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MINLP formulation and solution strategies for network-constrained unit commitment with nonlinear AC models

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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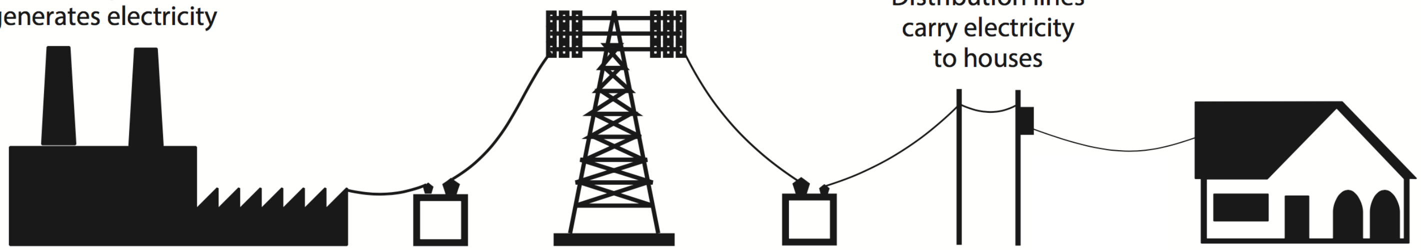
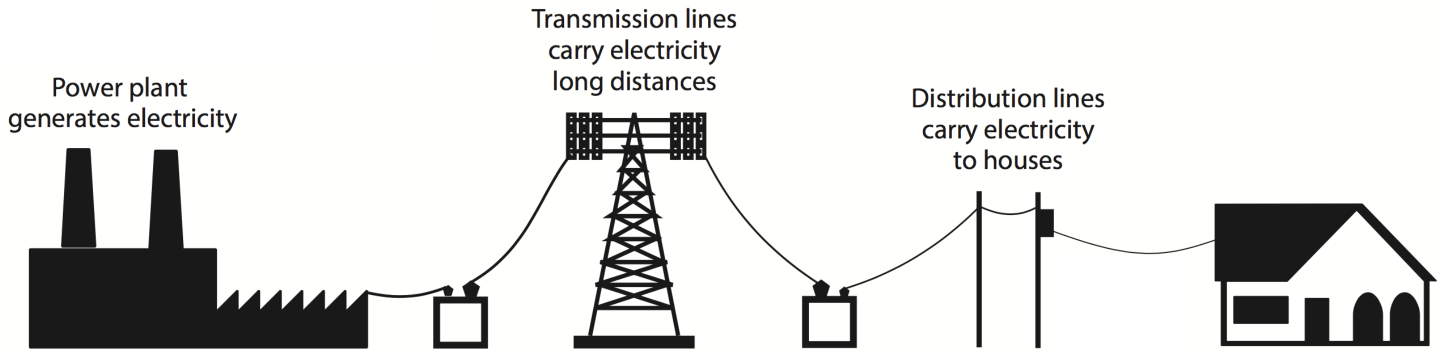
Electricity Supply and Delivery



[Source: National Energy Education Development Project]

Generation | Transmission | Distribution

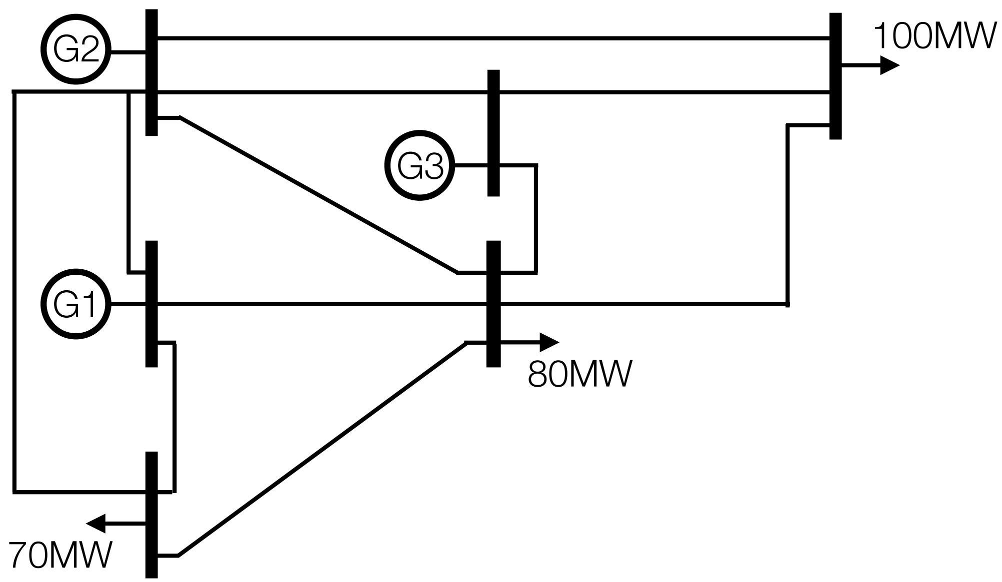
Electricity Supply and Delivery



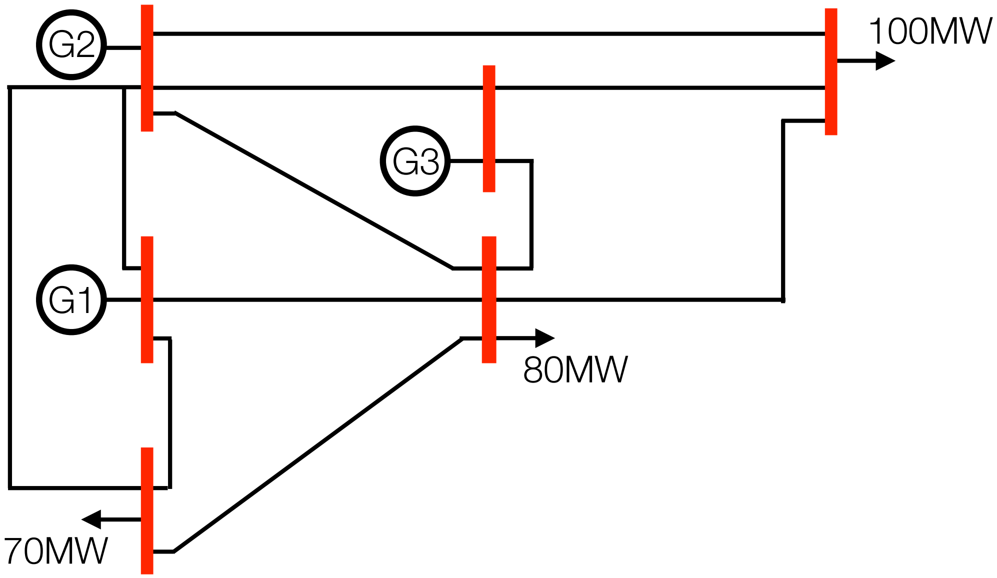
[Source: National Energy Education Development Project]

Generation	Transmission	Distribution
Wholesale: Independent System Operators (FERC)		Retail (States)

