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Global Solution Strategies for the Network-Constrained Unit Commitment Problem with AC Transmission Constraints

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***Abstract*—We propose a novel global solution algorithm for the network-constrained unit commitment problem that incorporates a nonlinear alternating current (AC) model of the transmission network, which is a nonconvex mixed-integer nonlinear programming (MINLP) problem. Our algorithm is based on the multi-tree global optimization methodology, which iterates between a mixed-integer lower-bounding problem and a nonlinear upper-bounding problem. We exploit the mathematical structure of the unit commitment problem with AC power flow constraints (UC-AC) and leverage second-order cone relaxations, piecewise outer approximations, and optimization-based bounds tightening to guarantee a globally optimal solution at convergence. Numerical results on four benchmark problems illustrate the effectiveness of our algorithm, both in terms of convergence rate and solution quality.**

A. Notation

Sets

$\mathcal{B}$	Set of all buses $\{1, ..., B\}$
$\mathcal{B}_b$	Set of all buses that are connected to bus b
$\mathcal{C}$	Set of all cycles in a cycle basis for the network
$\mathcal{G}$	Set of all generators $\{1, ..., G\}$
$\mathcal{G}_b$	Set of all generators at bus b
$\mathcal{L}$	Set of all branches (transmission lines)
$\mathcal{L}_c$	Set of branches in cycle c
$\mathcal{L}_b^{in}$	Set of all *inbound* branches to bus b
$\mathcal{L}_b^{out}$	Set of all *outbound* branches from bus b
$\mathcal{S}_g$	Set of startup segments of generator g $\{1, ..., S_g\}$
$\mathcal{SC}$	Set of all synchronous condensers $\{1, ..., SC\}$
$\mathcal{SC}_b$	Set of all synchronous condensers at bus b
$\mathcal{T}$	Set of time periods $\{1, ..., T\}$

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Parameters

$A_{g,n}$	Coefficients ($n = 0, 1, 2$) of quadratic production cost function of generator g
B_b^{sh}	Shunt susceptance at bus b
B_l	Imag. part of branch l admittance matrix
G_b^{sh}	Shunt conductance at bus b
G_l	Real part of branch l admittance matrix
$K_{g,\tau}^{su}$	Startup cost of generator g
K_g^{sd}	Shutdown cost of generator g
$P_{b,t}^D$	Real power demand at bus b, time t
P_t^R	System reserve requirement at time t
$P_g^{G,min}$	Min. real power output of generator g
$P_g^{G,max}$	Max. real power output of generator g
$Q_{b,t}^D$	Reactive power demand at bus b, time t
$Q_g^{G,min}$	Min. reactive power output of generator g
$Q_g^{G,max}$	Max. reactive power output of generator g
$Q_{sc}^{SC,min}$	Min. output of synchronous condenser sc
$Q_{sc}^{SC,max}$	Max. output of synchronous condenser sc
RD_g	Ramp-down limit of generator g
RU_g	Ramp-up limit of generator g
S_l^{max}	Apparent power limit on branch l
SD_g	Shutdown capability of generator g
SU_g	Startup capability of generator g
$T_{g,\tau}^{su}$	Startup cost function time segment for generator g
T_g^u	Min. uptime of generator g
T_g^d	Min. downtime of generator g
V_b^{min}	Min. voltage magnitude at bus b
V_b^{max}	Max. voltage magnitude at bus b

Variables

$\delta_{g,\tau,t}$	Startup cost segment indicator
$\theta_{l,t}$	Voltage phase angle difference between ends (bus b and bus k) of branch l at time t, $\theta_{b,k,t}$

$c_{b,k,t}$ Second-order cone variable
$c^p_{g,t}$ Production cost for generator g at time t
f^p Total production cost
f^{sd} Total shutdown cost
f^{su} Total startup cost
$p^G_{g,t}$ Real power output of generator g at time t
$p^f_{l,t}$ Real power flow *from* branch l, at time t
$p^t_{l,t}$ Real power flow *to* branch l, at time t
$q^G_{g,t}$ Reactive power output of generator g at time t
$q^f_{l,t}$ Reactive power flow *from* branch l, at time t
$q^t_{l,t}$ Reactive power flow *to* branch l, at time t
$q^{SC}_{sc,t}$ Reactive power output of synchronous condenser sc at time t
$r^a_{g,t}$ Real power reserve provided by generator g at time t
$s_{b,k,t}$ Second-order cone variable
$u_{g,t}$ Startup status, equal to 1 if generator g starts up at time t, 0 otherwise
$v_{b,t}$ Voltage magnitude at bus b at time t, $v^2_{b,t} = (v^r_{b,t})^2 + (v^j_{b,t})^2$
$v^j_{b,t}$ Imag. part of voltage phasor at bus b, time t
$v^r_{b,t}$ Real part of voltage phasor at bus b, time t
$w_{g,t}$ Shutdown status, equal to 1 if generator g shuts down at time t, 0 otherwise
$y_{g,t}$ Unit on/off status, equal to 1 if generator g is on-line at time t, 0 otherwise

I. Introduction

RECENTLY the Federal Energy Regulatory Commission (FERC) reported that uplift – out-of-market payments that result when out-of-merit generation costs are incurred to relieve a constraint – can arise due to the inability of independent system operators (ISOs) to fully model the steady-state physics on an alternating current (AC) network [1]. According to a recent National Academies report [2], solving this problem “could significantly improve the modeling and efficient dispatch of resources during the commitment, dispatching, and pricing processes.” Recent work on the day-ahead unit commitment problem, which was led by MISO technical staff in [3], attests to the importance and non-trivial complexity of incorporating AC network constraints due to the performance challenges introduced by denser matrices and additional nonlinearities.

Because of these modeling difficulties, current practice is to perform unit commitment using DC approximations (or copper plate) to represent the transmission network. These approximations do not allow rigorous treatment of AC power flow constraints. As a result, certain resources are consistently committed outside of the market to address unforeseen reliability issues; this results in concentrated uplift payments [1]. Such resources are often required for reactive power compensation in order to provide system voltage control that enables more efficient delivery and utilization of real power [4]. Because such reliability requirements are largely unmodeled in day-ahead unit commitment, more cost effective resources are displaced for these out-of-merit commitments. Alternatively, in the real-time market, operators may have to manually commit and dispatch reliability units while also manually re-dispatching or de-committing other resources, e.g., exceptional dispatches in CAISO [5], out-of-merit generation in NYISO [6], and balancing operating reserves in PJM [7].

To address these concerns, this paper focuses on solution of the unit commitment problem with AC power flow constraints (UC-AC). Solving real-world operations and market settlement with alternating current optimal power flow (ACOPF) is not trivial. Due to the scale of real-world power systems, network-constrained unit commitment problems can be extremely large and computationally challenging to solve. Coupling this with nonconvex AC power flow constraints leads to a mixed-integer nonlinear programming (MINLP) problem that is NP-hard [8–10]. If the continuous relaxation of the MINLP is a convex optimization problem, we refer to it as a *convex* MINLP. Otherwise, the problem is referred to as a *nonconvex* MINLP. With this definition, the UC-AC is a nonconvex MINLP. Algorithms exist to address both convex and nonconvex MINLP problems. However, tailored solution strategies are often required to achieve desired computational performance. In this paper, we present the first known global optimization approach that can successfully solve the UC-AC on a set of small- to medium-sized test problems.

