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{xz}^2 + \dot{\varepsilon}_{yz}^2} \end{aligned}$$

where s is the ice surface and,

$$\dot{\varepsilon}_{i,j} = \frac{1}{2}\left(\partial_j u_i + \partial_i u_j\right), \quad i,j \in \{x,y,z\}, \;\; \dot{\boldsymbol{\varepsilon}}_1 = \begin{bmatrix} 2\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{xz} \end{bmatrix}, \; \dot{\boldsymbol{\varepsilon}}_2 = \begin{bmatrix} \dot{\varepsilon}_{yx} \\ \dot{\varepsilon}_{xx} + 2\dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{yz} \end{bmatrix}$$

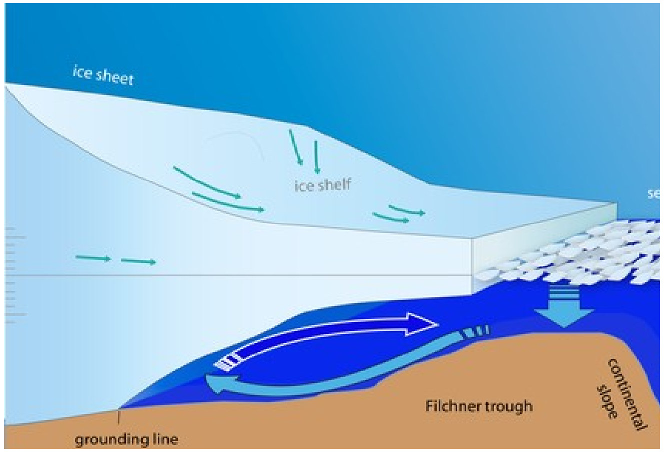
Remark The nonlinear viscosity μ is singular when $\dot{\varepsilon}_e = 0$, however, $\mu\, \dot{\boldsymbol{\varepsilon}}_1$ is not singular and the PDE is well defined.

Viscosity regularization: $\qquad \dot{\varepsilon}_e^{-\left(1-\frac{1}{n}\right)} \approx \left(\sqrt{\dot{\varepsilon}_e^2 + \delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$

(Numerical) Modeling Issues

- Computationally challenging, due to complexity of models, of geometries and large domains

  - design of linear/nonlinear solvers, preconditioners, etc.
  - mesh adaptivity especially close to the grounding line.



- Boundary conditions / coupling (e.g. with ocean)

  - Floating/calving
  - Basal friction at the bedrock,
  - Subglacial hydrology,
  - Heat exchange / phase change.

- Initialization / parameter estimation.

- Uncertainty quantification.

Implementation Overview (Felix)

- Felix (Finite Element Land Ice eXperiments) is a C/C++ finite element implementation of land ice models. It relies on Trilinos for data structure, for the solution of linear/nonlinear solvers and for adjoint/UQ capabilities.
- Models currently implemented are SIA, SSA, L1L2 and FO, which have been tested against Ismip-Hom experiments and CISM simulations.
- The nonlinear systems are solved using Newton method with exact Jacobian + continuation of regularization parameters to increase robustness.
- It is interfaced with the land ice modulus of MPAS (climate library, implements ocean and atmosphere models). Realistic simulation done for ice2sea projects.
- Even if adjoint and UQ capabilities are in early development, Felix can leverage on several trilinos packages which introduce great flexibility. Among these we have:
  - Dakota, MOOCHO (Optimization / UQ)
  - Sacado (Automatic Differentiation)

[1] *Software currently developed under the DOE project PISCEES*

[2] *www.trilinos.sandia.org (albany), www.lifev.org*

[3] *Perego, Gunzburger, Burkardt,* Journal of Glaciology, 2012

Inverse Problem

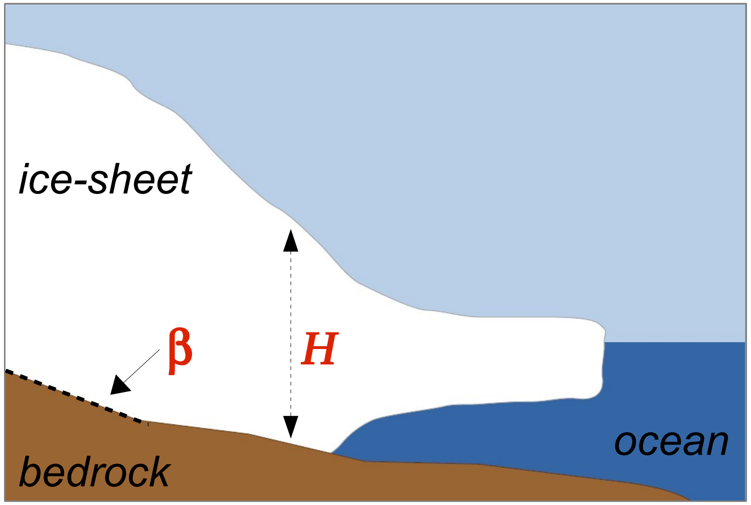
Estimation of ice-sheet initial state

(w/ G. Stadler, UT, and S. Price, LANL)

Problem: what is the initial thermo-mechanical state of the ice sheet?

Available data/measurements:

- *ice extension and surface topography*
- ***surface velocity***
- ***Surface Mass Balance*** *(SMB: accumulation/melt rate)*
- ***ice thickness H (very noisy)***



Fields to be estimated :

- ***ice thickness H***
- ***basal friction β***

Additional information:

- *ice fulfills* ***nonlinear Stokes equation***
- *ice is almost* ***at thermo-mechanical equilibrium***

Assumption (for now):

- *given* ***temperature field***

Inverse Problem

Estimation of ice-sheet initial state

G. Stadler (UT), M. P. and S. Price (LANL)

How to prescribe ice-sheet mechanical equilibrium:

divergence flux

$$\frac{\partial H}{\partial t} = -\mathrm{div}\,(\mathbf{U}H) + \tau_s, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz.$$

Surface Mass Balance

At equilibrium: $\mathrm{div}\,(\mathbf{U}H) = \tau_s$

Boundary condition at ice-bedrock interface:

$$(\sigma\mathbf{n} + \beta\mathbf{u})_{||} = \mathbf{0} \quad on \quad \Gamma_\beta$$

Bibliography*:

Arthern, Gudmundsson, J. Glaciology. 2010

Price, Payne, Howat and Smith, PNAS 2011

Morlighem Thesis 2011

Brinkerhoff, Meierbachtol, Johnson, Harper, Annals of Glaciology, 2011

Morlighem et al. A mass conservation approach for mapping glacier ice thickness, 2013

Pollard DeConto, TCD 2012

Petra, Zhu, Stadler, Hughes, Ghattas, J. Glaciology , 2012.