AC Transmission Network

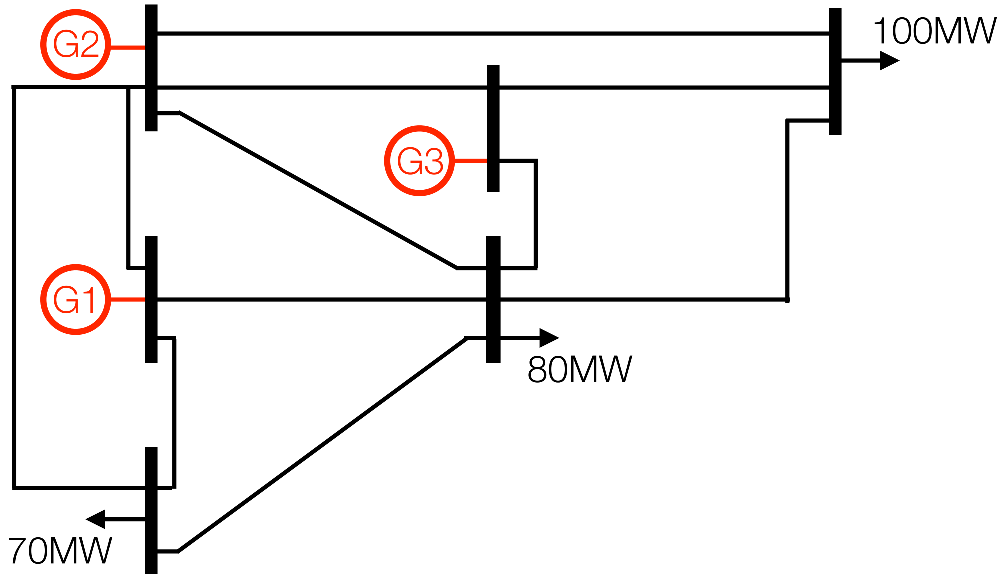


AC Transmission Network



6 Buses (or Nodes)

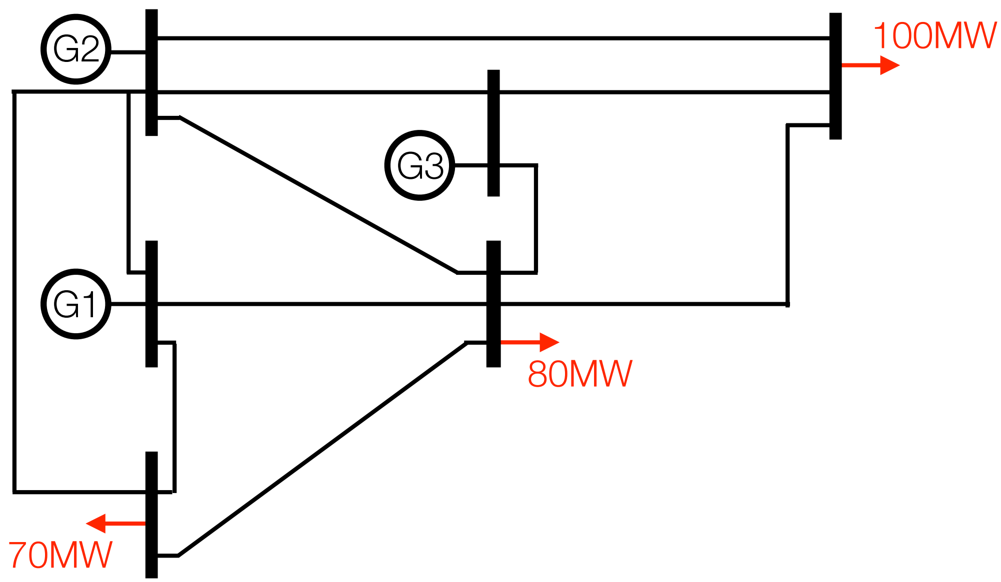
AC Transmission Network



6 Buses (or Nodes)

3 Generators (or Thermal Units)

AC Transmission Network

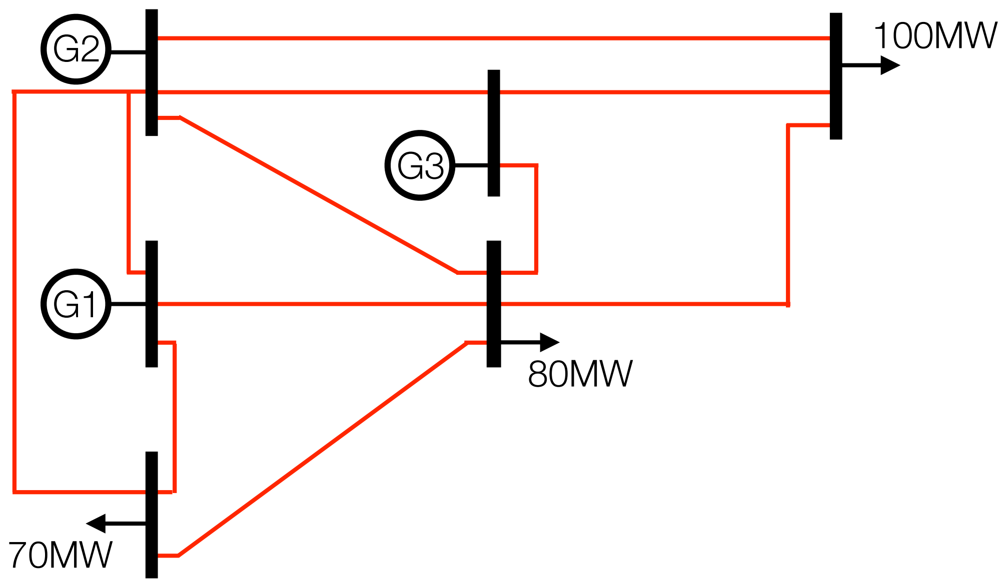


6 Buses (or Nodes)

3 Generators (or Thermal Units)

3 Demands (or Loads)

AC Transmission Network



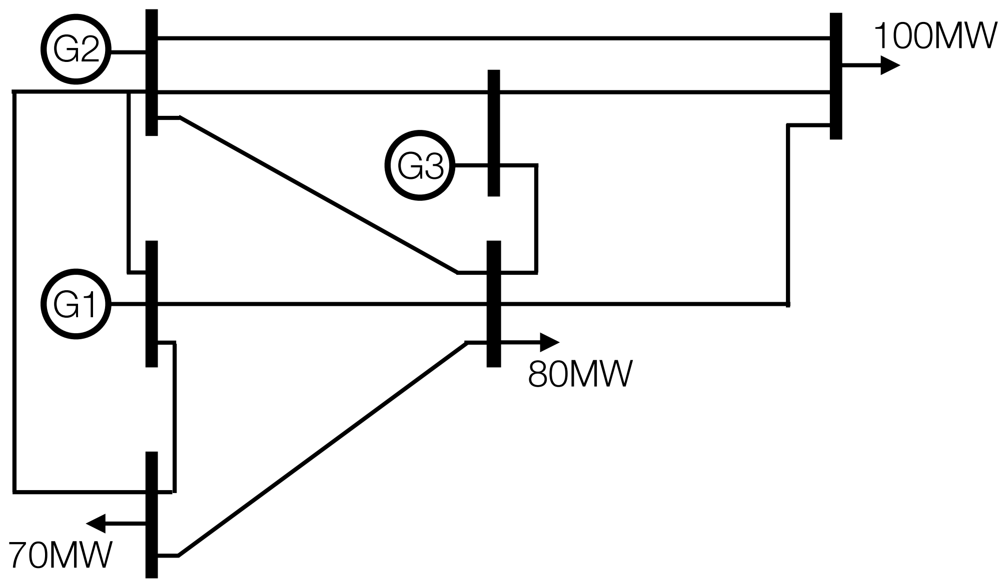
6 Buses (or Nodes)

3 Generators (or Thermal Units)

3 Demands (or Loads)

11 Branches (or Transmission Lines)

AC Optimal Power Flow (OPF) Problem



6 Buses (or Nodes)

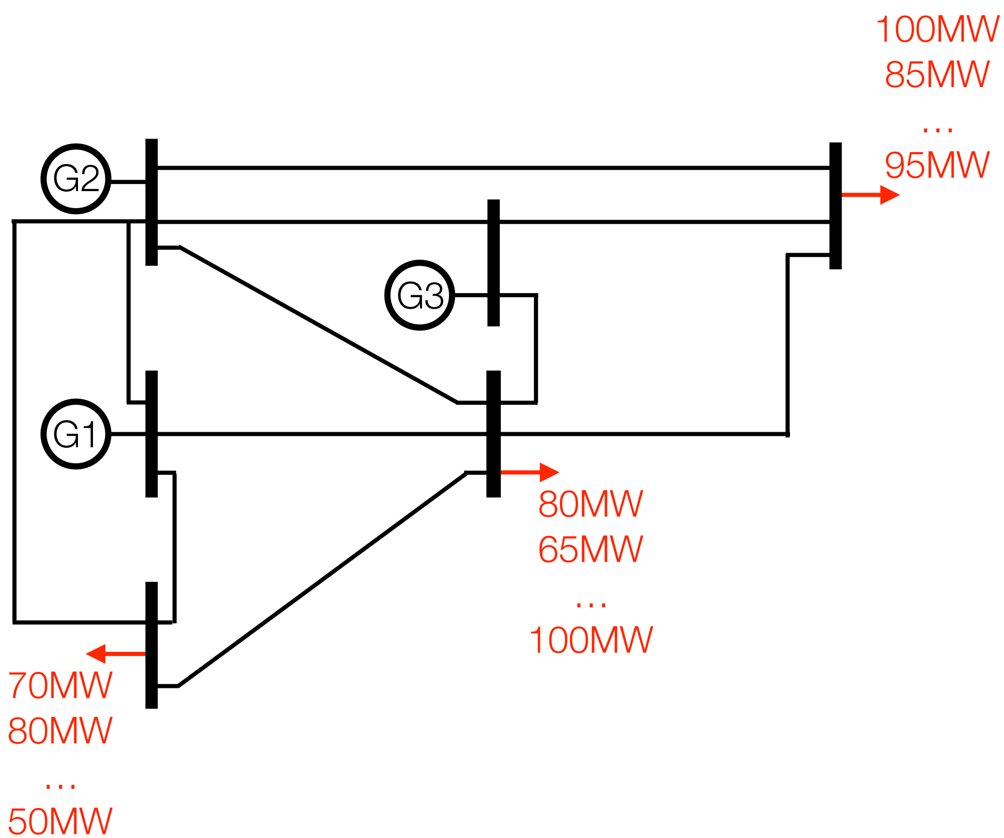
3 Generators (or Thermal Units)

3 Demands (or Loads)

11 Branches (or Transmission Lines)

ACOPF: seeking an optimal operation condition (power production for each generator) by minimizing a certain cost (generation cost) subject to network physical limits and constraints

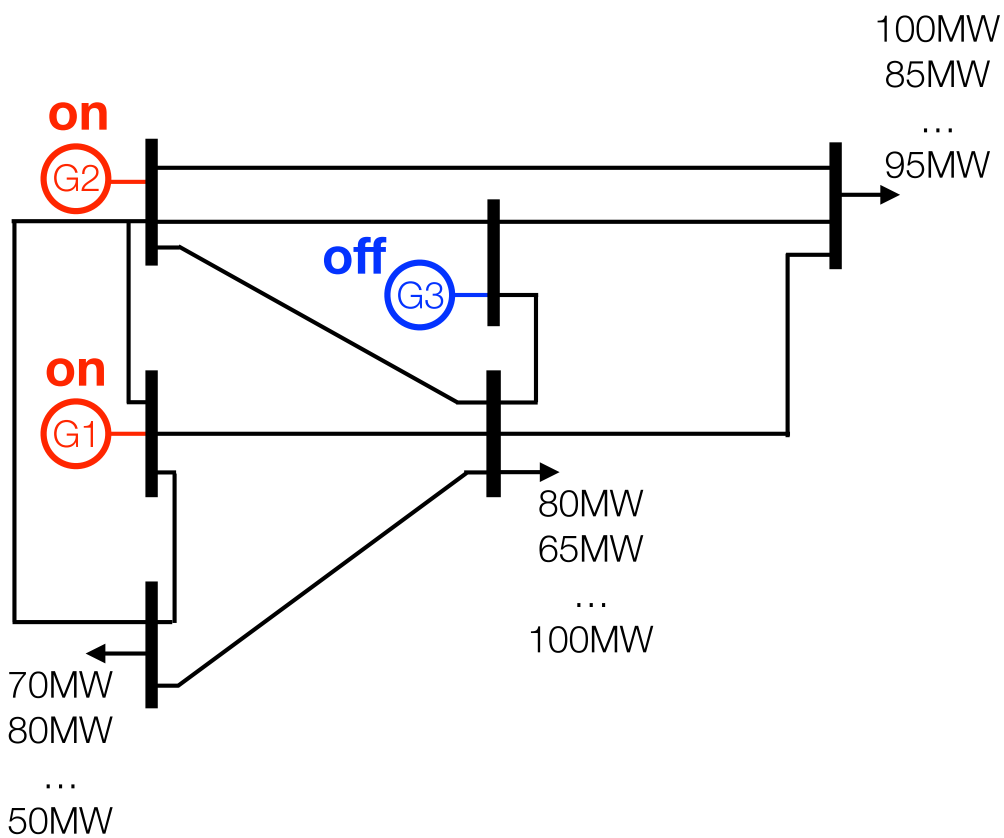
Network-Constrained Unit Commitment (NCUC) Problem



Multiple Time Periods:

24, 48, 96 hours

Network-Constrained Unit Commitment (NCUC) Problem



Multiple Time Periods:

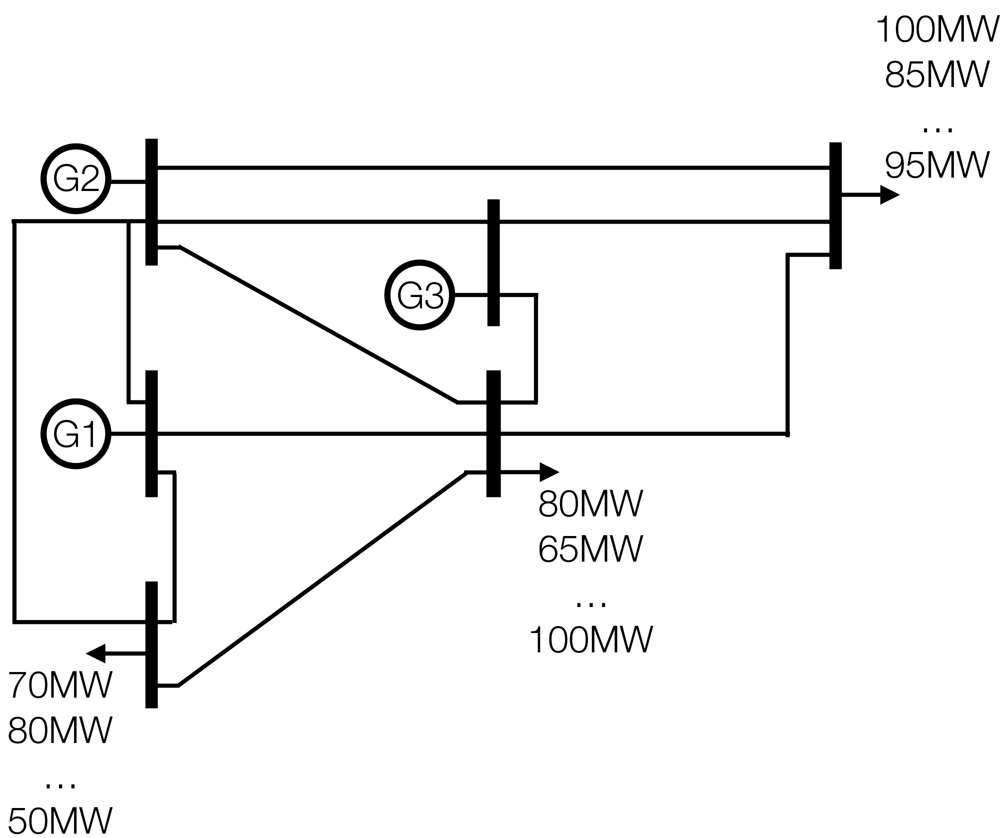
24, 48, 96 hours

Generator States:

on: $P \in [P_{min}, P_{max}]$, $Q \in [Q_{min}, Q_{max}]$

off: $P = 0$; $Q = 0$

Network-Constrained Unit Commitment (NCUC) Problem



Multiple Time Periods:

24, 48, 96 hours

Generator States:

on: $P \in [P_{min}, P_{max}]$, $Q \in [Q_{min}, Q_{max}]$

off: $P = 0$; $Q = 0$

NCUC: seeking an optimal **schedule** for each generator by minimizing a certain cost (generation cost) subject to network physical limits and constraints

ISO OPF Applications

- Real-time economic dispatch
  - Run every hour (with 5-15 minute dispatch)
  - Generator commitment (discrete decisions) fixed
  - Solve for generator setpoints
  - LP/QP formulation: DCOPF model
    - DCOPF: Linearized model, real power flows only, might include losses
    - AC feasibility tests

- Day-ahead unit commitment
  - Determine the On/Off schedule and set points for generation units (based on forecast)
  - Generator startup/shutdown/up/down time constraints, ramping limits
  - MIP formulation: NCUC+DCOPF model
    - DCOPF: Linearized model, real power flows only, might include losses
    - AC feasibility tested on solution
  - Locational marginal pricing (LMP) determines spot prices for wholesale market
    - Dual variable on the real power balance at network buses
    - Considers marginal unit cost, network congestion, and power losses

- Desire for global solutions to these problems with nonlinear AC transmission models

Network-Constrained Unit Commitment (NCUC) Problem

NCUC is very challenging

Binary variables and transmission models

- mixed-integer nonlinear programming (MINLP)