Deterministic MINLP algorithms can be classified into single-tree and multi-tree methods. Single-tree deterministic algorithms, i.e., the well-known Branch-and-Bound (BB) methods [11, 12], seek a global optimum by searching a single tree using a systematic enumeration strategy consisting of three primary steps: branching, bounding, and selection. BB-based global optimization strategies have been well-studied and specialized, yielding strategies such as Branch-and-Reduce [13], Reduced Space Branch-and-Bound [14], Branch-and-Contract [15], Branch-and-Cut [16], and Branch-and-Sandwich [17]. These approaches are suitable for general, nonconvex MINLP problems of small or medium size, but become computationally intractable with increasing numbers of discrete variables, e.g., such as those arising in UC-AC.

In contrast, multi-tree methods [18] iteratively solve a sequence of related lower-bounding (master problem) and upper-bounding (subproblem) problems. For convex MINLP problems, many multi-tree solution strategies – including Generalized Benders Decomposition (GBD) [19], Outer Approximation (OA) [20, 21], and Exact Cutting Plane (ECP) methods [22] – are effective, and have been applied to a broad range of MINLPs in various application domains. Extensions exist for nonconvex MINLPs. While many are heuristic, e.g., see [23, 24], Li et al. [25] propose a rigorous, nonconvex GBD (NGBD) method with piecewise convex relaxations that yields a sequence of nondecreasing lower bounds and nonincreasing upper bounds where monotonic convergence of the bounds is guaranteed. Bonami, Kılınç and Linderoth noted that recent advancements in respective mixed-integer linear programming (MILP) and nonlinear programming (NLP) problem classes have unfortunately resulted in "far more modest" improvements in general algorithms for even convex MINLPs [26], illustrating the need for specialized approaches.

The classic OA approach, a multi-tree technique, was originally developed to solve convex MINLPs. This approach solves a sequence of MILP master and convex NLP subproblems and yields a globally optimal solution for a convex MINLP in a finite number of iterations for a given ϵ-tolerance on the optimality gap [20, 21]. The MILP master problem is a relaxation of the original MINLP that provides a provable lower bound on the MINLP along with a candidate integer solution. Fixing the integers in the MINLP yields a convex NLP subproblem that provides a valid upper bound and a candidate solution (for both continuous and integer variables) to the overall MINLP. In this classic approach, the master problem is further refined (i.e., relaxation strengthened) though the addition of linear outer approximations of convex constraints in the MINLP. The algorithm iterates between the master problem and the NLP subproblem, and terminates when the gap between the lower and upper bounds is sufficiently small. Constraints can also be added to the master problem to remove previously visited integer solutions (using so-called *integer cuts*). These methods have been extended to nonconvex problems where global convergence of the MINLP can be achieved as long as global solutions of the NLP subproblems are ensured [27]. Kesavan et al. [28] develop decomposition algorithms for nonconvex MINLP that finds the global solution on finite termination by solving convex underestimators in the BB search. Similar multi-tree solution strategies for nonconvex MINLPs have also been successfully used in various applications [29–31].

Here, we extend our efforts in [32] and propose a multi-tree method based on OA for the UC-AC problem. The master problem is a mixed-integer second-order cone program (MISOCP) constructed using second-order cone (SOC) relaxations of the nonconvex AC transmission constraints [33]. As the algorithm iterates, the master problem is further refined with piecewise outer approximations to strengthen the tightness of the relaxation and the *lower-bound* computation. The algorithm from [32] is used to find a global solution of the nonconvex NLP subproblem in the *upper-bound* computation. Furthermore, we incorporate optimization-based bounds tightening (OBBT) techniques that are valid in both master and subproblem iterations and, because our proposed approach provides global solution of the NLP subproblem, we are able to include *integer cuts* in the master problem that remove previously visited solutions from the feasible space as the algorithm iterates. To the best of our knowledge, this is the first global solution algorithm successfully applied to the UC-AC problem, identifying solutions with quality certificates (optimality gaps) in time-limited environments.

The remainder of this paper is organized as follows. We begin in Section II by discussing relevant work to solving the UC-AC problem. In Section III we introduce the unit commitment formulation with AC transmission constraints (UC-AC). In Section IV we outline the necessary problem relaxations and the global optimization algorithm. In Section V we report numerical results on the range of currently available test systems. We then conclude in Section VI with a summary of our contributions and directions for future work.

II. Related Work

There is a growing body of literature on algorithms [34–47] for the solution of the UC-AC problem. A recent study by Aghaei et al. noted that the UC-AC problem is presently intractable for commercial MINLP solvers, including BARON, SBB, and DICOPT [42]. In this section we review the most relevant works in further detail.

Amjady et al. [46] leverage a Signomial convexification technique with second order approximations of the trignometric functions in the ACOPF constraint set. Without refinement, branching, or additional cuts, relaxations on their own do not provide a guarantee of global optimality to the original MINLP. Other comparable convexification approaches include the formulations by Bai and Wei [39] and Madani et al. [47], where these studies use semidefinite programming (SDP) to relax the ACOPF constraint set and the 0/1 variables. For these studies, when a solution is non-integral in the unit commitment variables, a rounding procedure is applied to determine a feasible, near-optimal solution. Such rounding heuristics are useful but not sufficient to guarantee a global solution. The other aforementioned methods solve the ACOPF subproblem – or network and voltage security subproblems – with local solution methods, and in some cases apply linearization techniques [44, 45, 48].

Approaches that make use of local solutions of the NLP subproblem (e.g., recent GBD examples include [42–45]) are not guaranteed to find global solutions in a finite number of steps. This is because, as indicated in [49], when applying GBD to nonconvex problems, global solution of the NLP subproblem is required to ensure valid cuts.

On the contrary, Sifuentes et al. [38] argued that such suboptimal outcomes due to nonconvexities can be reduced with constraint specifications (e.g., small angle difference constraints). However, more recent work by Wu et al. [50] indicates that such assumptions do not preclude the occurrence of multiple, local optima. As such, Frank et al. [51] develop a nonconvex GBD approach to solve AC-DC distribution system design problems where a global solver is leveraged for the nonconvex subproblems.

In contrast to GBD methods that have been applied to the UC-AC problem to-date, our *lower-bounding* problem incorporates a relaxation and outer-approximation of the full ACOPF constraint set. This master problem can be arbitrarily refined to provide improved integer solutions (although this was not necessary in our test cases). Furthermore, we apply our approach from [32] to determine a global solution of the *upper-bounding* problem, ensuring that the global solution is identified if the gap closes. Moreover, we are guaranteed finite termination by enumeration in the worst case.

III. UC-AC Problem Formulation

We now introduce our UC-AC problem formulation. We first present the core UC model in Section III-A, which is based on the compact three-binary (3BIN) formulation introduced in [52]. We then present the rectangular power-voltage (RPQV) model [53] in Section III-B to represent the steady-state operations of the nonlinear AC transmission network. We integrate these constraint sets to represent the UC-AC problem, resulting in a nonconvex MINLP. A tailored solution technique for this model is proposed in the following section.

A. Unit Commitment Model

We use the term *UC skeleton* when referring to a unit commitment model consisting only of a cost function, operating constraints, and any associated continuous and binary variables with no network representation. We summarize several key components of the *3BIN* formulation here; refer to [52] for further details.

