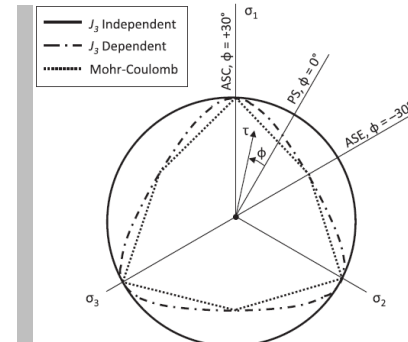
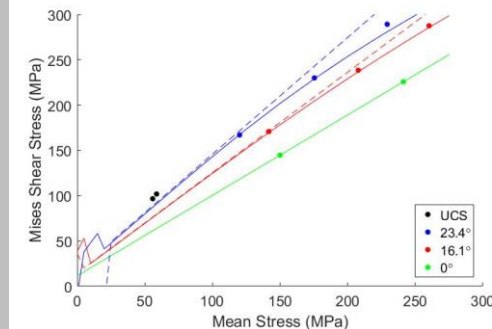


Exceptional service in the national interest



Failure of Sierra White granite under general states of stress

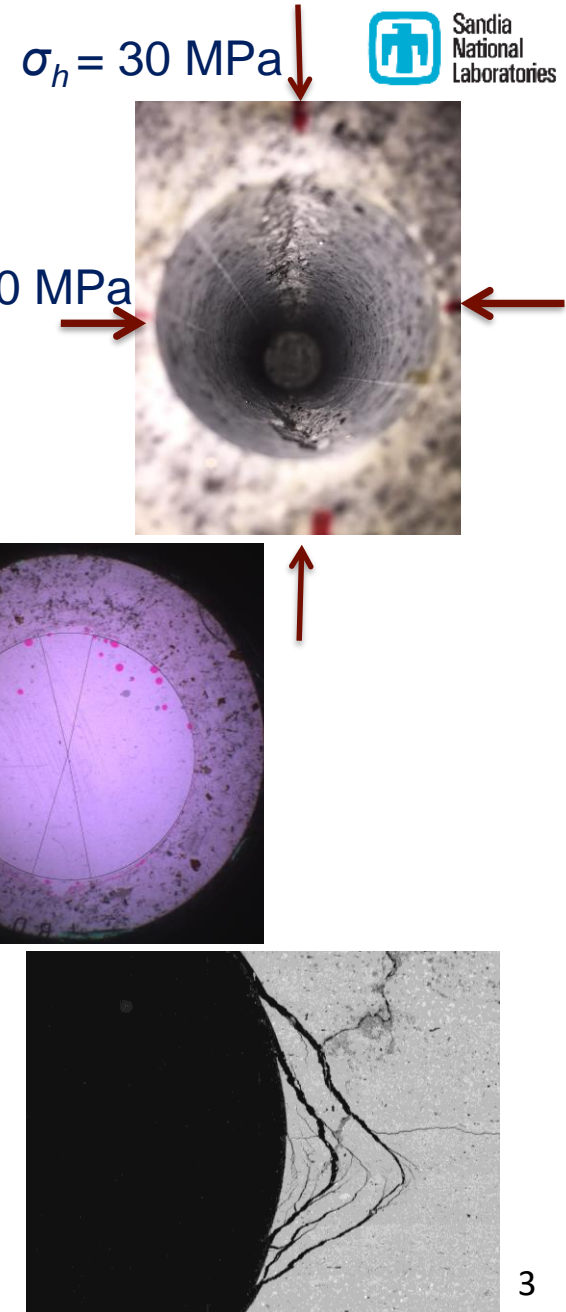
M.D. Ingraham, T.A. Dewers, M.Y. Lee, O.G. Holdman, C.S.N. Cheung, B.C. Haimson

Introduction

- Impetus
- 3D stress state
 - Invariants
 - 3D Failure Surfaces
- Stress-Strain response
 - Rock Type
 - TCM Response
- Bifurcation condition
 - Assumptions
 - Results
- Constitutive model for failure
 - mMNLD
 - Mogi
- Conclusions