Large scale

- real-world network: ~100 generators, ~1000 buses and branches
- multiple time periods: 24, 48, and 96 hours

Solution time limit

- Solved in minutes for rescheduling, global solutions

Outline

Network-Constrained Unit Commitment (NCUC) Problem

- ‘State-of-the-art’ UC skeleton
- Nonlinear AC transmission model
- Mixed-Integer Quadratically Constrained Quadratic Programming (MIQCQP)

Global Solution Strategy for NCUC

- Iterative algorithm based on adding integer cuts
- Solution for non-convex, nonlinear subproblems
- ‘Convex’ relaxations for mixed-integer master problems

Numerical Results

- Benchmark problems with 6, 24, and 118 buses

NCUC Problem Formulation

UC Skeleton + Transmission Model

Unit Commitment Skeleton

UC Skeleton

Cost Functions
production, startup, shutdown

Operation Constraints
minimum up/downtime, ramping up/down limits

Reserve Constraints
minimum power reservation

Bounds
power generation limits

+

Transmission Model

Unit Commitment Skeleton

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$$\min \quad f^p + f^{su} + f^{sd}$$

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \le c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

$$f^{su} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\tau \in \mathcal{S}_g} K_{g,\tau}^{su} \delta_{g,\tau,t}$$

$$f^{sd} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_g^{sd} w_{g,t}$$

$$\sum_{t'=t-T_g^u}^{t} u_{g,t'} \le y_{g,t} \qquad \forall\, g,\ t$$

$$\sum_{t'=t-T_g^d}^{t} w_{g,t'} \le 1 - y_{g,t} \quad \forall\, g,\ t$$

$$\sum_{b \in \mathcal{B}} P_{b,t}^D + P_t^R \le \sum_{g \in \mathcal{G}} P_{g,t}^a \quad \forall\, t$$

$$P_g^{G,min} y_{g,t} \le P_{g,t}^G \le P_{g,t}^a \le P_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_g^{G,min} y_{g,t} \le Q_{g,t}^G \le Q_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_{sc}^{SC,min} y_{sc,t} \le Q_{sc,t}^{SC} \le Q_{sc}^{SC,max} y_{sc,t} \quad \forall\, sc,\ t$$

$$\delta_{g,\tau.t} \le \sum_{t'=t-T_{g,\tau}^{su}}^{t+1-T_{g,\tau+1}^{su}} w_{g,t'} \quad \forall\, g,\ t,\ \tau$$

$$u_{g,t} = \sum_{\tau \in \mathcal{S}_g} \delta_{g,\tau,t} \qquad \forall\, g,\ t$$

$$y_{g,t} - y_{g,t-1} = u_{g,t} - w_{g,t} \quad \forall\, g,\ t$$

Cost Functions

Production Cost

Startup Cost

Shutdown Cost

Operation Constraints

Minimum Startup/Shutdown Time

Reserve Constraints

Minimum Power Reservation

Bounds

Power Generation Limits

Logical Constraints

Relations between Binaries

3-Binary Formulation (Morales, 2013)

Transmission Model

UC Skeleton		Transmission Model
Cost Functions production, startup, shutdown	+	**Power Balance** generation and demand
Operation Constraints minimum up/downtime, ramping up/down limits ✔		**Power Flow** real/reactive power power losses in the system
Reserve Constraints minimum power reservation		**Safety Constraints** thermal limit on transmission line
Bounds power generation limits		**Bounds** voltage, phase angle, …

Transmission Model

DC Power Flow

linear equations, LP (MILP)

approximation

poor solution quality

AC Power Flow

nonlinear equations, NLP (MINLP)

system complexities

simultaneous solutions

Transmission Model

Power Balance
generation and demand

Power Flow
real/reactive power
power losses in the system

Safety Constraints
thermal limit on transmission line

Bounds
voltage, phase angle, …

Transmission Model

DC Power Flow

linear equations, LP (MILP)

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Transmission Model

Power Balance
generation and demand

Power Flow
real/reactive power
power losses in the system

Safety Constraints
thermal limit on transmission line

Bounds
voltage, phase angle, …

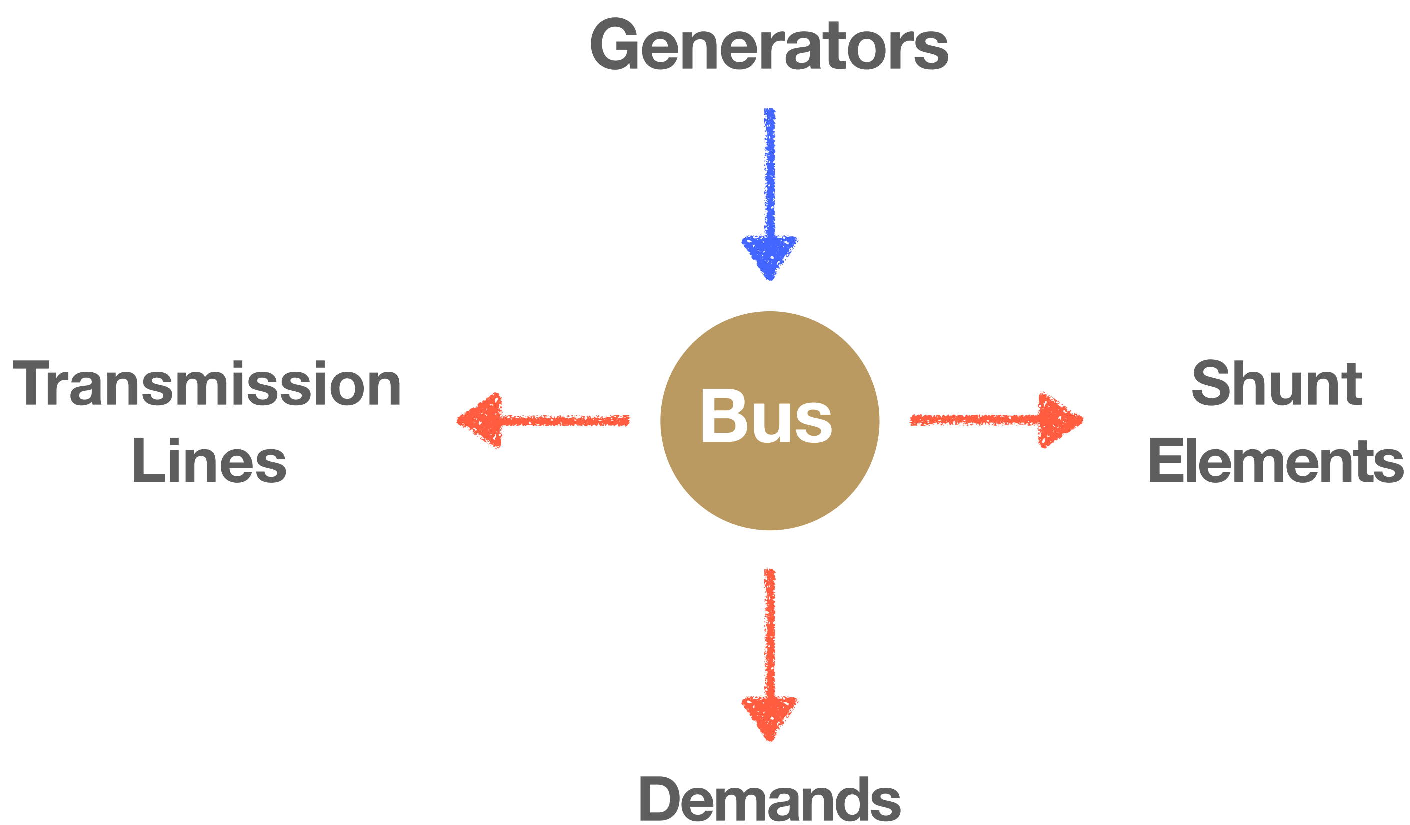
Rectangular Power-Voltage (PQV) Model

Power Balance

$$\sum_{l\in\mathcal{L}_b^{in}} P^t_{l,t} + \sum_{l\in\mathcal{L}_b^{out}} P^f_{l,t} + G_b^{sh} v_{b,t}^2 + P^D_{b,t} - \sum_{g\in\mathcal{G}_b} P^G_{g,t} = 0 \qquad \forall\, b,\ t$$

$$\sum_{l\in\mathcal{L}_b^{in}} Q^t_{l,t} + \sum_{l\in\mathcal{L}_b^{out}} Q^f_{l,t} - B_b^{sh} v_{b,t}^2 + Q^D_{b,t} - \sum_{g\in\mathcal{G}_b} Q^G_{g,t} - \sum_{sc\in\mathcal{SC}_b} \underline{Q^{SC}_{sc,t}} = 0 \qquad \forall\, b,\ t$$

Synchronous Condensers



Rectangular Power-Voltage (PQV) Model

Power Balance

$$\sum_{l\in\mathcal{L}_b^{in}} P^t_{l,t} + \sum_{l\in\mathcal{L}_b^{out}} P^f_{l,t} + G^{sh}_b v^2_{b,t} + P^D_{b,t} - \sum_{g\in\mathcal{G}_b} P^G_{g,t} = 0 \qquad \forall\, b,\ t$$

$$\sum_{l\in\mathcal{L}_b^{in}} Q^t_{l,t} + \sum_{l\in\mathcal{L}_b^{out}} Q^f_{l,t} - B^{sh}_b v^2_{b,t} + Q^D_{b,t} - \sum_{g\in\mathcal{G}_b} Q^G_{g,t} - \sum_{sc\in\mathcal{SC}_b} Q^{SC}_{sc,t} = 0 \qquad \forall\, b,\ t$$

Power Flow

$$P^f_{l,t} = G^{ff}_l v^2_{b,t} + G^{ft}_l (v^r_{b,t} v^r_{k,t} + v^j_{b,t} v^j_{k,t}) - B^{ft}_l (v^r_{b,t} v^j_{k,t} - v^j_{b,t} v^r_{k,t}) \qquad \forall\, l,\ t$$

$$Q^f_{l,t} = -B^{ff}_l v^2_{b,t} - B^{ft}_l (v^r_{b,t} v^r_{k,t} + v^j_{b,t} v^j_{k,t}) - G^{ft}_l (v^r_{b,t} v^j_{k,t} - v^j_{b,t} v^r_{k,t}) \qquad \forall\, l,\ t$$

$$P^t_{l,t} = G^{tt}_l v^2_{k,t} + G^{tf}_l (v^r_{k,t} v^r_{b,t} + v^j_{k,t} v^j_{b,t}) - B^{tf}_l (v^r_{k,t} v^j_{b,t} - v^j_{k,t} v^r_{b,t}) \qquad \forall\, l,\ t$$

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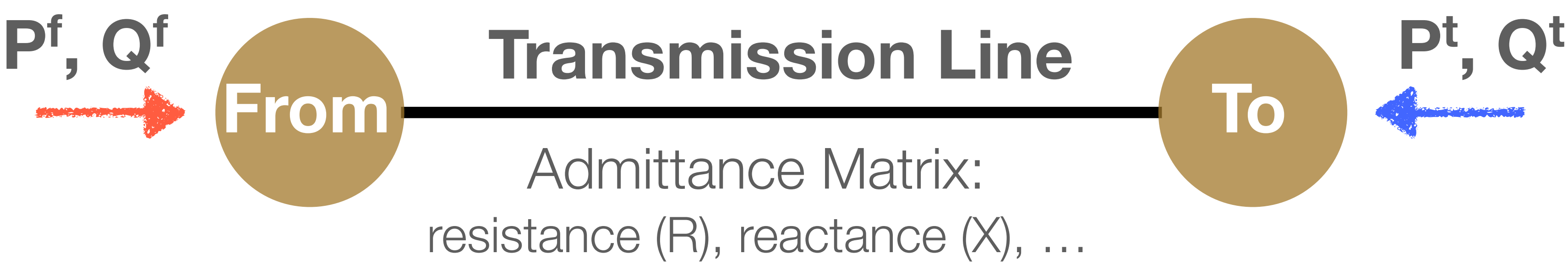

Rectangular Power-Voltage (PQV) Model

Power Balance

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

Power Flow

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - G_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - B_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^t = -B_l^{tt} v_{k,t}^2 - B_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - G_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

Safety Constraints

$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

Apparent Power

Rectangular Power-Voltage (PQV) Model

Power Balance

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

Power Flow

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

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$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

