SAND2017-13275PE

Exceptional service in the national interest

Sandia National Laboratories

Quantum simulation on near-term hardware

Is there any hope?

Mohan Sarovar
Sandia National Laboratories, Livermore

UC Berkeley AMO seminar, November 2017

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

Analog quantum simulation

- Emulate one quantum system by engineering another to mimic it
- Feynman's original idea for *quantum supremacy*
  Feynman, Int. J. Theor. Phys. **21**, 467 (1982)
- Already an experimental reality:

nature physics ARTICLES
PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NPHYS2232

Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas

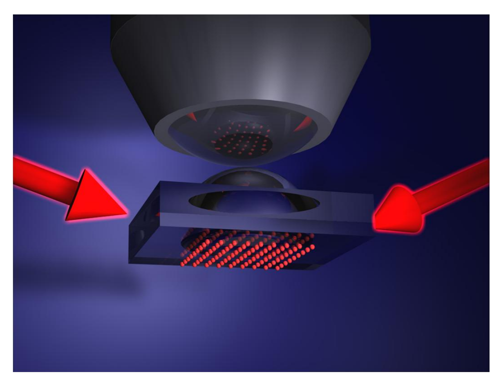
S. Trotzky[1,2,3]*, Y-A. Chen[1,2,3], A. Flesch[4]*, I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7,8] and I. Bloch[1,2,3]

nature Vol 465|3 June 2010|doi:10.1038/nature09071

LETTERS

Quantum simulation of frustrated Ising spins with trapped ions

K. Kim[1], M.-S. Chang[1], S. Korenblit[1], R. Islam[1], E. E. Edwards[1], J. K. Freericks[2], G.-D. Lin[3], L.-M. Duan[3] & C. Monroe[1]



Greiner Lab, Harvard

ARTICLE
doi:10.1038/nature09994

Quantum simulation of antiferromagnetic spin chains in an optical lattice

Jonathan Simon[1], Waseem S. Bakr[1], Ruichao Ma[1], M. Eric Tai[1], Philipp M. Preiss[1] & Markus Greiner[1]

New Journal of Physics
The open-access journal for physics

Quantum simulation of the transverse Ising model with trapped ions

K Kim[1,5,6], S Korenblit[1], R Islam[1], E E Edwards[1], M-S Chang[1,7], C Noh[2,8], H Carmichael[2], G-D Lin[3,9], L-M Duan[3], C C Joseph Wang[4], J K Freericks[4] and C Monroe[1]

Analog quantum simulation

Complex control requirements for any non-trivial quantum simulation experiment

Analog quantum simulation has no natural notion of error correction

How does one assess when an analog quantum simulation result is robust to noise/imperfections?

Rep. Prog. Phys. **75** (2012) 082401 (18pp) doi:10.1088/0034-4885/75/8/082401

Can one trust quantum simulators?

Philipp Hauke[1], Fernando M Cucchietti[1,2], Luca Tagliacozzo[1], Ivan Deutsch[3,4] and Maciej Lewenstein[1,5]

But is it so bad?

Physicist's intuition/consolation

1. For many questions we "care about" (coarse-grained observables) microscopic imperfections in the model (Hamiltonian) should not matter. After all, *no real material corresponds perfectly to an ideal model*.

2. Real materials are at finite temperature, so if we introduce an equilibration/ thermalization mechanism then the emergent properties may be stable.

Aims:

1. To formalize (1)
2. To enable (2)

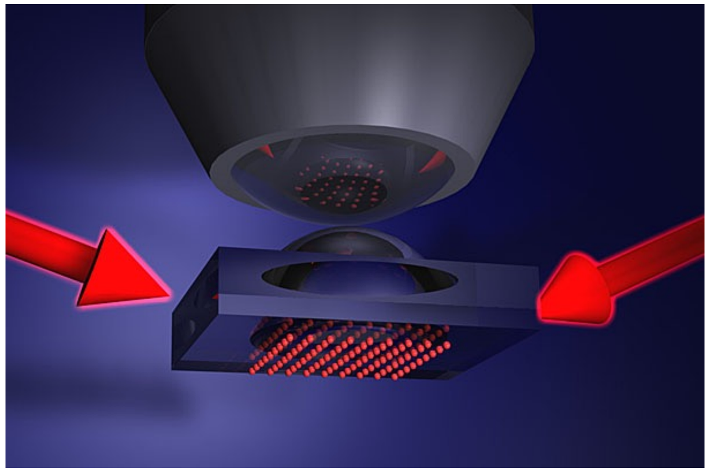
Outline

1. Parameter space compression and simulation sensitivity

2. Engineering thermalization in quantum many-body models

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions

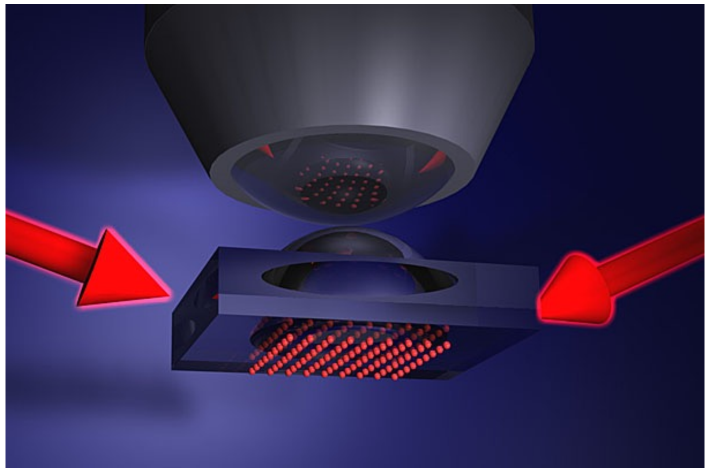


Greiner Lab, Harvard

$$\longrightarrow \{p_m\}_m$$

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



Greiner Lab, Harvard

$$\longrightarrow \{p_m\}_m$$

Ideal many-body Hamiltonian dependent on K parameters

$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k \qquad \lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$$

Observable of interest

$$O = \sum_m \theta_m P_m$$

Produces a parameterized probability distribution

$$p_m(\lambda) = \mathrm{tr}\left(P_m \rho_{\mathrm{th}}(T)\right)$$

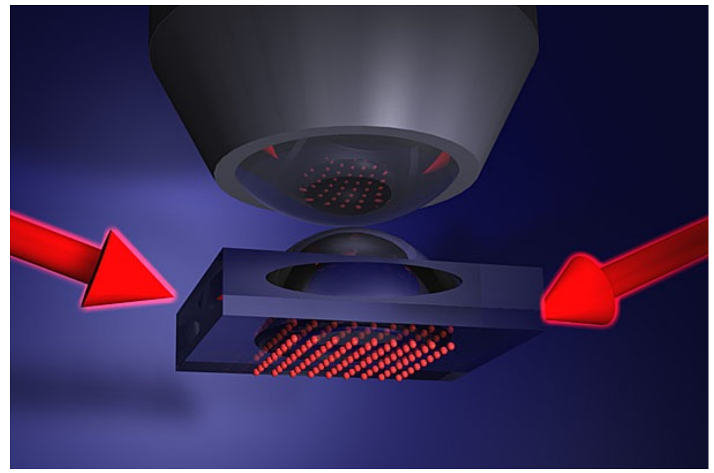
Or ground state

$$\rho_{\mathrm{th}}(T) = \frac{e^{\beta H(\lambda)}}{\mathcal{Z}}$$

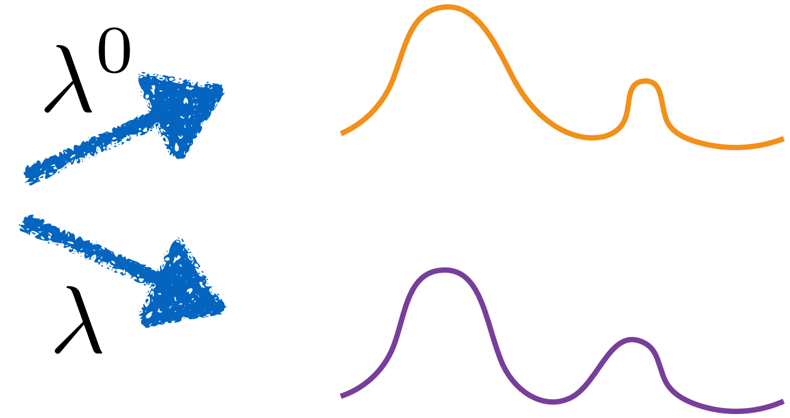
$$\beta = \frac{1}{k_B T}$$

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



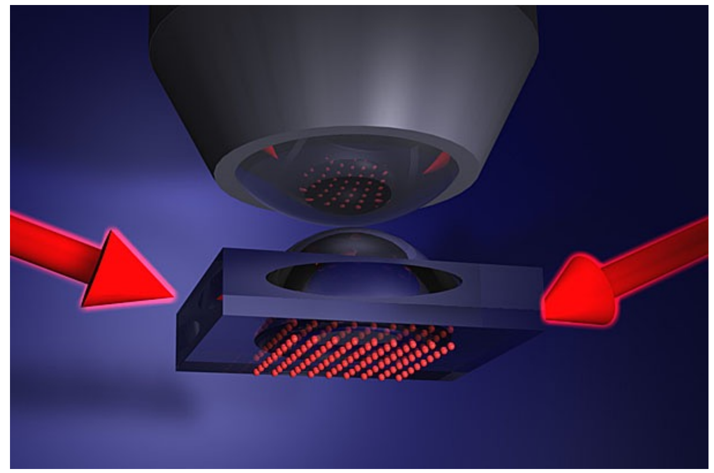
Greiner Lab, Harvard



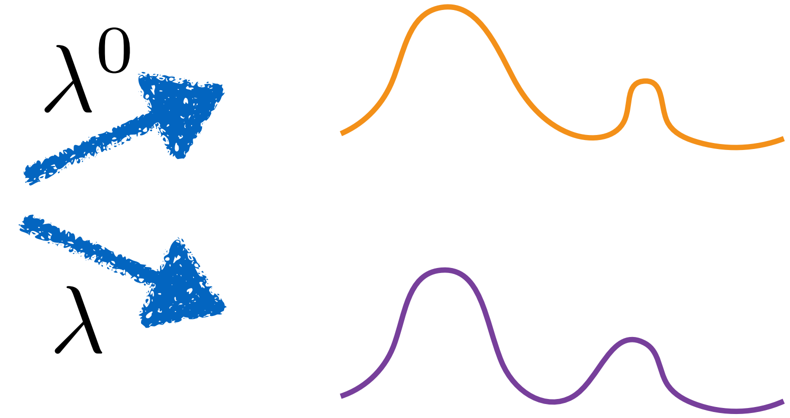
Robustness: how different are the output distributions under perturbations of the Hamiltonian parameters?

Quantifying parameter sensitivity

A quantum simulator produces parameterized probability distributions



Greiner Lab, Harvard



Robustness: how different are the output distributions under perturbations of the Hamiltonian parameters?

Kullback-Leibler divergence: a measure of difference between probability distributions

$$D_{\mathrm{KL}}(p(\lambda)||p(\lambda^0)) = \sum_m p_m(\lambda) \log \frac{p_m(\lambda)}{p_m(\lambda^0)}$$

$$= \frac{1}{2}\Delta\lambda^{\mathsf{T}} F(\lambda^0)\Delta\lambda + \mathcal{O}(||\Delta\lambda||^3)$$ (small deviations)

Fisher information matrix (FIM)

$$\Delta\lambda = \lambda - \lambda^0$$

Quantifying parameter sensitivity

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j}\bigg|_{\lambda=\lambda^0}$$

Spectral analysis of the FIM

$$\boxed{F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger}$$

Eigendecomposition of FIM

$$D_{\mathrm{KL}}(p(\lambda)||p(\lambda^0) \approx \frac{1}{2}\Delta\lambda^{\mathsf{T}} F(\lambda^0)\Delta\lambda \ = \sum_{k=1}^{K} \frac{\zeta_k}{2} \|v_k^\dagger \Delta\lambda\|^2$$

Quantum simulation of this model is <u>*trivially robust*</u> if all $\zeta_k \approx 0$

Quantifying parameter sensitivity

Spectral analysis of the FIM

$$\boxed{F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger}$$

Eigendecomposition of FIM

Key observation:

Eigenvalues of FIM prescribe an ordering of parameter influence, *i.e.* if ζ_k is large, then the parameter

$$\sum_j \lambda_j v_k^j$$

has a large influence on observations

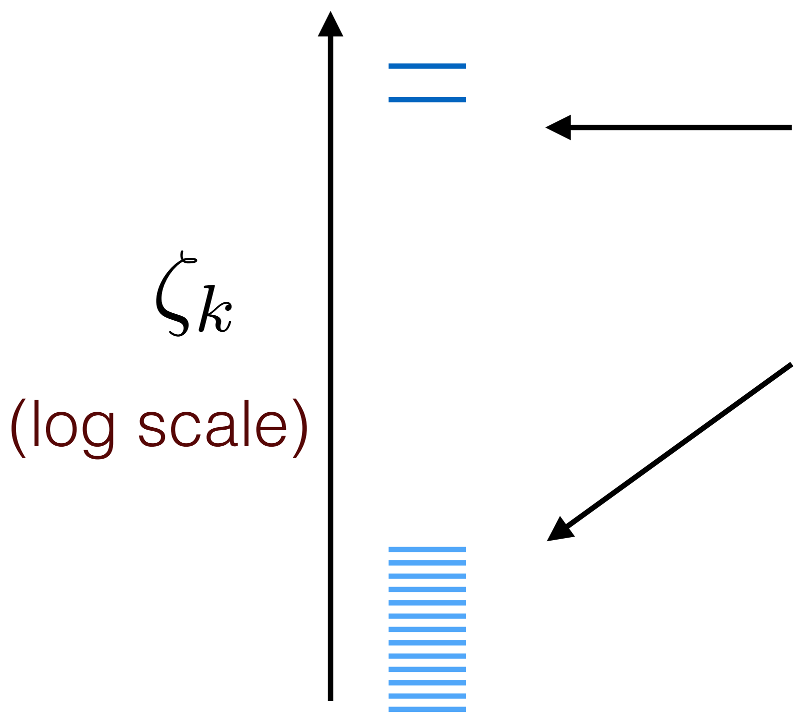
So a *non-trivial* way for a quantum simulation to be robust is that only a few eigenvalues matter, and they correspond to parameters that can be controlled well.

