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Energy-Stable Galerkin Reduced Order Models for Nonlinear Compressible Flow

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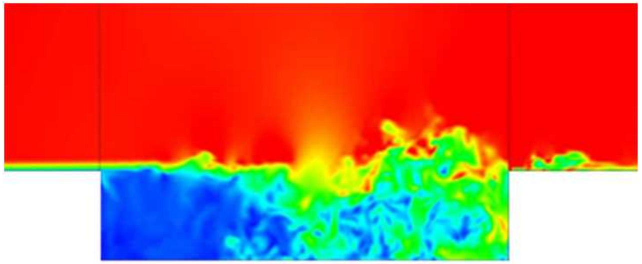
Motivation

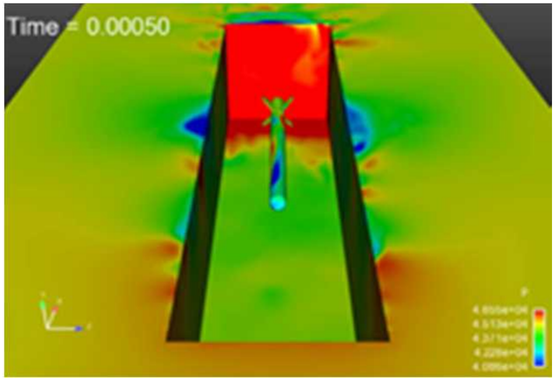
- Despite improved algorithms and powerful supercomputers, **“high-fidelity” models** are often too expensive for use in a design or analysis setting.

- ***Targeted application area in which this situation arises:*** compressible cavity flow problem.

→ **Large Eddy Simulations (LES)** with very fine meshes and long times are required to predict accurately dynamic pressure loads in cavity.

These simulations take ***weeks*** even when run in parallel on state-of-the-art supercomputers!





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Proper Orthogonal Decomposition (POD)/ Galerkin Method to Model Reduction

High-Fidelity Simulations:

- Snapshot 1
- Snapshot 2
- ⋮
- Snapshot K

Step 1 →

Modal Decomposition (POD):

$$\mathbf{x}(t) \approx \mathbf{\Phi}_M \mathbf{x}_M(t)$$

Step 2 →

Galerkin Projection of LTI FOM:

$$\mathbf{\Phi}_M^T \left[\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\right]$$

- ***Snapshot matrix***: $\boldsymbol{X} = (\boldsymbol{x}^1, \ldots, \boldsymbol{x}^K) \in \mathbb{R}^{NxK}$
- ***SVD:*** $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$
- ***Truncation:*** $\boldsymbol{\Phi}_M = (\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_M) = \boldsymbol{U}(:, 1{:}M)$

N = # of dofs in high-fidelity simulation
K = # of snapshots
M = # of dofs in ROM
$(M << N, M << K)$

"Small" ROM LTI System:

$$\begin{aligned}\dot{\mathbf{x}}_M(t) &= \mathbf{\Phi}_M^T \mathbf{A} \mathbf{\Phi}_M \mathbf{x}_M(t) + \mathbf{\Phi}_M^T \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}_M(t) &= \mathbf{C}\mathbf{\Phi}_M \mathbf{x}_M(t)\end{aligned}$$

Discrete vs. Continuous Galerkin Projection

Discrete Projection

Governing PDEs
$$\dot{\boldsymbol{q}} = \mathcal{L}\boldsymbol{q}$$

↓

CFD model
$$\dot{\boldsymbol{q}}_N = \boldsymbol{A}_N \boldsymbol{q}_N$$

↓

Discrete modal basis $\boldsymbol{\Phi}$

↓

Projection of CFD model (matrix operation)

↓

ROM
$$\dot{\boldsymbol{a}}_M = \boldsymbol{\Phi}^T \boldsymbol{A}_N \boldsymbol{\Phi} \boldsymbol{a}_M$$

If PDEs are linear or have polynomial non-linearities, projection can be calculated in **offline stage** of MOR.