Bounds

$$(v_b^{min})^2 \le v_{b,t}^2 \le (v_b^{max})^2 \qquad \forall\, b,\ t$$

$$v_{b,t}^2 \equiv (v_{b,t}^r)^2 + (v_{b,t}^j)^2 \qquad \forall\, b,\ t$$

NCUC Problem Formulation

UC Skeleton: 3-Binary

Cost Functions
production, startup, shutdown

Operation Constraints
minimum up/downtime, ramping
up/down limits

Reserve Constraints
minimum power reservation

Bounds
power generation limits

+

Transmission Model: AC

Power Balance
generation and demand

Power Flow
real/reactive power
power losses in the system

Safety Constraints
thermal limit on transmission line

Bounds
voltage, phase angle, …

NCUC Problem Formulation: MIQCQP

UC Skeleton: 3-Binary

$$\min \quad f^p + f^{su} + f^{sd}$$

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \le c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

$$f^{su} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\tau \in \mathcal{S}_g} K_{g,\tau}^{su} \delta_{g,\tau,t}$$

$$f^{sd} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_g^{sd} w_{g,t}$$

$$\sum_{t'=t-T_g^u}^{t} u_{g,t'} \le y_{g,t} \qquad \forall\, g,\ t$$

$$\sum_{t'=t-T_g^d}^{t} w_{g,t'} \le 1 - y_{g,t} \quad \forall\, g,\ t$$

$$\sum_{b \in \mathcal{B}} P_{b,t}^D + P_t^R \le \sum_{g \in \mathcal{G}} P_{g,t}^a \quad \forall\, t$$

$$P_g^{G,min} y_{g,t} \le P_{g,t}^G \le P_{g,t}^a \le P_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

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$$Q_{sc}^{SC,min} y_{sc,t} \le Q_{sc,t}^{SC} \le Q_{sc}^{SC,max} y_{sc,t} \quad \forall\, sc,\ t$$

$$\delta_{g,\tau.t} \le \sum_{t'=t-T_{g,\tau}^{su}}^{t+1-T_{g,\tau+1}^{su}} w_{g,t'} \quad \forall\, g,\ t,\ \tau$$

$$u_{g,t} = \sum_{\tau \in \mathcal{S}_g} \delta_{g,\tau,t} \qquad \forall\, g,\ t$$

$$y_{g,t} - y_{g,t-1} = u_{g,t} - w_{g,t} \quad \forall\, g,\ t$$

+

Transmission Model: AC

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - G_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - B_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

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$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(v_b^{min})^2 \le v_{b,t}^2 = (v_{b,t}^r)^2 + (v_{b,t}^j)^2 \le (v_b^{max})^2 \qquad \forall\, b,\ t$$

Mixed-Integer Quadratically-Constrained Quadratic Programming (MIQCQP)

Outline

Network-Constrained Unit Commitment (NCUC) Problem

- ‘State-of-the-art’ UC skeleton
- Nonlinear AC transmission model
- Mixed-Integer Quadratically Constrained Quadratic Programming (MIQCQP)

Global Solution Strategy for NCUC

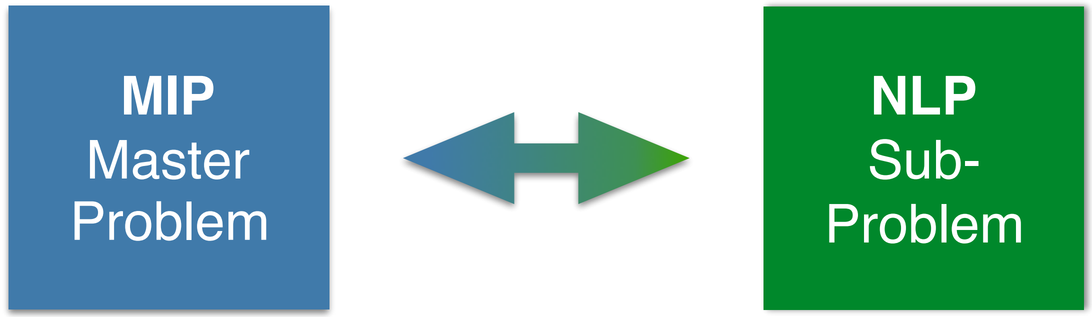
- Iterative algorithm based on adding integer cuts
- Solution for non-convex, nonlinear subproblems
- ‘Convex’ relaxations for mixed-integer master problems

Numerical Results

- Benchmark problems with 6, 24, and 118 buses

Global Optimization Framework for NCUC

Multi-Tree Algorithm



Iterations between Master Problem and Subproblem

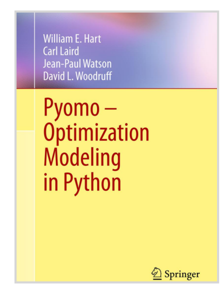
Master Problem: lower bound, relaxation of NCUC problem

Subproblem: upper bound, discrete decisions from the master problem

Implemented in Pyomo

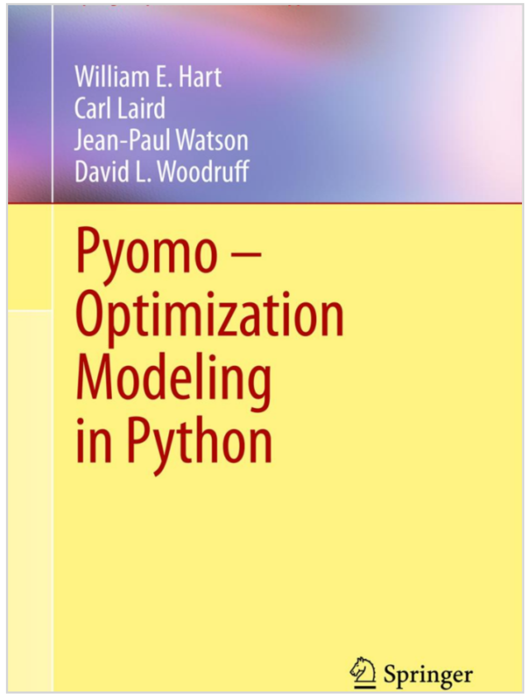
Flexible Modeling Language Based in Python

MIP Solver: GUROBI 6.5.2, NLP Solver: IPOPT 3.12.6



Modeling Power Systems with Pyomo

Algebraic Modeling → Pyomo → Existing Solvers



Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

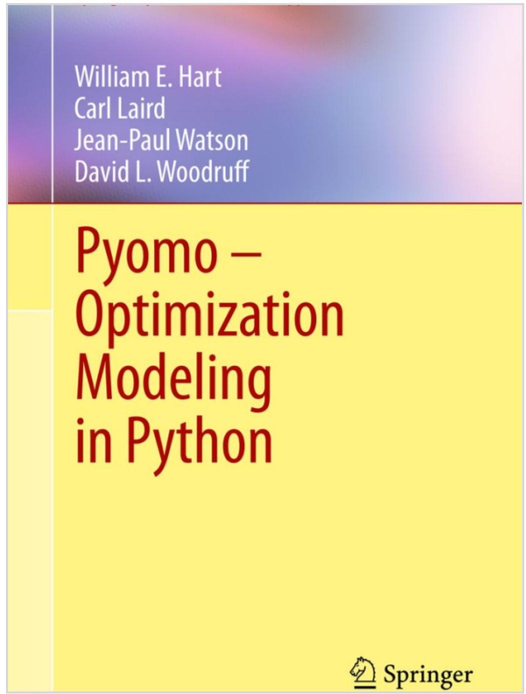
Modeling Power Systems with Pyomo

Algebraic Modeling → Pyomo → IPOPT

Power System Model Formulation & Solution System Analysis → Algebraic Modeling

Matpower & Other Input Formats (PSSE) → Power System Model Formulation & Solution System Analysis