Inverse Problem

Estimation of ice-sheet initial state

G. Stadler (UT), M. P. and S. Price (LANL)

Problem: find initial conditions such that the ice is almost at thermo-mechanical equilibrium given the geometry and the SMB, and matches available observations.

Optimization Problem:
find β and H that minimizes the functional $\mathcal{J}$

$$\begin{aligned}
\mathcal{J}(\beta, H) &= \int_\Sigma \frac{1}{\sigma_u^2} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds && \text{surface velocity mismatch} \\
&+ \int_\Sigma \frac{1}{\sigma_\tau^2} |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds && \text{SMB mismatch} \\
&+ \int_\Sigma \frac{1}{\sigma_H^2} |H - H^{obs}|^2 \, ds && \text{thickness mismatch} \\
&+ \mathcal{R}(\beta, H) && \text{regularization terms.}
\end{aligned}$$

Common (surface velocity mismatch); *Novel* (surface velocity, SMB and thickness mismatch)

subject to ice sheet model equations (FO or Stokes)

$\boldsymbol{U}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

Inverse Problem
Estimation of ice-sheet initial state

- Settings of the preliminary experiments:

1) Constraint: FO model.

2) No coupling with temperature solver (temperature field is given).

3) Tikhonov regularization both for β and H. .

- Optimization:

Optimization Package Moocho (Trilinos).

Sequential Quadratic Programming using *LBFGS* for approximating the reduced Hessian.

The first derivatives of the constraint and the cost functional are provided by LifeV.

Inverse Problem

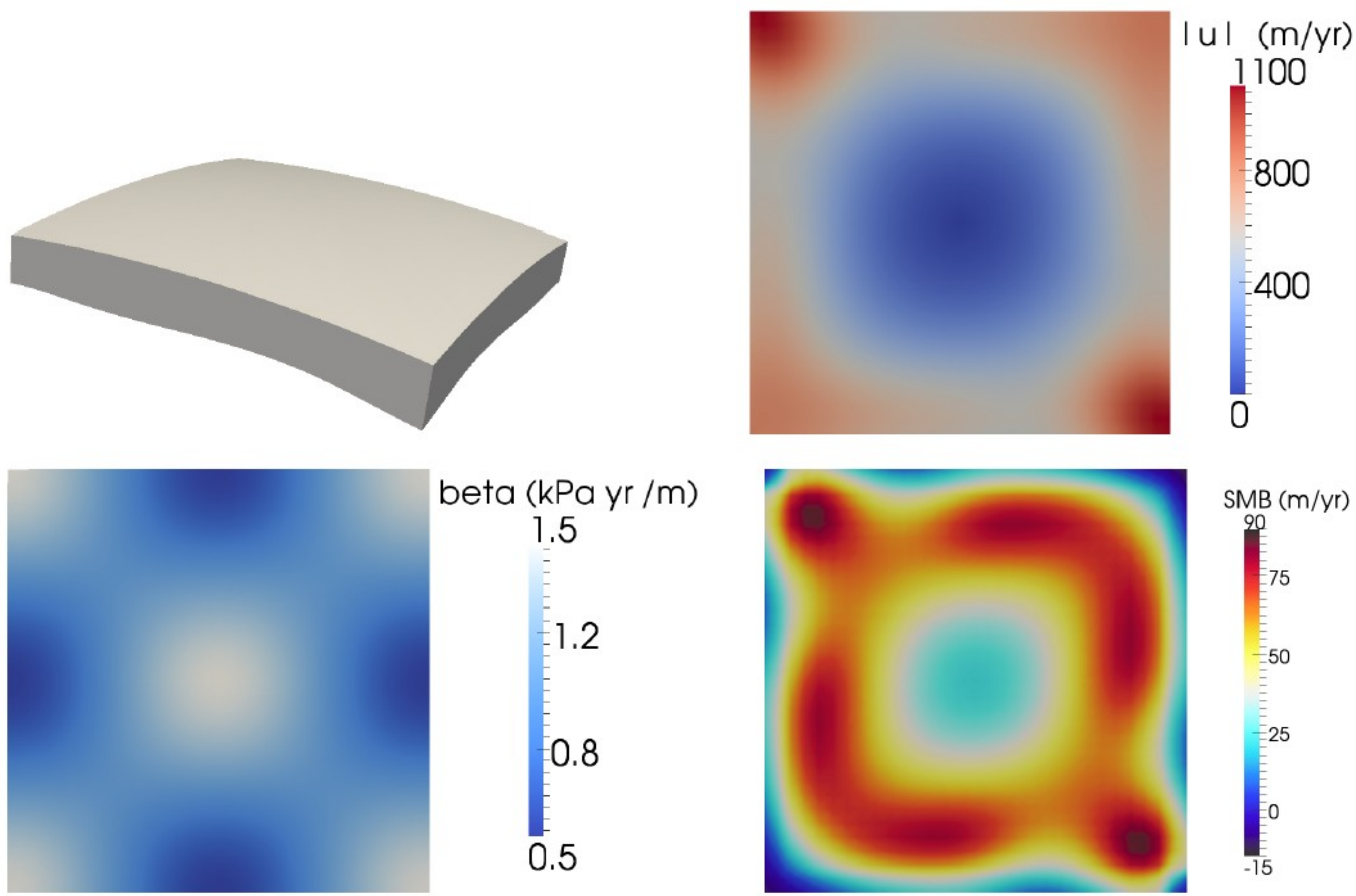
Estimation of ice-sheet initial state

Algorithm and Software tools used

Algorithm	Software Tools
Basal non-uniform triangular mesh	***Triangle***
Linear Finite Elements on tetrahedra	***LifeV***
Quasi-Newton optimization (L-BFGS)	**Rol**
Nonlinear solver (Newton method)	**NOX**
Krylov Linear Solvers	**AztecOO/IfPack**

Inverse Problem
Estimation of ice-sheet initial state

Synthetic test case, settings and forward problem.