This talk focuses on →

Continuous Projection

Governing PDEs
$$\dot{\boldsymbol{q}} = \mathcal{L}\boldsymbol{q}$$

↓

CFD model
$$\dot{\boldsymbol{q}}_N = \boldsymbol{A}_N \boldsymbol{q}_N$$

↓

Continuous modal basis* $\boldsymbol{\phi}_j(\boldsymbol{x})$

↓

Projection of governing PDEs (numerical integration)

↓

ROM
$$\dot{a}_j = (\boldsymbol{\phi}_j, \mathcal{L}\boldsymbol{\phi}_k) a_k$$

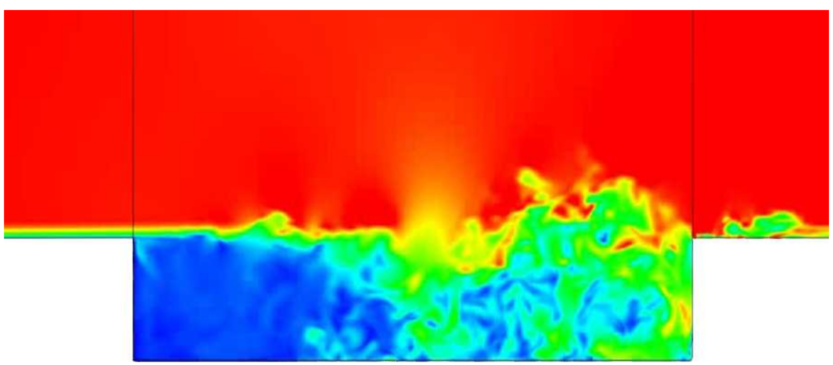
* Continuous functions space is defined using finite elements.

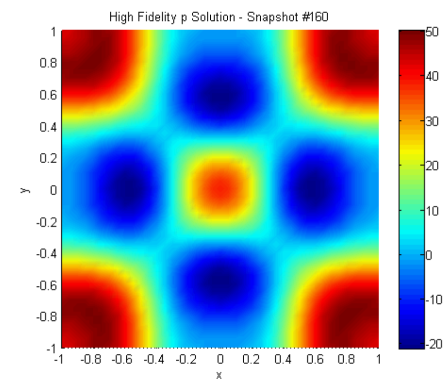
Stability Issues of POD/Galerkin ROMs

Full Order Model (FOM)	**Reduced Order Model (ROM)**
$\dot{\boldsymbol{q}}(t) = \mathcal{L}\boldsymbol{q}(t) + \mathcal{N}(\boldsymbol{q}(t))$	$\dot{\boldsymbol{q}}_M(t) = \boldsymbol{A}_M\boldsymbol{q}_M(t) + \boldsymbol{N}_M(\boldsymbol{q}_M(t))$

Problem: FOM stable $\not\Rightarrow$ ROM stable!

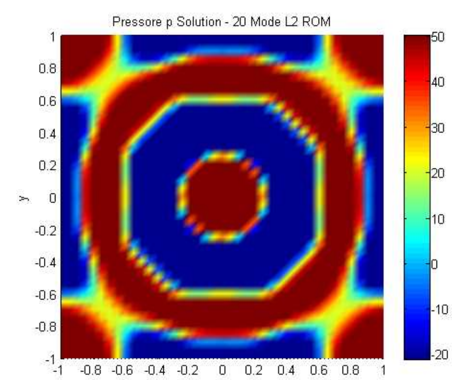
- There is no *a priori* stability guarantee for POD/Galerkin ROMs.
- Stability of a ROM is commonly evaluated *a posteriori* – **RISKY!**
- Instability of POD/Galerkin ROMs is a **real** problem in some applications…





Top right: FOM

Bottom right: ROM



…e.g., compressible flows, high-Reynolds number flows.

Stability Preserving ROM Approaches: Literature Review

Approaches for building stability-preserving POD/Galerkin ROMs found in the literature fall into **two categories:**

This talk.

1. ROMs which derive ***a priori*** a stability-preserving model reduction framework (usually specific to an equation set).

Can have an intrusive implementation

- ROMs based on projection in special 'energy-based' (not L^2) inner products, e.g., Rowley *et al*. (2004), Barone & Kalashnikova *et al.* (2009), Serre *et al*. (2012).

2. ROMs which stabilize an unstable ROM through an ***a posteriori*** post-processing stabilization step applied to the algebraic ROM system.

Can have inconsistencies between ROM and FOM physics

- Approaches in which an optimization problem that stabilizes an unstable ROM is formulated and solved, e.g., Amsallem *et al.* (2012), Bond *et al.* (2008), Kalashnikova *et al.* (2014).
- ROMs with increased numerical stability due to inclusion of 'stabilizing' terms in the ROM equations, e.g., Wang *et al.* (2012).

Energy-Stability

- **Practical Definition:** Numerical solution does not "blow up" in finite time.
- **More Precise Definition:** Numerical discretization does not introduce any spurious instabilities inconsistent with natural instability modes supported by the governing continuous PDEs.