Integrated Python-based toolchain

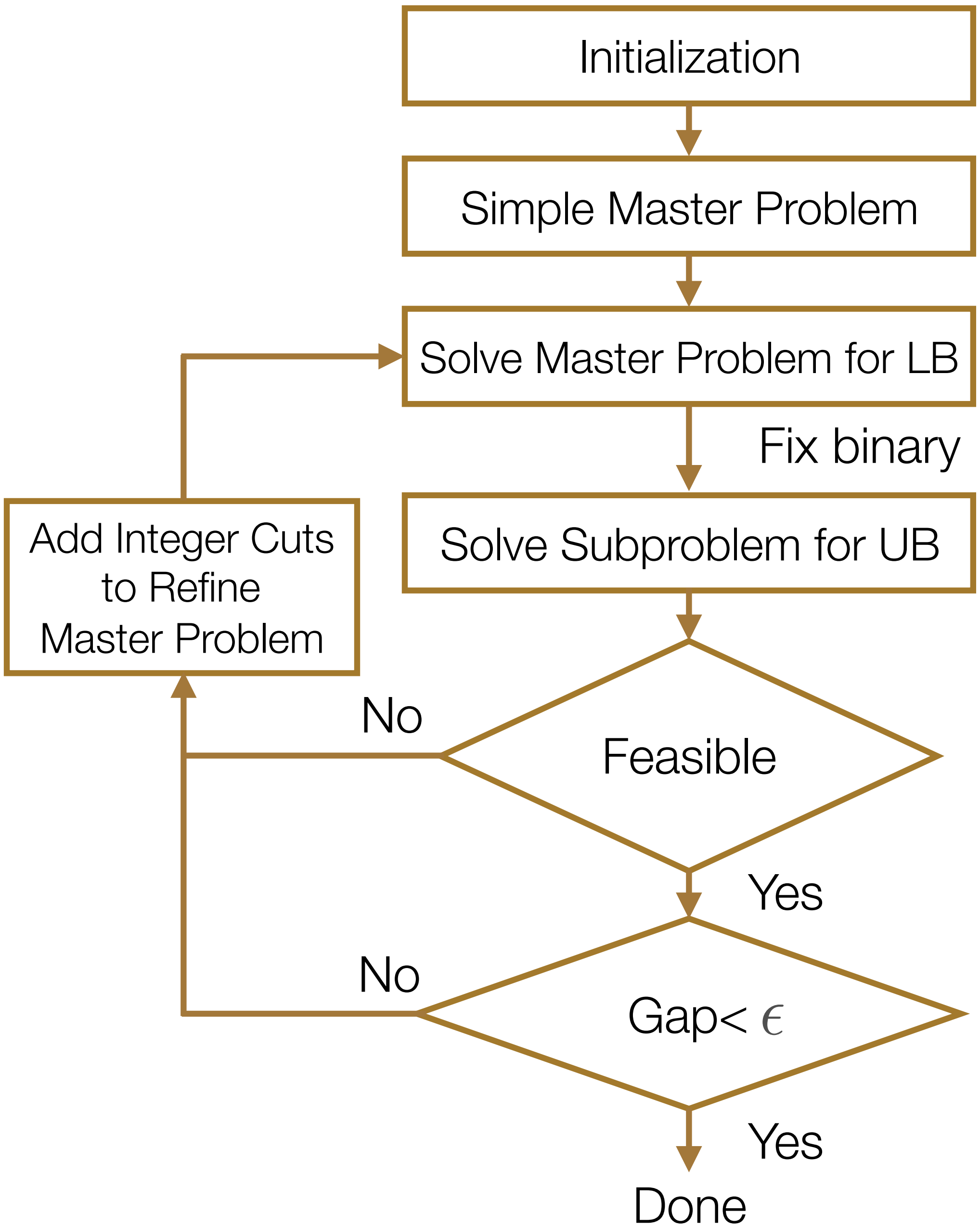


Hackebeil, Hart, Laird, Siirola, Watson, Woodruff, and many others…

ACOPF Solution with Pyomo and Ipopt

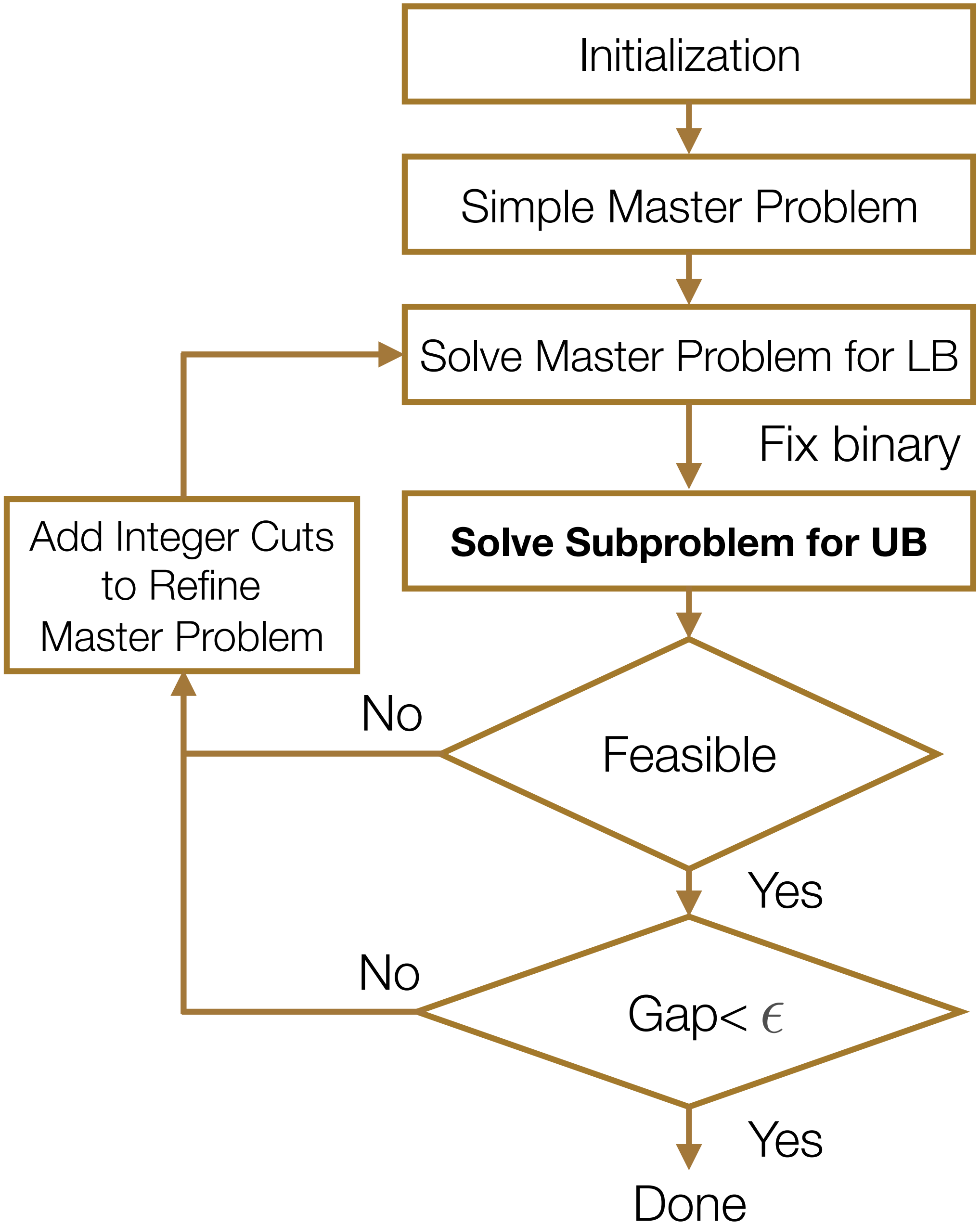
Case Name	Number of Variables	Solution Time (CPU seconds)
case30	46	2.410E-05
case9	95	-2.931E-04
case9Q	95	-4.806E-04
case6ww	105	-7.390E-05
case14	197	5.000E-03
case30Q	399	-3.940E-05
case_ieee30	399	-8.082E-04
case24_ieee_rts	416	-5.421E-03
case39	465	2.282E-04
case57	767	8.225E-04
case118	1,831	3.700E-02
case2383wp	28,456	2.200E+00
case2737sop	33,742	1.139E-03
case2736sp	33,807	-5.044E-03
case2746wop	34,013	-1.544E-04
case2746wp	34,063	-6.135E-03
case3012wp	35,242	-1.976E-02
case3120sp	36,247	-1.149E-02

Global Optimization Framework for NCUC



Global Optimization Framework for NCUC

Sandia National Laboratories



Tight Relaxation?

Solving nonlinear, non-convex subproblem to global optimality?

NCUC Problem Formulation: MIQCQP

UC Skeleton

$$\min \quad f^p + f^{su} + f^{sd}$$

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \leq c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

$$f^{su} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\tau \in \mathcal{S}_g} K_{g,\tau}^{su} \delta_{g,\tau,t}$$

$$f^{sd} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_g^{sd} w_{g,t}$$

$$\sum_{t'=t-T_g^u}^{t} u_{g,t'} \leq y_{g,t} \qquad \forall\, g,\ t$$

$$\sum_{t'=t-T_g^d}^{t} w_{g,t'} \leq 1 - y_{g,t} \quad \forall\, g,\ t$$

$$\sum_{b \in \mathcal{B}} P_{b,t}^D + P_t^R \leq \sum_{g \in \mathcal{G}} P_{g,t}^a \quad \forall\, t$$

$$P_g^{G,min} y_{g,t} \leq P_{g,t}^G \leq P_{g,t}^a \leq P_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_g^{G,min} y_{g,t} \leq Q_{g,t}^G \leq Q_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_{sc}^{SC,min} y_{sc,t} \leq Q_{sc,t}^{SC} \leq Q_{sc}^{SC,max} y_{sc,t} \quad \forall\, sc,\ t$$

$$\delta_{g,\tau.t} \leq \sum_{t'=t-T_{g,\tau}^{su}}^{t+1-T_{g,\tau+1}^{su}} w_{g,t'} \quad \forall\, g,\ t,\ \tau$$

$$u_{g,t} = \sum_{\tau \in \mathcal{S}_g} \delta_{g,\tau,t} \qquad \forall\, g,\ t$$

$$y_{g,t} - y_{g,t-1} = u_{g,t} - w_{g,t} \quad \forall\, g,\ t$$

+

Transmission Model

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - G_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - B_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^t = -B_l^{tt} v_{k,t}^2 - B_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - G_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \leq (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \leq (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(v_b^{min})^2 \leq v_{b,t}^2 = (v_{b,t}^r)^2 + (v_{b,t}^j)^2 \leq (v_b^{max})^2 \qquad \forall\, b,\ t$$

Mixed-Integer Quadratically-Constrained Quadratic Programming (MIQCQP)

Subproblem: Multi-Period AC OPF Problem

Sandia National Laboratories

UC Skeleton

$$\min \quad f^p + f^{su} + f^{sd}$$

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \leq c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

$$\cancel{f^{su} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\tau \in \mathcal{S}_g} K_{g,\tau}^{su} \delta_{g,\tau,t}}$$

$$\cancel{f^{sd} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_g^{sd} w_{g,t}}$$

$$\cancel{\sum_{t'=t-T_g^u}^{t} \alpha_{g,t'} \leq y_{g,t} \quad \forall\, g,\ t}$$

$$\cancel{\sum_{t'=t-T_g^d}^{t} w_{g,t'} \leq 1 - y_{g,t} \quad \forall\, g,\ t}$$

$$\cancel{\sum_{b \in \mathcal{B}} P_{b,t}^D + P_t^R \leq \sum_{g \in \mathcal{G}} P_{g,t}^a \quad \forall\, t}$$

$$P_g^{G,min} y_{g,t} \leq P_{g,t}^G \leq P_{g,t}^a \leq P_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_g^{G,min} y_{g,t} \leq Q_{g,t}^G \leq Q_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_{sc}^{SC,min} y_{sc,t} \leq Q_{sc,t}^{SC} \leq Q_{sc}^{SC,max} y_{sc,t} \quad \forall\, sc,\ t$$

$$\cancel{\delta_{g,\tau.t} \leq \sum_{t'=t-T_{g,\tau}^{su}}^{t+1-T_{g,\tau+1}^{su}} w_{g,t'} \quad \forall\, g,\ t,\ \tau}$$

$$\cancel{\alpha_{g,t} = \sum_{\tau \in \mathcal{S}_g} \delta_{g,\tau,t} \quad \forall\, g,\ t}$$

$$\cancel{y_{g,t} - y_{g,t-1} = \alpha_{g,t} - w_{g,t} \quad \forall\, g,\ t}$$

$$+$$

Transmission Model

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - G_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - B_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^t = -B_l^{tt} v_{k,t}^2 - B_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - G_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \leq (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \leq (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(v_b^{min})^2 \leq v_{b,t}^2 = (v_{b,t}^r)^2 + (v_{b,t}^j)^2 \leq (v_b^{max})^2 \qquad \forall\, b,\ t$$

Fixed Binary Variables

Quadratically-Constrained Quadratic Programming (QCQP)

AC Transmission Model

Power Balance

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} \underline{v_{b,t}^2} + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} \underline{v_{b,t}^2} + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

Power Flow

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft} (\underline{v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j}) - B_l^{ft} (\underline{v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r}) \qquad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft} (\underline{v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j}) - G_l^{ft} (\underline{v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r}) \qquad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf} (\underline{v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j}) - B_l^{tf} (\underline{v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r}) \qquad \forall\, l,\ t$$

$$Q_{l,t}^t = -B_l^{tt} v_{k,t}^2 - B_l^{tf} (\underline{v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j}) - G_l^{tf} (\underline{v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r}) \qquad \forall\, l,\ t$$

Safety Constraints

$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \le (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

Bounds

$$(v_b^{min})^2 \le v_{b,t}^2 \le (v_b^{max})^2 \qquad \forall\, b,\ t$$

$$v_{b,t}^2 \equiv \underline{(v_{b,t}^r)^2 + (v_{b,t}^j)^2} \qquad \forall\, b,\ t$$

Convex Relaxations of AC Transmission Model

Semi-Definite Programming (SDP) (Bai, 2008, Levaei, 2012)

- Empirically exact for a wide range of problems
- No sufficient condition ensuring an exact SDP relaxation
- High computational burden and difficult to incorporate within UC model

Quadratic Convex (QC) Relaxations (Hijazi, 2013, Coffrin, 2015)

- Not uniformly better compared with SDP relaxations
- Tight relaxation depends on small voltage phase angle differences

Second-Order Cone Programming (SOCP) (Jabr, 2006, Kocuk, 2015)

- Competitive performance on many problems
- Computational efficiency

SOCP Relaxation of AC Transmission Model

Second-Order Cone Programming (SOCP) Relaxation
(Jabr, 2006, Kocuk, 2015)

$$c_{b,b,t} = (v^r_{b,t})^2 + (v^j_{b,t})^2 = v^2_{b,t}$$

$$c_{b,k,t} = v^r_{b,t}v^r_{k,t} + v^j_{b,t}v^j_{k,t} = |v_{b,t}||v_{k,t}|\cos\theta_{b,k,t}$$

$$s_{b,k,t} = v^r_{b,t}v^j_{k,t} - v^r_{k,t}v^j_{b,t} = -|v_{b,t}||v_{k,t}|\sin\theta_{b,k,t}$$

SOCP Relaxation of AC Transmission Model

Power Balance

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} c_{b,b,t} + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \quad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} c_{b,b,t} + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \quad \forall\, b,\ t$$

Power Flow

$$P_{l,t}^f = G_l^{ff} c_{b,b,t} + G_l^{ft} c_{b,k,t} - B_l^{ft} s_{b,k,t} \quad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} c_{b,b,t} - B_l^{ft} c_{b,k,t} - G_l^{ft} s_{b,k,t} \quad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} c_{k,k,t} + G_l^{tf} c_{b,k,t} + B_l^{tf} s_{b,k,t} \quad \forall\, l,\ t$$

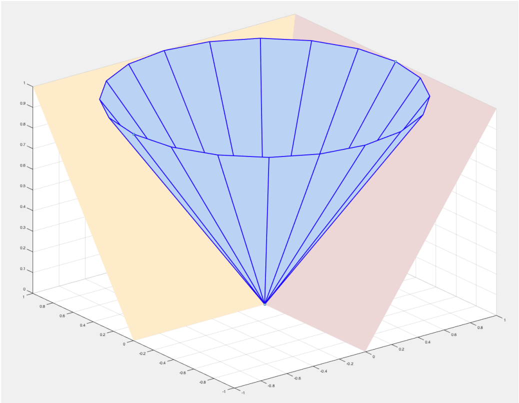
$$Q_{l,t}^t = -B_l^{tt} c_{k,k,t} - B_l^{tf} c_{b,k,t} + G_l^{tf} s_{b,k,t} \quad \forall\, l,\ t$$