1) Cost Function: The total cost in UC is the sum of three major components – production costs, startup costs, and shutdown costs – as follows:

$$f^p + f^{su} + f^{sd}.$$

We assume that the production cost f^p is a quadratic monotonically non-decreasing function of real power generation; in practice, this is often replaced with a piecewise approximation. Computation of f^p in the quadratic case is accomplished by imposing the constraints

$$A_{g,2}(p^G_{g,t})^2 + A_{g,1}p^G_{g,t} + A_{g,0}y_{g,t} \leq c^p_{g,t} \qquad \forall\, g,\ t \quad (1)$$

$$f^p = \sum\nolimits_{g\in\mathcal{G}} \sum\nolimits_{t\in\mathcal{T}} c^p_{g,t} \quad (2)$$

where $A_{g,2}$, $A_{g,1}$, and $A_{g,0}$ are known cost coefficients in (\$/MW^{2}h), (\$/MWh) and (\$/h) associated with a specific generator g.

To formulate the total startup cost, f^{su}, we first introduce a new binary variable $\delta_{g,\tau,t}$, which indicates the startup type τ of generator g at time period t. In particular, $\delta_{g,\tau,t}$ takes the value of 1 if the generator g starts up at time t and has been previously offline within $[T^{su}_{g,\tau}, T^{su}_{g,\tau+1})$ hours. The logical constraints between $w_{g,t}$, $u_{g,t}$, and $\delta_{g,\tau,t}$ are given as

$$\delta_{g,\tau.t} \leq \sum\nolimits_{t'=t-T^{su}_{g,\tau}}^{t+1-T^{su}_{g,\tau+1}} w_{g,t'} \qquad \forall\, g,\ t,\ \tau\in[1,S_g) \quad (3)$$

$$u_{g,t} = \sum\nolimits_{\tau\in\mathcal{S}_g} \delta_{g,\tau,t} \qquad \forall\, g,\ t \quad (4)$$

where S_g is the number of startup types for generator g, and $u_{g,t}$ and $w_{g,t}$ indicate startup and shutdown of generator g in time t, respectively. Note that $w_{g,t}$ with positive time index t are variables, otherwise $w_{g,t}$ are treated as constants to demonstrate previous system status.

For a thermal unit, the startup cost is assumed to be a monotonically increasing step function with respect to the generator's previous off-line time. The total startup cost is given by

$$f^{su} = \sum\nolimits_{g\in\mathcal{G}} \sum\nolimits_{t\in\mathcal{T}} \sum\nolimits_{\tau\in\mathcal{S}_g} K^{su}_{g,\tau}\delta_{g,\tau,t} \quad (5)$$

where $K^{su}_{g,\tau}$ is the cost of startup type τ for generator g. Given logical constraints (3) and (4), and the monotonically non-decreasing startup cost function, it can be shown that $\delta_{g,\tau,t}$ will always solve to a binary value. In other words, instead of explicitly defining $\delta_{g,\tau,t}$ as a binary, it can be relaxed as a continuous variable within range $[0,1]$.

The shutdown cost of generator g is assumed to be independent of its previous on-line states, and the total shutdown cost is:

$$f^{sd} = \sum\nolimits_{g\in\mathcal{G}} \sum\nolimits_{t\in\mathcal{T}} K^{sd}_g w_{g,t}. \quad (6)$$

2) Operating Constraints: According to operating restrictions, a thermal unit must stay in one state (either on-line or off-line) for a certain period of time before its state can be changed again. Such time periods vary between different generator types. To enforce this requirement, we have to introduce minimum uptime and downtime constraints

$$\sum\nolimits_{t'=t-T_g^u+1}^{t} u_{g,t'} \leq y_{g,t} \qquad \forall\, g,\ t \quad (7)$$

$$\sum\nolimits_{t'=t-T_g^d+1}^{t} w_{g,t'} \leq 1 - y_{g,t} \qquad \forall\, g,\ t \quad (8)$$

where $u_{g,t}$ and $w_{g,t}$ with positive time index t are unknown variables, otherwise they are treated as constants to indicate previous system status. Additional constraints are required to denote the logical correlation between $u_{g,t}$, $w_{g,t}$, and $y_{g,t}$ in

$$y_{g,t} - y_{g,t-1} = u_{g,t} - w_{g,t} \qquad \forall\, g,\ t. \quad (9)$$

Note that these constraints ensure that a generator cannot start up and shut down within the same time period. Given the fact that $y_{g,t}$ is a binary variable, imposing constraints (7), (8) and (9) together guarantees that $u_{g,t}$ and $w_{g,t}$ take binary values only. Consequently, $u_{g,t}$, $w_{g,t}$, and $\delta_{g,\tau,t}$, though initially defined as binaries, can be relaxed as continuous within $[0,1]$, leaving the $y_{g,t}$ as the only binary variables in our *UC skeleton* formulation. The spinning reserve constraint is defined as

$$P_t^R \leq \sum\nolimits_{g\in\mathcal{G}} r_{g,t} \qquad \forall\, t \quad (10)$$

and determines the extra generating capacity available by generators included in the commitment solution at time t; typically, the spinning reserve is defined as a fraction of the current total power demand. The upper- and lower-bounds of generator output is dependent on its operating state; the real power productions are constrained by $[P_g^{G,min}, P_g^{G,max}]$, the startup and shutdown capabilities SD_g and SU_g, and state indicators $y_{g,t}$, $u_{g,t}$, and $w_{g,t}$ where both real power generation $p_{g,t}$ and spinning reserve $r_{g,t}$ are accounted for in

$$p_{g,t} + r_{g,t} \leq (P_g^{G,max} - P_g^{G,min}) y_{g,t} - (P_g^{G,max} - SU_g) u_{g,t} \qquad \forall\, g,\ t \quad (11)$$

$$p_{g,t} + r_{g,t} \leq (P_g^{G,max} - P_g^{G,min}) y_{g,t} - (P_g^{G,max} - SD_g) w_{g,t+1} \qquad \forall\, g,\ t \quad (12)$$

when $T_g^u = 1$, and

$$p_{g,t} + r_{g,t} \leq (P_g^{G,max} - P_g^{G,min}) y_{g,t} - (P_g^{G,max} - SU_g) u_{g,t} - (P_g^{G,max} - SD_g) w_{g,t+1} \qquad \forall\, g,\ t \quad (13)$$

when $T_g^u \geq 2$. The real power production is also constrained by ramp-up and ramp-down limits, which are given as

$$p_{g,t} + r_{g,t} - p_{g,t-1} \leq RU_g \qquad \forall\, g,\ t \quad (14)$$

$$-p_{g,t} + p_{g,t-1} \leq RD_g \qquad \forall\, g,\ t. \quad (15)$$

Then, the reactive power productions are only constrained by $[Q_g^{G,min}, Q_g^{G,max}]$ and $y_{g,t}$ in

$$Q_g^{G,min} y_{g,t} \leq q_{g,t}^G \leq Q_g^{G,max} y_{g,t} \qquad \forall\, g,\ t. \quad (16)$$