Numerical solutions must maintain proper **energy balance**.

- Stability of ROM is intimately tied to choice of **inner product** for the Galerkin projection.
- Stability-preserving inner product derived using the **energy method**:
  - Bounds numerical solution energy in a physical way.
  - Borrowed from spectral methods community.
  - Analysis is straightforward for ROMs constructed via **continuous projection.**

Can show: if a Galerkin ROM is constructed in an ***energy inner product***, the ROM system energy will be bounded in a way that is consistent with the behavior of the exact solution to the PDEs, i.e., the ROM will be ***energy-stable***.

Linearized Compressible Flow Equations

Energy-Stability for Linearized PDEs:
FOM linearly stable ⇒ ROM built in energy inner product linearly stable ($Re(\lambda) < 0$)
(WCCM X talk and paper: Kalashnikova & Arunajatesan, 2012).

Linearized compressible Euler/Navier-Stokes equations are appropriate when a compressible fluid system can be described by small-amplitude perturbations about a steady-state mean flow.

- Linearization of full compressible Euler/Navier-Stokes equations obtained as follows:
  1. Decompose fluid field as **steady mean** plus **unsteady fluctuation**

$$\boldsymbol{q}(\boldsymbol{x},t) = \overline{\boldsymbol{q}}(\boldsymbol{x}) + \boldsymbol{q}'(\boldsymbol{x},t)$$

  2. Linearize full nonlinear compressible Navier-Stokes equations around steady mean to yield **linear hyperbolic/incompletely parabolic** system

$$\dot{\boldsymbol{q}}' + \boldsymbol{A}_i(\overline{\boldsymbol{q}})\frac{\partial \boldsymbol{q}'}{\partial xi} + \frac{\partial}{\partial xj}\left[\boldsymbol{K}_{ij}(\overline{\boldsymbol{q}})\frac{\partial \boldsymbol{q}'}{\partial xi}\right] = \boldsymbol{0}$$

Energy-Stable ROMs for Linearized Compressible Flow

Linearized compressible Euler/Navier-Stokes equations are **symmetrizable** (Barone & Kalashnikova, 2009; Kalashnikova & Arunajatesan, 2012).

- There exists a symmetric positive definite matrix $\boldsymbol{H} \equiv \boldsymbol{H}(\overline{\boldsymbol{q}})$ (system "**symmetrizer**") s.t.:
  - The convective flux matrices $\boldsymbol{H}\boldsymbol{A}_i$ are symmetric
  - The following augmented viscosity matrix is symmetric positive semi-definite

$$\boldsymbol{K}^s = \begin{pmatrix} \boldsymbol{HK}_{11} & \boldsymbol{HK}_{12} & \boldsymbol{HK}_{13} \\ \boldsymbol{HK}_{21} & \boldsymbol{HK}_{22} & \boldsymbol{HK}_{23} \\ \boldsymbol{HK}_{31} & \boldsymbol{HK}_{32} & \boldsymbol{HK}_{33} \end{pmatrix}$$

Symmetry Inner Product (weighted L^2 inner product):

$$(\boldsymbol{q}_1, \boldsymbol{q}_2)_{\boldsymbol{H}} = \int_{\Omega} \boldsymbol{q}_1 \boldsymbol{H} \boldsymbol{q}_2 d\Omega$$

- If ROM is built in **symmetry inner product**, Galerkin approximation will satisfy the same energy expression as continuous PDEs:

$$\left|\left|\boldsymbol{q}'_M(\boldsymbol{x},t)\right|\right|_{\boldsymbol{H}} \leq e^{\beta t} \left|\left|\boldsymbol{q}'_M(\boldsymbol{x},0)\right|\right|_{\boldsymbol{H}} \quad \left(\Rightarrow \frac{dE_M}{dt} \leq 0 \text{ for uniform base flow}\right)$$

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Symmetrizers for Several Hyperbolic/ Incompletely Parabolic Systems

- ***Wave equation:*** $\ddot{u} = a^2 \frac{\partial^2 u}{\partial x^2}$ or $\dot{\boldsymbol{q}} = \boldsymbol{A}\frac{\partial \boldsymbol{q}}{\partial x}$ where $\boldsymbol{q} = \left(\dot{u}, \frac{\partial u}{\partial x}\right)$ $\Rightarrow \boldsymbol{H} = \begin{pmatrix} 1 & 0 \\ 0 & a^2 \end{pmatrix}$