Bounds

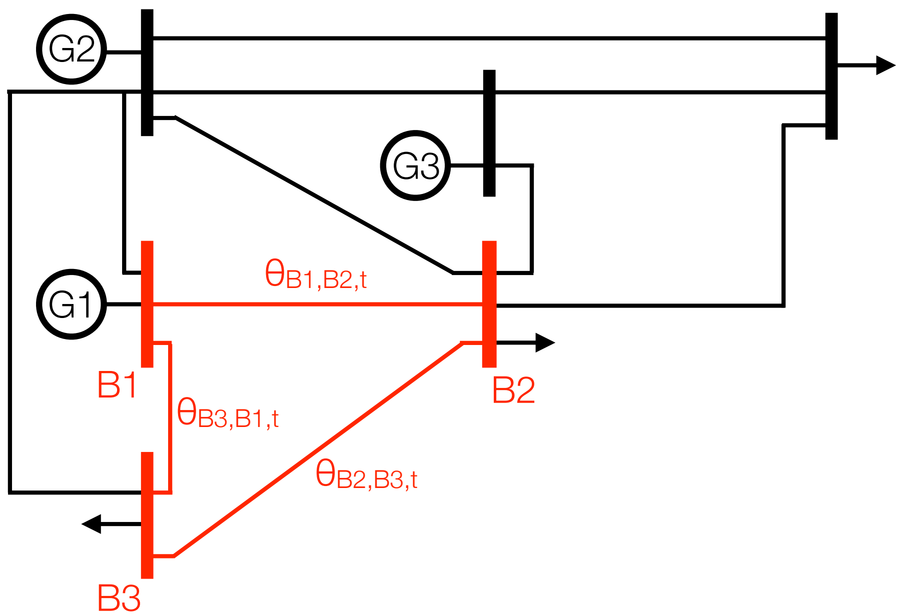
$$(v_b^{min})^2 \leq c_{b,b,t} \leq (v_b^{max})^2 \quad \forall\, b,\ t$$

Second-Order Cone Constraints

$$c_{b,k,t}^2 + s_{b,k,t}^2 \leq c_{b,b,t} c_{k,k,t} \quad \forall\, l,\ t$$



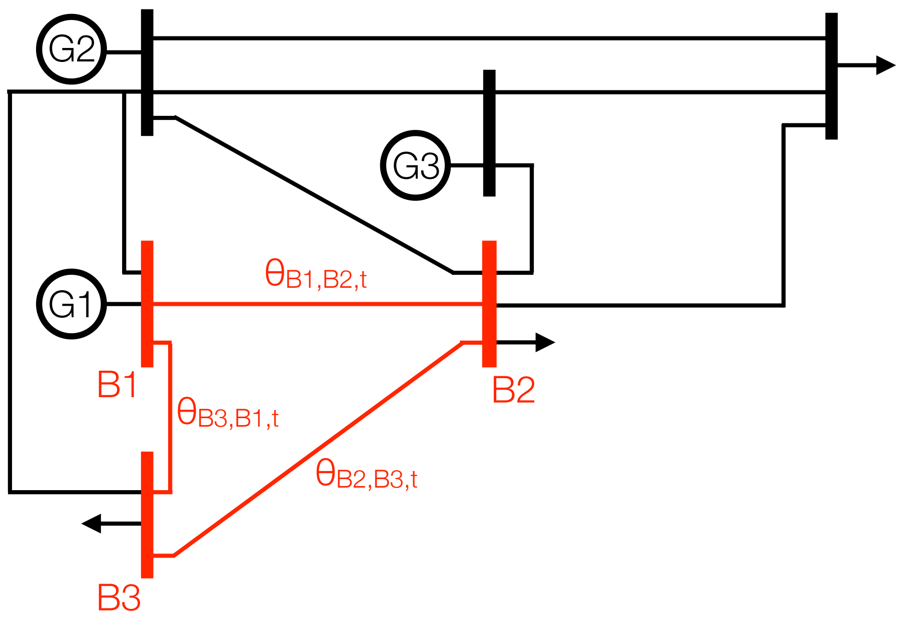
Strong SOCP Relaxation



Cycle Constraints:
the sum of angle differences on each cycle equals zero

$$\theta_{B1,B2,t} + \theta_{B2,B3,t} + \theta_{B3,B1,t} = 0$$

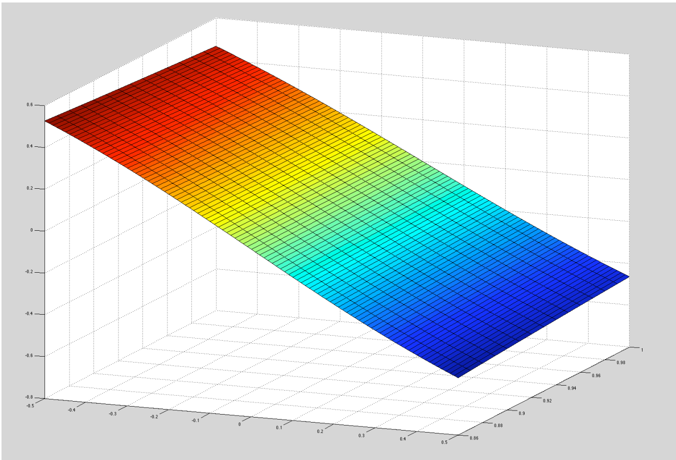
Strong SOCP Relaxation



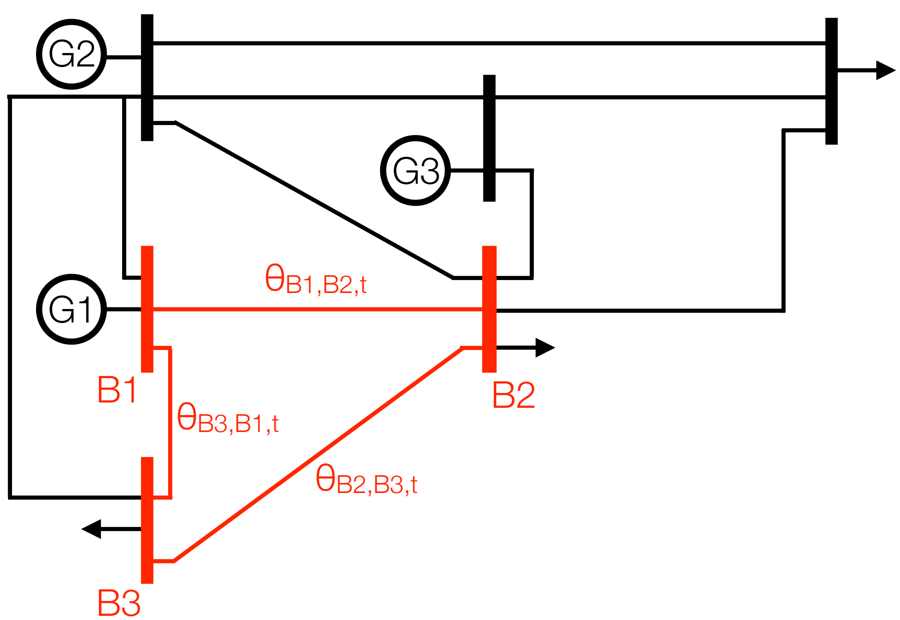
Cycle Constraints:
the sum of angle differences on each cycle equals zero

$$\theta_{B1,B2,t} + \theta_{B2,B3,t} + \theta_{B3,B1,t} = 0$$

$$\theta_{b,k,t} = -\arctan\left(\frac{s_{b,k,t}}{c_{b,k,t}}\right)$$



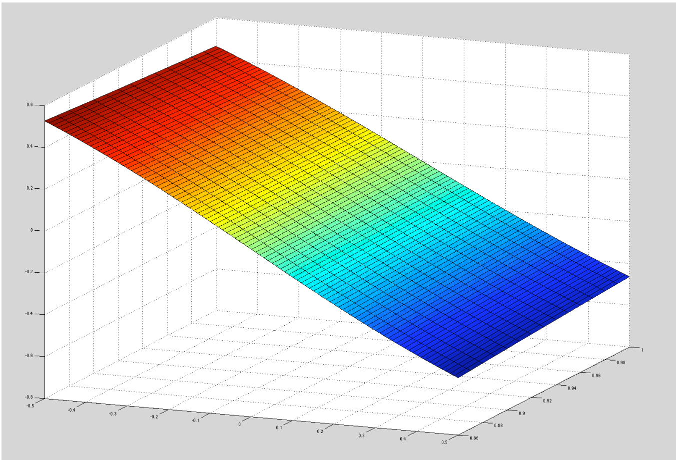
Strong SOCP Relaxation



Cycle Constraints:
the sum of angle differences on each cycle equals zero

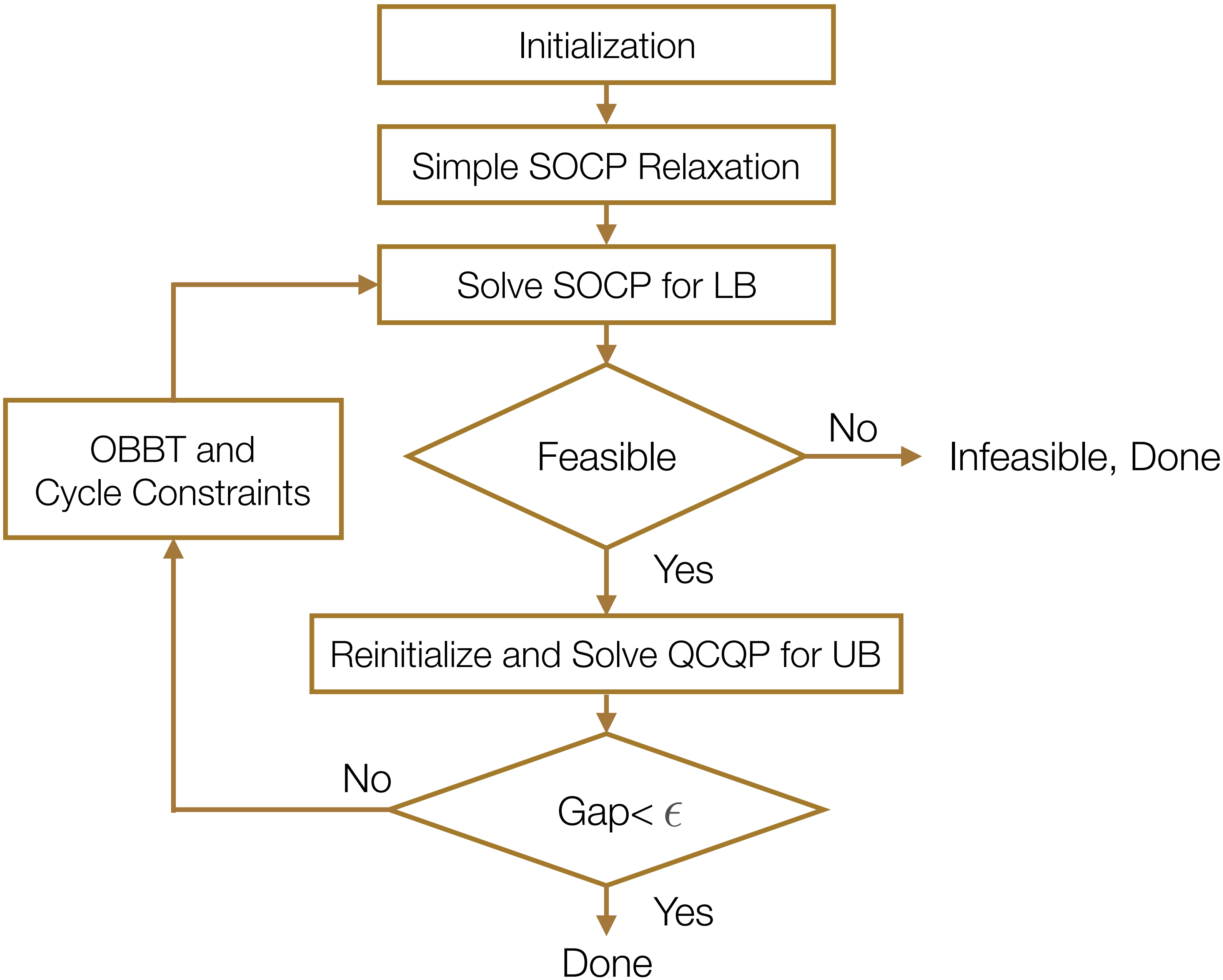
$$\theta_{B1,B2,t} + \theta_{B2,B3,t} + \theta_{B3,B1,t} = 0$$

$$\theta_{b,k,t} = -\arctan(\frac{s_{b,k,t}}{c_{b,k,t}})$$



Convex Relaxation of arctan:
Linear Over- and Under-Estimators
Optimality-Based Bound Tightening (OBBT)
Gradually Adding Cycle Constraints