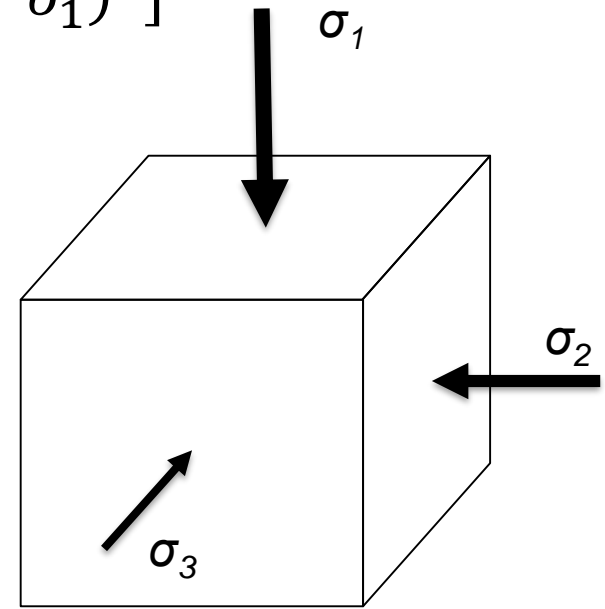
Impetus

- In situ stress state vital for subsurface engineering projects, hazard predictions
- Most common measurement technique utilizes borehole methods
 - Integrate density logs for vertical stress
 - Leak off tests for minimum horizontal stress
 - Maximum horizontal stress must be calculated from borehole breakouts
- DOE Jason 2014 Report – errors in stress measurement can be as high as 30-40% in complicated terranes
- This work undertaken to build computational models that can replicate laboratory (tests performed at UW-Madison) and field measurements.



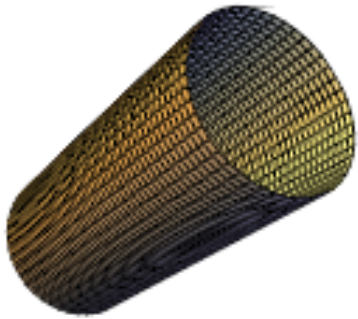
3D Stress State: Invariants

- $\Theta = \tan^{-1} \left[\frac{\sigma_1 - 2\sigma_2 + \sigma_3}{\sqrt{3}(\sigma_1 - \sigma_3)} \right]$
- $\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$
- $\tau = \sqrt{\frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$
- $I_1 = 3\sigma_m$
- $\tau = \sqrt{J_2}$
- $\sin 3\Theta = -\frac{\sqrt{27}J_3}{2\tau^3}$