- ***Shallow water equations:*** $\dot{\boldsymbol{q}}' + \boldsymbol{A}_i(\overline{\boldsymbol{q}})\frac{\partial \boldsymbol{q}'}{\partial xi} = \boldsymbol{0}$ $\Rightarrow \boldsymbol{H} = \begin{pmatrix} \bar{\phi} & 0 & 0 \\ 0 & \bar{\phi} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- ***Linearized compressible Euler:*** $\dot{\boldsymbol{q}}' + \boldsymbol{A}_i(\overline{\boldsymbol{q}})\frac{\partial \boldsymbol{q}'}{\partial xi} = \boldsymbol{0}$ $\Rightarrow \boldsymbol{H} = \begin{pmatrix} \bar{\rho} & 0 & 0 \\ 0 & \alpha^2\gamma\bar{\rho}^2\bar{p} & \bar{\rho}\alpha^2 \\ 0 & 0 & \frac{(1+\alpha^2)}{\gamma\bar{p}} \end{pmatrix}$

- ***Linearized compressible Navier-Stokes:*** $\dot{\boldsymbol{q}}' + \boldsymbol{A}_i(\overline{\boldsymbol{q}})\frac{\partial \boldsymbol{q}'}{\partial xi} + \frac{\partial}{\partial xj}\left[\boldsymbol{K}_{ij}(\overline{\boldsymbol{q}})\frac{\partial \boldsymbol{q}'}{\partial xi}\right] = \boldsymbol{0}$

$$\Rightarrow \boldsymbol{H} = \begin{pmatrix} \bar{\rho} & 0 & 0 \\ 0 & \frac{\bar{\rho}R}{\overline{\overline{T}(\gamma-1)}} & 0 \\ 0 & 0 & \frac{R\bar{T}}{\bar{\rho}} \end{pmatrix}$$

- Barone & Kalashnikova, *JCP*, 2009.
- Kalashnikova & Arunajatesan, *WCCM X*, 2012.
- Kalashnikova *et al.*, *SAND report*, 2014.

Continuous Projection Implementation: "Spirit" Code

"Spirit" ROM Code = 3D parallel C++ POD/Galerkin test-bed ROM code that uses data-structures and eigensolvers from **Trilinos** to build energy-stable ROMs for compressible flow problems → stand-alone code that can be synchronized with any high-fidelity code!

- POD modes defined using piecewise smooth finite elements.
- Gauss quadrature rules of sufficient accuracy are used to compute exactly inner products with the help of the `libmesh` library.
- Physics in Spirit:
  - ***Linearized compressible Euler*** (L^2, energy inner product).
  - ***Linearized compressible Navier-Stokes*** (L^2, energy inner product).
  - ***Nonlinear isentropic compressible Navier-Stokes*** (L^2, stagnation energy, stagnation enthalpy inner product).
  - ***Nonlinear compressible Navier-Stokes*** (L^2, energy inner product).

First, testing of ROMs for these physics

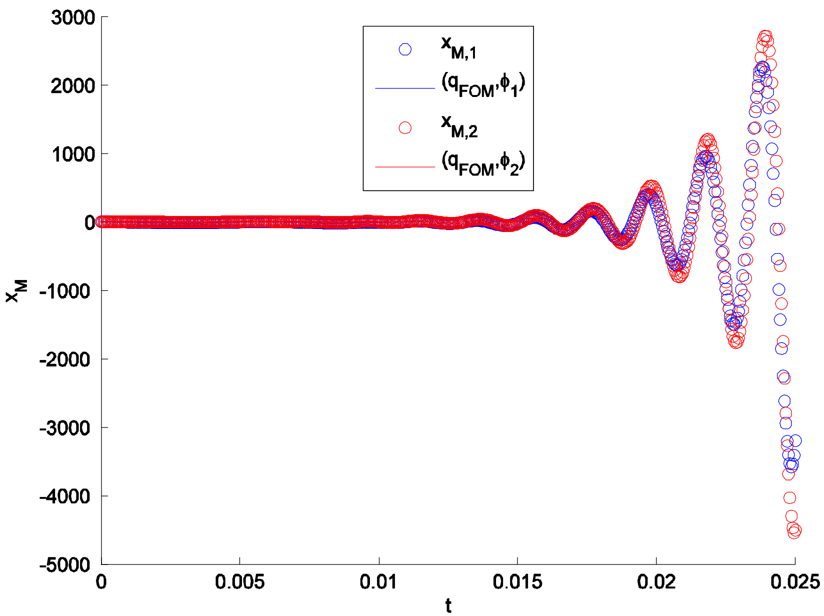
"SIGMA CFD" High-Fidelity Code = Sandia in-house finite volume flow solver derived from LESLIE3D (Genin & Menon, 2010), an LES flow solver originally developed in the Computational Combustion Laboratory at Georgia Tech.