Quantifying parameter sensitivity

Spectral analysis of the FIM

$$\boxed{F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger}$$

Eigendecomposition of FIM



We call this a sloppy quantum simulation model

c.f.

Parameter Space Compression Underlies Emergent Theories and Predictive Models
Benjamin B. Machta *et al.*
Science **342**, 604 (2013);
DOI: 10.1126/science.1238723

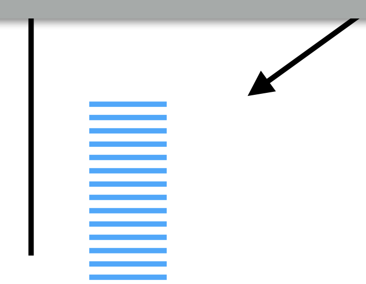
Quantifying parameter sensitivity

Spectral analysis of the FIM

$$\boxed{F = \sum_{k=1}^{K} \zeta_k v_k v_k^{\dagger}}$$

Eigendecomposition of FIM

Are quantum many-body models sloppy?
If so, why?

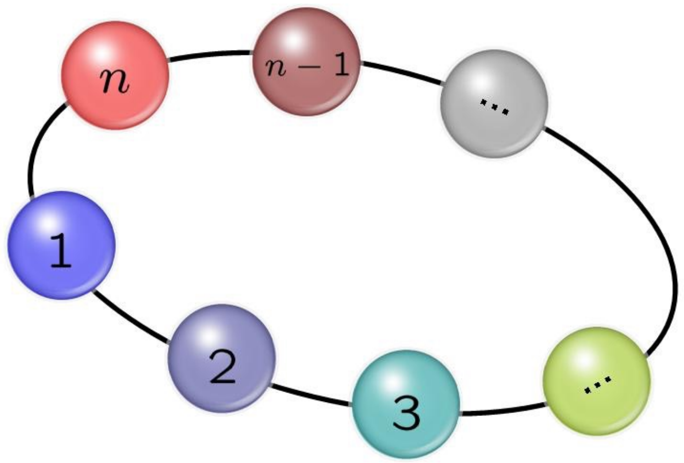


We call this a sloppy quantum simulation model

c.f.

Parameter Space Compression Underlies Emergent Theories and Predictive Models
Benjamin B. Machta *et al.*
Science **342**, 604 (2013);
DOI: 10.1126/science.1238723

Transverse-field Ising model



$$H_{\text{1D Ising}} = \sum_{i=1}^{n} B_i \sigma_x^i + \sum_{i=1}^{n-1} J_i \sigma_z^i \sigma_z^{i+1} + J_n \sigma_z^n \sigma_z^1$$

Uniform antiferromagnetic model parameter regime, so we want:

$$B_i = B^0 \qquad \forall i$$

$$J_i = J^0 > 0 \qquad \forall i$$

Quantum critical point at

$$B^0 = \frac{J^0}{2}$$

Observables of interest

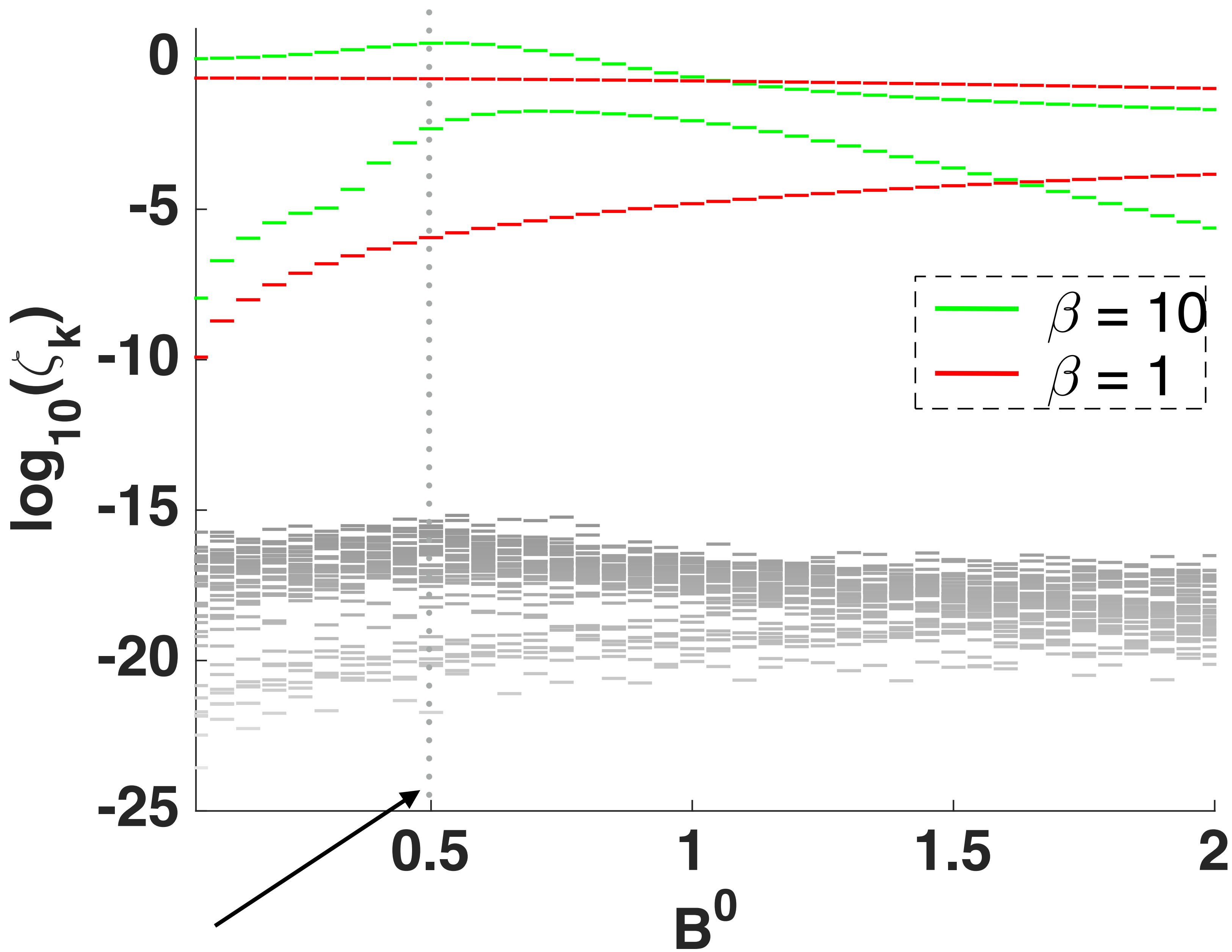
$$S_z = \sum_i \langle \sigma_z^i \rangle$$ Net magnetization

$$C_z(i,j) = \langle \sigma_z^i \sigma_z^j \rangle$$ Correlation functions

Transverse-field Ising model

Net magnetization observable

$$n = 10, J^0 = 1$$



Stiff parameters

Sloppy parameters

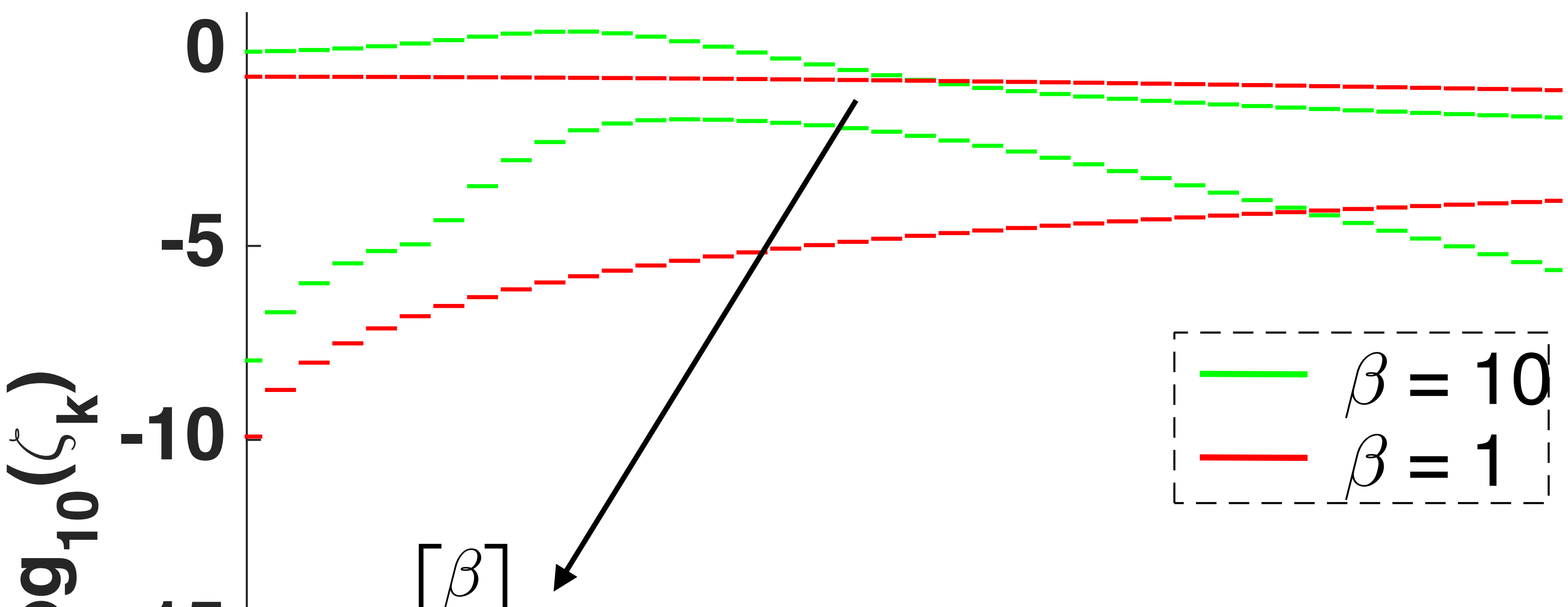
Quantum critical point

Transverse-field Ising model

Net magnetization observable

$$n = 10, J^0 = 1$$

What is the dominant stiff parameter?



Stiff parameters

$$\begin{bmatrix} \beta \\ \vdots \\ \beta \\ \gamma \\ \vdots \\ \gamma \end{bmatrix} \quad \Rightarrow v_1^\dagger \Delta\lambda = \beta \sum_i \Delta B_i + \gamma \sum_i J_i$$