3D Failure Surfaces

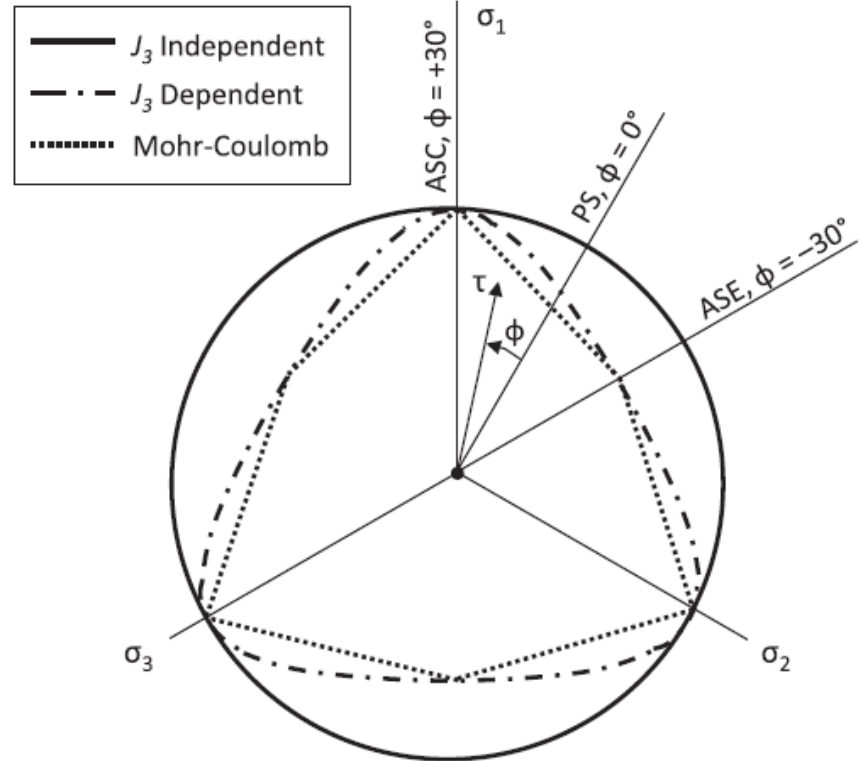
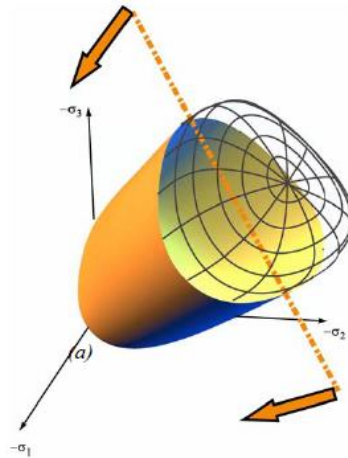
Von Mises