Numerical Experiment: 2D Inviscid Pressure Pulse

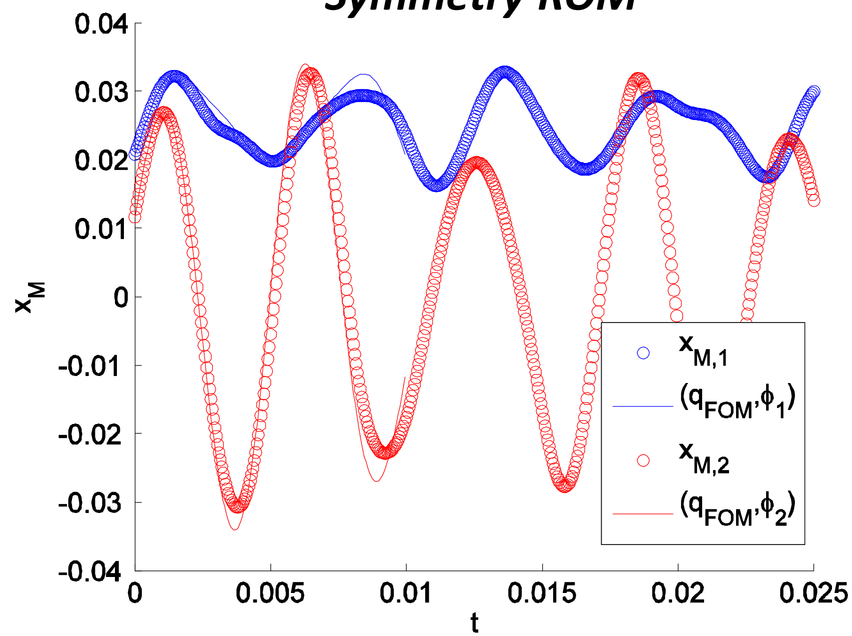
- Inviscid pulse in a uniform base flow (linear dynamics).
- High-fidelity simulation run on mesh with 3362 nodes, up to time $t = 0.01$ seconds.
- 200 snapshots of solution used to construct $M = 20$ mode ROM in L^2 and symmetry inner products.

$x_{M,i}(t)$ vs.$(\boldsymbol{q}'_{CFD}, \boldsymbol{\phi}_i)$ for $i = 1,2$

L^2 ROM



Symmetry ROM



Numerical Experiment: 2D Inviscid Pressure Pulse (cont'd)

- Inviscid pulse in a uniform base flow (linear dynamics).
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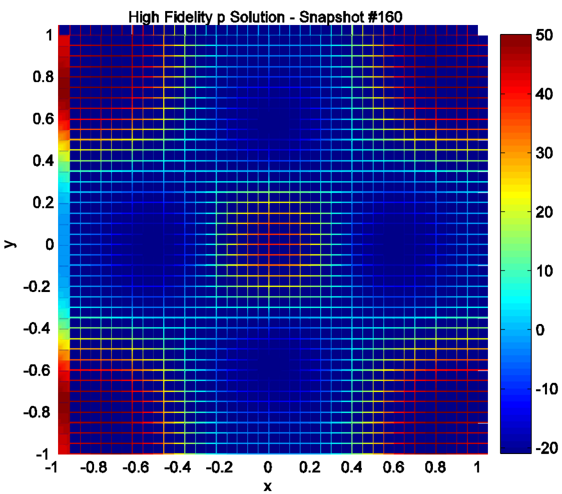
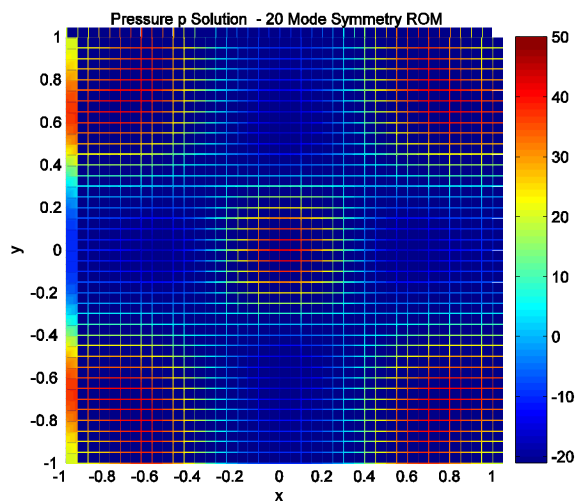
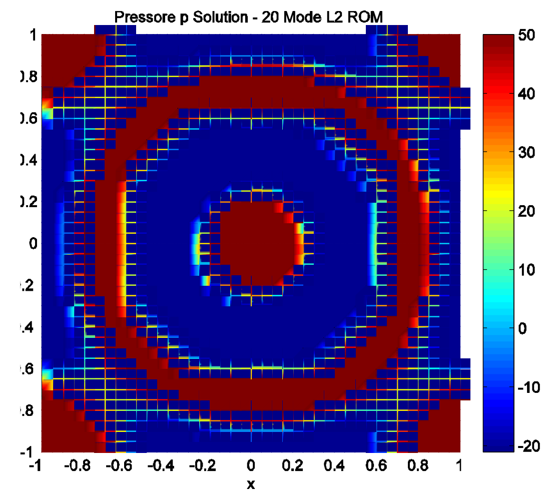
p': High-fidelity



