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Investigating an implicit thermo-mechanical model implemented in FELIX

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Introduction:

We present an implicit three-dimensional thermal solver for polythermal ice based on the enthalpy formulation proposed by Aschwanden et. al. (2012) [1], with the addition of the gravity-driven moisture drainage model proposed by Hewitt and Schoof (2017) [2]. The thermal solver is implicitly coupled with the Blatter-Pattyn ice sheet flow model allowing ice velocity and temperature to be solved together directly at steady state.

Unlike the more traditional approach where the temperature diffusion and advection are splitted into a 2D horizontal advection and a 1d vertical diffusion operator, our solver is fully three dimensional.

We show results on simplified geometries and consider the case where the ice at the bed is frozen, melting or refreezing. We compare the basal temperature computed with our model for the Greenland ice sheet with the temperature map by MacGregor et al. [3].

Mathematical Model:

We model the temperature and porosity (water content) in terms of the enthalpy defined as

$$h = \rho c (T - T_0) + \rho_w L \phi$$

enthalpy temperature latent heat porosity (water content)

Relations between enthalpy, temperature and porosity are summarized in the table:

$$h_m := \rho c (T_m - T_0)$$

melting point enthalpy and temperature

	cold ice $h < h_m$	temperate ice $h \geq h_m$
T	$T = T_0 + \frac{1}{\rho c} h$	$T = T_m$
ϕ	0	$\frac{1}{\rho_w L}(h - h_m)$

Steady state **enthalpy** equation reads:

$$\mathbf{u} \cdot \nabla h + \nabla \cdot \mathbf{q} = \tau : \dot{\epsilon}$$

ice velocity dissipation heat

$$\mathbf{q} = \begin{cases} -k \nabla T & \text{cold} \\ -k \nabla T_m + \rho_w L \mathbf{j} & \text{temperate} \end{cases}$$

total flux

$$\mathbf{j} = \frac{k_0}{\eta_w} \phi^\alpha (\rho_w - \rho) \mathbf{g}$$

water flux

At the bed interface we have the **Stefan condition**:

$$\rho_w L\, m = G + \tau_b \cdot \mathbf{u} - k \nabla T \cdot \mathbf{n}$$

melting rate geothermal heat flux frictional heating

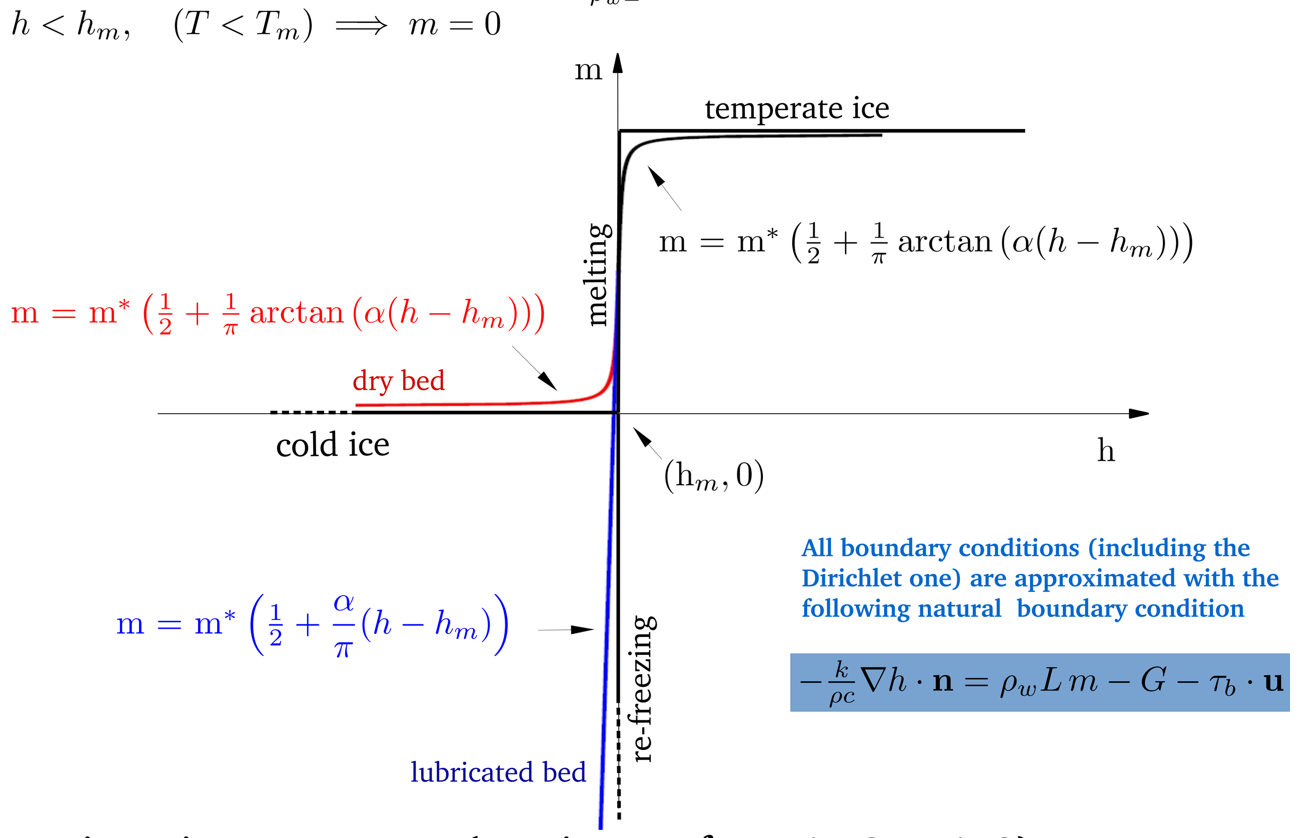
The enthalpy equation is coupled with an ice flow model. Temperature affects the ice viscosity and the vertical velocity b.c. through melting at the ice bed, and ice velocity is responsible for temperature advection and for the friction and dissipation heat terms. In our case, we couple the enthalpy equation with the FO equations.

