Failure of Sierra White granite under general states of stress

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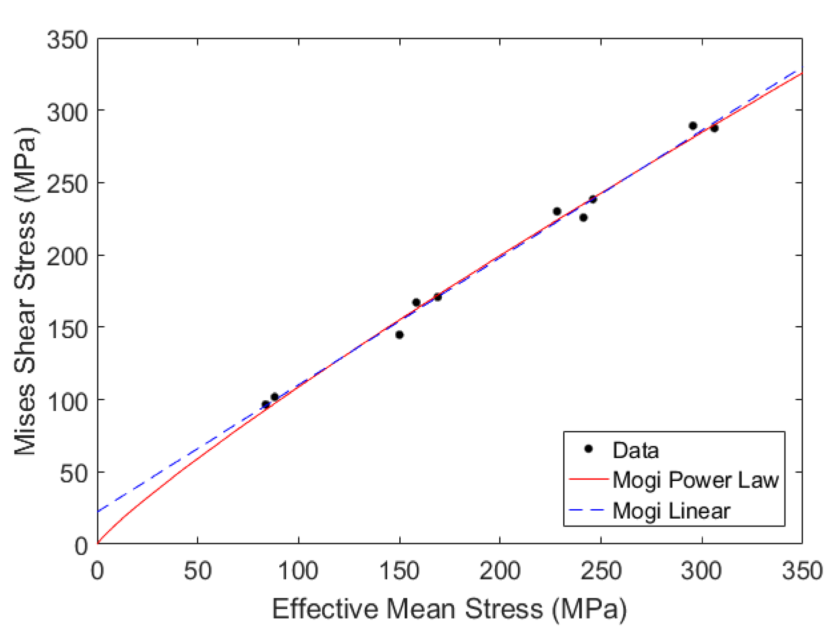
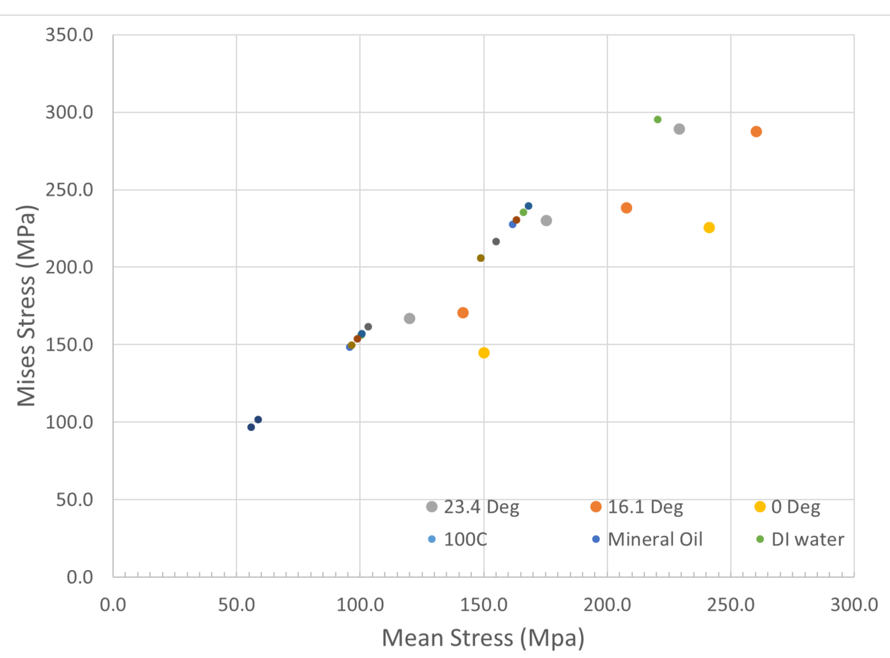
Abstract: The effect of the intermediate principal stress on the failure of Sierra White granite was investigated by performing tests under true triaxial states of stress. Tests were performed under constant Lode angle conditions with Lode angles ranging from 0 to 30°, pure shear to axisymmetric compression. Results show that the failure of Sierra White granite is heavily dependent on the intermediate principal stress which became more dramatic as the mean stress increased. An analysis of the shear bands formed at failure was performed using an associated flow rule and the Rudnicki and Rice (1975) localization criteria. The localization analysis showed excellent agreement with experimental results.