p': Symmetry ROM



p': L^2 ROM



time of snapshot 160

Nonlinear Compressible Flow Equations

Energy-Stability for Nonlinear PDEs:
ROM built in energy inner product will preserve stability of an equilibrium point at 0 for the governing nonlinear system of PDEs (Rowley, 2004; Kalashnikova *et al., 2014*).

- Compressible isentropic Navier-Stokes equations (cold flows, moderate Mach #):

$$\frac{Dh}{Dt} + (\gamma - 1)h\nabla \cdot \boldsymbol{u} = 0$$
$$\frac{D\boldsymbol{u}}{Dt} + \nabla h - \frac{1}{Re}\Delta \boldsymbol{u} = \boldsymbol{0}$$

h = enthalpy
$\boldsymbol{u}$ = velocity vector
ρ = density
T = temperature
$\boldsymbol{\tau}$ = viscous stress tensor

- Full compressible Navier-Stokes equations:

$$\rho\frac{D\boldsymbol{u}}{Dt} + \frac{1}{\gamma M^2}\nabla(\rho T) - \frac{1}{Re}\nabla \cdot \boldsymbol{\tau} = \boldsymbol{0}$$
$$\frac{D\rho}{Dt} + \rho\nabla \cdot \boldsymbol{u} = 0$$
$$\rho\frac{DT}{Dt} + (\gamma - 1)\rho T\nabla \cdot \boldsymbol{u} - \frac{\gamma}{PrRe}\nabla \cdot (\kappa\nabla T) - \left(\frac{\gamma(\gamma - 1)M^2}{Re}\right)\nabla\boldsymbol{u} \cdot \boldsymbol{\tau} = 0$$

Energy-Stable ROMs for Nonlinear Compressible Flow (Isentropic NS)

In (Rowley, 2004), Rowley *et al.* showed that energy inner product for the compressible isentropic Navier-Stokes equations can be defined following a transformation of these equations.

- Transformed compressible isentropic Navier-Stokes equations:

$$\frac{Dc}{Dt} + \frac{\gamma - 1}{2} c \nabla \cdot \boldsymbol{u} = 0$$
$$\frac{D\boldsymbol{u}}{Dt} + \frac{2}{\gamma - 1} c \nabla c - \frac{1}{Re} \Delta \boldsymbol{u} = \boldsymbol{0}$$

c = speed of sound ($c^2 = (\gamma - 1)h$)
$\boldsymbol{u}$ = velocity

- Family of inner products:

$$(\boldsymbol{q}_1, \boldsymbol{q}_2)_\alpha = \int_\Omega \left(\boldsymbol{u}_1 \cdot \boldsymbol{u}_2 + \frac{2\alpha}{\gamma - 1} c_1 c_2 \right) d\Omega$$

$$\alpha = \begin{cases} 1 \Rightarrow ||\boldsymbol{q}||_\alpha = \text{stagnation enthalpy} \\ \frac{1}{\gamma} \Rightarrow ||\boldsymbol{q}||_\alpha = \text{stagnation energy} \end{cases}$$

If Galerkin projection step of model reduction is performed in α inner product, then the Galerkin projection will **preserve the stability of an equilibrium point at the origin** (Rowley, 2004).

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Energy-Stable ROMs for Nonlinear Compressible Flow (Full NS)

Present work extends ideas in (Rowley, 2004) to **full compressible Navier-Stokes equations**.