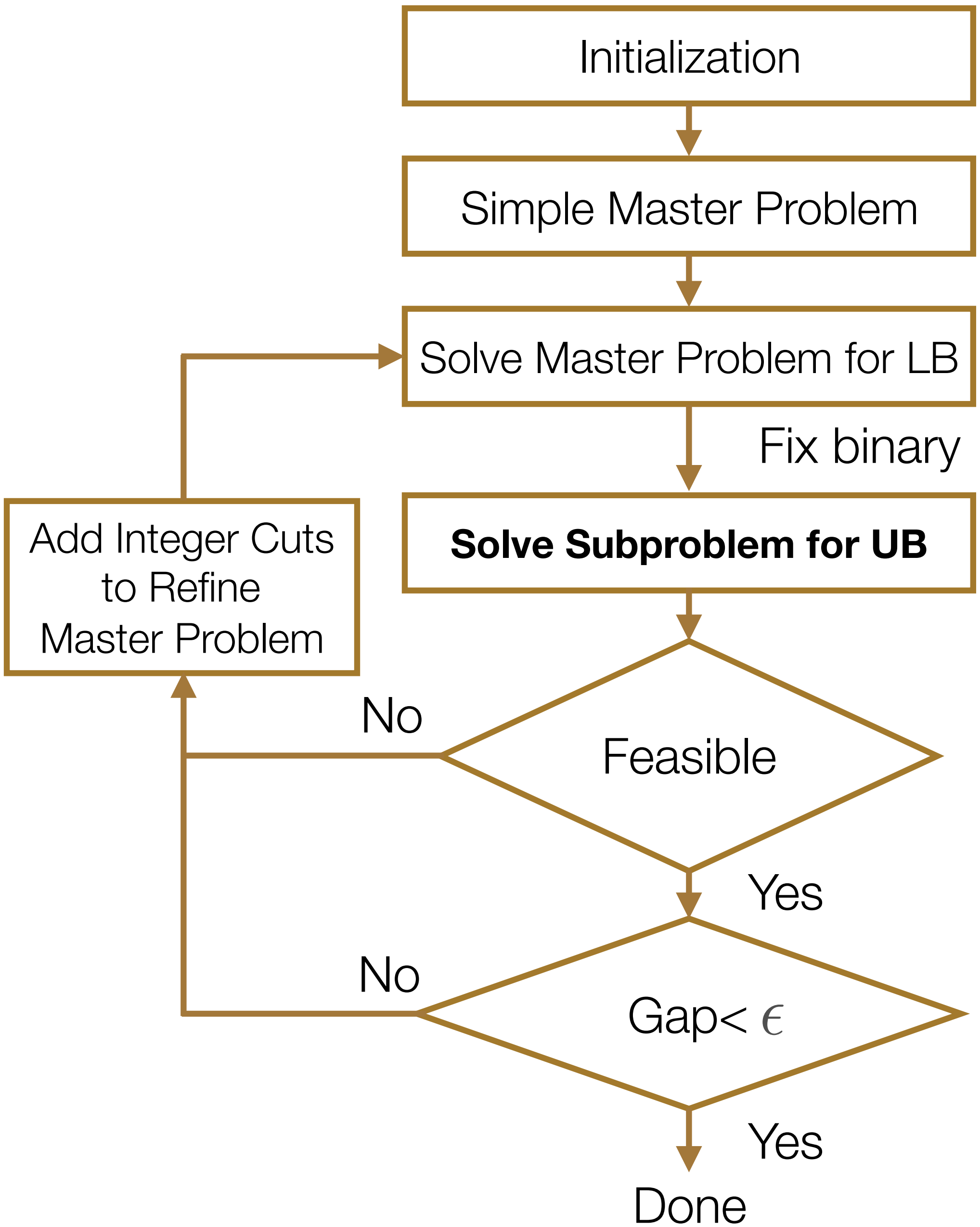
Global Sub-Algorithm for Subproblem



Numerical Results (Sub-Algorithm)

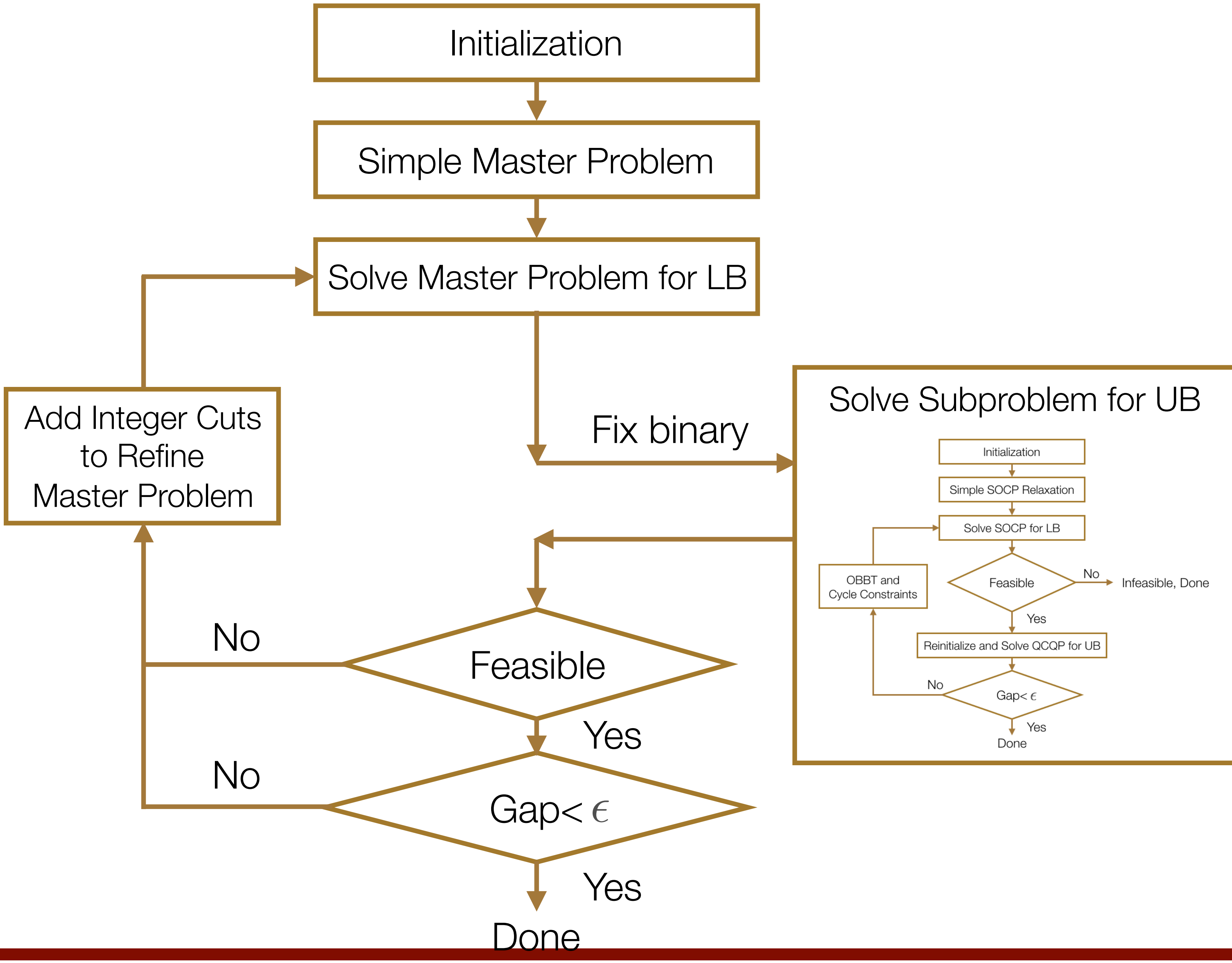
Case Name	Optimal Solution	Optimality Gap (%)	CPU Time (s)	Iterations
Case6ww	3126.36	8×10^{-3}	0.85	4
Case14	8081.52	4×10^{-3}	1.00	3
Case30	574.52	7×10^{-3}	1.18	6
Case39	41864.18	6×10^{-3}	1.60	3
Case57	41737.79	9×10^{-3}	9.16	13
Case118	129660.69	4×10^{-3}	37.2	26
Case300	719725.10	8×10^{-3}	257.0	36
NESTA Case6ww	3143.97	0.0	1.04	7
NESTA Case14	244.05	2×10^{-3}	0.51	3
NESTA Case30	204.97	0.0	2.53	11
NESTA Case39	96505.52	1×10^{-2}	9.89	13
NESTA Case57	1143.27	9×10^{-3}	9.65	20
NESTA Case118	3718.64	0.0	48.9	41
NESTA Case300	16891.28	0.0	136.6	45

Global Optimization Framework for NCUC

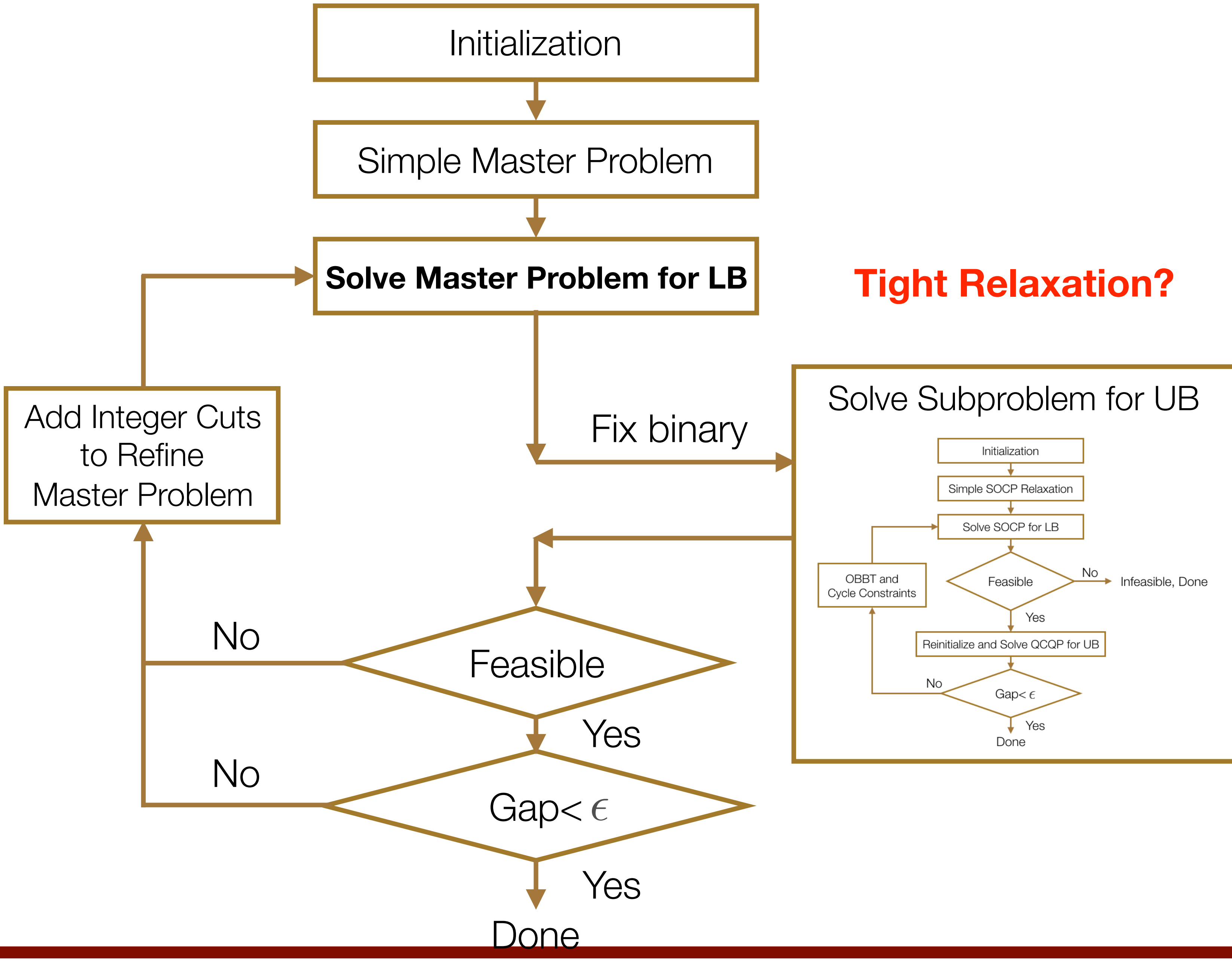


Solving nonlinear, non-convex subproblem to global optimality? ✔

Global Optimization Framework for NCUC



Global Optimization Framework for NCUC



NCUC Problem Formulation: MIQCQP

UC Skeleton

$$\min \quad f^p + f^{su} + f^{sd}$$

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \leq c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