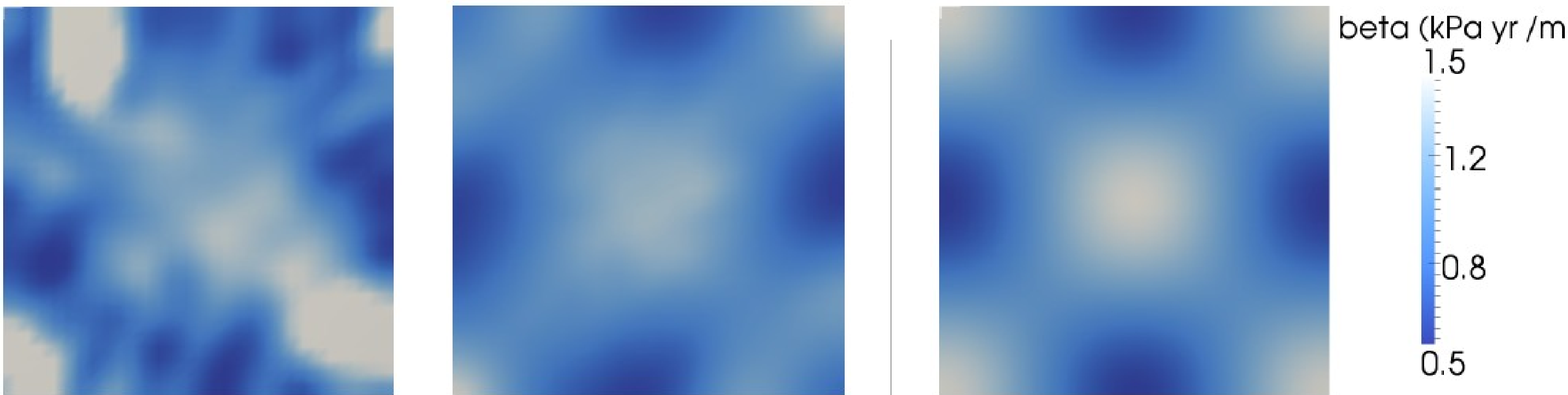


We add noise to the divergence flux, surface velocity and bedrock topography obtained with the forward simulation and use them as “true” SMB surface velocity and bedroc topography.

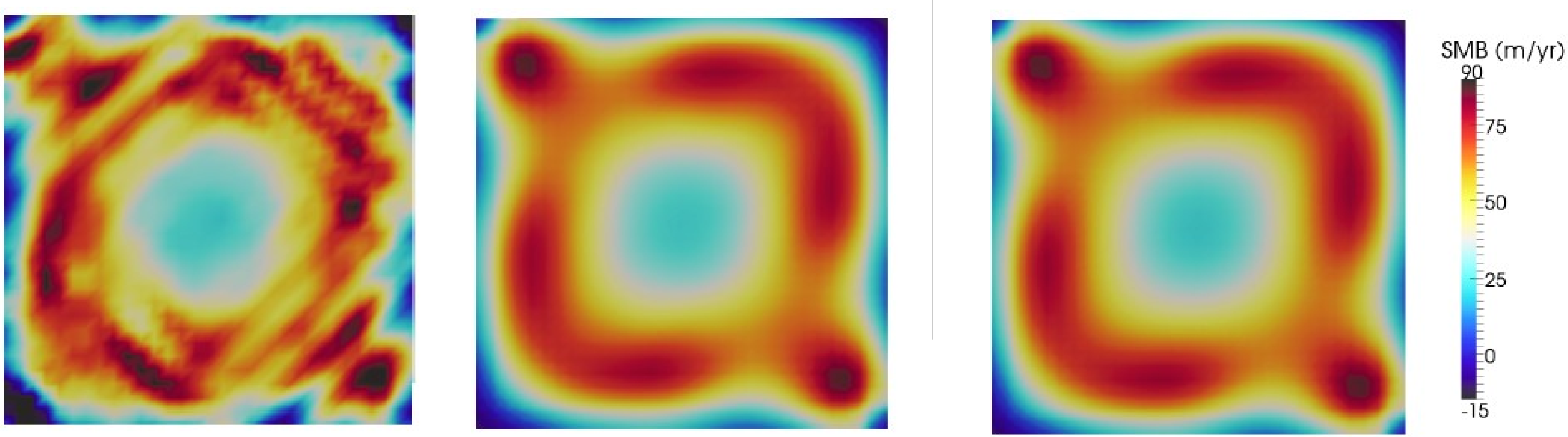
Inverse Problem
Estimation of ice-sheet initial state

Synthetic test case, inversion results, β and SMB.

Common appr. | Novel appr. | Target



Recovered basal friction. | Exact basal friction.



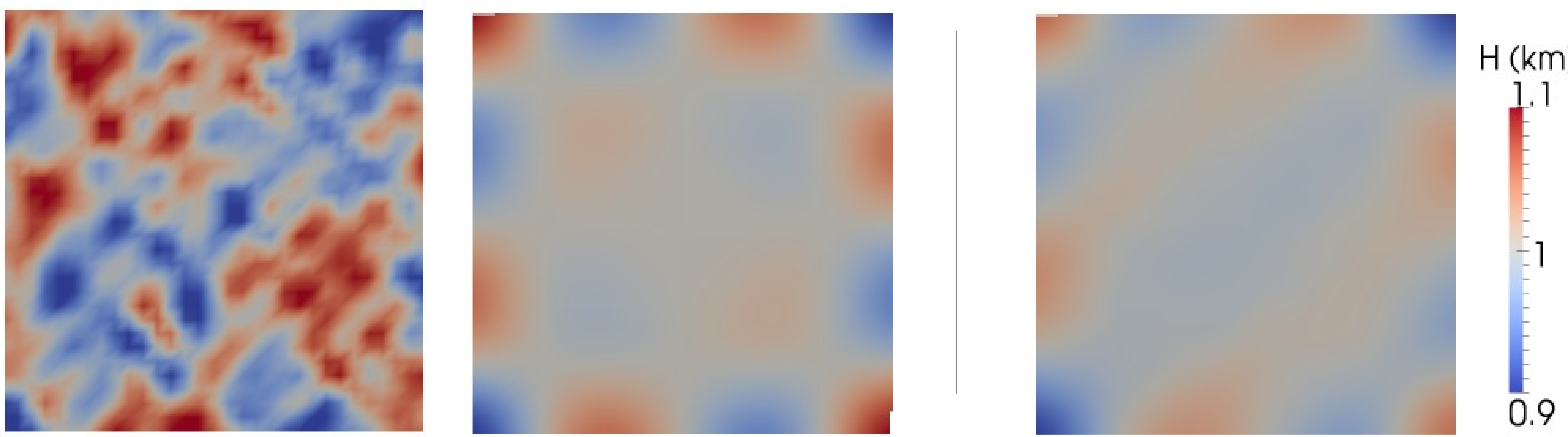
SMB needed for equilibrium | Target SMB

Inverse Problem

Estimation of ice-sheet initial state

Synthetic test case, inversion results,thickness.