- First, full compressible Navier-Stokes equations are **transformed** into the following variables:

$$a = \sqrt{\rho}, \quad \boldsymbol{b} = a\boldsymbol{u}, \; d = ae$$

e =internal energy

- Next, the following **"total energy"** inner product is defined:

$$(\boldsymbol{q}_1, \boldsymbol{q}_2)_{TE} = \int_{\Omega} (\boldsymbol{b}_1 \cdot \boldsymbol{b}_2 + a_1 d_2 + a_2 d_1) d\Omega$$

→ Norm induced by total energy inner product is the total energy of the fluid system:

$$||\boldsymbol{q}||_{TE} = \int_{\Omega} \left(\rho e + \frac{1}{2}\rho u_i u_i\right) d\Omega$$

If Galerkin projection step of model reduction is performed in total energy inner product, then the Galerkin projection will **preserve the stability of an equilibrium point at the origin** (Kalashnikova *et al.*, 2014)

☺ Transformed equations have only **polynomial non-linearities** (projection of which can be computed in offline stage of MOR and stored).

☹ Transformation introduces **higher order polynomial** non-linearities.

☺ Efficiency of online stage of MOR can be recovered using **interpolation** (e.g., DEIM, gappy POD).

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Numerical Experiment: Viscous Laminar Cavity

- Viscous cavity problem at $M = 0.6$, $Re = 1500$ (laminar regime).
- **High-fidelity simulation**: DNS based on full nonlinear compressible Navier-Stokes equations with $\approx 117{,}000$ nodes (right).
- 500 snapshots collected, every $\Delta t_{snap} = 1 \times 10^{-4}$ seconds.
- Snapshots used to construct $M = 15$ mode ROM for nonlinear compressible Navier-Stokes equations in $\boldsymbol{L^2}$ **and total energy inner products**.
- $M = 15$ mode POD bases capture $\approx 99\%$ of snapshot energy.

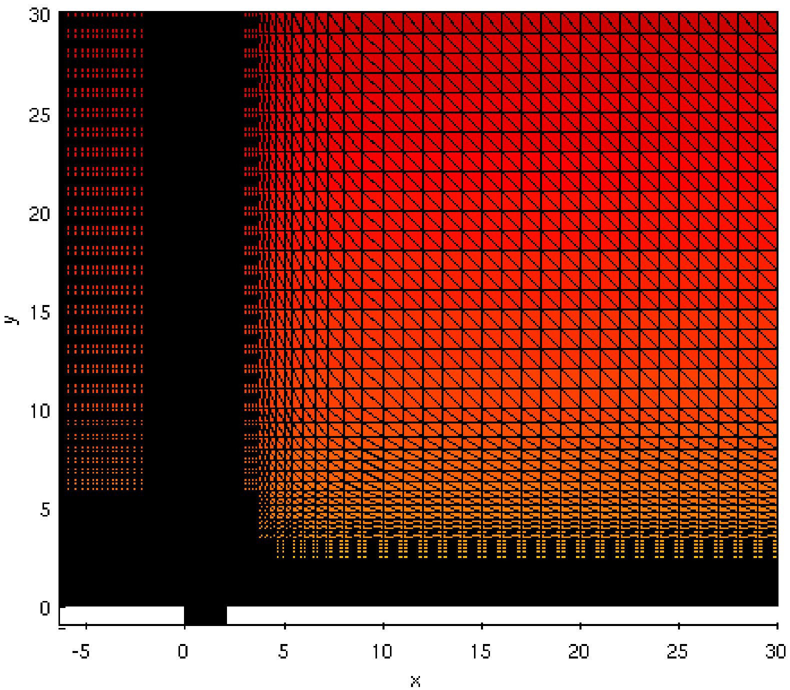

Figure above: viscous laminar cavity problem domain/mesh.

Numerical Experiment: Viscous Laminar Cavity (cont'd)