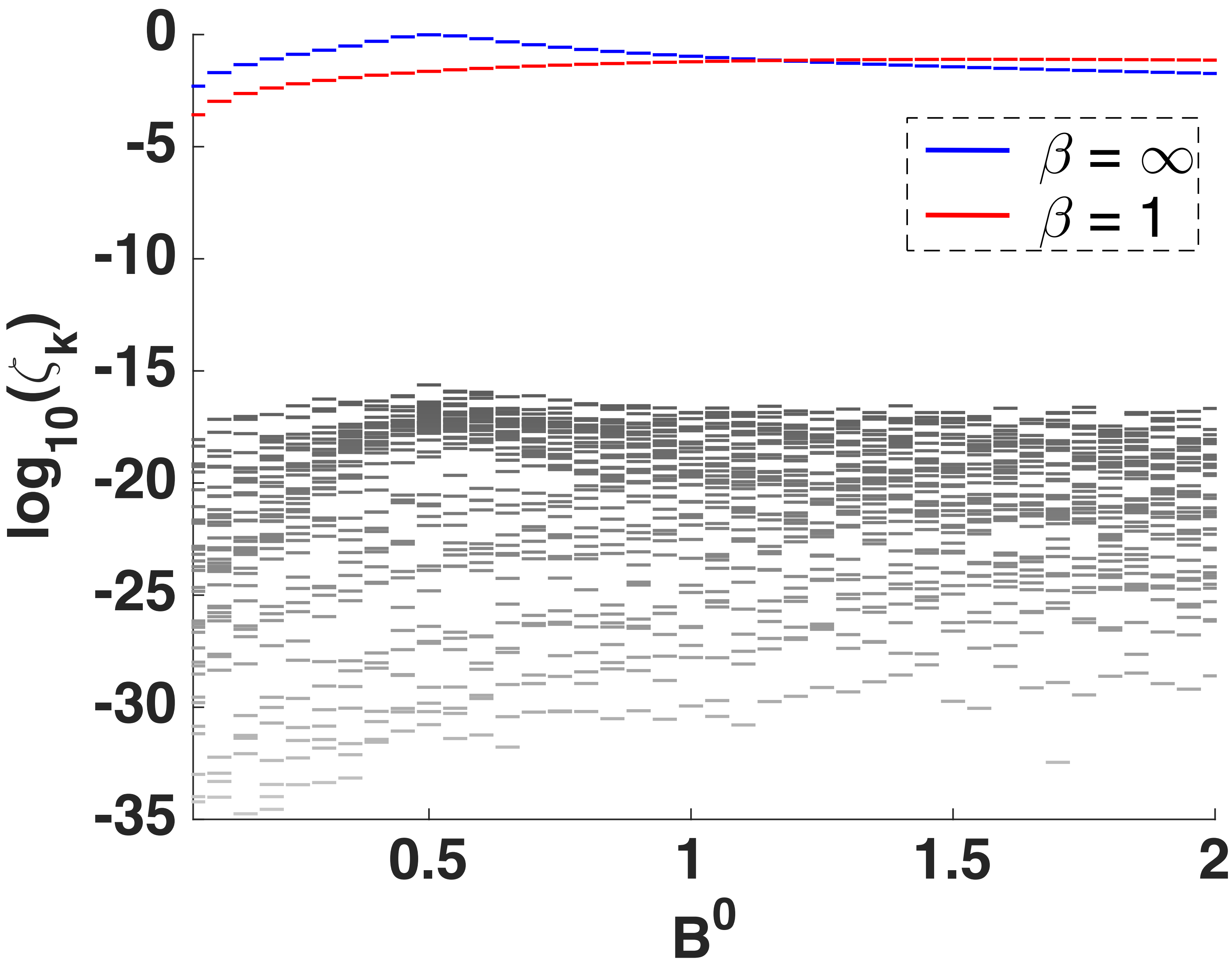
Fluctuations in B_i and J_i do not matter, as long as these (spatially) average to zero

Transverse-field Ising model

Correlation function observable

$C_z(2,6)$

$n = 10, J^0 = 1$



Stiff parameter

Sloppy parameters

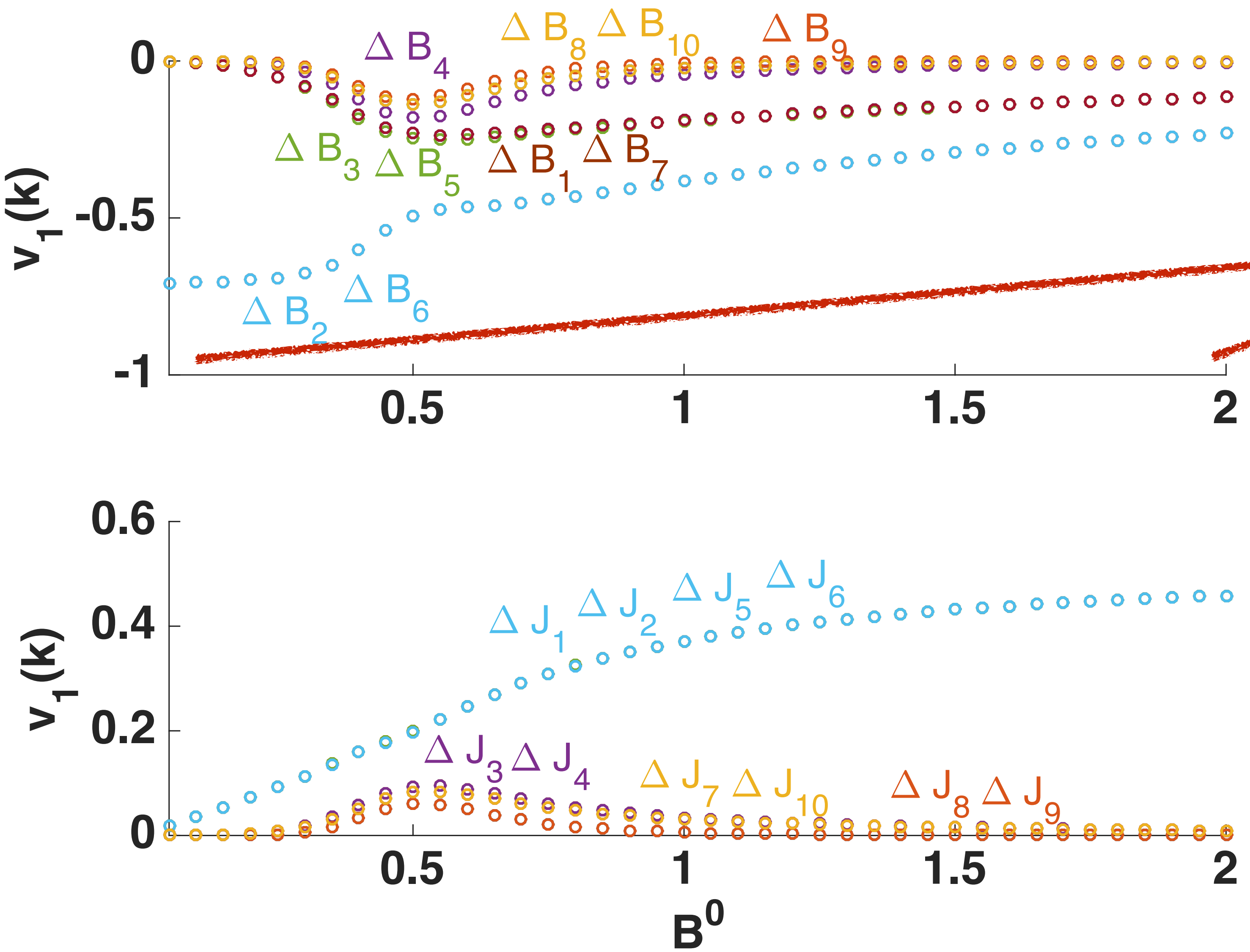
Transverse-field Ising model

Correlation function observable

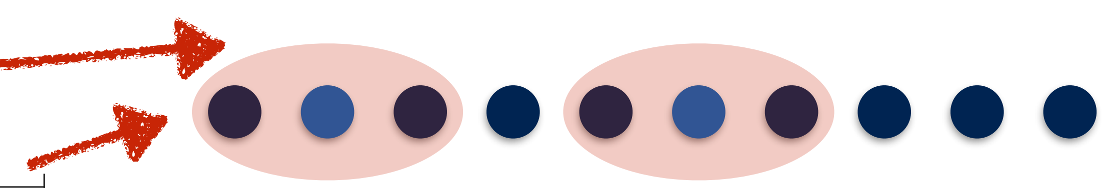
$C_z(2,6)$

$n = 10, J^0 = 1$

Composition of stiff parameter



$$\boxed{v_1^\dagger \Delta\lambda = \sum_i v_1^i \Delta\lambda_i}$$



Away from critical point:

stiff parameter only depends on “local” parameters

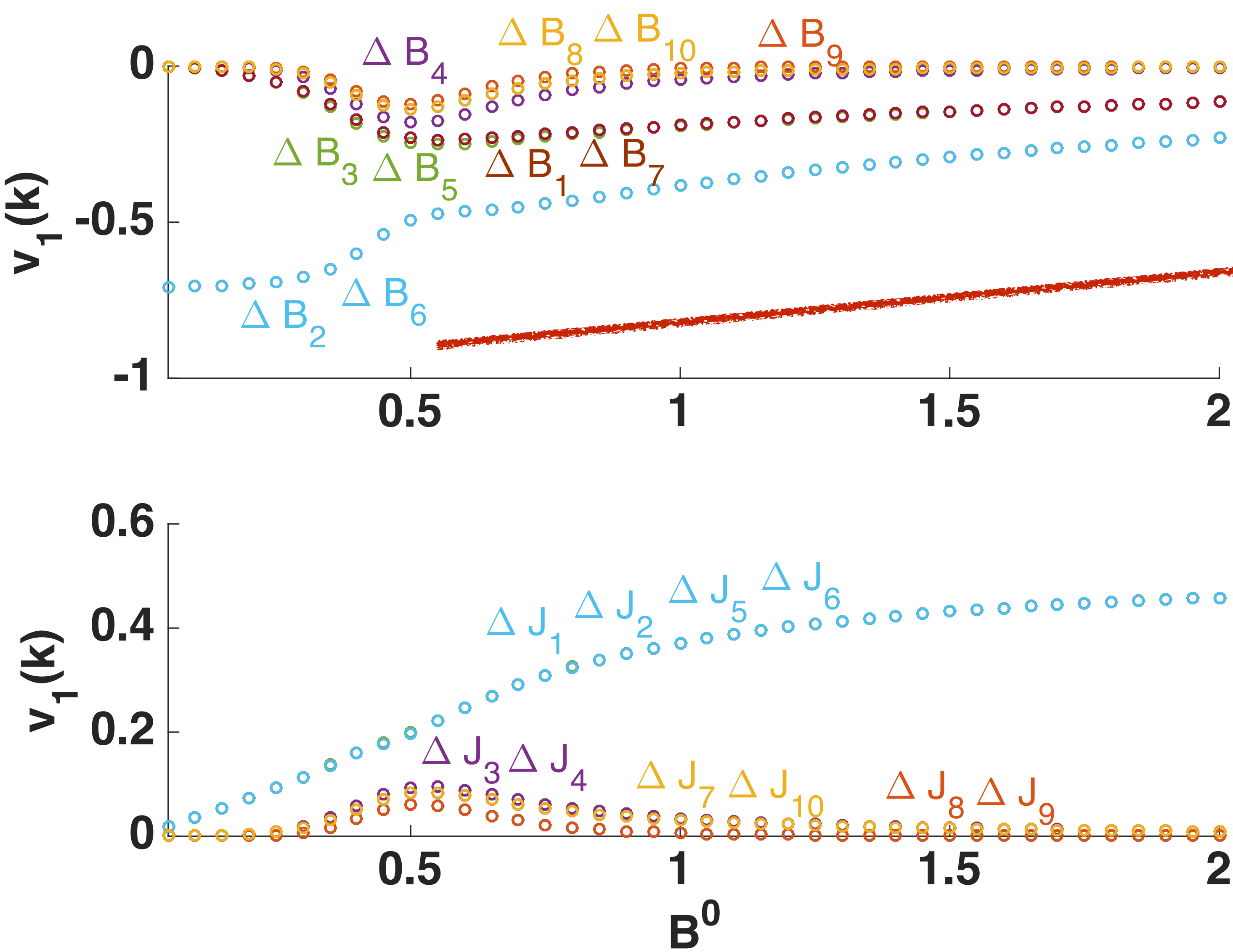
Transverse-field Ising model

Correlation function observable

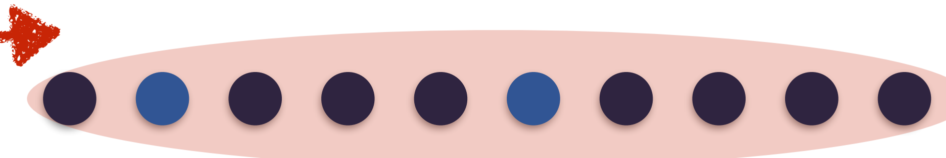
$$C_z(2,6)$$

$$n = 10, J^0 = 1$$

Composition of stiff parameter



$$\boxed{v_1^\dagger \Delta\lambda = \sum_i v_1^i \Delta\lambda_i}$$



Around the critical point:

stiff parameter is a complex linear combination of all the parameters (c.f. collective phenomena, correlations depend on the whole system)

Transverse-field Ising model

Lessons learnt:

- The observable matters. Collective, global phenomena can be very robust to local fluctuations.
- Quantum phase transitions can lead to complex dependence of stiff parameters on microscopic parameters.
- Generally: sloppiness increases with temperature.
- Specifically for Ising model: detect phase transition by measuring magnetic susceptibility as opposed to measuring a correlation function.

Other models

We have studied many other quantum-many body models of interest:

1. 1D and 2D transverse-field Ising models with varying boundary conditions
2. Several flavors of the Heisenberg model
3. J1-J2-anti-ferromagnetic Heisenberg model (non-nearest neighbor interactions)
4. Fermi-Hubbard model
5. Random/disordered transverse-field Ising model (not sloppy)

See:
Reliability of analog quantum simulation,
M. S., J. Zhang, L. Zeng. EPJ Quantum Information 4,1 (2017)
arXiv:1603.09283

Why so sloppy?

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j}\bigg|_{\lambda=\lambda^0}$$

Use rank as a proxy for sloppiness

low rank => sloppy

Why so sloppy?

Fisher Information matrix (FIM)

$$F_{ij}(\lambda^0) = \sum_{m=1}^{M} \frac{1}{p_m(\lambda)} \frac{\partial p_m(\lambda)}{\partial \lambda_i} \frac{\partial p_m(\lambda)}{\partial \lambda_j} \Bigg|_{\lambda=\lambda^0}$$

Write as:

$$F = V \Lambda^{-1} V^{\mathsf{T}}$$

$$\underset{K \times M}{V} = \begin{pmatrix} \frac{\partial p_1(\lambda)}{\partial \lambda_1} & \frac{\partial p_2(\lambda)}{\partial \lambda_1} & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_1} \\ \vdots & & & \vdots \\ \frac{\partial p_1(\lambda)}{\partial \lambda_K} & & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_K} \end{pmatrix} \qquad \underset{M \times M}{\Lambda} = \begin{pmatrix} p_1(\lambda) & & \\ & \ddots & \\ & & p_M(\lambda) \end{pmatrix}$$

Λ is full-rank, therefore $\mathrm{rank}(F) \leq \mathrm{rank}(V)$

What dictates the rank of V?