$$\theta = \frac{\pi}{4} + \frac{1}{2} asin\left[\frac{\frac{2}{3}(1+\nu)(\beta+\mu) - N_{II}(1-2\nu)}{\sqrt{4-3N_{II}^2}}\right]$$

$$N_{II} = \frac{(\sigma-\sigma_2)}{\tau}$$





Mogi Criterion:

The linear Mogi failure criterion provides a reasonable estimate of the response of Sierra White granite to true triaxial states of stress. The upper figure plots the Mises equivalent shear stress versus the effective mean normal stress. It is evident that these results are aligned quite well with the single line, described by Mogi's linear failure criterion where *a*=22.21 and *b*=0.880 (correlation coefficient of r^2=0.9928).

The power law version of the Mogi failure criterion also fits the data from this study quite well with the fitting parameters of α=1.923 and n=0.8762, and a correlation coefficient of r^2=0.9917. Note the small difference between the correlation coefficient for the linear and power law fits.

Localization:

To determine the band angles (example below) the equations above were used, and normality was assumed, i.e. the increment of strain is normal to the yield surface, or β=μ. The other parameters in these equations, ν= Poisson's Ratio, σ=mean stress, τ= shear stress, and σ2=intermediate principal stress were determined from the experiments.

$$\sigma = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$$

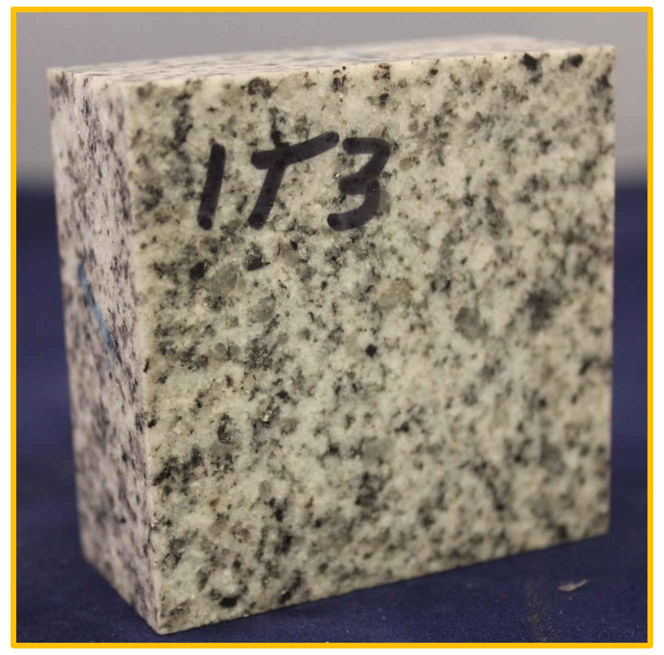
$$\tau = \sqrt{\frac{1}{6}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]}$$

$$\theta = \tan^{-1}\left[\frac{\sigma_1-2\sigma_2+\sigma_3}{\sqrt{3}(\sigma_1-\sigma_3)}\right]$$