Synchronous condensers are not modeled with startup/shutdown costs and their reactive power output is constrained by $[Q_{sc}^{SC,min}, Q_{sc}^{SC,max}]$

$$Q_{sc}^{SC,min} \leq q_{sc,t}^{SC} \leq Q_{sc}^{SC,max} \qquad \forall\, sc,\ t. \quad (17)$$

B. AC Transmission Network Model

In electric power system analysis, the *RPQV* model is widely-used to represent an AC transmission network; this approach explicitly models real and reactive power flows in terms of complex voltages in the rectangular form. A transmission line is denoted as $l\equiv(b,k)$, where b is the index of the bus at the *from* end and k is the index of the bus at the *to* end of branch l. For integration into our *UC skeleton*, the *RPQV* model is given by

$$\sum\nolimits_{l\in\mathcal{L}_b^{in}} p_{l,t}^t + \sum\nolimits_{l\in\mathcal{L}_b^{out}} p_{l,t}^f + G_b^{sh} v_{b,t}^2 + P_{b,t}^D - \sum\nolimits_{g\in\mathcal{G}_b} p_{g,t}^G = 0 \qquad \forall\, b,\ t \quad (18)$$

$$\sum\nolimits_{l\in\mathcal{L}_b^{in}} q_{l,t}^t + \sum\nolimits_{l\in\mathcal{L}_b^{out}} q_{l,t}^f - B_b^{sh} v_{b,t}^2 + Q_{b,t}^D - \sum\nolimits_{g\in\mathcal{G}_b} q_{g,t}^G - \sum\nolimits_{sc\in\mathcal{SC}_b} q_{sc,t}^{SC} = 0 \qquad \forall\, b,\ t \quad (19)$$

$$p_{l,t}^f = G_l^{ff} v_{b,t}^2 + G_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - B_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t \quad (20)$$

$$q_{l,t}^f = -B_l^{ff} v_{b,t}^2 - B_l^{ft}(v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j) - G_l^{ft}(v_{b,t}^r v_{k,t}^j - v_{b,t}^j v_{k,t}^r) \qquad \forall\, l,\ t \quad (21)$$

$$p_{l,t}^t = G_l^{tt} v_{k,t}^2 + G_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - B_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t \quad (22)$$

$$q_{l,t}^t = -B_l^{tt} v_{k,t}^2 - B_l^{tf}(v_{k,t}^r v_{b,t}^r + v_{k,t}^j v_{b,t}^j) - G_l^{tf}(v_{k,t}^r v_{b,t}^j - v_{k,t}^j v_{b,t}^r) \qquad \forall\, l,\ t \quad (23)$$

$$(V_b^{min})^2 \leq v_{b,t}^2 \leq (V_b^{max})^2 \qquad \forall\, b,\ t \quad (24)$$

$$(p_{l,t}^f)^2 + (q_{l,t}^f)^2 \leq (S_l^{max})^2 \qquad \forall\, l,\ t \quad (25)$$

$$(p_{l,t}^t)^2 + (q_{l,t}^t)^2 \leq (S_l^{max})^2 \qquad \forall\, l,\ t \quad (26)$$

where $v_{b,t}^2 \equiv (v_{b,t}^r)^2 + (v_{b,t}^j)^2$; see [32] for details on computing G_l and B_l branch admittance submatrices. Note that the *RPQV* problem is nonconvex due to bilinear terms and nonconvex quadratics.

C. UC-AC Problem Formulation

The UC-AC is a nonconvex MINLP formulations that combines the UC skeleton with the nonlinear ACOPF constraints, giving:

$$\begin{aligned} &\min f^p + f^{su} + f^{sd} \\ &\text{s.t.} \\ &(1) - (26) \\ &y_{g,t}, u_{g,t}, w_{g,t} \in \{0,1\} \quad \forall\, g,\ t \end{aligned} \tag{27}$$

In the next section we exploit the special mathematical structure of this problem to solve the problem globally.

IV. UC-AC Global Solution Framework

The UC-AC is a nonconvex MINLP, and our proposed algorithm is a *nested* multi-tree method where both the outer and inner algorithm are based on a nonconvex OA approach that solves a sequence of lower-bounding master problems and upper-bounding subproblems. In this section, we first provide a high-level explanation of the nested multi-tree approach used to solve the UC-AC MINLP problem, followed by a detailed description of the master and NLP subproblems and the algorithm definition. Here, we denote $d = [y, u, w]$ to represent the discrete decisions (i.e., generator commitment variables), and x to represent the continuous variables in the UC-AC problem.

A. Overview

Figure 1 shows the multi-tree approach for the UC-AC problem. The algorithm iterates between a master problem and an NLP subproblem, and each pair of such solves comprise a major iteration q for candidate solution denoted as $[d^q, x^q]$. The high-level description of the

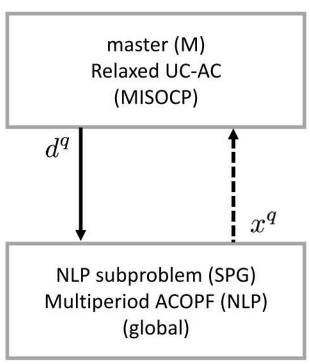

Fig. 1: High-level description of the multi-tree approach for global solution of the UC-AC MINLP problem.

Outer Algorithm is as follows:

The master problem (M) is a relaxation of the UC-AC problem where the AC power flow constraints are relaxed using the SOC representation from [33]. The initial solution of (M) provides a lower bound on the UC-AC problem and a candidate solution for the binary variables (the generator commitments) given by d^q for iteration q. Fixing these variables in the UC-AC MINLP problem yields a nonconvex NLP that represents a multi-period ACOPF problem given by (SPG). This NLP subproblem, if feasible, provides an upper bound, z_U^q, and a candidate solution to the UC-AC, $[d^q, x^q]$. If the gap between the upper and lower bound is sufficiently small, then the solution has been found, i.e., $z^* = z_U^q$ for $[d^*, x^*] = [d^q, x^q]$.

To further accelerate exploration of the generator commitments, it is also desirable to add cuts to (M) that remove previously visited solutions d^q from the feasible space. With these *integer cuts* (see Section IV-D1), the solution z_L^q of (M) is not a true lower bound to the original MINLP, and to ensure convergence with this approach, it is required that we find a globally optimal solution to the NLP subproblem (SPG) for each candidate binary solution d^q. Note that, in the limit, this will result in full enumeration, ensuring convergence of the discrete decision space in a finite number of iterations. However, for the applications and test cases presented in this work, only a few outer iterations were required to close the gap.

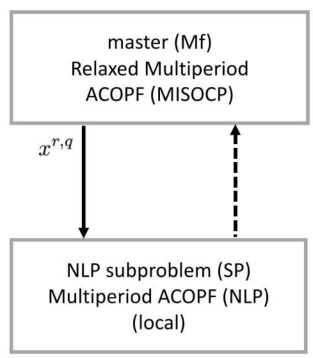

Fig. 2: High-level description of the multi-tree approach for global solution of the NLP subproblem (SPG).