Quantum simulation model symmetries

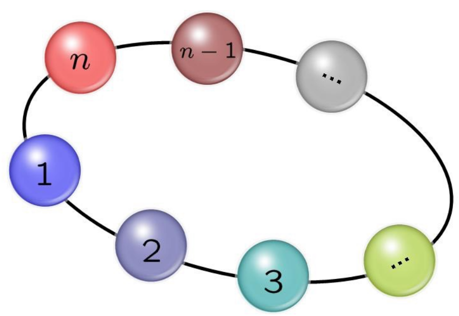
Let G be the group of symmetries of the quantum simulation model, *i.e.*

$$[U_g, H(\lambda)] = 0 \qquad \forall g$$
$$[U_g, O] = 0$$

$\{U_g\}_g$ a faithful unitary representation of G

e.g.

Translational invariance



Recall

$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k$$

$$O = \sum_m \theta_m P_m$$

$$p_m(\lambda) = \mathrm{tr}\left(P_m \rho_{\mathrm{th}}(T)\right)$$

Quantum simulation model symmetries

Let G be the group of symmetries of the quantum simulation model, *i.e.*

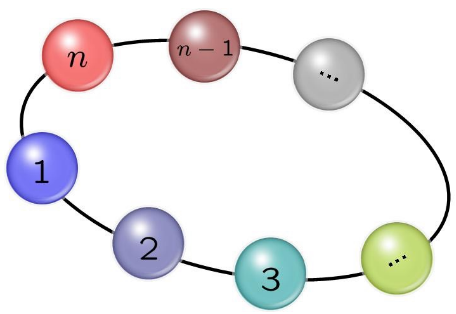
$$[U_g, H(\lambda)] = 0 \qquad \forall g$$

$$[U_g, O] = 0$$

$\{U_g\}_g$ a faithful unitary representation of G

e.g.

Translational invariance



Suppose

$$U_g H_k U_g^\dagger = H_j$$

e.g.

$$H_{1\text{D Ising}} = \sum_{i=1}^{n} B_i \sigma_x^i + \sum_{i=1}^{n-1} J_i \sigma_z^i \sigma_z^{i+1} + J_n \sigma_z^n \sigma_z^1$$

Recall

$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k$$

$$O = \sum_m \theta_m P_m$$

$$p_m(\lambda) = \mathrm{tr}\left(P_m \rho_{\mathrm{th}}(T)\right)$$

Quantum simulation model symmetries

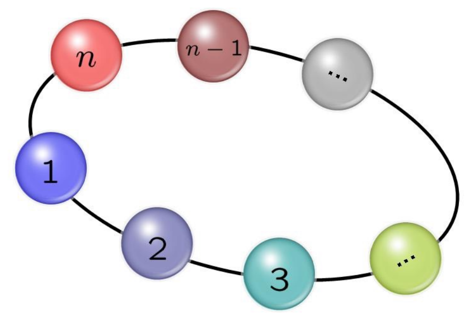
Let G be the group of symmetries of the quantum simulation model, *i.e.*

$$[U_g, H(\lambda)] = 0 \qquad \forall g$$
$$[U_g, O] = 0$$

$\{U_g\}_g$ a faithful unitary representation of G

e.g.

Translational invariance



Suppose

$$U_g H_k U_g^\dagger = H_j$$

Then

$$\frac{\partial p_m(\lambda)}{\partial \lambda_k} = \frac{\partial p_m(\lambda)}{\partial \lambda_j} \qquad \forall m$$

$$\begin{pmatrix} \frac{\partial p_1(\lambda)}{\partial \lambda_1} & \frac{\partial p_2(\lambda)}{\partial \lambda_1} & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_1} \\ \vdots & & & \vdots \\ \frac{\partial p_1(\lambda)}{\partial \lambda_K} & & \cdots & \frac{\partial p_M(\lambda)}{\partial \lambda_K} \end{pmatrix}$$

Repeated rows in V

Model symmetries reduce rank of F

Quantum simulation model symmetries

Model symmetries reduce rank of V, and hence of F

Algorithm for computing an upper bound on the rank of F:

1. For all 1<= k <= K, compute the orbit of H_k under the symmetry group

   Orbit: $\{U_g H_k U_g^\dagger | g \in G\}$

2. Number of distinct orbits will be the number of unique rows in V, and therefore a bound on the rank of V and F

Recall

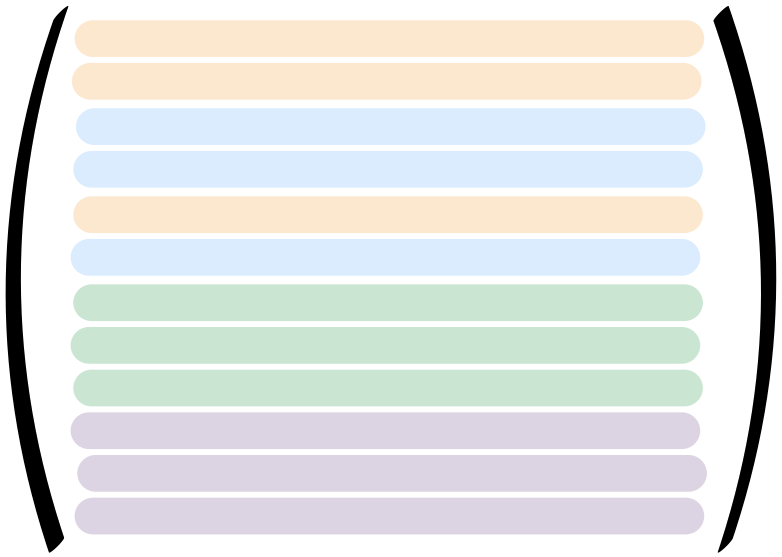
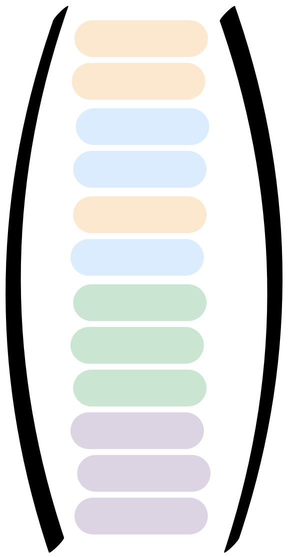
$$H(\lambda) = \sum_{k=1}^{K} \lambda_k H_k$$

Quantum simulation model symmetries

Such a symmetry analysis allows us to extract the form of the stiff/influential parameter as well

$$F = V\Lambda^{-1}V^{\mathsf{T}}$$

Eigenvectors of F

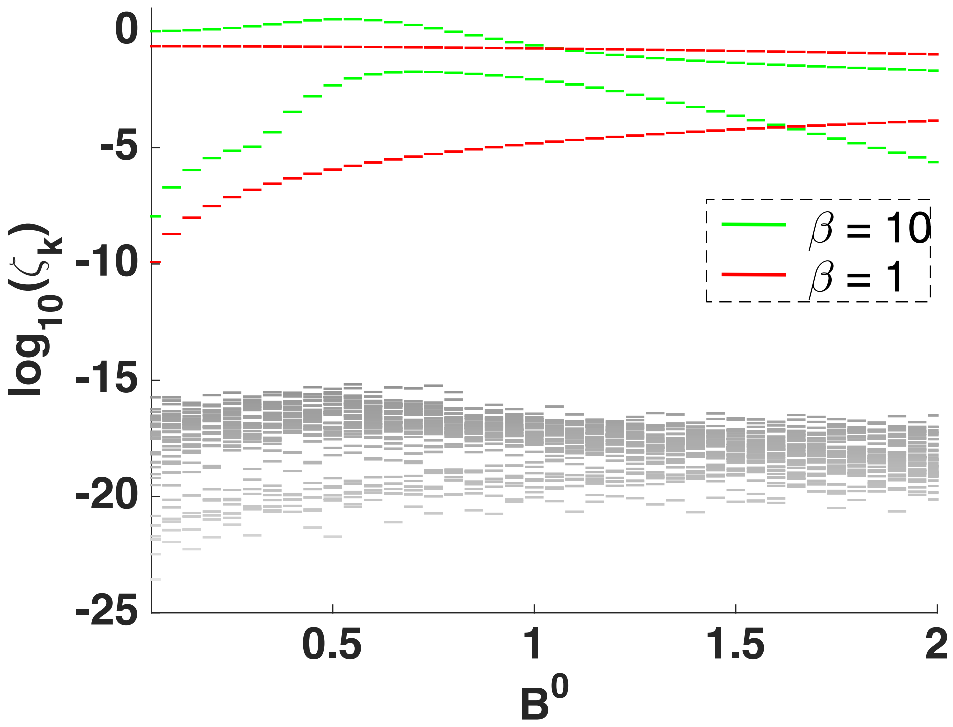
$V =$  $\Rightarrow$ $v_i =$ 

$$v_k^\dagger \Delta\lambda = \sum_s \mu_s^k(\lambda^0, \beta) \sum_{l:H_l \in \mathrm{Orbit}\ s} \Delta\lambda_l$$

Sum over orbits

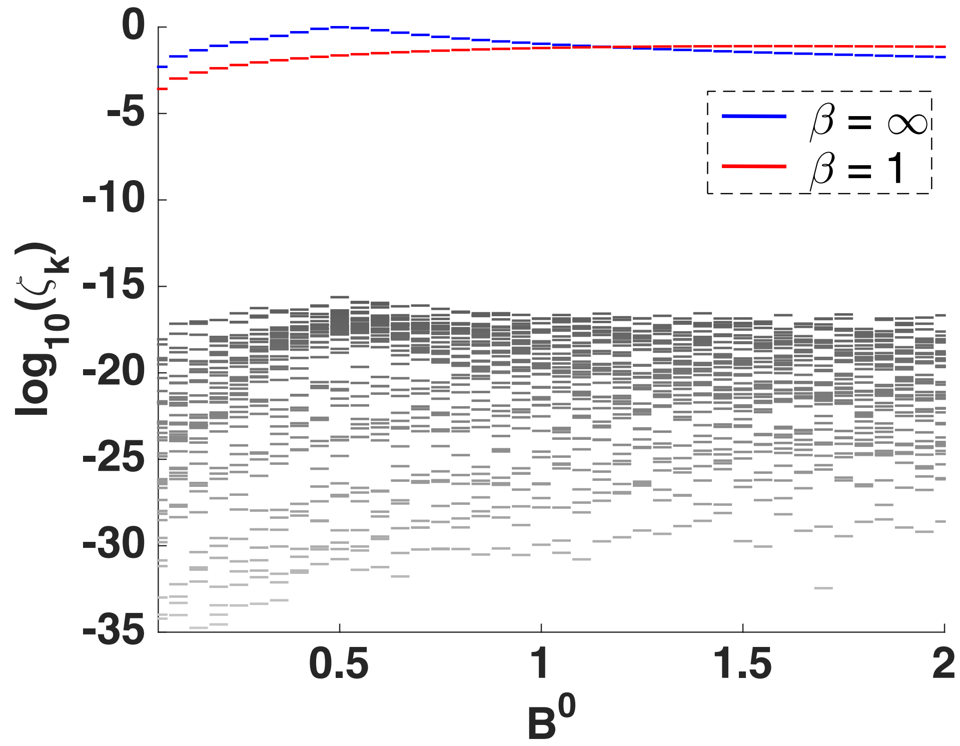
Transverse-field Ising model *revisited*

Net magnetization observable



Rank(F) <= 2

Correlation function observable



Rank(F) <= 1

For any value of the model parameters, temperature, and any number of spins

<u>This is a very sloppy model, and hence potentially robustly simulatable.</u>

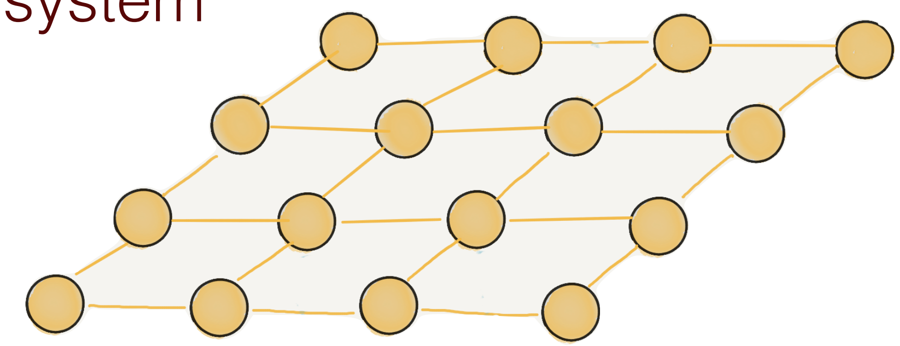
Outline

1. Parameter space compression and simulation sensitivity

2. Engineering thermalization in quantum many-body models

Simulation of many-body systems

Many-body lattice system



Engineered Hamiltonian

$$H$$

How do we prepare a thermal state at temperature T? $\rho(\beta) = \frac{e^{-\beta H}}{\mathcal{Z}}$

The apparatus has a physical temperature, but this is always relevant.