Mohr-Coulomb

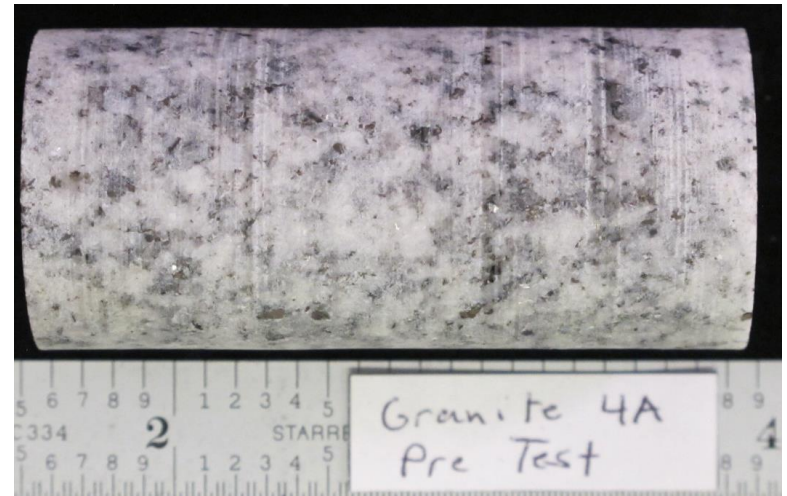


J_3 Dependent



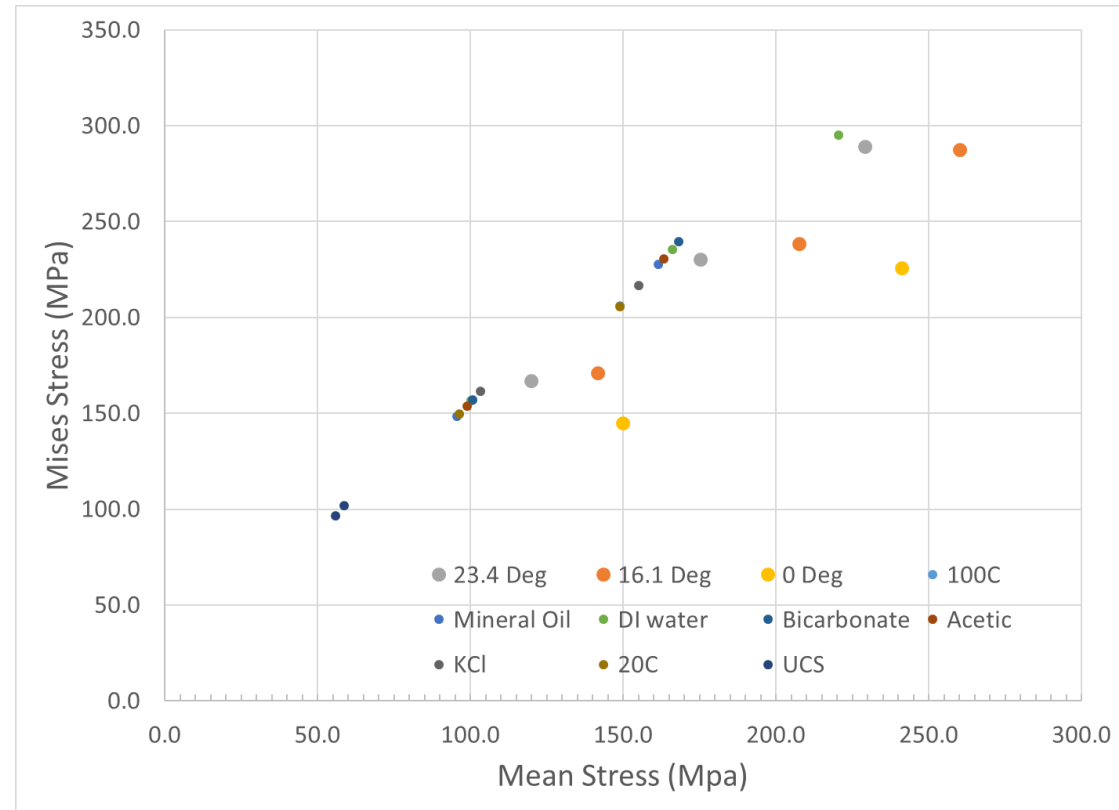
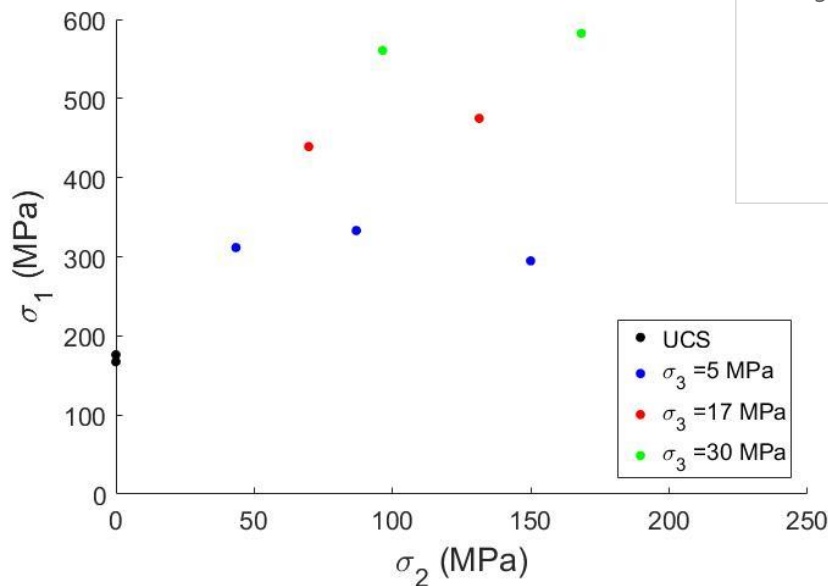
Sierra White Granite

- UCS – 176.2 Mpa
- Density – 2.64 g/cc
- Grain size 1-3 mm
- Youngs Modulus – 48.5 Gpa
- Poisson's Ratio – 0.22
- Comprised of oligoclase, quartz, orthoclase, biotite, muscovite, and other trace.



TCM Response

- Tests were performed on granite cylinders to investigate the thermal and chemical effects
 - Chemical and thermal effects showed a small change in strength, but it is minor compared to σ_2 effect.