For global solution of the multi-period ACOPF in (SPG) we apply the approach of [32], and for completeness, Figure 2 shows this algorithm. This strategy is also a multi-tree approach, and hence we refer to the overall algorithm as a *nested* multi-tree approach. Recall that the candidate generator commitments d^q are fixed for this problem. Similar to the *Outer Algorithm* in Figure 1, this approach iterates between the master and the NLP subproblem, and each pair of such solves constitutes a minor iteration r on iteration q. The high-level description of the *Inner Algorithm* is as follows:

The master problem (Mf) is a MISOCP relaxation of the problem (SPG) (d^q fixed). Therefore, in (Mf) the only binary variables are those corresponding to piecewise outer approximations. The master problem (Mf) is solved to find a lower bound for (SPG), and the solution $x^{r,q}$ from (Mf) is used to initialize the NLP subproblem (SP). This NLP subproblem, if feasible provides an upper bound, $z_U^{r,q}$, and a candidate solution $x^{r,q}$. Note that the NLP subproblem (SP) in Figure 2 is the same formulation as (SPG) in Figure 1, however, in this case we only seek a local solution of the NLP subproblem (SP).

Since we do not add integer cuts to the master problem (Mf), it is a true relaxation of (SPG), and closure of the gap between the upper and lower bounds is sufficient to indicate convergence. At each iteration r, the master problem is progressively refined by the addition and/or tightening of piecewise outer approximations, as well as optimization-based bounds tightening (OBBT), as discussed later in Sections IV-D.

Note that for both *Outer* and *Inner Algorithms*, the respective master problems (M) and (Mf) can be further refined with any selection of piecewise outer approximations (see Sections IV-D2 and IV-D3) and with domain reduction techniques, e.g., OBBT (see Section IV-D4).

B. Problem Formulations

This section provides a description of the problem formulations (M), (SPG), (SP), and (Mf) used in the global algorithm. The master problem (M) for the UC-AC problem is based on the SOC relaxation of the power flow equations from [33]. We replace the quadratic and bilinear terms in (27) for all $l \equiv (b,k)$ and t with

$$c_{b,b,t} := (v^r_{b,t})^2 + (v^j_{b,t})^2$$

$$c_{b,k,t} := v^r_{b,t} v^r_{k,t} + v^j_{b,t} v^j_{k,t}$$

$$s_{b,k,t} := v^r_{b,t} v^j_{k,t} - v^r_{k,t} v^j_{b,t}$$

and introduce a second-order cone relaxation of the condition

$$c^2_{b,k,t} + s^2_{b,k,t} = c_{b,b,t} c_{k,k,t} \tag{28}$$

as

$$c^2_{b,k,t} + s^2_{b,k,t} \le c_{b,b,t} c_{k,k,t}. \tag{29}$$

1) Master Problem (M): With the definitions above, the problem formulation for (M) is given as follows:

$$z_L := \min f^p + f^{su} + f^{sd} \tag{M.1}$$

s.t.

$$(1) – (17), (25), (26) \tag{M.2}$$

$$\sum_{l \in \mathcal{L}_b^{in}} p^t_{l,t} + \sum_{l \in \mathcal{L}_b^{out}} p^f_{l,t} + G^{sh}_b c_{b,b,t} + P^D_{b,t} - \sum_{g \in \mathcal{G}_b} p^G_{g,t} = 0 \quad \forall\, b,\ t \tag{M.3}$$

$$\sum_{l \in \mathcal{L}_b^{in}} q^t_{l,t} + \sum_{l \in \mathcal{L}_b^{out}} q^f_{l,t} - B^{sh}_b c_{b,b,t} + Q^D_{b,t} - \sum_{g \in \mathcal{G}_b} q^G_{g,t} - \sum_{sc \in \mathcal{SC}_b} q^{SC}_{sc,t} = 0 \quad \forall\, b,\ t \tag{M.4}$$

$$p^f_{l,t} = G^{ff}_l c_{b,b,t} + G^{ft}_l c_{b,k,t} - B^{ft}_l s_{b,k,t} \quad \forall\, l,\ t \tag{M.5}$$

$$q^f_{l,t} = -B^{ff}_l c_{b,b,t} - B^{ft}_l c_{b,k,t} - G^{ft}_l s_{b,k,t} \quad \forall\, l,\ t \tag{M.6}$$

$$p^t_{l,t} = G^{tt}_l c_{k,k,t} + G^{tf}_l c_{k,b,t} - B^{tf}_l s_{k,b,t} \quad \forall\, l,\ t \tag{M.7}$$

$$q^t_{l,t} = -B^{tt}_l c_{k,k,t} - B^{tf}_l c_{k,b,t} - G^{tf}_l s_{k,b,t} \quad \forall\, l,\ t \tag{M.8}$$

$$(V^{min}_b)^2 \le c_{b,b,t} \le (V^{max}_b)^2 \quad \forall\, b,\ t \tag{M.9}$$

$$c_{b,k,t} = c_{k,b,t} \quad \forall\, l,\ t \tag{M.10}$$

$$s_{b,k,t} = -s_{k,b,t} \quad \forall\, l,\ t \tag{M.11}$$

$$c^2_{b,k,t} + s^2_{b,k,t} \le c_{b,b,t} c_{k,k,t} \quad \forall\, l,\ t \tag{M.12}$$

$$y_{g,t}, u_{g,t}, w_{g,t} \in \{0,1\} \quad \forall\, g,\ t \tag{M.13}$$

2) NLP Subproblems (SPG) and (SP): The same NLP subproblem is used in both the outer and the inner multi-tree algorithms, however, for (SPG), a global solution is required. The NLP subproblem is formed by fixing the binary variables d=$[y,u,w]$ (generator commitments) in the original MINLP formulations for the UC-AC. This produces a multi-period ACOPF formulation. For any iteration j, problem for fixed $d^{(j)}$=$[y^{(j)}, u^{(j)}, w^{(j)}]$ is given as:

$$z_U := \min f^p + f^{su} + f^{sd}$$

s.t.

$$(1) – (26) \tag{SP}$$

where

$$y_{g,t} := y^{(j)}_{g,t}, u_{g,t} := u^{(j)}_{g,t}, w_{g,t} := w^{(j)}_{g,t} \quad \forall\, g,\ t$$

3) Master Problem (Mf): Problem (Mf) is the master problem used in the inner multi-tree approach for obtaining globally optimal solutions to the NLP subproblem (SPG) from the outer problem. It is based on the same SOC relaxation that is used for problem

Algorithm 1 *Outer Algorithm* for UC-AC

1: **Initialization.**
Iteration q=0,
$z_L^* \leftarrow -\infty$. $z_U^* \leftarrow +\infty$. $(d^*, x^*) \leftarrow \emptyset$.
2: **Solve the Master Problem** (M)**.**
Solve problem (M) to compute its objective value z_L^q and binary solution d^q.
(a) If (M) is infeasible, then (d^*, x^*) is the optimal solution (unless $(d^*, x^*) \equiv \emptyset$, then the UC-AC problem is infeasible). Terminate.
(b) If $z_L^* > z_L^q$, then $z_L^* \leftarrow z_L^q$.
3: **Solve for the Upper-Bound.**
Solve the NLP subproblem (SPG) (with fixed d^q) to global optimality using **Algorithm 2**. Let z_U^q and (d^q, x^q) be the optimal objective value and solution.
(a) If feasible and $z_U^* < z_U^q$, then update the candidate solution: $z_U^* \leftarrow z_U^q$ and $(d^*, x^*) \leftarrow (d^q, x^q)$.
4: **Convergence Check**
(a) If gap $(z_U^* - z_L^*)/z_L^* < \epsilon_O$, the optimal solution (d^*, x^*) has been identified. Terminate.
(b) Otherwise add *integer cut* (IC) for d^q to (M). Further refinements possible as in 4(b) of Alg. 2.
5: **Iterate** $q \leftarrow q + 1$. **Go to Step 2**.