Common appr. | Novel appr. | Target



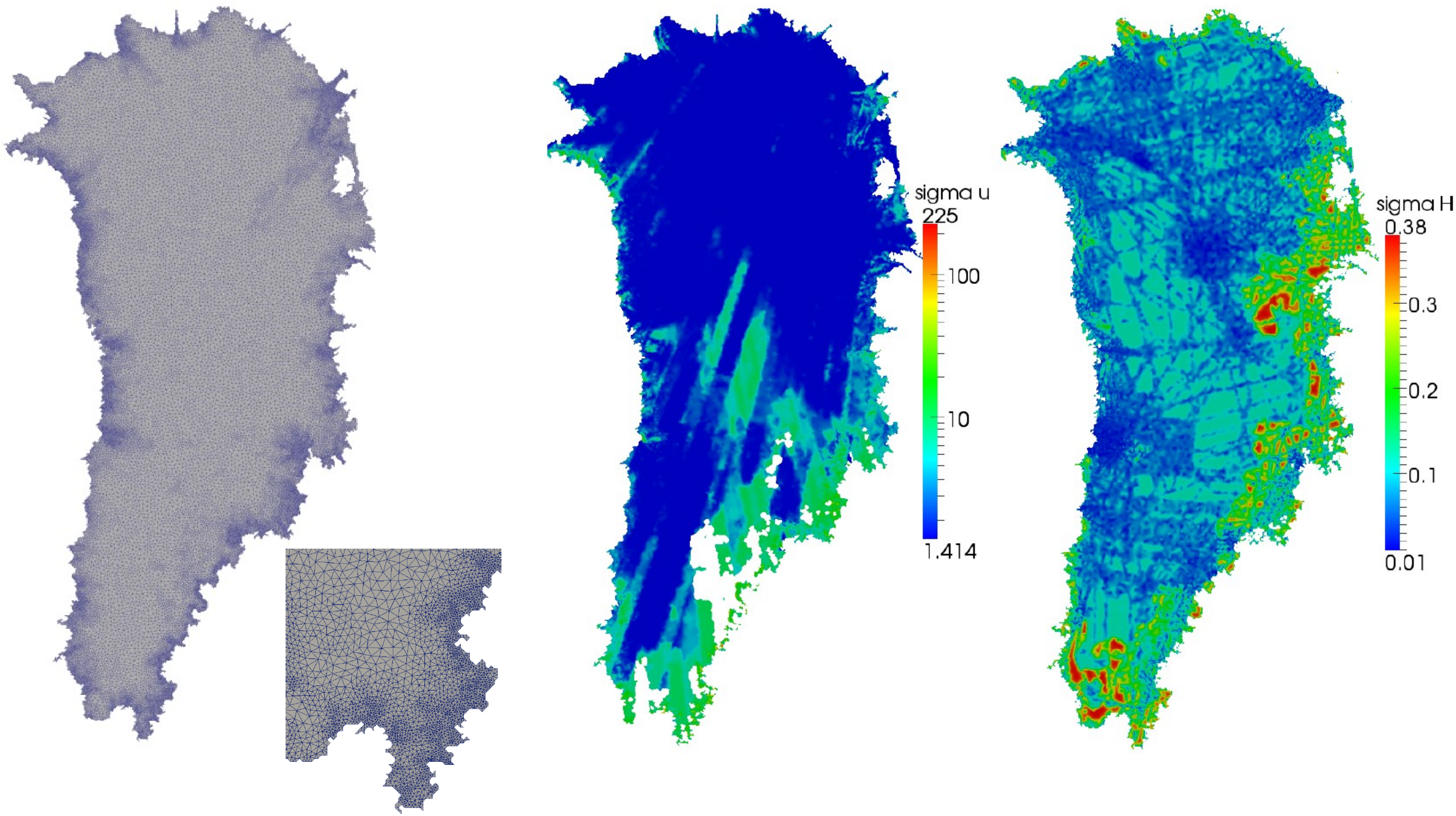
recovered thickness

exact thickness

Inverse Problem

Estimation of ice-sheet initial state of Greenland ice sheet

Grid and RMS of velocity and thickness observations

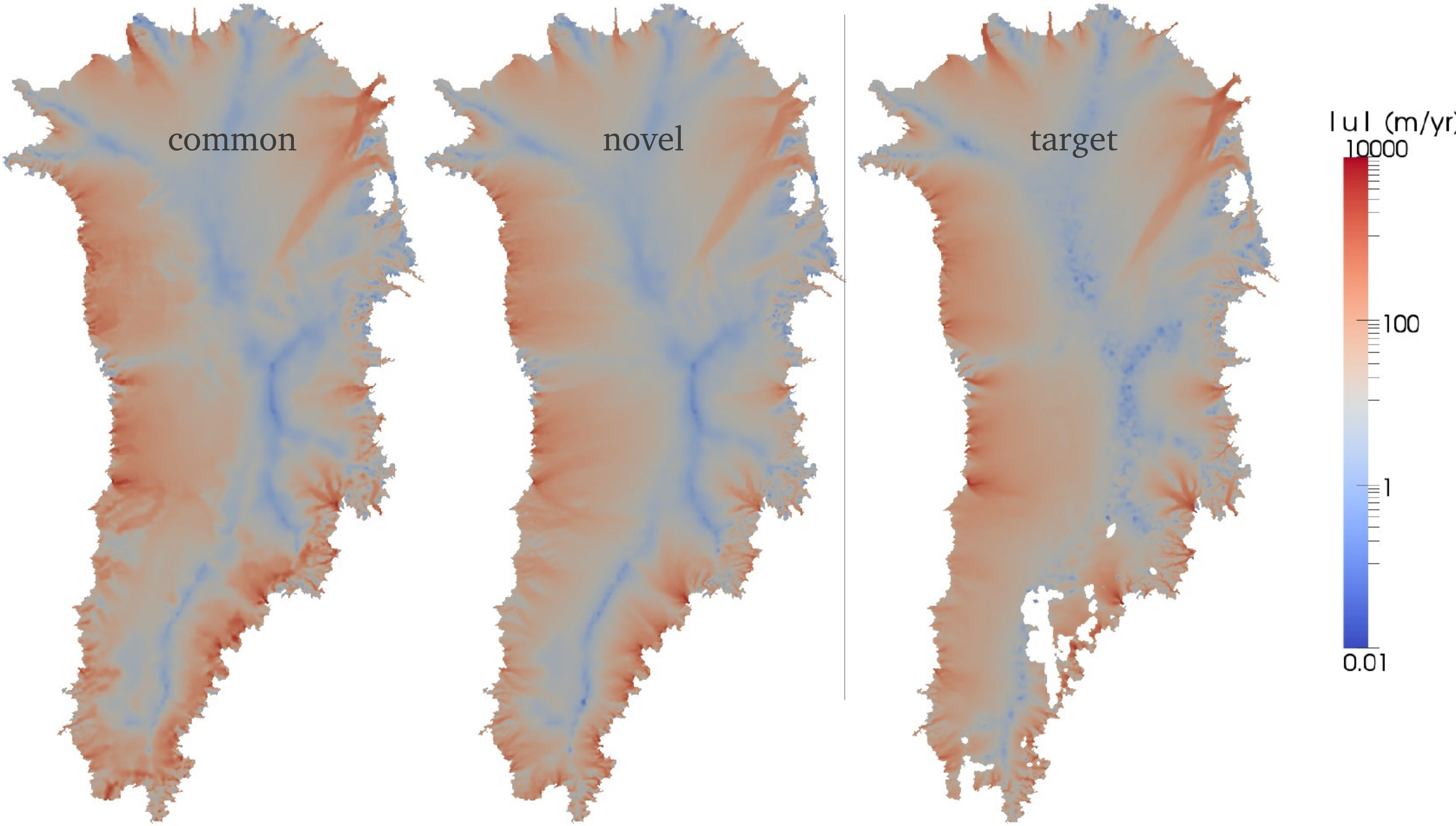


Inverse Problem
Estimation of ice-sheet initial state of Greenland ice sheet

Inversion results: surface mass balance (SMB)