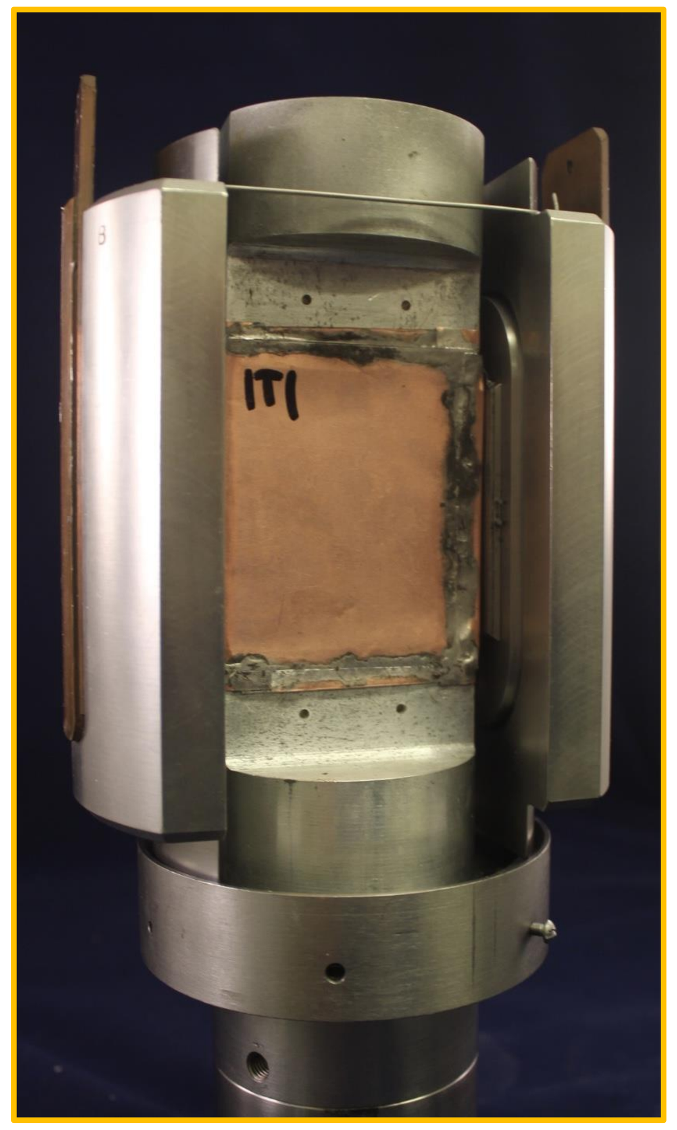
$$\sigma_{me} = \frac{(\sigma_1+\sigma_3)}{2}$$