(M), however, the generator commitments d=$[y, u, w]$ are fixed. Problem (Mf) for any iteration j with fixed $d^{(j)}$=$[y^{(j)}, u^{(j)}, w^{(j)}]$ is given by:

$$z_{L_{fixed}} \coloneqq \min f^p + f^{su} + f^{sd}$$

s.t.

$$(\text{M.2}) - (\text{M.12}) \qquad (\text{Mf})$$

where

$$y_{g,t} \coloneqq y_{g,t}^{(j)}, u_{g,t} \coloneqq u_{g,t}^{(j)}, w_{g,t} \coloneqq w_{g,t}^{(j)} \quad \forall\, g,\ t$$

C. Global Solution Algorithm

In this section, we formally present the *nested* multi-tree algorithm. Algorithm 1 presents the *Outer Algorithm* for the solution of the UC-AC problem, and Algorithm 2 presents the *Inner Algorithm* for global solution of the NLP subproblem from the *Outer Algorithm*. For implementation details on the *integer cuts* (IC), piecewise outer relaxations (UE), (OE), and (CC), and OBBT referred to in the presented algorithms, please see the following section.

D. Algorithm Details

Algorithm 2 *Inner Algorithm* for (SPG)

1: **Initialization.**
For outer iteration q and fixed binary d^q:
Inner iteration $r = 0$.
$z_{L_{fixed}}^* \leftarrow -\infty$. $z_U^q \leftarrow +\infty$. $x^{q,r} \leftarrow \emptyset$.
2: **Solve for the Lower-Bound.**
Solve problem (Mf) (with fixed d^q) to find lower bound $z_{L_{fixed}}^r$ solution $x^{q,r}$.
(a) If (Mf) is infeasible then the subproblem (SPG) is infeasible. Return to **Step 3** in **Algorithm 1**.
(b) If $z_{L_{fixed}}^* > z_{L_{fixed}}^r$, then $z_{L_{fixed}}^* \leftarrow z_{L_{fixed}}^r$.
3: **Solve for the Upper-Bound.**
Solve problem (SP) (initialized from $x^{q,r}$) to compute its objective value $z_{U_{fixed}}^r$ and solution $x_{fixed}^{q,r}$. If $z_U^q < z_{U_{fixed}}^r$, then $z_U^q \leftarrow z_{U_{fixed}}^r$ and $x^q \leftarrow x_{fixed}^{q,r}$.
4: **Convergence Check.**
(a) If $(z_U^q - z_{L_{fixed}}^*)/z_{L_{fixed}}^* < \epsilon_I$ (optimality tolerance), then x^q is optimal. Return z_U^q and x^q to **Step 3** in **Algorithm 1**.
(b) Else perform OBBT on selected variables and add or refine partitions for piecewise outer relaxations (UE), (OE), and (CC).
5: **Iterate** $r \leftarrow r + 1$. **Go to Step 2**.

1) Integer Cuts: At each iteration q of the *Outer Algorithm* we add integer cuts that remove previously visited solutions d^q. These cuts are given by,

$$\sum_{(g,t)\in\mathcal{B}^{(q)}} y_{g,t} - \sum_{(g,t)\in\mathcal{N}^{(q)}} y_{g,t} \leq |\mathcal{B}^{(q)}| - 1 \qquad (\text{IC})$$

for $q = 1 \ldots Q - 1$ where $\mathcal{B}^{(q)} = \{g, t | y_{g,t}^{(q)} = 1\}$ and $\mathcal{N}^{(q)} = \{g, t | y_{g,t}^{(q)} = 0\}$ This enhancement ensures that distinct solutions are obtained during each major iteration q of our global solution algorithm. As a result, since the solutions to problem (M) are enumerated with these integer cuts, the below refinements (in Sections IV-D2 and IV-D3) can be ignored in Step 4(b) of the *Outer Algorithm.*

2) "Reverse Cone": For any solution of (M) or (Mf), we may have that equation (28) is violated, i.e.,

$$c_{b,b,t}c_{k,k,t} - (c_{b,k,t}^2 + s_{b,k,t}^2) > \varepsilon$$

for any l and t due to the second-order cone relaxation of (28). Therefore, we introduce piecewise relaxations of

$$c_{b,k,t}^2 + s_{b,k,t}^2 \geq c_{b,b,t}c_{k,k,t}, \qquad (33)$$

as necessary in each iteration of the *Inner Algorithm.* To describe these relaxations, we define new variables

$$cs_{b,k,t} \coloneqq c_{b,k,t}^2 + s_{b,k,t}^2$$

$$cc_{b,k,t} \coloneqq c_{b,b,t} c_{k,k,t}$$

where we construct piecewise over-estimators for $c_{b,k,t}^2 + s_{b,k,t}^2$ and piecewise under-estimators for $c_{b,b,t} c_{k,k,t}$ to obtain an adjustable approximation of (33).

Specifically, as first introduced in [32], we extend the bivariate partitioning scheme in [54]. We denote our partitioning variables as $cs_{b,k,t,}^{i,j}$ and $cc_{b,k,t}^{i,j}$, where $[\underline{c}_{b,k,t}^i, \overline{c}_{b,k,t}^i]$ refers to the i-th interval for $c_{b,k,t} \in [\underline{c}_{b,k,t}, \overline{c}_{b,k,t}]$ and $[\underline{s}_{b,k,t}^j, \overline{s}_{b,k,t}^j]$ refers to the j-th interval for $s_{b,k,t} \in [\underline{s}_{b,k,t}, \overline{s}_{b,k,t}]$.

The piecewise over-estimators for $cs_{b,k,t}$ are

$$cs_{b,k,t}^{i,j} \leq (\underline{c}_{b,k,t}^i + \overline{c}_{b,k,t}^i) c_{b,k,t}^{i,j} + (\underline{s}_{b,k,t}^j + \overline{s}_{b,k,t}^j) s_{b,k,t}^{i,j}$$
$$- (\underline{c}_{b,k,t}^i \overline{c}_{b,k,t}^i) c_{b,k,t}^{i,j} + \underline{s}_{b,k,t}^j \overline{s}_{b,k,t}^j) \sigma_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t$$
$$cs_{b,k,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k,t}^{cs}} cs_{b,k,t}^{i,j} \quad \forall\, l,\ t$$
$$\underline{c}_{b,k,t}^i \sigma_{b,k,t}^{i,j} \leq c_{b,k,t}^{i,j} \leq \overline{c}_{b,k,t}^i \sigma_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t$$
$$c_{b,k,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k}^{cs}} c_{b,k,t}^{i,j} \quad \forall\, l,\ t$$
$$\underline{s}_{b,k,t}^j \sigma_{b,k,t}^{i,j} \leq s_{b,k,t}^{i,j} \leq \overline{s}_{b,k,t}^j \sigma_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t \tag{OE}$$
$$s_{b,k,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k}^{cs}} s_{b,k,t}^{i,j} \quad \forall\, l,\ t$$
$$\sum\nolimits_{(i,j) \in \Omega_{b,k}^{cs}} \sigma_{b,k,t}^{i,j} = 1 \quad \forall\, l,\ t$$
$$\sigma_{b,k,t}^{i,j} \in \{0,1\} \quad \forall\, (i,j),\ l,\ t$$