Strong dependence on σ_2

Bifurcation Condition

$$\alpha = \frac{\pi}{4} + \frac{1}{2} \sin^{-1} \left[\frac{\frac{2}{3}(1+\nu)(\beta + \mu) - N_{II}(1-2\nu)}{\sqrt{4 - 3N_{II}^2}} \right]$$

$$N_{II} = \frac{\sigma_m - \sigma_2}{\tau}$$

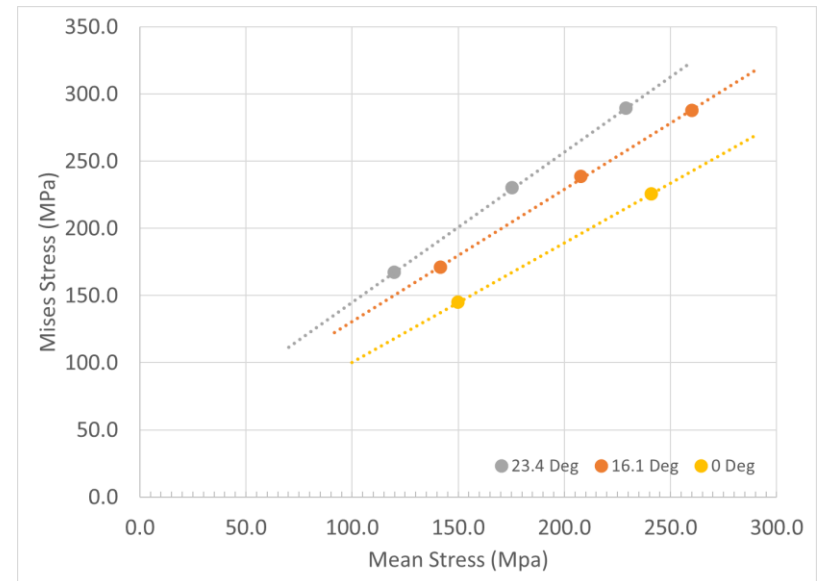
ν = Poisson's Ratio

α = Band angle

β = Plastic potential

μ = Slope of the yield surface

Bifurcation in this form developed by
Rudnicki and Olsson (1998)

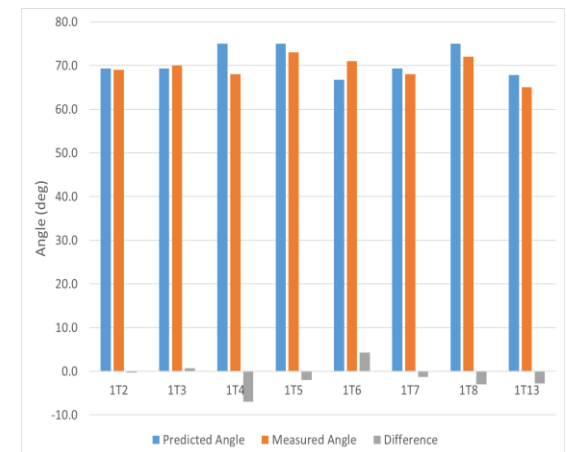


■ Assumptions

- Normality of the yield surface ($\beta = \mu$)
- Failure surface is an acceptable proxy for yield surface
- In the region of interest the yield surface was represented by a straight line for each Lode angle

Bifurcation Results

	σ_m (MPa)	τ (MPa)	θ°	α_m	α_p
1A	58.7	101.8	30.0	NA	87.2
3A	55.8	96.6	30.0	NA	87.2
1T2	260.2	287.5	16.1	69	69.4
1T3	141.7	170.8	16.1	70	69.3
1T4	120.1	167.0	23.4	68	75.0
1T5	175.4	230.0	23.4	73	75.0
1T6	150.0	144.7	0.0	71	66.7
1T7	207.8	238.4	16.1	68	69.3
1T8	229.2	289.2	23.4	72	75.0
1T9	149.1	143.8	0.0	60	66.7
1T13	241.2	225.7	0.0	65	67.8