$$f^{su} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\tau \in \mathcal{S}_g} K_{g,\tau}^{su} \delta_{g,\tau,t}$$

$$f^{sd} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_g^{sd} w_{g,t}$$

$$\sum_{t'=t-T_g^u}^{t} u_{g,t'} \leq y_{g,t} \qquad \forall\, g,\ t$$

$$\sum_{t'=t-T_g^d}^{t} w_{g,t'} \leq 1 - y_{g,t} \quad \forall\, g,\ t$$

$$\sum_{b \in \mathcal{B}} P_{b,t}^D + P_t^R \leq \sum_{g \in \mathcal{G}} P_{g,t}^a \quad \forall\, t$$

$$P_g^{G,min} y_{g,t} \leq P_{g,t}^G \leq P_{g,t}^a \leq P_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_g^{G,min} y_{g,t} \leq Q_{g,t}^G \leq Q_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_{sc}^{SC,min} y_{sc,t} \leq Q_{sc,t}^{SC} \leq Q_{sc}^{SC,max} y_{sc,t} \quad \forall\, sc,\ t$$

$$\delta_{g,\tau.t} \leq \sum_{t'=t-T_{g,\tau}^{su}}^{t+1-T_{g,\tau+1}^{su}} w_{g,t'} \quad \forall\, g,\ t,\ \tau$$

$$u_{g,t} = \sum_{\tau \in \mathcal{S}_g} \delta_{g,\tau,t} \qquad \forall\, g,\ t$$

$$y_{g,t} - y_{g,t-1} = u_{g,t} - w_{g,t} \quad \forall\, g,\ t$$

$+$

Transmission Model

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t$$

$$P_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - G_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - B_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$Q_{l,t}^t = -B_l^{tt} v_{k,t}^2 - B_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - G_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t$$

$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \leq (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \leq (S_{l,t}^{max})^2 \qquad \forall\, l,\ t$$

$$(v_b^{min})^2 \leq v_{b,t}^2 = (v_{b,t}^r)^2 + (v_{b,t}^j)^2 \leq (v_b^{max})^2 \qquad \forall\, b,\ t$$

Mixed-Integer Quadratically-Constrained Quadratic Programming (MIQCQP)

Relaxation NCUC-RQ

UC Skeleton

$$\min \quad f^p + f^{su} + f^{sd}$$

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \le c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

$$f^{su} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \sum_{\tau \in \mathcal{S}_g} K_{g,\tau}^{su} \delta_{g,\tau,t}$$

$$f^{sd} = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} K_g^{sd} w_{g,t}$$

$$\sum_{t'=t-T_g^u}^{t} u_{g,t'} \le y_{g,t} \qquad \forall\, g,\ t$$

$$\sum_{t'=t-T_g^d}^{t} w_{g,t'} \le 1 - y_{g,t} \quad \forall\, g,\ t$$

$$\sum_{b \in \mathcal{B}} P_{b,t}^D + P_t^R \le \sum_{g \in \mathcal{G}} P_{g,t}^a \quad \forall\, t$$

$$P_g^{G,min} y_{g,t} \le P_{g,t}^G \le P_{g,t}^a \le P_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_g^{G,min} y_{g,t} \le Q_{g,t}^G \le Q_g^{G,max} y_{g,t} \quad \forall\, g,\ t$$

$$Q_{sc}^{SC,min} y_{sc,t} \le Q_{sc,t}^{SC} \le Q_{sc}^{SC,max} y_{sc,t} \quad \forall\, sc,\ t$$

$$\delta_{g,\tau.t} \le \sum_{t'=t-T_{g,\tau}^{su}}^{t+1-T_{g,\tau+1}^{su}} w_{g,t'} \quad \forall\, g,\ t,\ \tau$$

$$u_{g,t} = \sum_{\tau \in \mathcal{S}_g} \delta_{g,\tau,t} \qquad \forall\, g,\ t$$

$$y_{g,t} - y_{g,t-1} = u_{g,t} - w_{g,t} \quad \forall\, g,\ t$$

$+$

Relaxed Transmission Model

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} c_{b,b,t} + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \qquad \forall\, b,\ t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} c_{b,b,t} + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \quad \forall\, b,\ t$$

$$P_{l,t}^f = G_l^{ff} c_{b,b,t} + G_l^{ft} c_{b,k,t} - B_l^{ft} s_{b,k,t} \quad \forall\, l,\ t$$

$$Q_{l,t}^f = -B_l^{ff} c_{b,b,t} - B_l^{ft} c_{b,k,t} - G_l^{ft} s_{b,k,t} \quad \forall\, l,\ t$$

$$P_{l,t}^t = G_l^{tt} c_{k,k,t} + G_l^{tf} c_{b,k,t} + B_l^{tf} s_{b,k,t} \quad \forall\, l,\ t$$

$$Q_{l,t}^t = -B_l^{tt} c_{k,k,t} - B_l^{tf} c_{b,k,t} + G_l^{tf} s_{b,k,t} \quad \forall\, l,\ t$$

$$(P_{l,t}^f)^2 + (Q_{l,t}^f)^2 \le (S_{l,t}^{max})^2 \quad \forall\, l,\ t$$

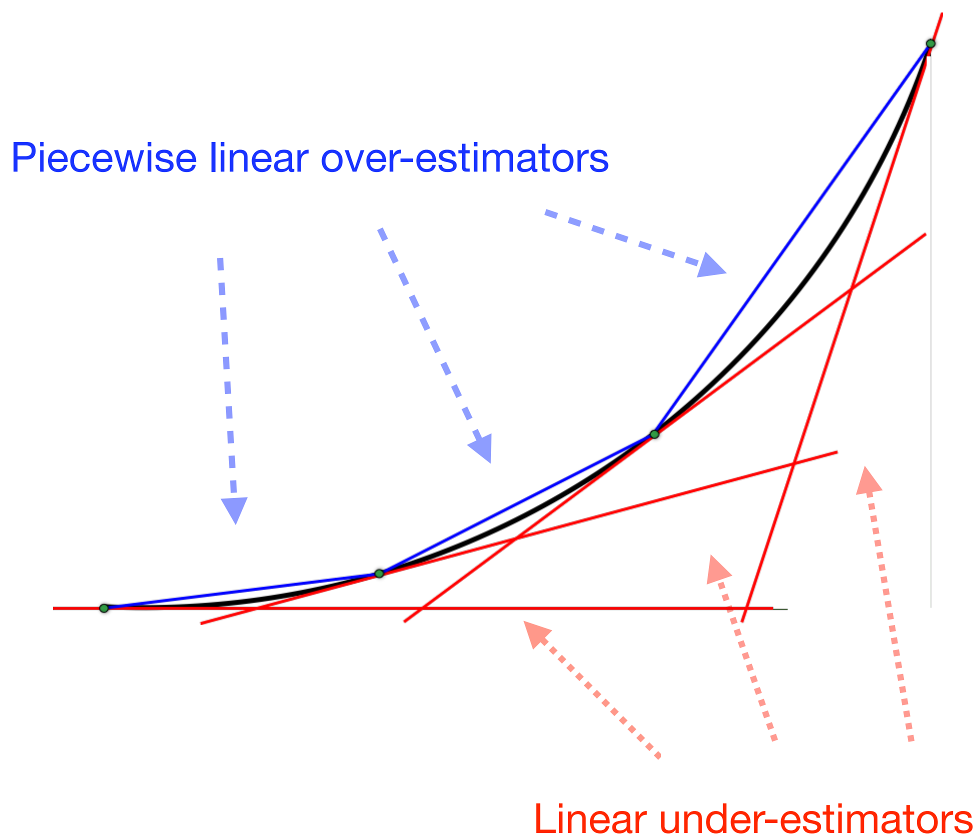
$$(P_{l,t}^t)^2 + (Q_{l,t}^t)^2 \le (S_{l,t}^{max})^2 \quad \forall\, l,\ t$$

$$(v_b^{min})^2 \le c_{b,b,t} \le (v_b^{max})^2 \quad \forall\, b,\ t$$

$$c_{b,k,t}^2 + s_{b,k,t}^2 \le c_{b,b,t} c_{k,k,t} \quad \forall\, l,\ t$$

Mixed-Integer Quadratically-Constrained Quadratic Programming (MIQCQP)

Piecewise relaxations



Quadratic Cost Functions

$$A_g^2 (P_{g,t}^G)^2 + A_g^1 P_{g,t}^G + A_g^0 y_{g,t} \le c_{g,t}^p \quad \forall\, g,\ t$$

$$f^p = \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} c_{g,t}^p,$$

Linear Under-Estimators

$$B_{g,t,n}^* P_{g,t}^G + C_{g,t,n}^* + A_g^0 y_{g,t} \le c_{g,t}^p \quad \forall\, g,\ t,\ n$$

$$f^p = \sum_{g \in G} \sum_{t \in T} c_{g,t}^p$$

Mixed-Integer Master Problems

	Formulation	Cost Function	Thermal Limits	Second-Order Cone Constraints
NCUC-RQ	‘Convex’ MIQCQP	Quadratic	Quadratic	Quadratic
NCUC-R	MISOCP	Linear Under-Estimators	—	Quadratic
NCUC-RL	MILP	Linear Under-Estimators	—	Linear Outer Approximations

Outline

Network-Constrained Unit Commitment (NCUC) Problem

- 'State-of-the-art' UC skeleton
- Nonlinear AC transmission model
- Mixed-Integer Quadratically Constrained Quadratic Programming (MIQCQP)

Global Solution Strategy for NCUC

- Iterative algorithm based on adding integer cuts
- Solution for non-convex, nonlinear subproblems
- 'Convex' relaxations for mixed-integer master problems

Numerical Results

- Benchmark problems with 6, 24, and 118 buses

Numerical Results

Sandia National Laboratories

Case	Formulation	Upper Bound	Lower Bound	Gap (%)	CPU Time (s)	Iteration
Case6	(NCUC-RQ)	101,763	101,655	0.11%	3.6	1
	(NCUC-R)	101,763	101,624	0.14%	3.7	1
	(NCUC-RL)-5	101,763	101,479	0.28%	11.8	1
	(NCUC-RL)-10	101,763	101,665	0.10%	47.0	1
RTS-79	(NCUC-RQ)	895,096	893,967	0.13%	266.4	1
	(NCUC-R)	895,040	894,247	0.09%	3274	16
	(NCUC-RL)-5	–	820,736	–	3755	30*
	(NCUC-RL)-10	–	886,783	–	36000*	19
RTS-96	(NCUC-RQ)	886,362	885,707	0.07%	321.0	1
	(NCUC-R)	886,759	885,759	0.11%	146.4	1
	(NCUC-RL)-5	–	813,401	–	4309	30*
	(NCUC-RL)-10	–	878,793	–	36000*	13
Case118	(NCUC-RQ)	835,926	833,057	0.34%	8480	1
	(NCUC-R)	835,996	833,207	0.33%	15705	3
	(NCUC-RL)-5	–	647,112	–	36000*	1
	(NCUC-RL)-10	–	823,684	–	36000*	1

NCUC-RQ: relaxation with quadratic cost functions and thermal limits
NCUC-R: relaxation with linear under-estimators of cost functions
NCUC-RL5 /10: linear outer approximations with different segment points

Numerical Results

Case	Formulation	Upper Bound	Lower Bound	Gap (%)	CPU Time (s)	Iteration
Case6	(NCUC-RQ)	101,763	101,655	0.11%	3.6	1
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	(NCUC-RL)-10	101,763	101,665	0.10%	47.0	1
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NCUC-RQ: relaxation with quadratic cost functions and thermal limits
NCUC-R: relaxation with linear under-estimators of cost functions
NCUC-RL5 /10: linear outer approximations with different segment points

Conclusion and Future Work

Network-Constrained Unit Commitment (NCUC) Problem

‘State-of-the-art’ UC skeleton

Nonlinear AC transmission model

Tailored Global Solution Strategy for NCUC

Global solution for non-convex, nonlinear subproblems

Three ‘convex’ relaxations for mixed-integer master problems

Efficient solution on benchmark problems with 6, 24, and 118 buses

Future Work

Possible enhancements of the tailored global algorithm

Modeling and optimization tool for general NCUC problems

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