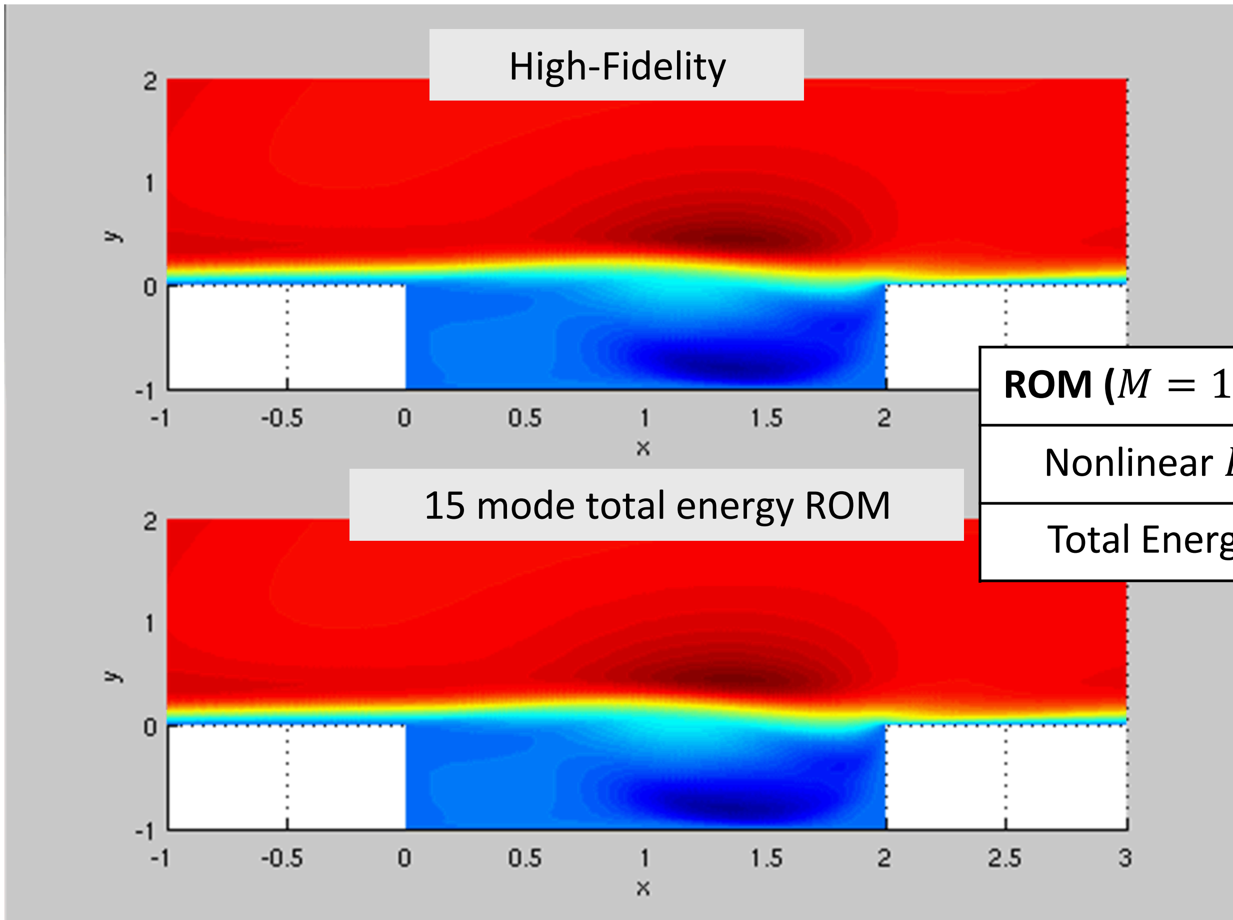

Figure above: u-component of velocity as a function of time t

- L^2 ROM exhibited instability for for $M > 5$ modes.

ROM ($M = 15$ modes)	**Error (L^2 norm)**
Nonlinear L^2 ROM	NaN
Total Energy ROM	5.52×10^{-2}

- In contrast, total energy ROM remained stable and agreed well with high-fidelity solution!

Summary & Future Work

- A Galerkin model reduction approach in which the ***continuous*** equations are projected onto the basis modes in a continuous inner product is proposed.
- It is shown that the choice of inner product for the Galerkin projection step is crucial to stability of the ROM.
  - For ***linearized compressible flow***, Galerkin projection in the "symmetry" inner product leads to a ROM that is energy-stable for any choice of basis.
  - For ***nonlinear compressible flow***, an inner product that induces the total energy of the fluid system is developed. A ROM constructed in this inner product will preserve the stability of an equilibrium point at 0 for the system.
- Results are promising for a nonlinear problem involving ***compressible viscous laminar flow*** over an open cavity: a total energy ROM remains stable whereas an L^2 ROM exhibits an instability.

Ongoing/Future Work

- Improve efficiency of nonlinear ROMs through interpolation (e.g., DEIM, gappy POD
- Studies of predictive capabilities of ROMs (robustness w.r.t. parameter changes).

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Thank You! Questions?

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Some references on these ideas:

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