where $(i,j) \in \Omega_{b,k,t}^{cs} \coloneqq [\underline{c}_{b,k,t}^i, \overline{c}_{b,k,t}^i] \times [\underline{s}_{b,k,t}^j, \overline{s}_{b,k,t}^j]$. Then, the piecewise under-estimators for $cc_{b,k,t}$ are

$$cc_{b,k,t}^{i,j} \leq \overline{cc}_{k,k,t}^{i,j} cc_{b,b,t}^{i,j} + \overline{cc}_{b,b,t}^{i,j} cc_{k,k,t}^{i,j}$$
$$- \overline{cc}_{b,b,t}^{i,j} \overline{cc}_{k,k,t}^{i,j} \varphi_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t$$
$$cc_{b,k,t}^{i,j} \leq \underline{cc}_{k,k,t}^{i,j} cc_{b,b,t}^{i,j} + \underline{cc}_{b,b,t}^{i,j} cc_{k,k,t}^{i,j}$$
$$- \underline{cc}_{b,b,t}^{i,j} \underline{cc}_{k,k,t}^{i,j} \varphi_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t$$
$$cc_{b,k,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k,t}^{cc}} cc_{b,k,t}^{i,j} \quad \forall\, l,\ t$$
$$\underline{c}_{b,b,t}^i \varphi_{b,k,t}^{i,j} \leq c_{b,b,t}^{i,j} \leq \overline{c}_{b,b,t}^i \varphi_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t \tag{UE}$$
$$c_{b,b,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k,t}^{cc}} c_{b,b,t}^{i,j} \quad \forall\, l,\ t$$
$$\underline{c}_{k,k,t}^i \varphi_{b,k,t}^{i,j} \leq c_{k,k,t}^{i,j} \leq \overline{c}_{k,k,t}^i \varphi_{b,k,t}^{i,j} \quad \forall\, (i,j),\ l,\ t$$
$$c_{k,k,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k,t}^{cc}} c_{k,k,t}^{i,j} \quad \forall\, l,\ t$$
$$\sum\nolimits_{(i,j) \in \Omega_{b,k,t}^{cc}} \varphi_{b,k,t}^{i,j} = 1 \quad \forall\, l,\ t$$
$$\varphi_{b,k,t}^{i,j} \in \{0,1\} \quad \forall\, (i,j),\ l,\ t$$

where $(i,j) \in \Omega_{b,k,t}^{cc} \coloneqq [\underline{c}_{b,b,t}^i, \overline{c}_{b,b,t}^i] \times [\underline{c}_{k,k,t}^j, \overline{c}_{k,k,t}^j]$. Note that unique $c_{b,b,t}^{i,j}$ and $c_{k,k,t}^{i,j}$ variables must be introduced for every line l where the under-estimators are constructed.

3) Cycle Constraints: In the second order cone relaxations used in (M) and (Mf), Kirchhoff's voltage law (KVL) is no longer guaranteed to be satisfied, but can be enforced through the *cycle constraints*,

$$\sum\nolimits_{(b,k) \in \mathcal{L}_c} \theta_{b,k,t} = 0 \tag{36}$$

for all t and

$$\theta_{b,k,t} = -\arctan(s_{b,k,t}/c_{b,k,t}) \tag{37}$$

for all l and t. As the *Inner Algorithm* iterates, these constraints are gradually enforced as needed in sub-problem (Mf) by addition and refinement of piecewise outer approximations. We construct the respective piecewise under- and over-estimators for each $\theta_{b,k,t} = -\arctan(s_{b,k,t}/c_{b,k,t})$ term, where

$$\theta_{b,k,t}^{i,j} \geq \alpha_n^{i,j} s_{b,k,t}^{i,j} + \beta_n^{UE} c_{b,k,t}^{i,j} + \gamma_n^{UE} \quad \forall n,\ (i,j),\ l,\ t$$
$$\theta_{b,k,t}^{i,j} \leq \alpha_n^{i,j} s_{b,k,t}^{i,j} + \beta_n^{OE} c_{b,k,t}^{i,j} + \gamma_n^{OE} \quad \forall n,\ (i,j),\ l,\ t$$
$$\theta_{b,k,t} = \sum\nolimits_{(i,j) \in \Omega_{b,k}^{cs}} \theta_{b,k,t}^{i,j} \tag{CC}$$
$$\sum\nolimits_{(b,k) \in \mathcal{L}_c} \theta_{b,k,t} = 0$$

where $n \in \{1,2\}$ and the parameters α, β, and γ are based on the planes constructed in [55]; please see [32] for implementation details.

4) Optimization-Based Bounds Tightening: The optimization-based bounds tightening (OBBT) is only computed for the second-order cone variables $c_{b,k,t}$ and $s_{b,k,t}$ to perform domain reduction on the initial lower-bounding subproblem (Mf). This approach results in two optimization routines per variable, i.e.,

$$\underline{c}_{b,k,t} \leftarrow \max\left(\underline{c}_{b,k,t}, \min\{c_{b,k,t} | c(\mathsf{Mf}), z_U^0 \leq z_U^*\}\right)$$
$$\overline{c}_{b,k,t} \leftarrow \min\left(\overline{c}_{b,k,t}, \max\{c_{b,k,t} | c(\mathsf{Mf}), z_U^0 \leq z_U^*\}\right)$$
$$\underline{s}_{b,k,t} \leftarrow \max\left(\underline{s}_{b,k,t}, \min\{s_{b,k,t} | c(\mathsf{Mf}), z_U^0 \leq z_U^*\}\right)$$
$$\overline{s}_{b,k,t} \leftarrow \min\left(\overline{s}_{b,k,t}, \max\{s_{b,k,t} | c(\mathsf{Mf}), z_U^0 \leq z_U^*\}\right)$$

for all l and t where $c(\mathsf{Mf})$ denotes the constraint set of (Mf). This procedure is computed selectively for $c_{b,k,t}$ and $s_{b,k,t}$ corresponding to large violations in second-order cone constraints (28).

V. Numerical Results

We now test our global UC-AC solution algorithm on four benchmark problems: a 6-bus test system (6-bus) with 3 generators [37], two 24-bus test systems – RTS-79 [56] and RTS-96 – each with 33 generators [57], and a modified IEEE 118-bus test system (IEEE-118mod) with 54 generators [37]. The scheduling horizon solution for all test cases is 24 hours at hourly time resolution, but we solve for 48 hours (by stacking the same 24-hour demand profile) to address end of time horizon impacts. Our global solution algorithm is implemented in *Pyomo*, a Python-based open-source optimization modeling language [58]. All computational experiments are conducted on a 64-bit server with 24 CPUs (Intel(R) Xeon(R) CPU E5-2697 v2 @ 2.70GHz) and 256 GB of RAM. All SOCP and MISOCP subproblems are solved using Gurobi 6.5.2 [59] limited to 24 threads. All NLP subproblems are solved with Ipopt 3.12.6 [60] using HSL's MA27 linear solver [61].