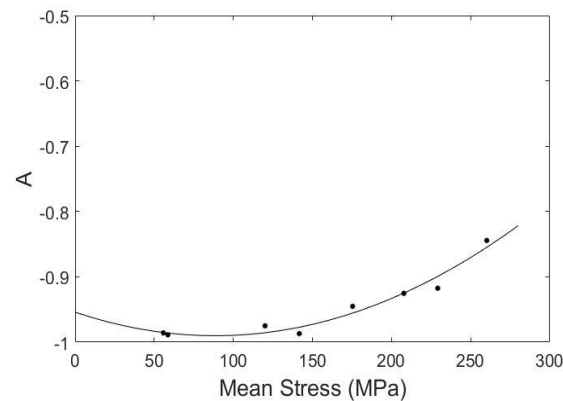
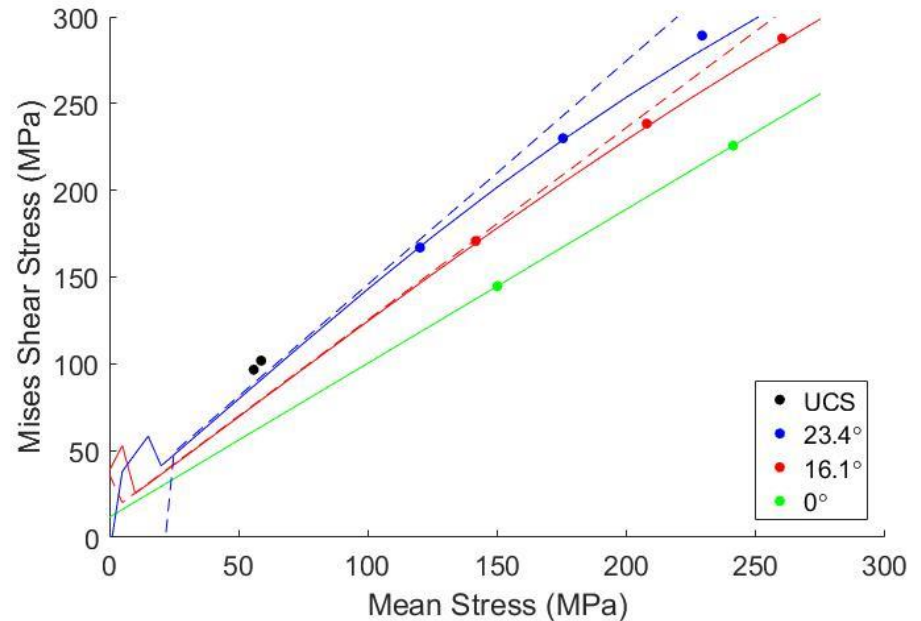


Modified Matsuoka-Nakai-Lade-Duncan

$$\sqrt{\frac{4}{27} A(\sigma) \sin(3) \left(\frac{\tau}{\tau_0(\sigma)} \right)^3 + \left(\frac{\tau}{\tau_0(\sigma)} \right)^2} - 1 = 0$$