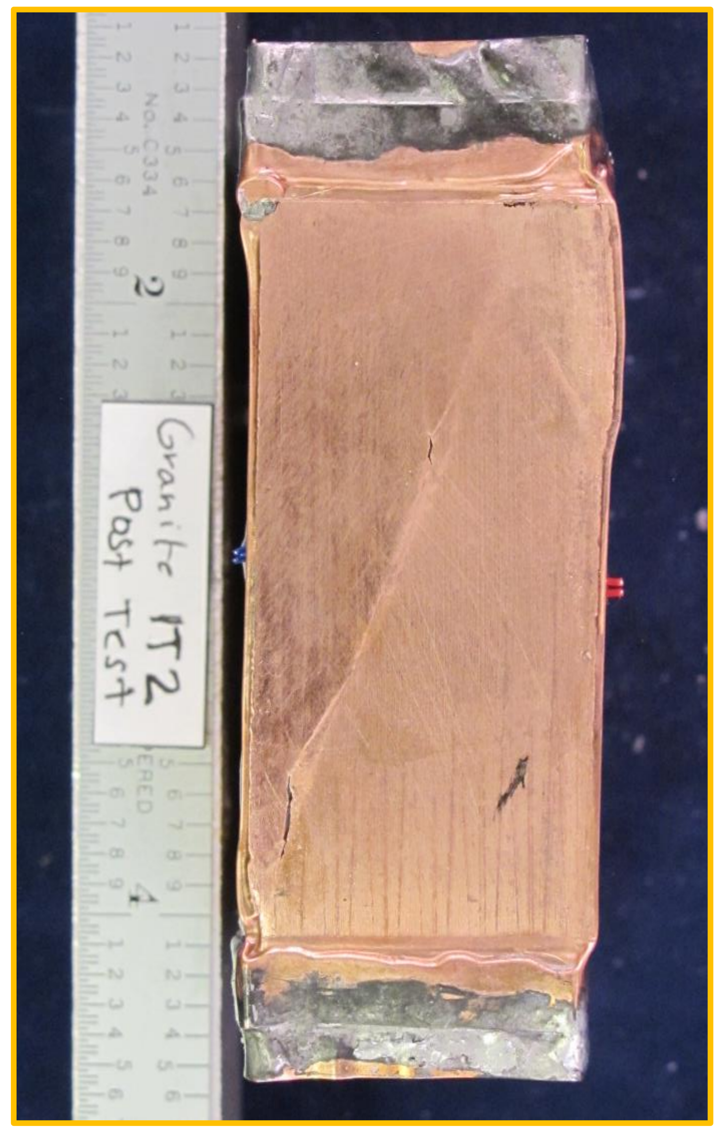
$$\tau = a + b\sigma_{me}$$

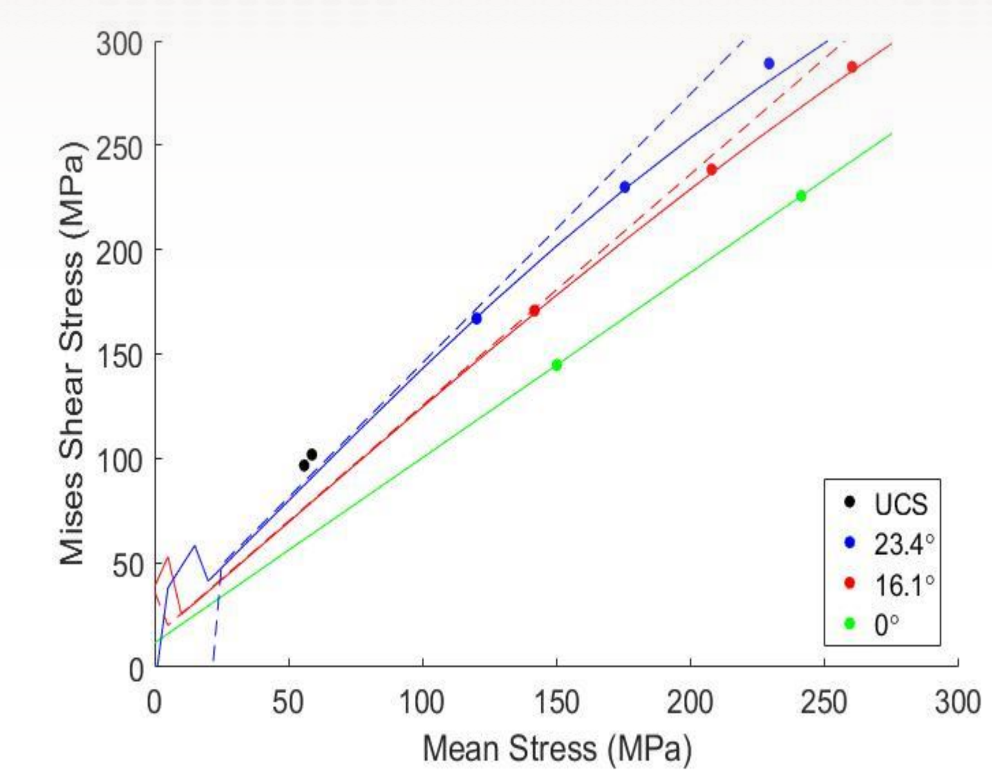
$$\tau = \alpha\sigma_{me}^n$$

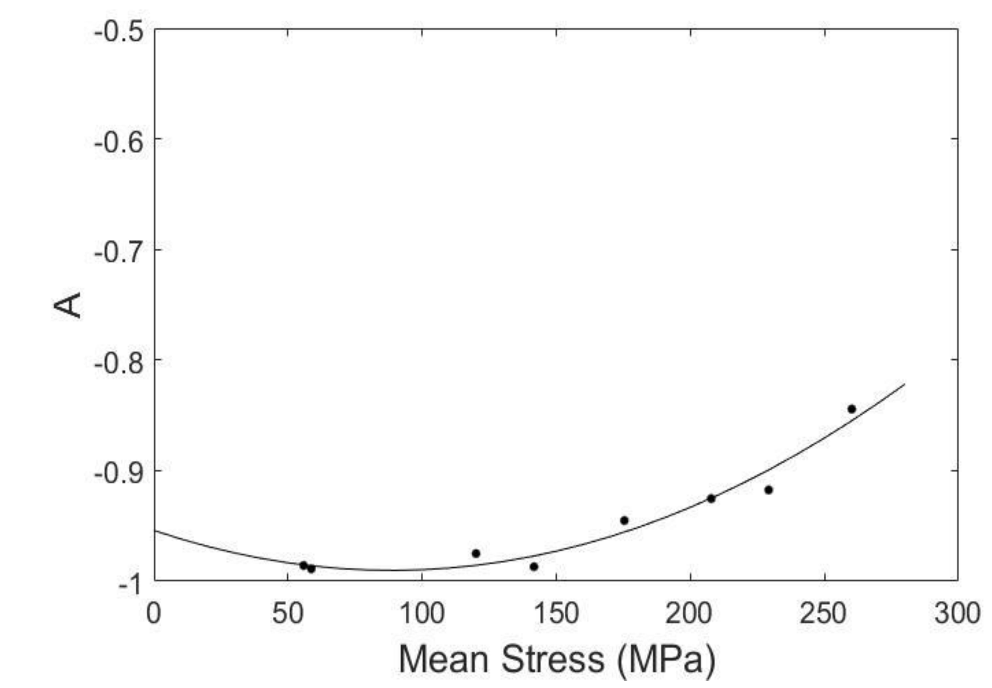


Specimen	Mean Stress (MPa)	Shear Stress (MPa)	Lode Angle	Measured Angle	Predicted Angle
1A	58.7	101.8	30.0	NA	87.2
3A	55.8	96.6	30.0	NA	87.2
1T2	260.2	287.5	16.1	69	69.4
1T3	141.7	170.8	16.1	70	69.3
1T4	120.1	167.0	23.4	68	75.0
1T5	175.4	230.0	23.4	73	75.0
1T6	150.0	144.7	0.0	71	66.7
1T7	207.8	238.4	16.1	68	69.3
1T8	229.2	289.2	23.4	72	75.0
1T9	149.1	143.8	0.0	60	66.7
1T13	241.2	225.7	0.0	65	67.8









Modified MNLD Criterion:

The Matsuoka-Nakai-Lade-Duncan Failure criterion as modified by Rudnicki (mMNLD) was fit to our experimental results. The fit was performed by first fitting the 0° Lode angle data to determine a functional form for $\tau_0(\sigma)$, because at θ=0 the first term of the equation disappears. This is then substituted back into the equation and solved for the rest of the data to determine the equation for *A(σ)*.

A plot of *A* vs the mean stress is shown in to the left. This function shows good agreement with the data. However, the quadratic function begins to decrease significantly below approximately 50 MPa mean stress. This causes the fitting function to become unstable. To improve the fit for *A*, more tests would need to be run at lower mean stresses, which would require applying tension to the samples. The result is that this criterion fits our experimental data very well even with non-ideal fitting functions. As shown by the fits in the plot (solid lines represent the MNLD criterion).

$$\tau_0(\sigma) = 0.8878\sigma + 11.604$$

$$A(\sigma) = 5E^{-6}\sigma^2 - 0.0008\sigma - 0.9541$$