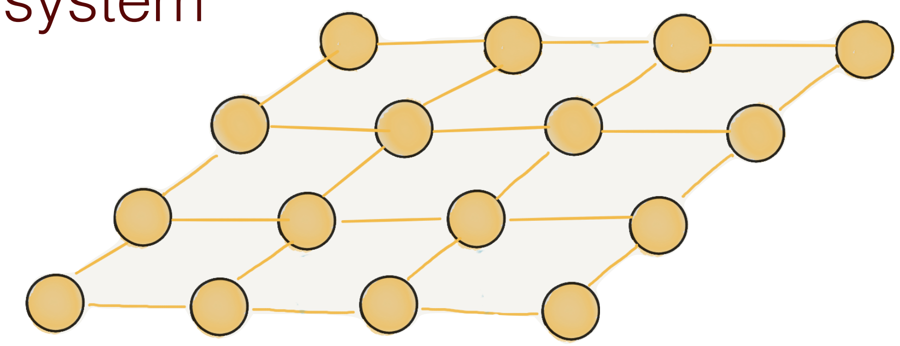
The errors/noise on the logical degrees of freedom may be non-equilibrating.

e.g. Spin lattice, and independent depolarizing channel on each spin

$$\rho(t) \to \frac{\mathbf{1}}{n}$$

How to get thermalization?

Many-body lattice system



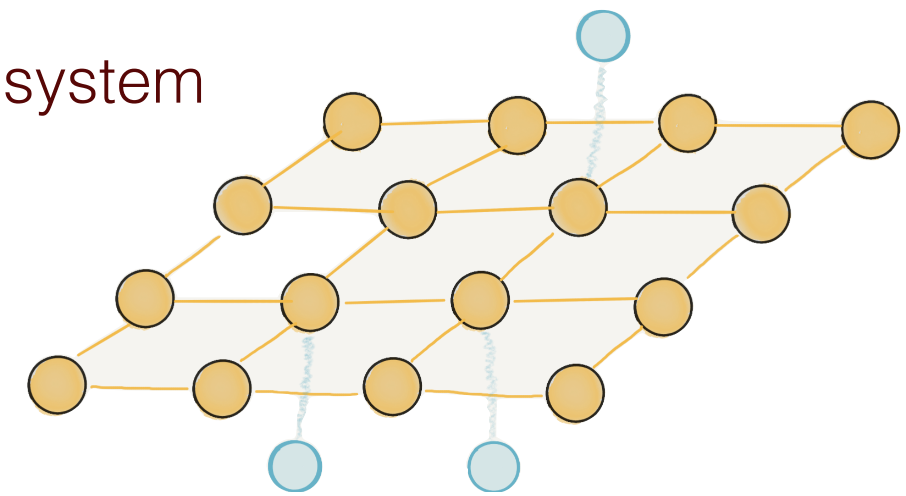
Engineered Hamiltonian

H

Need to *engineer* thermalization also

How to get thermalization?

Many-body lattice system



Engineered Hamiltonian

$$H$$

Engineered dissipative evolution

$$\dot{\rho(t)} = \mathcal{L}\rho(t)$$

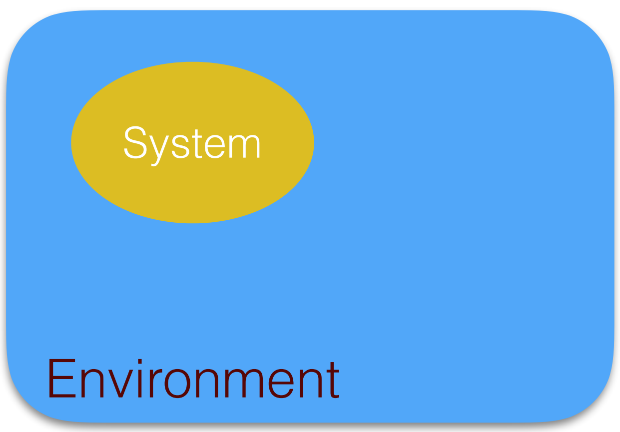
But what kind of evolution will result in a thermal state?

Only general characterization known [*e.g.* Breuer & Petruccione, The theory of open quantum systems]:

Born-Markov master equation + ergodicity + KMS conditions => thermal steady state

How to get thermalization?

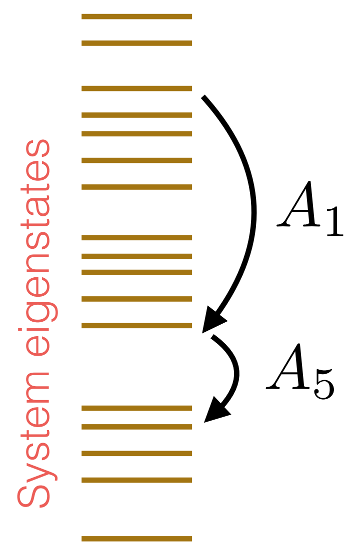
Born-Markov master equation + ergodicity + KMS conditions => thermal steady state



- Weak coupling to many degrees of freedom
- Fast relaxing environment

$$[A_\alpha, X] = [A_\alpha^\dagger, X] = 0$$

$$\Rightarrow X \propto 1$$



$$\frac{\gamma_{\alpha\beta}(-\omega)}{\gamma_{\beta\alpha}(\omega)} = e^{-\beta\omega}$$

Bath induced fluctuations and dissipation satisfy detailed balance

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{\omega,\alpha,\beta} \gamma_{\alpha\beta}(\omega)\left(A_\beta(\omega)\rho A_\alpha^\dagger(\omega) - \frac{1}{2}\{A_\alpha^\dagger(\omega)A_\beta(\omega)\}\right)$$

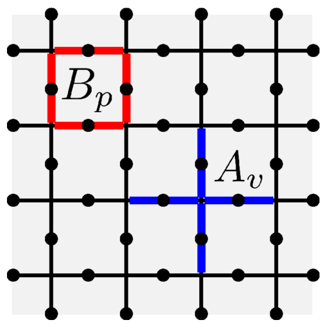
How to get thermalization?

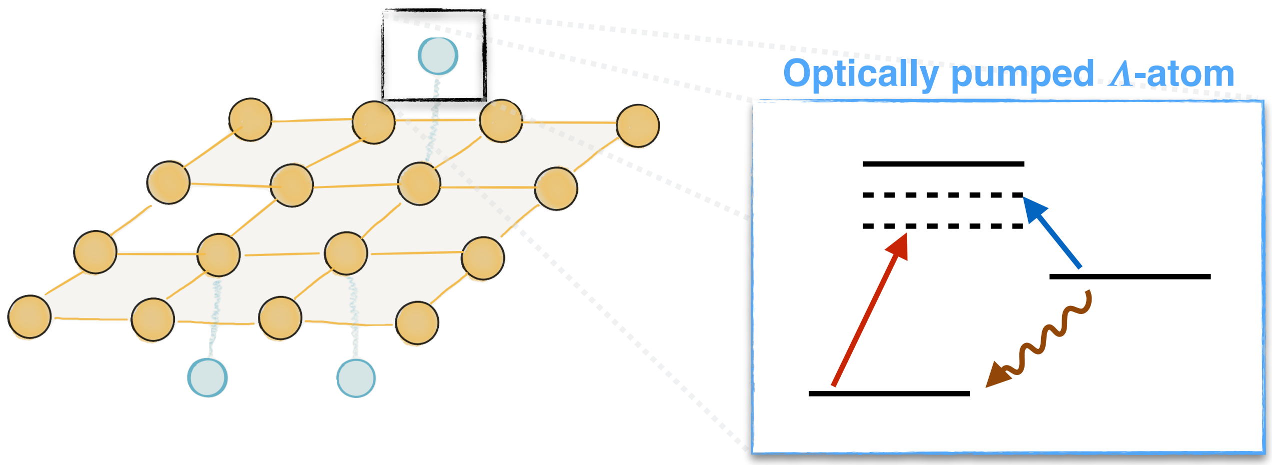
A constructive approach to thermalization for stabilizer Hamiltonians

K. Young, M.S. *et al.* J. Phys. B, **45,** 154012 (2012)

e.g. toric code

$$H_{TC} = -\lambda_e \sum_v A_v - \lambda_m \sum_p B_p$$





A number of special properties of stabilizer Hamiltonians make this work:

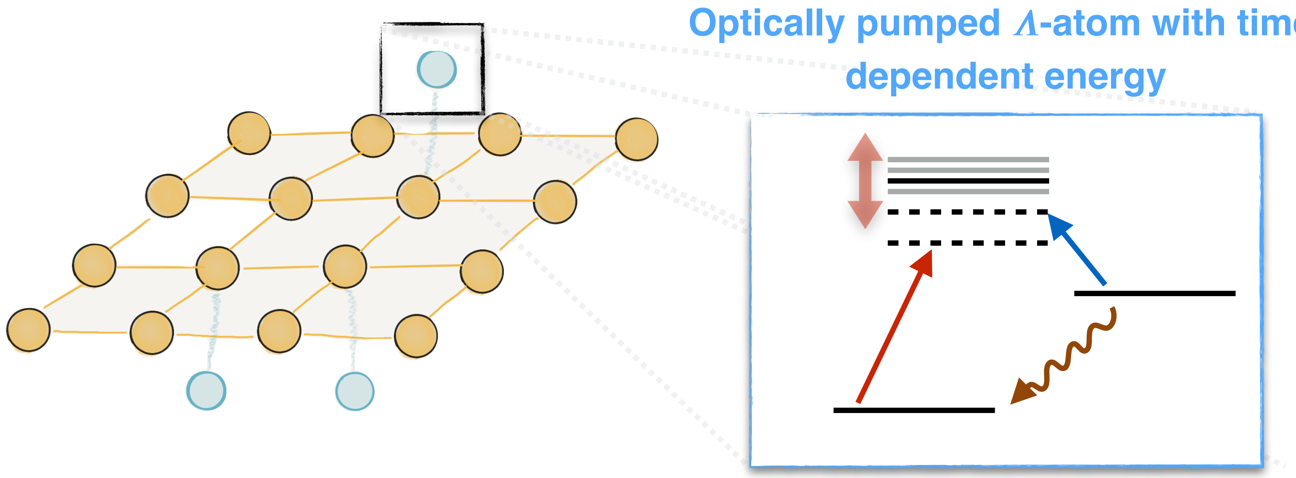
1. Easily predicted spectra => we can choose the energies of the ancillas to achieve resonant energy exchange

2. Local perturbations create energy excitations => so local couplings to ancilla move one up and down between eigenstates (ergodicity easy)

How to get thermalization?

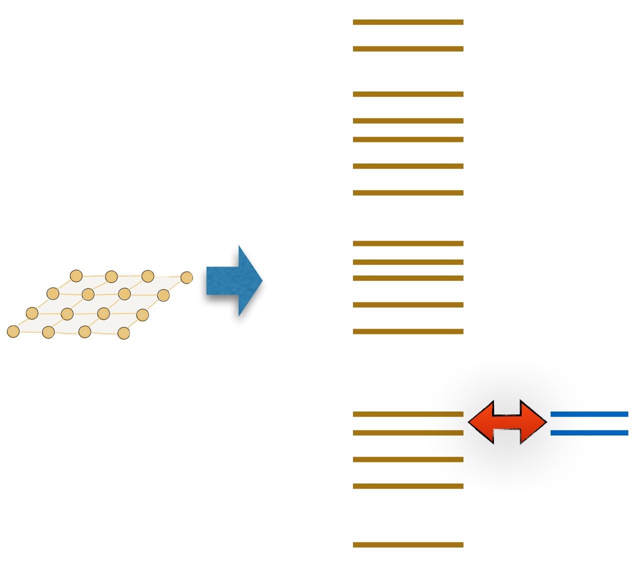
A generalization of this approach

M.S., Jonathan Moussa, C. Daniel Freeman



We assume ergodicity is satisfied by the system-ancilla couplings

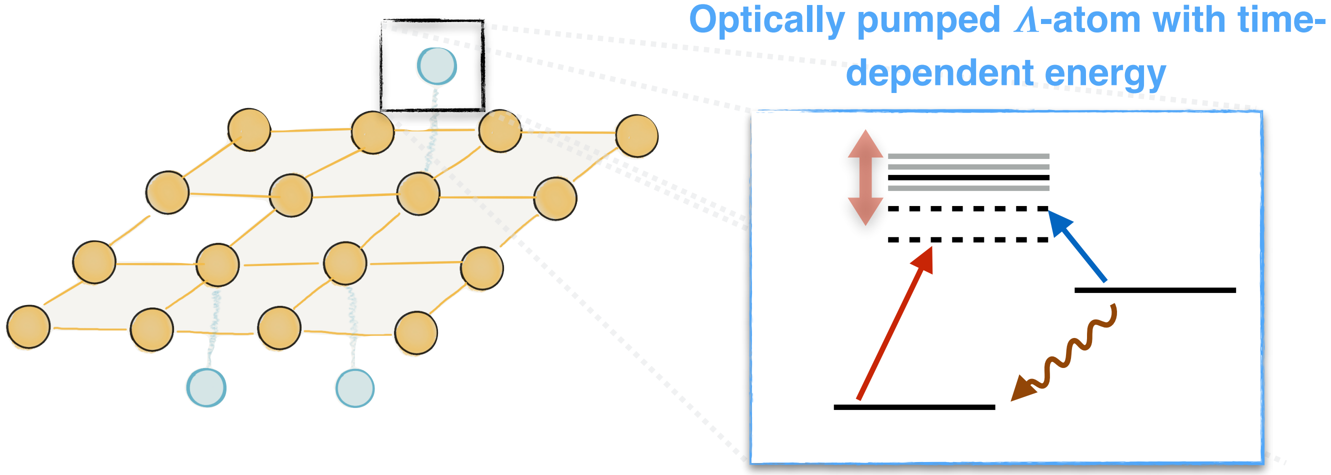
This mimics a macroscopic bath over some longer timescale



How to get thermalization?