$$\tau_0(\sigma) = 0.8878\sigma + 11.604$$

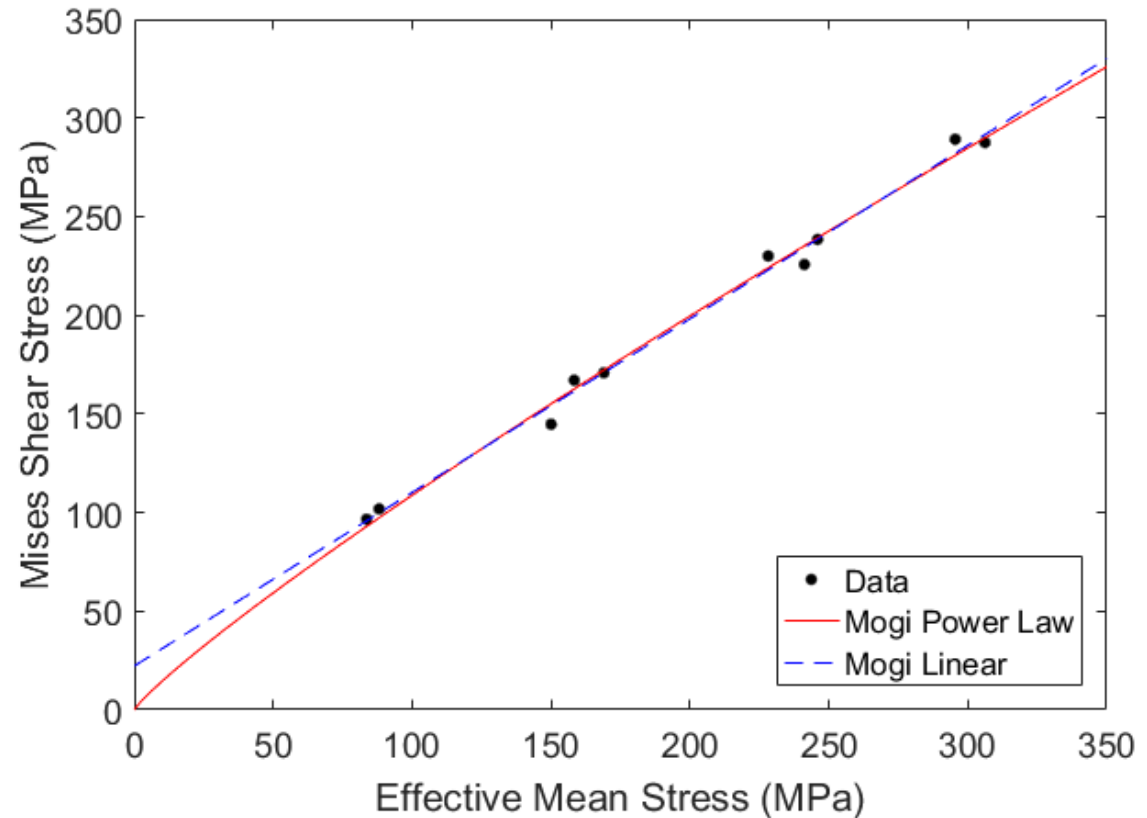
$$A(\sigma) = 5E^{-6}\sigma^2 - 0.0008\sigma - 0.9541$$



	Calculated A	Fit A	τ_0	Calculated τ	Actual τ	τ Difference
1A	-0.989	-0.984	63.76	100.2	101.8	1.6
3A	-0.989	-0.983	61.13	95.9	96.6	0.7
1T2	-0.844	-0.824	242.65	285.7	287.5	1.8
1T3	-0.987	-0.967	137.40	169.5	170.8	1.3
1T4	-0.975	-0.978	118.20	167.6	167.0	-0.6
1T5	-0.945	-0.941	167.32	229.0	230.0	1.0
1T7	-0.925	-0.905	196.07	236.8	238.4	1.6
1T8	-0.918	-0.875	215.07	281.0	289.2	8.2

Mogi Criterion

- $\sigma_{me} = \frac{(\sigma_1 + \sigma_3)}{2}$
- $\tau = a + b\sigma_{me}$
 - $a=22.21$
 - $b=0.880$
- $\tau = \alpha\sigma_{me}^n$
 - $\alpha=1.923$
 - $n=0.8762$



Correlation coefficient is nearly the same for both methods (0.993 and 0.992 respectively)

Conclusions

- Failure depends on σ_2
- Band angle predictions match well with experiments
 - Noteworthy considering assumptions made
- Results are well modeled by both mMNLD model, and both Mogi criteria
- Selection of model depends on data available and intended use for model (are invariants required?)

Acknowledgements

- Moo Lee, Mike Hileman, Perry Barrow, Aron Robbins, R. Charles Choens
- Work was funded under Sandia National Labs: Laboratory Directed Research and Development Program
- SAND2017-XXXXC