computed surface velocity

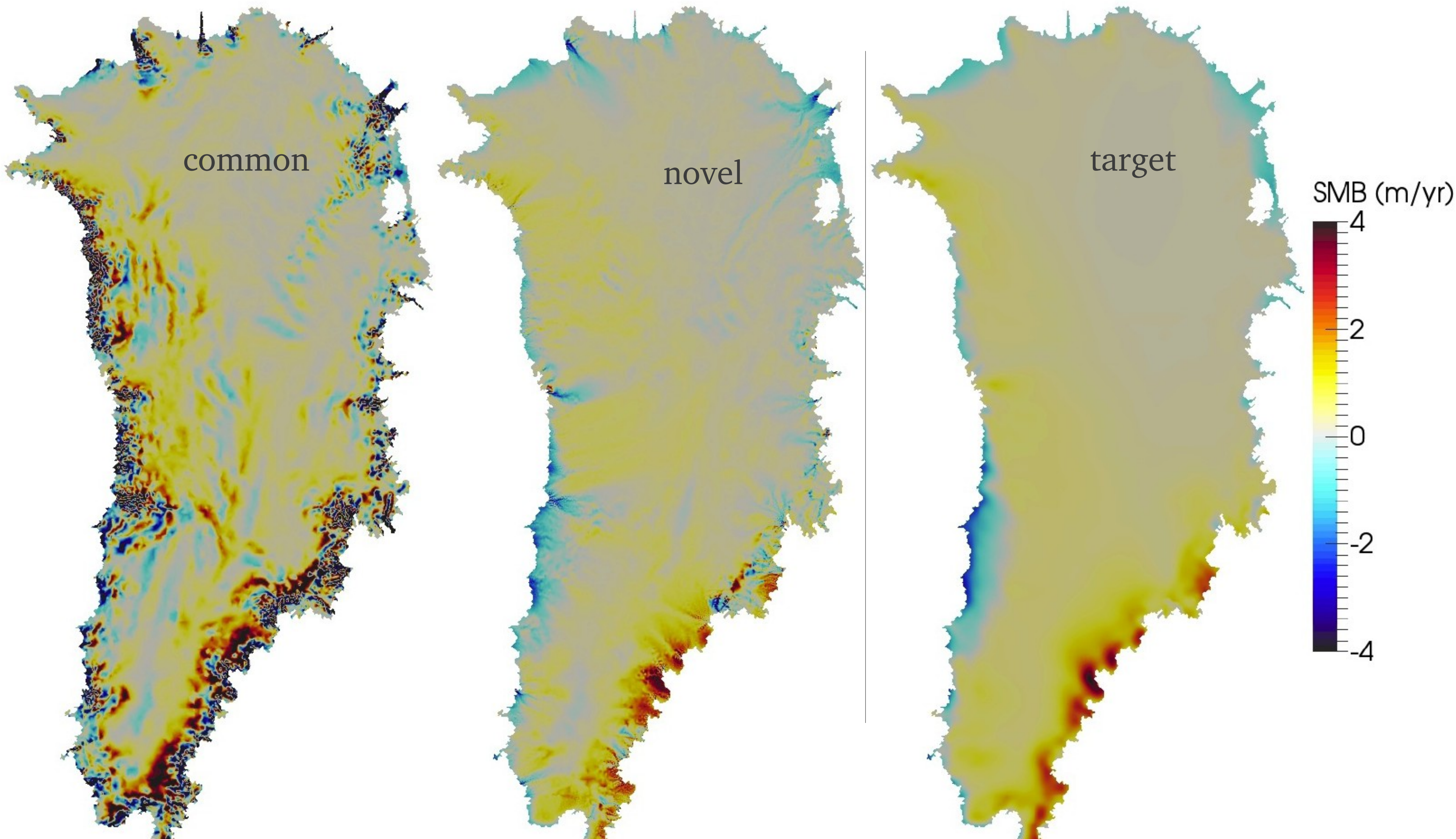
observed surface velocity



Inverse Problem
Estimation of ice-sheet initial state of Greenland ice sheet

Inversion results: surface mass balance (SMB)

SMB needed for equilibrium | SMB from climate model



Inverse Problem
Estimation of ice-sheet initial state of Greenland ice sheet

Estimated beta and change in topography.