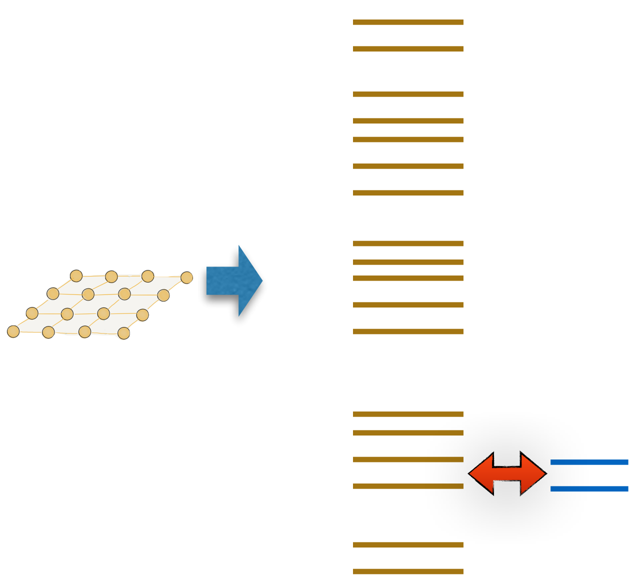
A generalization of this approach

M.S., Jonathan Moussa, C. Daniel Freeman



We assume ergodicity is satisfied by the system-ancilla couplings

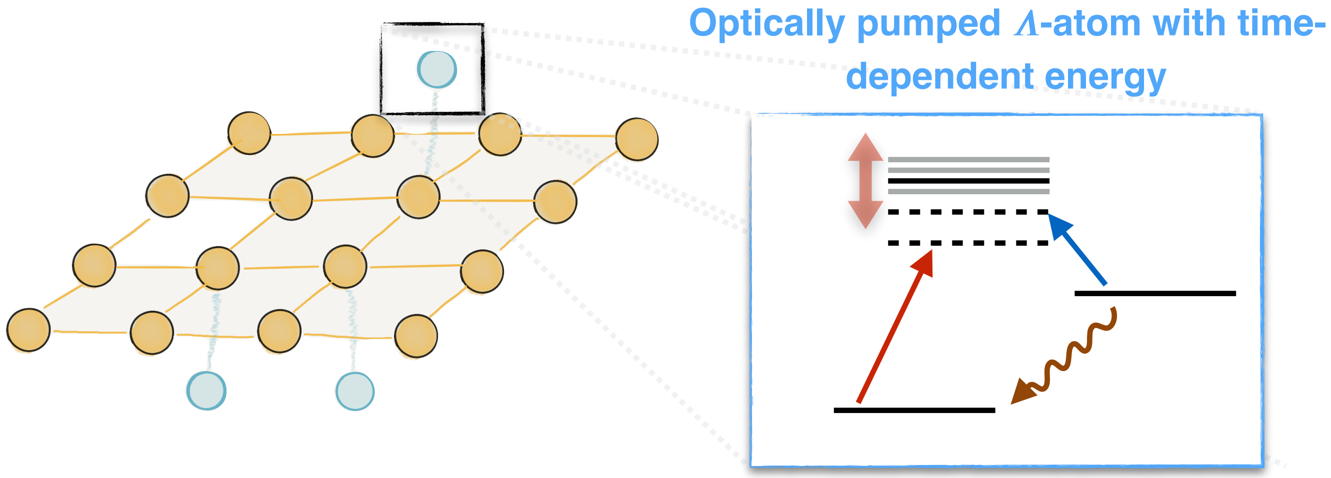
This mimics a macroscopic bath over some longer timescale



How to get thermalization?

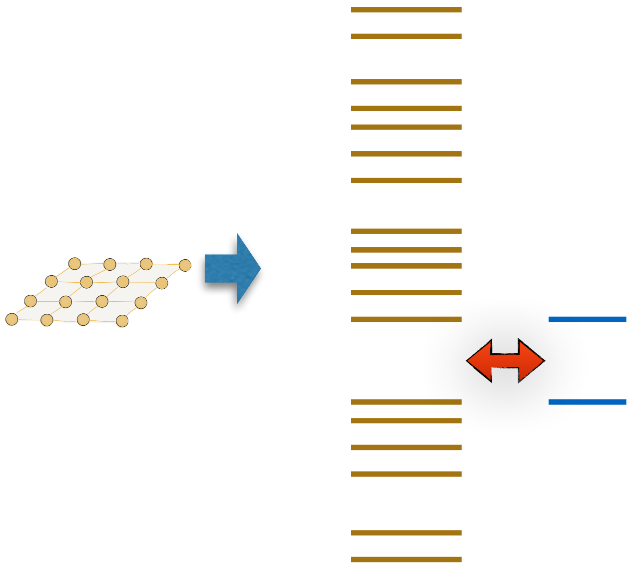
A generalization of this approach

M.S., Jonathan Moussa, C. Daniel Freeman



We assume ergodicity is satisfied by the system-ancilla couplings

This mimics a macroscopic bath over some longer timescale

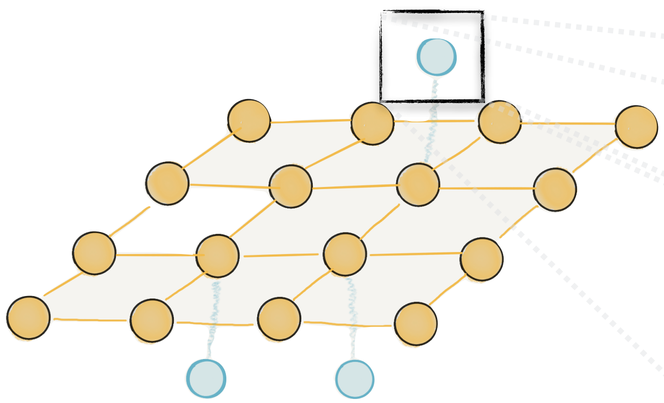


Ancillas will be resonant with different energy transitions in the system at different times, so as long as we maintain Boltzmann populations in the ancilla at all time, we will induce transitions that thermalize the system.

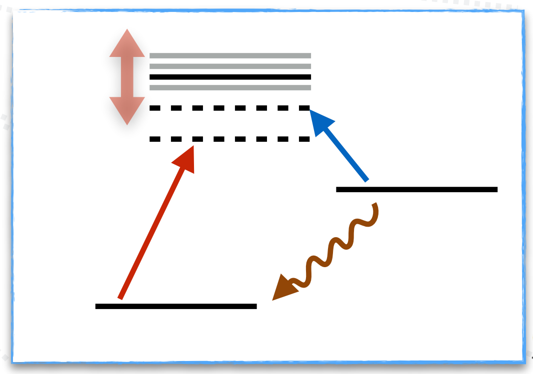
How to get thermalization?

Spin lattice example

M.S., Jonathan Moussa, C. Daniel Freeman



Optically pumped Λ-atom with time-dependent energy



$$\frac{dr^m(t)}{dt} = \mathcal{L}_m(t)[r^m(t)] = \gamma_+^m(t)\mathcal{D}[\tau_+^m]r^m(t) + \gamma_-^m(t)\mathcal{D}[\tau_-^m]r^m(t),$$

$$H_T(t) = H_{\text{sys}} - \sum_{m=1}^{M} \frac{\Omega_m(t)}{2}\tau_z^m + \sum_{m=1}^{M} g_m\left(\sigma_x^{k_m}\tau_x^m\right),$$

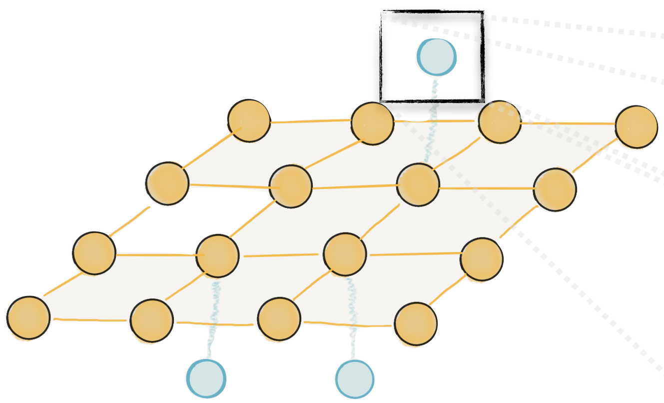
We require the parameter regime

$$\left|\frac{d\Omega_m(t)}{dt}\right| \ll g_m \sim \Gamma^m \ll ||H_{\text{sys}}||, \quad \forall m, t$$

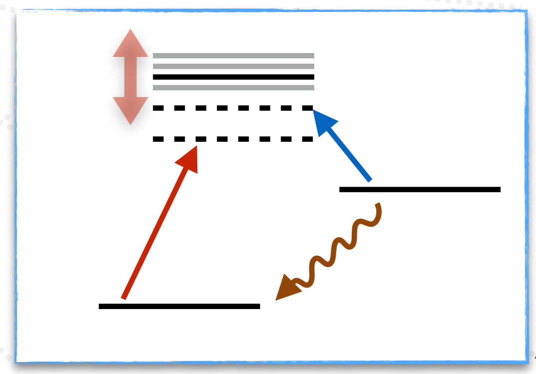
$$\Gamma^m \equiv \gamma_+^m + \gamma_-^m$$

How to get thermalization?

Spin lattice example



Optically pumped Λ-atom with time-dependent energy



$$\frac{dr^m(t)}{dt} = \mathcal{L}_m(t)[r^m(t)] = \gamma_+^m(t)\mathcal{D}[\tau_+^m]r^m(t) + \gamma_-^m(t)\mathcal{D}[\tau_-^m]r^m(t),$$

$$H_T(t) = H_{\text{sys}} - \sum_{m=1}^{M} \frac{\Omega_m(t)}{2}\tau_z^m + \sum_{m=1}^{M} g_m \left(\sigma_x^{k_m}\tau_x^m\right),$$

We require the parameter regime

$$\left|\frac{d\Omega_m(t)}{dt}\right| \ll g_m \sim \Gamma^m \ll ||H_{\text{sys}}||, \quad \forall m, t$$

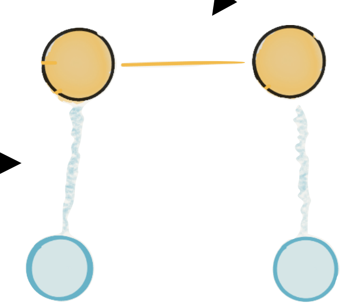
$$\Gamma^m \equiv \gamma_+^m + \gamma_-^m$$

In this regime we can derive a reduced master equation for the system alone that describes its evolution when the ancilla dynamics is averaged over.

A simple example

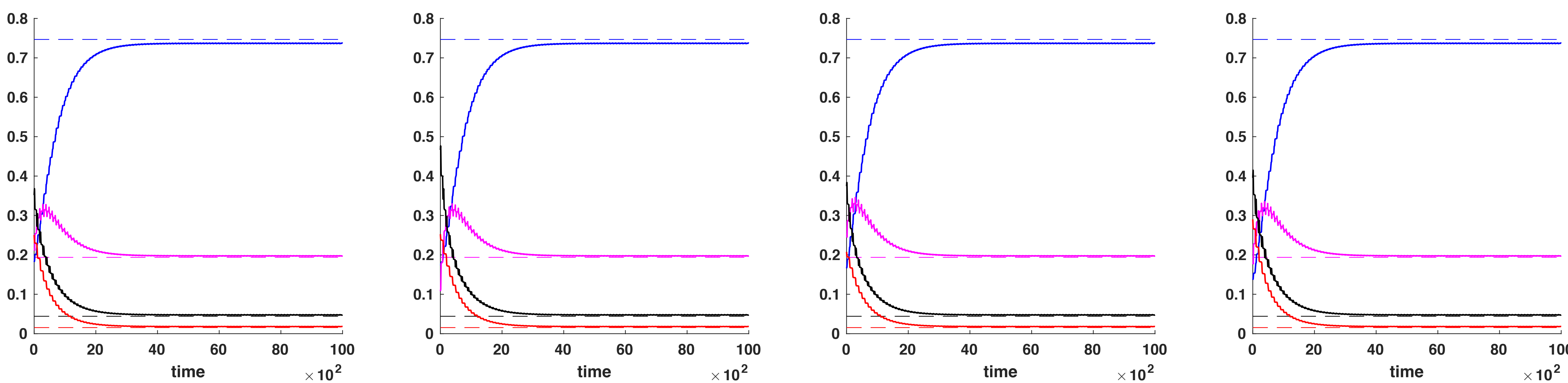
$$H_{\text{sys}} = -\frac{\omega_1}{2}\sigma_z^1 - \frac{\omega_2}{2}\sigma_z^2 + J_x\sigma_x^1\sigma_x^2 + J_y\sigma_y^1\sigma_y^2 + J_z\sigma_z^1\sigma_z^2$$

$$H_{\text{s-a}} = \sum_{i=1}^{2} g(\sigma_x^i \tau_x^1)$$



$$\Omega_i(t) = \Omega_{\min} + \Omega_{\max}\sin(\omega t)$$

Thermalization from four random initial states



Ongoing work

1. Bounds on thermalization time of this protocol

2. Comparison to discrete-time (gate-based) thermalization protocols (e.g. quantum Metropolis)

Thanks!