Numerical challenges:

The enthalpy model presents several strong nonlinearities because of the phase changes. In particular, at the bed interface, ice can be cold, melting, re-freezing. Further, in the interior of the domain the ice can be cold or temperate. The former satisfies an elliptic equation, while the latter satisfies an hyperbolic equation. Modeling these different cases is challenging from a numerical point of view, especially for implicit large-scale solvers.

enthalpy/melting graph at bed

$$h > h_m, \quad (T = T_m) \implies m = m^* := \frac{1}{\rho_w L}\left(G + \tau_b \cdot \mathbf{u} + k\, \partial_z T_m\right)$$
$$h < h_m, \quad (T < T_m) \implies m = 0$$



α: continuation parameter (e.g. it goes form 1e-2 to 1e2)

Here we depict the enthalpy/melting graph at the bed. We choose to approximate numerically the graph with a smooth one shown. Depending on whether the bed is lubricated or not, we follow the blue or the red curve. We perform a **parameter continuation** in order to get close to the original diagram.

We also perform continuation on parameter β to smoothly transition from cold ice to temperate ice:

$$\mathbf{q} = \begin{cases} (1-\theta)\left(-\frac{k}{\rho c}\nabla h\right) & \text{cold} \\ \theta\left(-\frac{k}{\rho c}\nabla h_m\right) + \rho_w L \mathbf{j} & \text{temperate} \end{cases} \qquad \theta = \left(\frac{1}{2} + \frac{1}{\pi}\arctan\left(\beta(h - h_m)\right)\right)$$

Implementation:

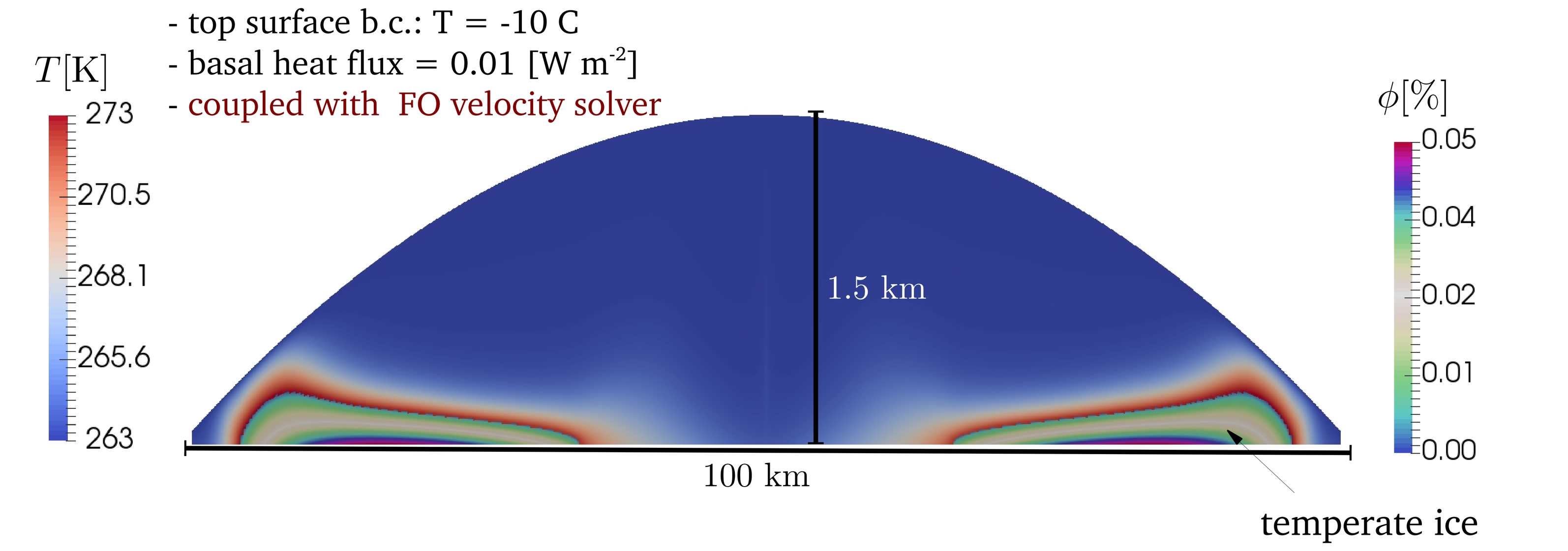
Code implemented in parallel C++ finite element library **Albany**, within the **FELIX** module [4,5].
FELIX implements FO ice flow model and features inverse/analysis capabilities for the estimation of poorly known fields such as basal friction and bed topography and for UQ. It is part of the DOE climate model (ACME) as a dycore of the land ice component of MPAS. We use Streamline Upwind method to stabilize the enthalpy equation.

Albany heavily relies on Trilinos suites, in particular on **NOX** and **LOCA** for nonlinear solvers and algorithms for parameter continuations, which are essential for addressing the nonlinearities in the enthalpy model and on **BELOS**, **ML** and **Ifpack** for linear solvers and preconditioners.

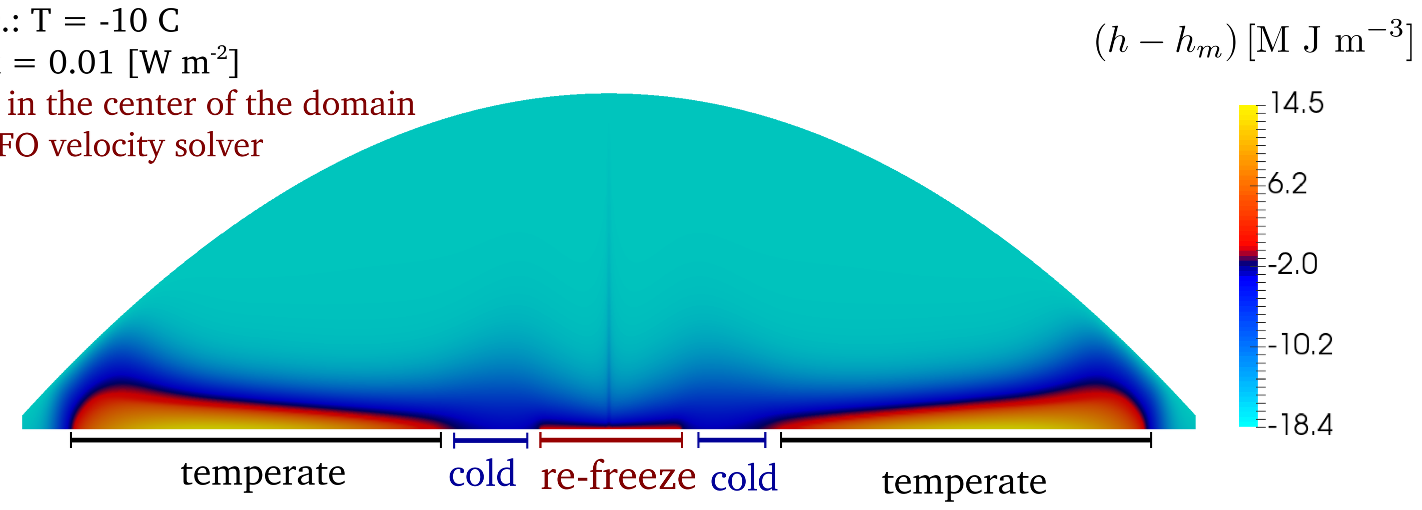
Results:

Dome problem: based on Hewitt and Schoof [2].

Settings
- top surface b.c.: T = -10 C
- basal heat flux = 0.01 [W m^{-2}]
- coupled with FO velocity solver



Settings
- top surface b.c.: T = -10 C
- basal heat flux = 0.01 [W m^{-2}]
- bed lubricated in the center of the domain
- coupled with FO velocity solver

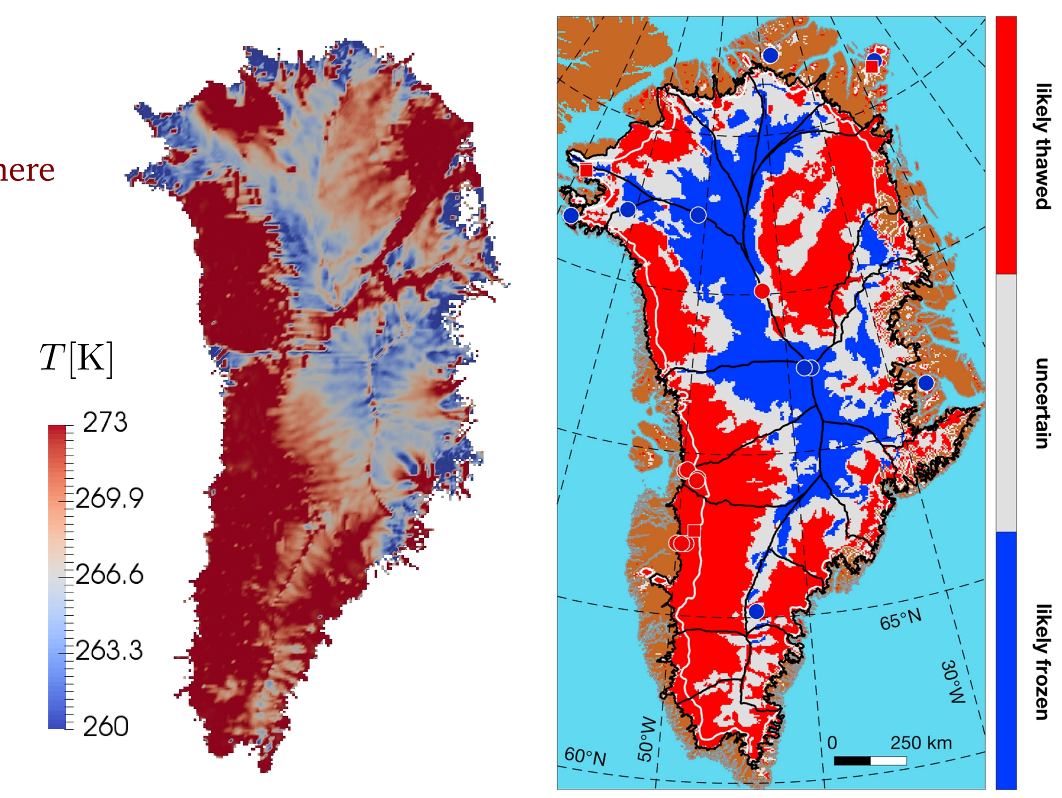


Towards Realistic Problems: 8km-resolution Greenland ice sheet. Comparison with literature.

Settings
- prescribed surface air temperature
- prescribed geothermal heat
- bed lubricated set to be lubricated where basal friction less 5 [kPa yr m^{-1}]
- prescribed FO velocity (not coupled)

Basal temperature

Thawed/frozen map from MacGregor et al. [3]



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