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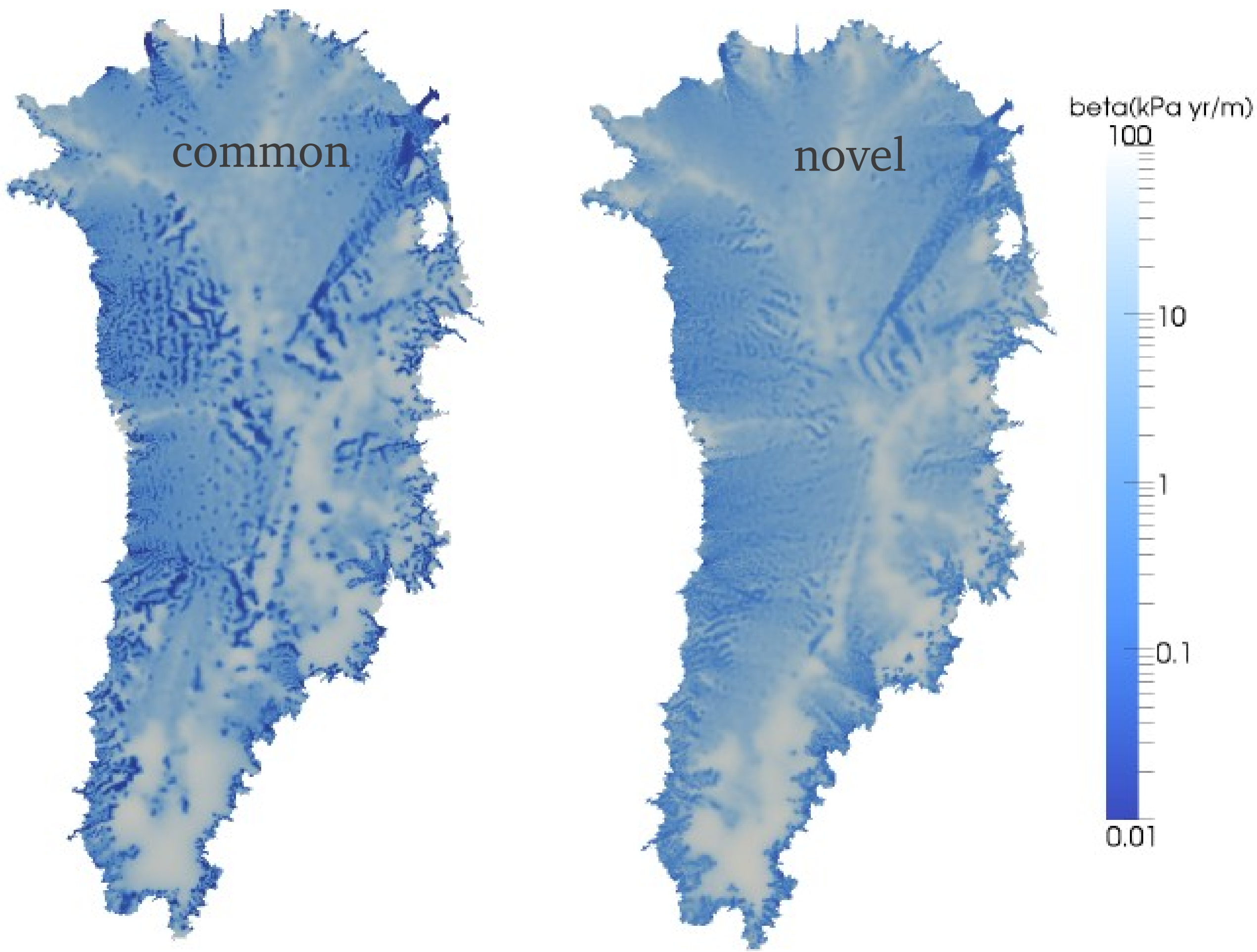
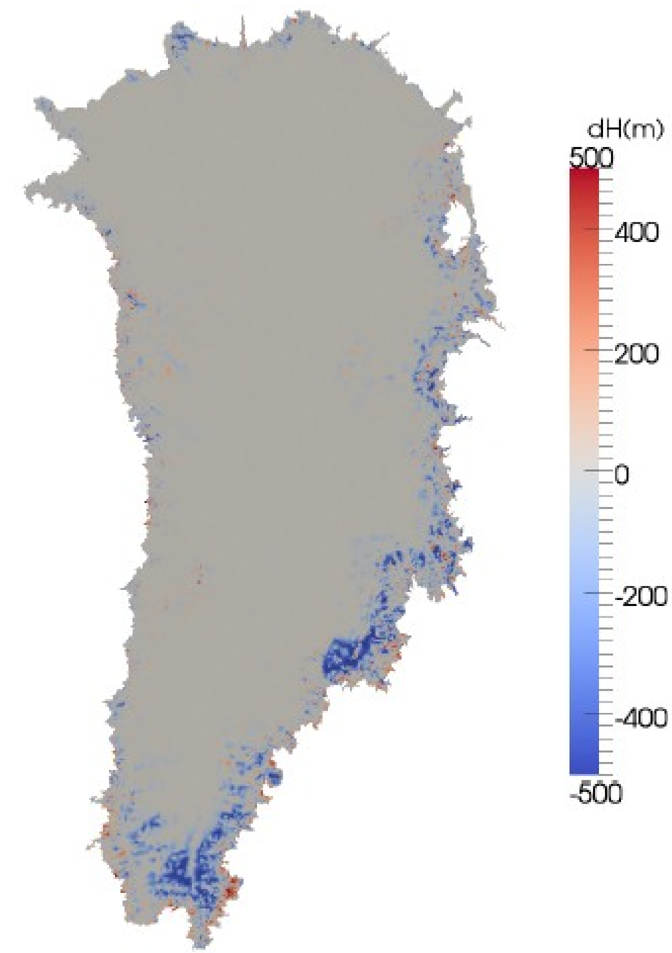
recovered basal friction



difference between recovered and observed thickness



Bayesian Inversion

(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Reduction of parameter space dimension

Difficulty in UQ approach: "***Curse of dimensionality***". The parameter space has O(30,000) parameters (or more).

- Reduce the dimension of the parameter space.

Method of choice: Karhunen-Loeve Expansion (**KLE**).
In our experiment, we reduce the dimension of parameter space to 5.

1. Assume analytic covariance kernel $C(r_1, r_2) = exp\left(-\frac{|r_1 - r_2|^2}{L^2}\right)$.
2. Perform eigenvalue decomposition of C.
3. Take the mean $\bar{\beta}$ to be the deterministic solution and expand β in basis of eigenvector $\{\phi_k\}$ of C, with random variables $\{\xi_k\}$

$$\beta(\omega) = \bar{\beta} + \sum_{k=1}^{K} \sqrt{\lambda_k} \phi_k \xi_k(\omega)$$

*Expansion done on $\log(\beta)$ to avoid negative values for β.