Collaborators

Reliability of analog quantum simulation

Jun Zhang, Lishan Zeng
Shanghai Jiao Tong University

Engineered thermalization

Jonathan Moussa
Sandia National Labs, Albuquerque

C. Daniel Freeman
Univ. California, Berkeley

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

Backup slides

Interlude: parameter space compression

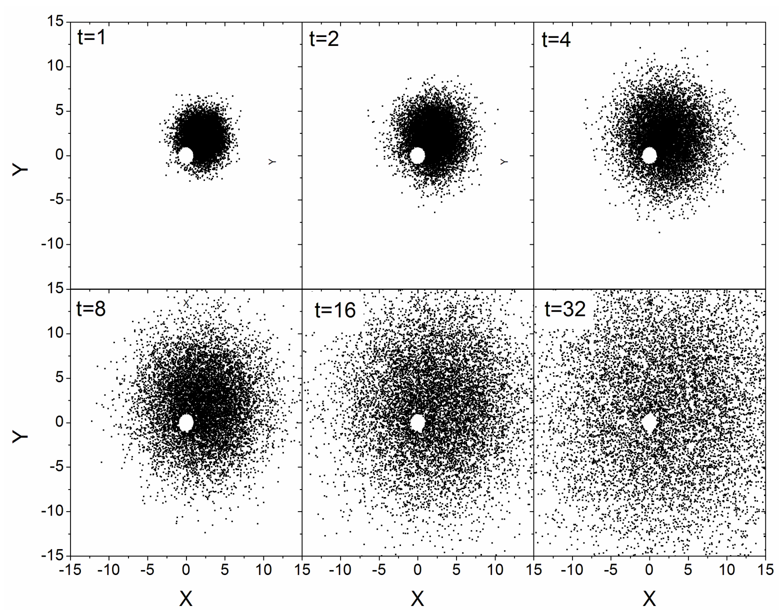
Sethna group, Cornell University

Why the unreasonable effectiveness of simple models in physics?

Science AAAS

Parameter Space Compression Underlies Emergent Theories and Predictive Models
Benjamin B. Machta *et al.*
Science **342**, 604 (2013);
DOI: 10.1126/science.1238723

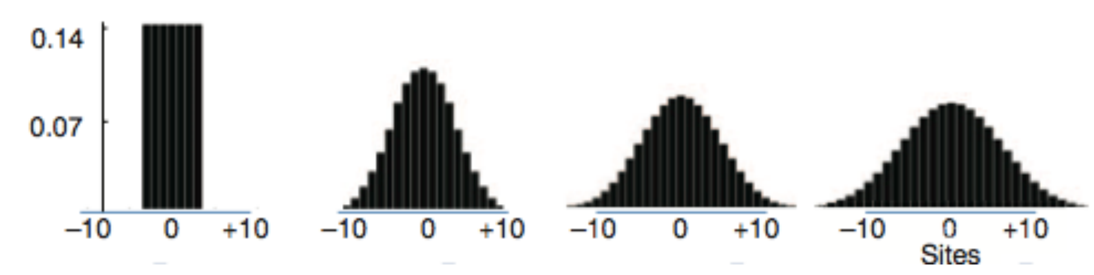
e.g. particle diffusion



Plante & Cucinotta

Equation for particle density

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) - V\frac{\partial}{\partial x}\rho(x,t) + R\rho(x,t)$$



Only 3 parameters!

PSC and Fisher Information

General setup

Model dependent on K parameters $\lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$

Interested in some set of observables $\vec{x}$

Parameterized distribution over observables $P_\lambda(\vec{x})$

PSC and Fisher Information

General setup

Model dependent on K parameters $\lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$

Interested in some set of observables $\vec{x}$

Parameterized distribution over observables $P_\lambda(\vec{x})$

Fisher Information matrix (FIM)

$$F_{ij}(\lambda) = \sum_{\vec{x}} \frac{1}{P_\lambda(\vec{x})} \left(\frac{\partial P_\lambda(\vec{x})}{\partial \lambda_i} \right) \left(\frac{\partial P_\lambda(\vec{x})}{\partial \lambda_j} \right)$$

$$\boxed{F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger}$$

Key observation:

Eigenvalues of FIM prescribe an ordering of parameter influence, *i.e.* if ζ_k is large, then the parameter

$$\sum_j \lambda_j v_k^j$$

has a large influence on observations

PSC and Fisher Information

General setup

Model dependent on K parameters $\lambda = (\lambda_1, \lambda_2, ..., \lambda_K)$

Interested in some set of observables $\vec{x}$

Parameterized distribution over observables $P_\lambda(\vec{x})$

Fisher Information matrix (FIM)

$$F_{ij}(\lambda) = \sum_{\vec{x}} \frac{1}{P_\lambda(\vec{x})} \left(\frac{\partial P_\lambda(\vec{x})}{\partial \lambda_i} \right) \left(\frac{\partial P_\lambda(\vec{x})}{\partial \lambda_j} \right)$$

$$\boxed{F = \sum_{k=1}^{K} \zeta_k v_k v_k^\dagger}$$

Key observation:

Another way to think about this:

Sensitivity analysis — FIM is the expected value of a Hessian:

$$F_{ij} = -\left\langle \frac{\partial^2 \log(p_m(\lambda))}{\partial \lambda_i \partial \lambda_j} \right\rangle$$

Parameter space compression

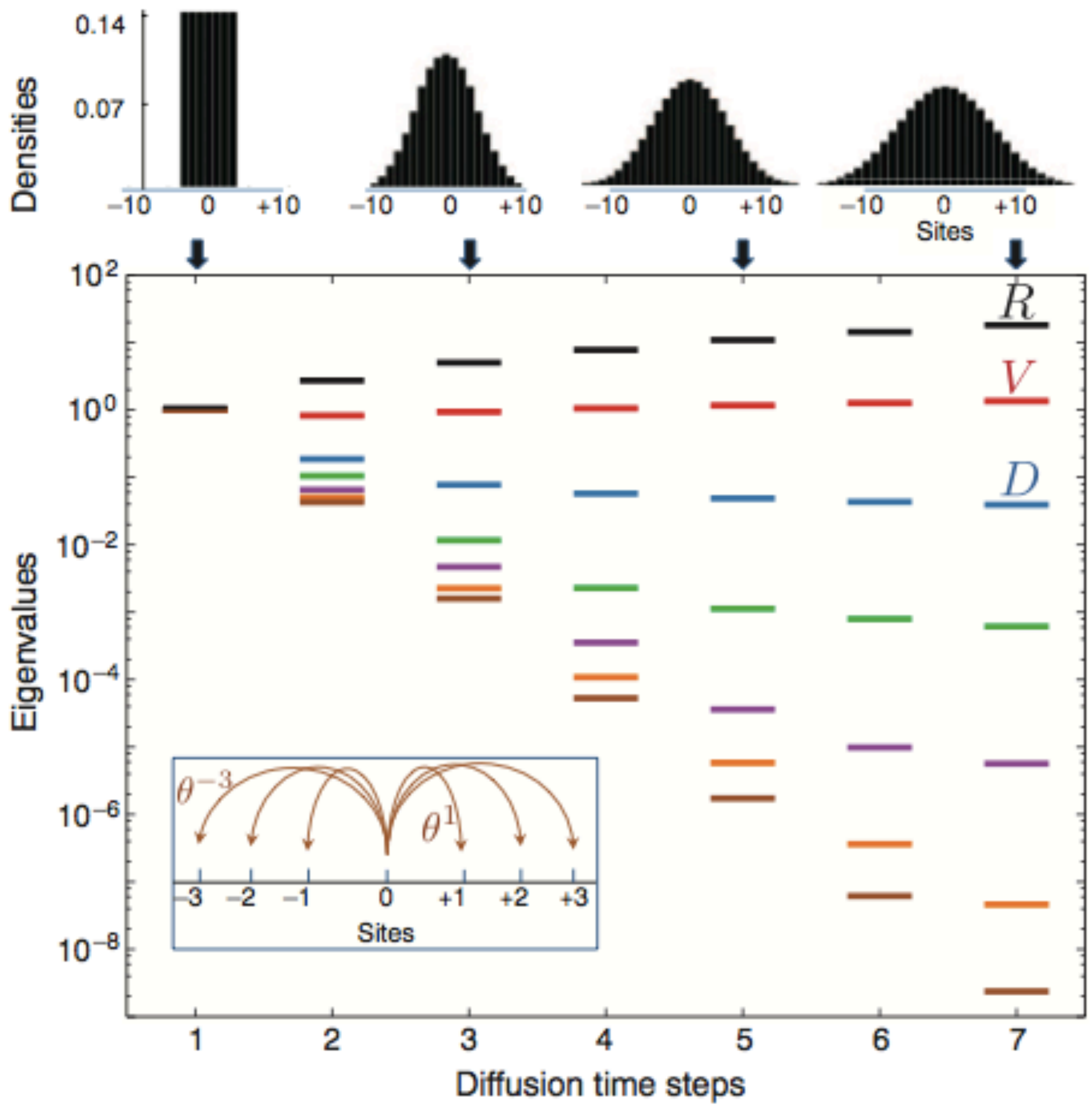
e.g. particle diffusion

Machta et al. Science, 342, 604 (2013)

Equation for particle density

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) - V\frac{\partial}{\partial x}\rho(x,t) + R\rho(x,t)$$

Discrete time/space diffusion in 1D



Observable here is the full particle density.

If we ask what the particle density is at every time step, then eigenvalues are clustered.

But if we only ask every few time steps, then eigenvalues separate, and a few important parameters emerge, and these are R, V, D.

Parameter space compression

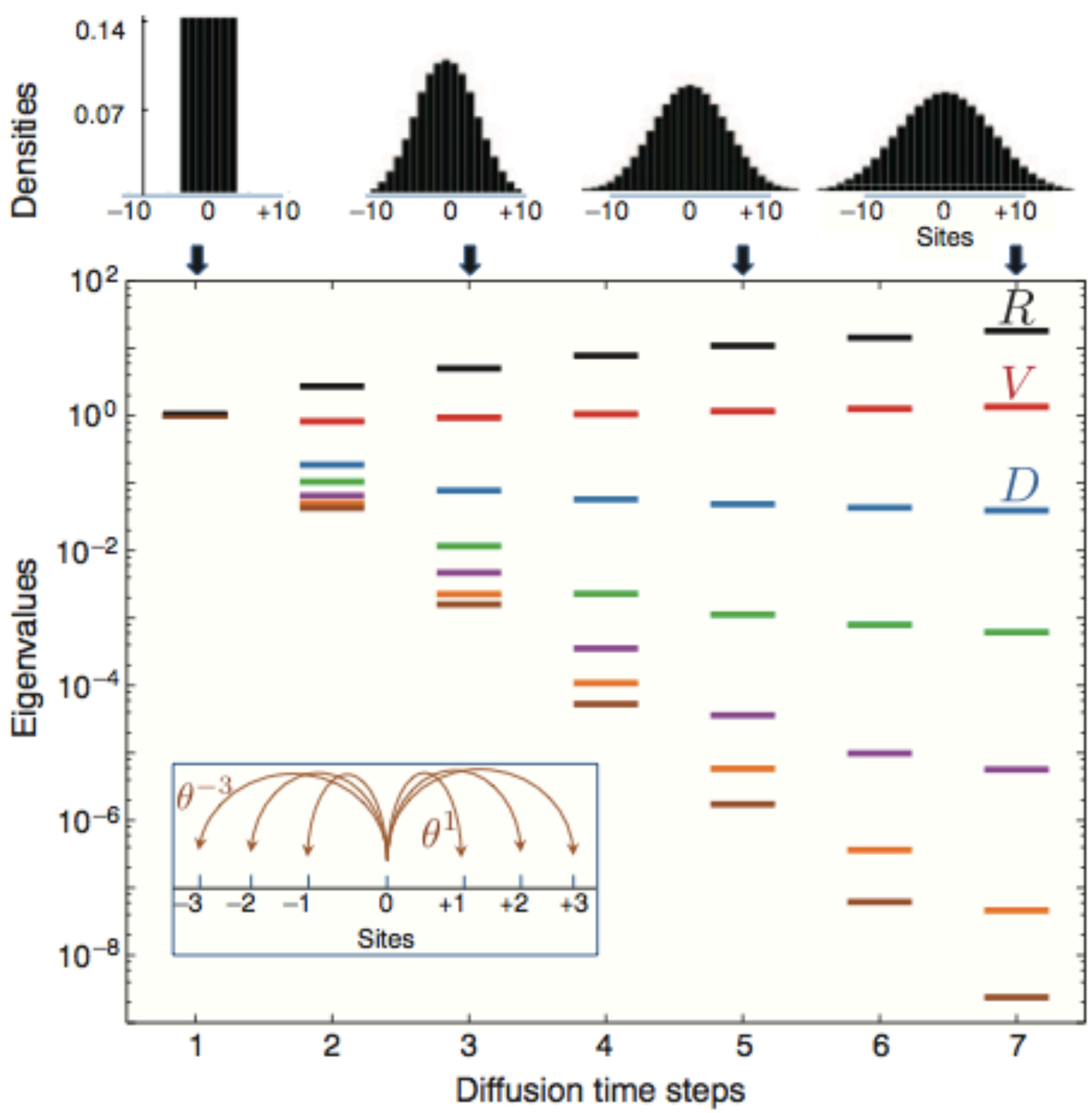
e.g. particle diffusion

Machta et al. Science, 342, 604 (2013)