$$\sqrt{\frac{4}{27}}A(\sigma)\sin(3\theta)\left(\frac{\tau}{\tau_0(\sigma)}\right)^3 + \left(\frac{\tau}{\tau_0(\sigma)}\right)^2 - 1 = 0$$

Analysis:

The upper figure to the left includes the best fit Rankine surface for each of the Lode angles (dashed line). These curves were determined by setting the value of *A* in the criterion described by Rudnicki to -1 resulting in an inverted Rankine surface. This provides a reasonable fit to the data at lower mean stresses. However, the Rankine criterion does not take into account the nonlinearity in the failure surface with increasing mean stress due to the effect of the intermediate principal stress. The *A* values are relatively constant (Figure 8) implying that the failure surface curvature remains fairly constant with increasing mean stress. In other words, the intermediate principal stress effect on failure does not change significantly with mean stress at the stress levels tested.

If the value of *A* is taken to the other extreme (*A*=0), the failure criterion becomes $\tau = \tau_0(\sigma)$. Thus, if the function selected for τ_0 is linear, then the failure criterion becomes a Drucker-Prager criterion. This means that the criterion for all Lode angles reduces to a single line, which is the existing fit for the 0° Lode angle shown in the figure as a solid green line.

Although the Mogi failure criteria are good approximations of failure, more complex equations like mMNLD tend to fit true triaxial data even better, and do not require that the intermediate principal stress dependence is removed from the failure criterion. This is especially true for specimens that undergo a brittle-ductile transition. The mMNLD criterion requires more complicated parameterization, but is effective and simple to use once tests are performed.

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