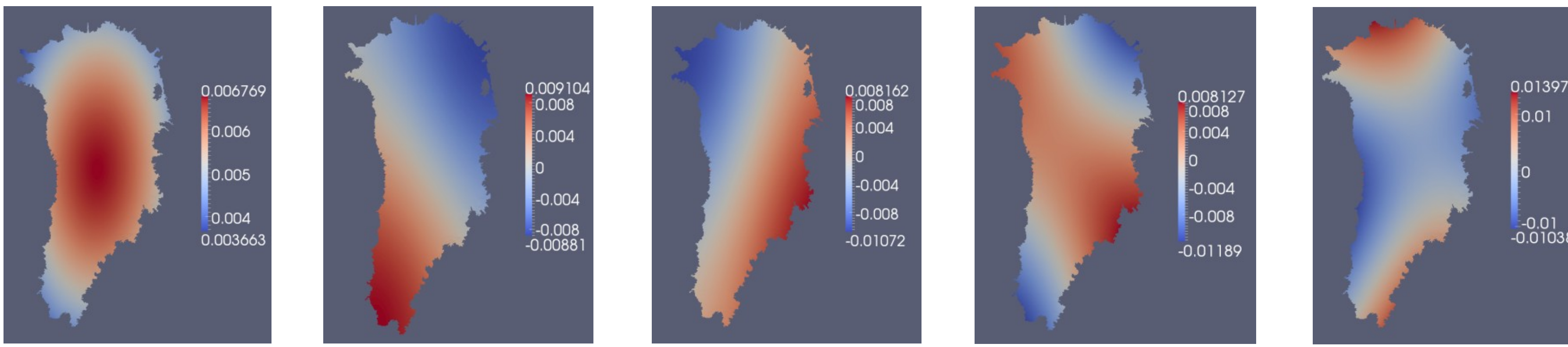
Development(?): parameter reduction based on physical knowledge.
(e.g. include *basal hydrology model*)

Bayesian Inversion

(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Reduction of parameter space dimension: Greenland modes

- 5 KLE modes capture 95% of covariance energy (parallel C++/Trilinos code **Anasazi**).



Only spatial correlation has been considered.

Ongoing development: Use eigenvectors of the inverse of the Hessian of the cost functional as modes.

Bayesian Inversion

(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Compute model surrogate and invert

- Mismatch (**ALBANY**): $\mathcal{J}(\beta) = \dfrac{1}{2}\displaystyle\int_{\Gamma} \dfrac{1}{\sigma_s^2} |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds.$

- ***Build Surrogate Model.*** Polynomial chaos expansion (**PCE**) was formed for the mismatch over random variables using uniform prior distributions. **DAKOTA**.

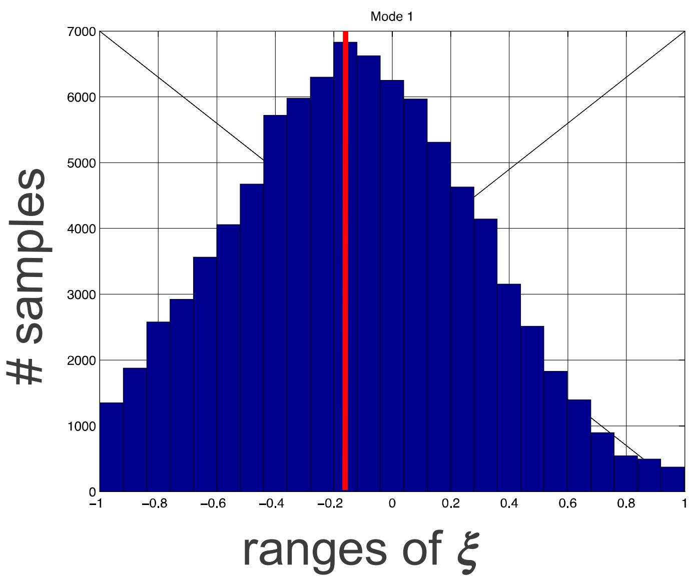
- **Inversion/Calibration**. Markov Chain Monte Carlo (**MCMC**) was performed on the PCE with 100K samples **QUESO**.

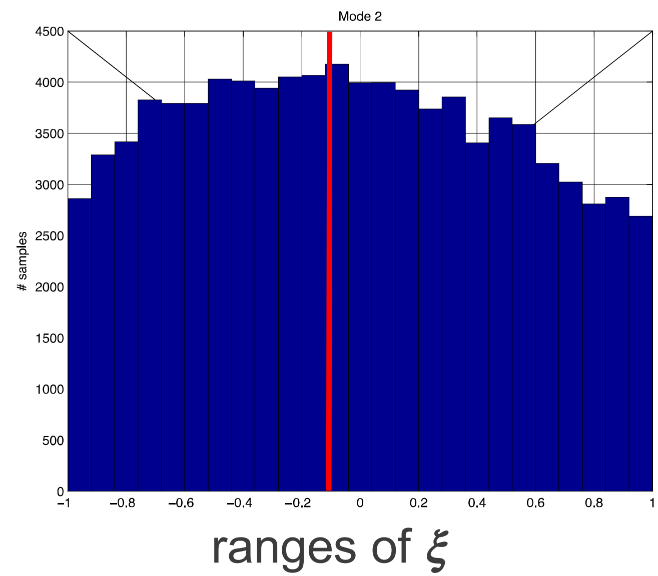
Development(?): use simple physical model (***e.g. L1L2 or SIA***) as the surrogate model.

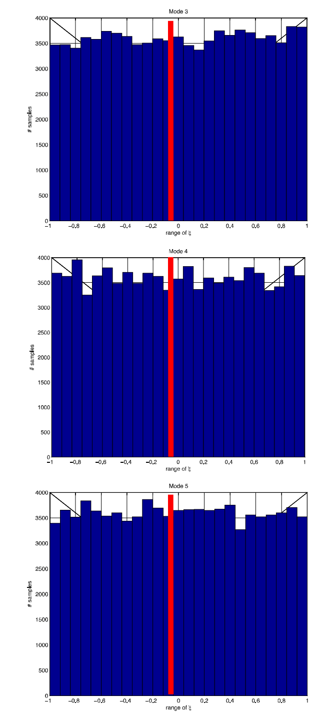
Bayesian Inversion
(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Numerical Results

Posterior distributions for the 5 KLE coefficients:







MAP solution: ξ = (-0.16, -0.08, 0, 0, 0)