Equation for particle density

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial^2}{\partial x^2}\rho(x,t) - V\frac{\partial}{\partial x}\rho(x,t) + R\rho(x,t)$$

Discrete time/space diffusion in 1D



Some terminology

Stiff parameters

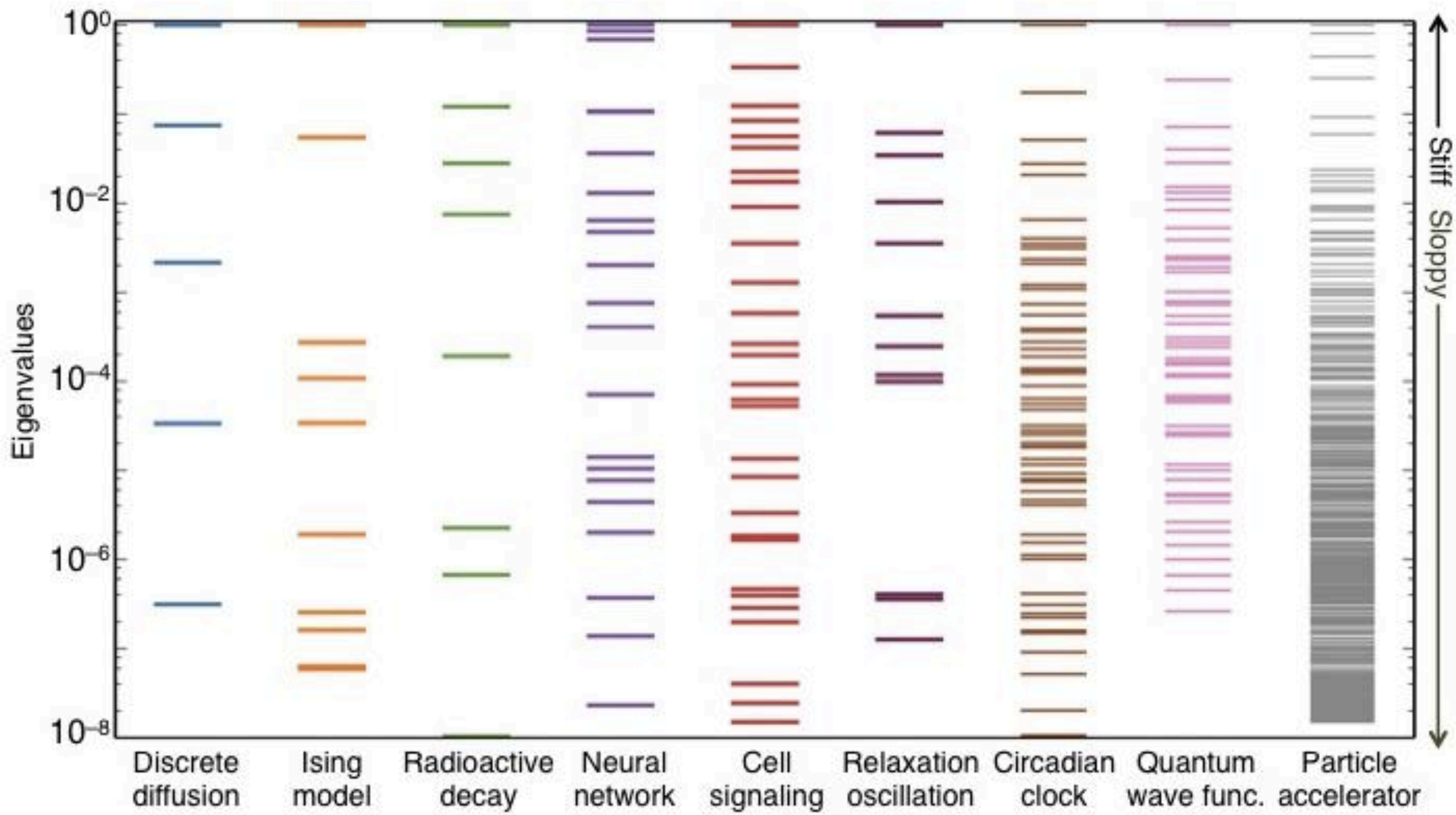
Sloppy parameters

Diffusion is a sloppy model

Sloppiness abounds!

This phenomenon has been demonstrated for many models in physics and biophysics.

Sethna *et al.*: Models of nature tend to be sloppy, and this is why science is even possible



From the Sethna group website

Scaling the analysis

The true value of a quantum simulator is to extract properties of models that are not classically tractable.

But the FIM can only be explicitly calculated for small versions of a model.

Strategies for scaling this analysis technique [arXiv:1603.09283]:

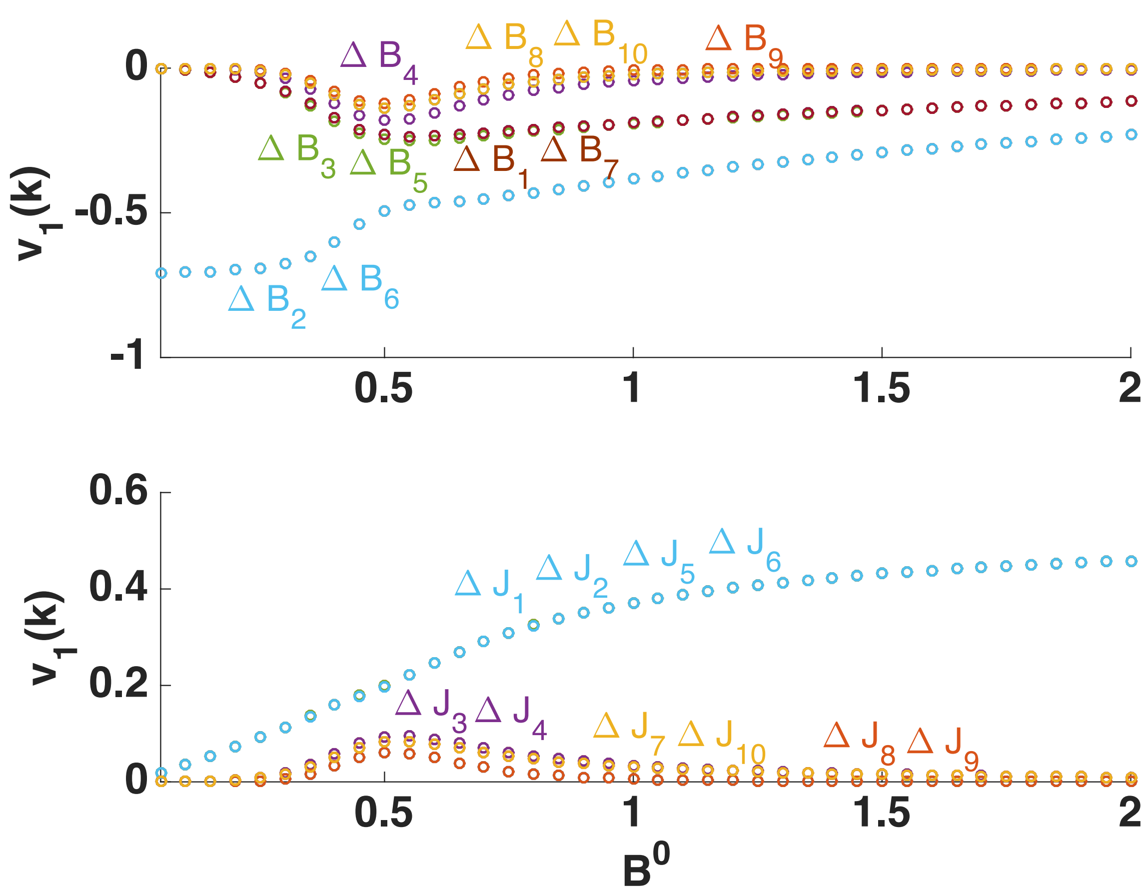
1. Symmetry analysis can be done analytically, independent of system size.
   e.g. translational invariance a powerful symmetry that implies sensitivity *only to* collective parameters for any model size.
2. Empirically, sloppiness and form of stiff parameters carries over from small-scale models to large-scale versions.
   e.g. TFIM with correlation function

Transverse field Ising model

Correlation function observable

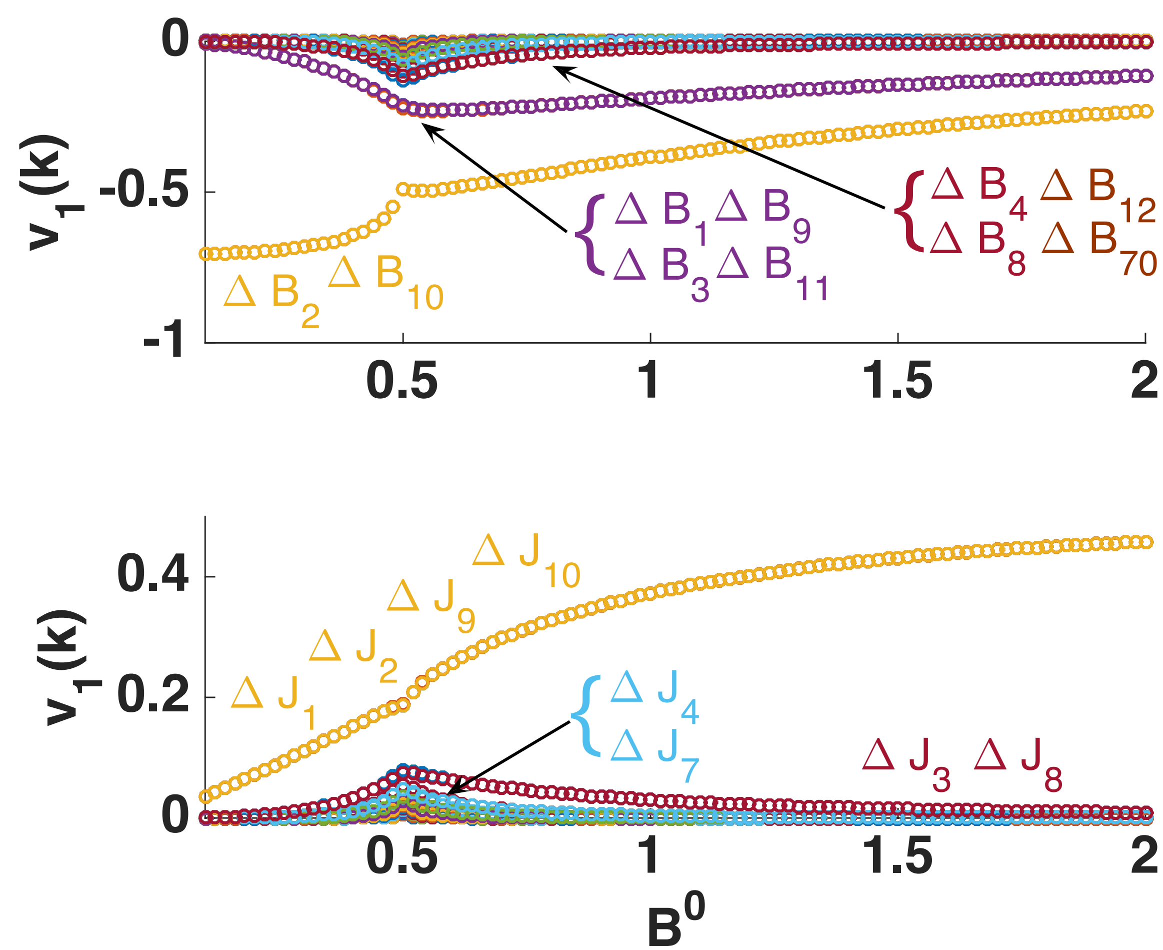
$n = 10, J^0 = 1$

$C_z(2,6)$



$n = 70, J^0 = 1$

$C_z(2,10)$



Qualitatively the same

Scaling the analysis

The true value of a quantum simulator is to extract properties of models that are not classically tractable.

But the FIM can only be explicitly calculated for small versions of a model.

Strategies for scaling this analysis technique [arXiv:1603.09283]:

1. Symmetry analysis can be done analytically, independent of system size.
   e.g. translational invariance a powerful symmetry that implies sensitivity *only to* collective parameters for any model size.
2. Empirically, sloppiness and form of stiff parameters carries over from small-scale models to large-scale versions.
   e.g. TFIM with correlation function
3. Conjecture:
   If a small-scale model is sloppy, then its large-scale version will also be sloppy.