While having a tight and compact formulation is one path toward obtaining improved performance in global solution frameworks, convergence speed is also a function of other characteristics of the underlying numerical problem that impact computational difficulty, including formulation size and degeneracy / symmetry in the solution space. Typically, there is a large subset of solutions that are within an ϵ-tolerance of an optimal-cost schedule. To balance computational burden and solution quality, we initially set the Gurobi MILP gap to 0.1%. Then, if the optimality gap of our global solution algorithm does not show improvement within N iterations, we tighten the MILP gap by a factor of 10.

In all of our computational experiments, we set $N{=}5$ with a total wall clock time limit of $14400s$ and a major iteration limit $q{=}30$. The optimality tolerance for both our global solution algorithm and its nested multi-tree algorithm are set to 0.1%. Note that in contrast to research on global solution of MILP models, in which accepted optimality tolerances are typically $1 \cdot 10^{-4}$, standards for global solution of MINLP models are typically within 1% – due to the relative increase in computational difficulty.

A. Computational Performance

Computational results for our global UC-AC solution algorithm on the 4 benchmark problems are reported in Table I. The second column reports the best obtained upper bound, which corresponds to the *best known solution* to the UC-AC problem. The third column reports the best obtained lower bound, which corresponds to the solution of the problem defined in (M). The relative optimality gap is shown in the fourth column, followed by the total wall clock time and the number of major iterations. All problems are solved to within a 0.5% global optimality gap under the wall clock time limit. For IEEE-118mod, we obtained a 0.34% optimality gap after the first iteration (in approximately $8400s$), which remains unchanged before the time limit is reached in major iteration $k{=}2$ with a 0.11% MIP gap for the lower-bounding problem.

Table II additionally reports IEEE-118mod results for our global UC-AC algorithm compared to results obtained with version 16.12.7 of the commercially available general MINLP solver BARON [62, 63], and heuristic UC-AC methods as reported in [48] and in [36]. BARON was unable to solve any of the UC-AC problems within a time limit of 10 hours using the default solver parameters, CLP/CBC for the LP/MILP subproblems, and IPOPT and FILTERSD for the NLP subproblems. It is possible that better performance could be obtained with BARON by leveraging commercial subproblem solvers and additional tuning.

Our global UC-AC upper bound solution is 0.92% less costly than the heuristic OA [48] solution and 1.84% less costly than the heuristic GBD [36] solution. We expect for the improvement attained by solving the UC-AC problem globally to increase with the size of the network and generator set. Even a percentage improvement in operating efficiency leads to a monetary savings of billions annually [53].

B. Globally Optimal Unit Commitment Schedules

In addition to the differences in optimal objective value, we also want to compare the commitment schedules obtained with global and local algorithms. The globally optimal generator commitment schedules for our test cases are reported in Table III, IV, and V. Due to symmetry (e.g., co-location of identical generating units at a given bus), there can be multiple global solutions with the same objective value (here, we have not included alternate solutions). Comparing these commitment schedules with those from the local results in [48], we observe that they are the same for the 6-Bus case, but differ for RTS-79 (where a slightly improved solution is found) and IEEE-118mod (where a significantly improved solution is found).

VI. Conclusions

Solving the UC-AC problem is fundamental to obtaining real-world operations and market settlements that fully incorporate the impact of alternating current network physics. We have introduced, to the best of our knowledge, the first globally optimal approach to solving this practically critical and computationally difficult problem

TABLE I: Numerical results for our global UC-AC solution algorithm

Case	Upper Bound ($)	Lower Bound ($)	Optimality Gap (%)	Wall Clock Time (s)	Iteration (k)
6-bus	101,763	101,740	0.02%	8.5	2
RTS-79	895,040	894,392	0.07%	1394	6
RTS-96	886,362	885,707	0.07%	321.0	1
IEEE-118mod	835,926	833,057	0.34%	14400†	2

† MIP gap of 0.11% for the master problem at the time limit.

TABLE II: Comparative results with alternative UC-AC approaches

	Upper Bound ($)			
Case	Our Global UC-AC	BARON UC-AC	*Castillo et al.* UC-AC [48]	*Fu et al.* UC-AC [36]
IEEE-118mod	835,926	n/a	843,591	851,274

TABLE III: Commitments for the 6-Bus System

Bus	Gen	Commitment (h)
B1	G1	1-24
B2	G2	1, 12-21
B6	G3	10-22

TABLE IV: Commitments for the 24-Bus Systems

Bus	Gen	Commitment (h)	
		RTS-79	RTS-96
B1	G1, G2	Ø	Ø
B1	G3, G4	8-23	8-23
B2	G5, G6	10	Ø
B2	G7	8-24	8-24
B2	G8	8-23	8-23
B7	G9	1-23	1-23
B7	G10	9-24	10-24
B7	G11	10-18	Ø
B13	G12	11-22	1-18
B13	G13	Ø	11-22
B13	G14	Ø	Ø
B14	G15	1-24	1-24
B15	G16-G18	10-15	Ø
B15	G19, G20	10-13	Ø
B15	G21	9-24	9-24
B16	G22	1-24	1-24
B18	G23	1-24	1-24
B21	G24	1-24	1-24
B22	G25-G30	1-24	1-24
B23	G31-G33	1-24	1-24

TABLE V: Commitments for the IEEE-118mod System

Gen	Commitment (h)	Gen	Commitment (h)
G1	Ø	G28	1-24
G2	Ø	G29	1-24
G3	Ø	G30	1-24
G4	1-10, 24	G31	Ø
G5	1-24	G32	Ø
G6	Ø	G33	Ø
G7	11-22	G34	7-24
G8	Ø	G35	1-24
G9	Ø	G36	1-24
G10	1-2, 12-24	G37	8-23
G11	1-24	G38	Ø
G12	Ø	G39	Ø
G13	Ø	G40	1-10, 22-24
G14	10-22	G41	Ø
G15	Ø	G42	Ø
G16	9-16	G43	1-24
G17	Ø	G44	Ø
G18	Ø	G45	1-24
G19	Ø	G46	Ø
G20	1-24	G47	Ø
G21	8-24	G48	Ø
G22	Ø	G49	Ø
G23	Ø	G50	Ø
G24	9-23	G51	9-13
G25	Ø	G52	14-23
G26	Ø	G53	7-24
G27	1-2, 13-24	G54	9-23

Although our obtained run times are still longer than those required for operations, our proposed approach can be used to quantify the (near-) global optimality of "off-line" solutions, as well as test and validate other algorithmic approaches including the heuristics and local solution techniques referenced in Section II.

A considerable fraction of the computational time associated with our algorithm is in the global solution of the subproblems. Efficient global solution of ACOPF subproblems is an ongoing focus in the literature [64, 65] and furthermore can be leveraged to produce valid cuts in the master problem. Optimization results on larger data sets will require further development that potentially leverages a variety of advancements in relaxation refinements, MISOCP solvers, cut generation, and decomposition techniques. For example, other future directions for research include incorporating symmetry-breaking methods, and decomposition and parallelization techniques to improve the efficiency of determining lower bounds. Improvements to the mixed-integer refinement problem in the nested algorithm include adaptive, non-uniform partitioning schemes. Security (e.g., N-1) considerations and parameter uncertainties do not alter the core UC-AC problem that needs to be solved, but does increase the dimensionality of the problem; such dimensionality increase is addressable through the aforementioned advancements, which are extensions left for future work.

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