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Bayesian Inversion

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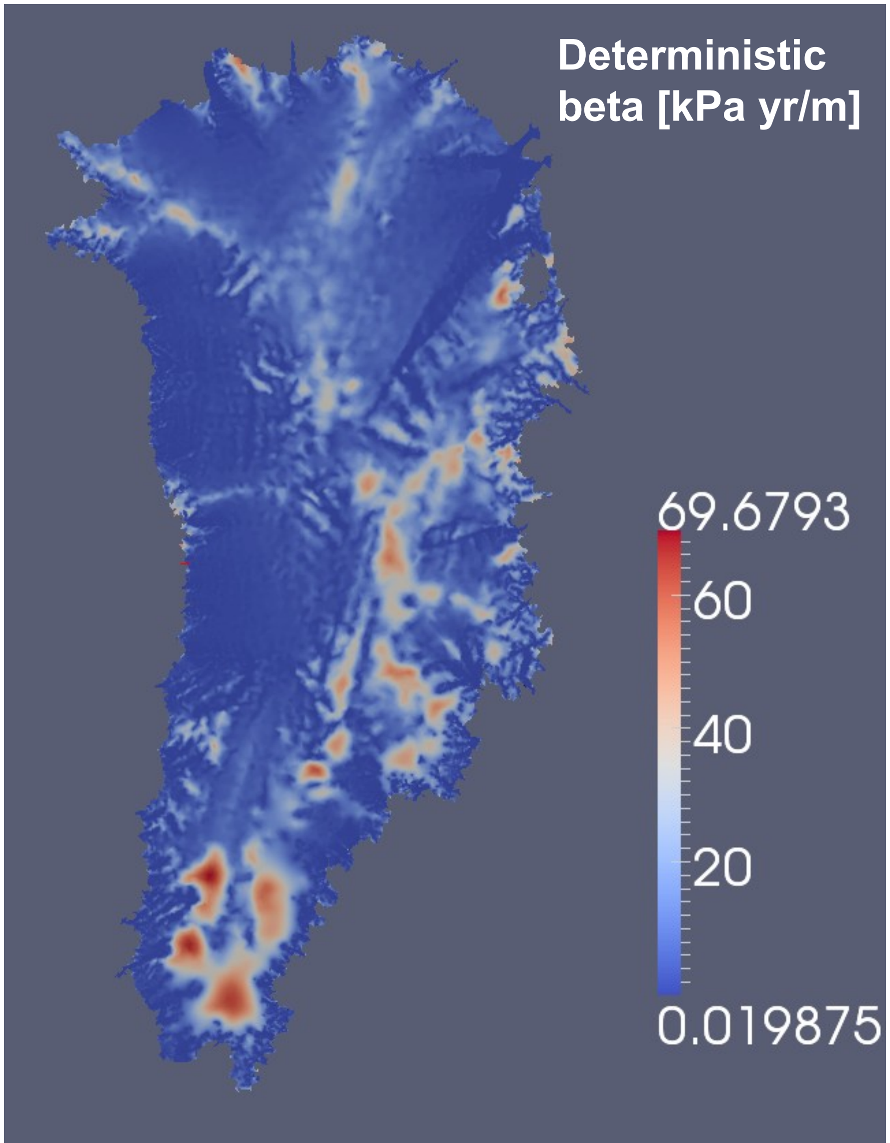
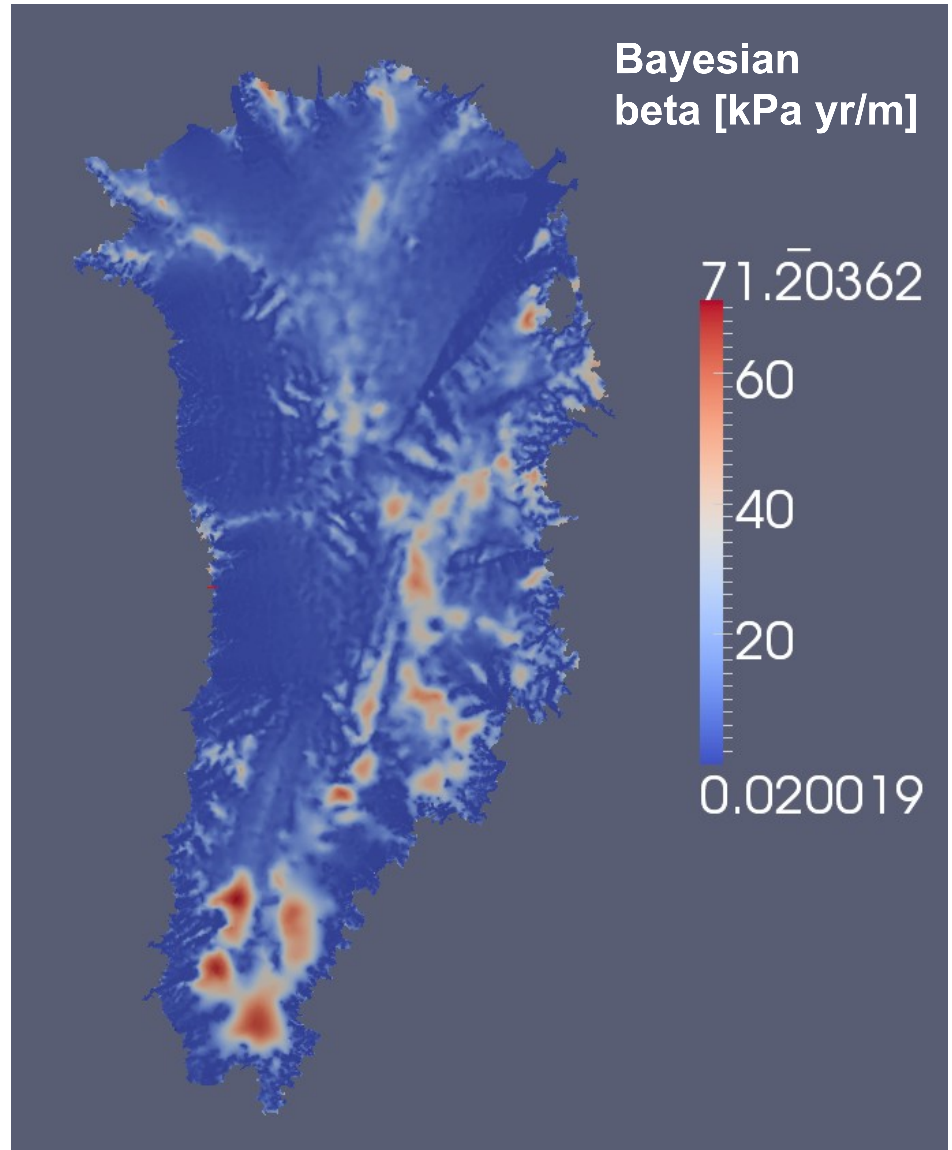
Numerical Results





Thank you for your attention