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Surface wave velocity estimation and their uncertainties using the multiwavelet transform: applications to surface wave tomography

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1. Introduction

We explore using the multiwavelet transform (MWT) as an alternate method to estimate surface wave speeds. The MWT decomposes a signal similarly to the conventional filter bank technique, but with two primary advantages: 1) the time-frequency localization is optimized in regards to the time-frequency tradeoff, and 2) we can use the MWT to estimate the uncertainty of the resulting surface wave group- and phase-velocities. The uncertainties of the surface wave speed measurements can then be propagated into, for example, tomographic inversions to provide uncertainties of resolved Earth structure. We apply our technique to ambient noise correlograms that were collected as part of the the University of Nevada Reno seismic network near the Nevada National Security Site . We find that the group- and phase-velocity estimates for numerous station pairs are comparable between the conventional filter bank method and the MWT method. To demonstrate the utility of the MWT, we apply the uncertainties of the estimated surface wave velocities to place error bounds on 1D surface wave tomographic inversion.

2. Similarity between filter banks and the wavelet transform

2a. Filter bank

To resolve surface wave dispersion, a common approach is to apply a series of band-limited filters:

$$\hat{S}(t, f_c) = \int_{t_0}^{T} H(t, f_c \pm \Delta f) s(t-\tau) d\tau$$

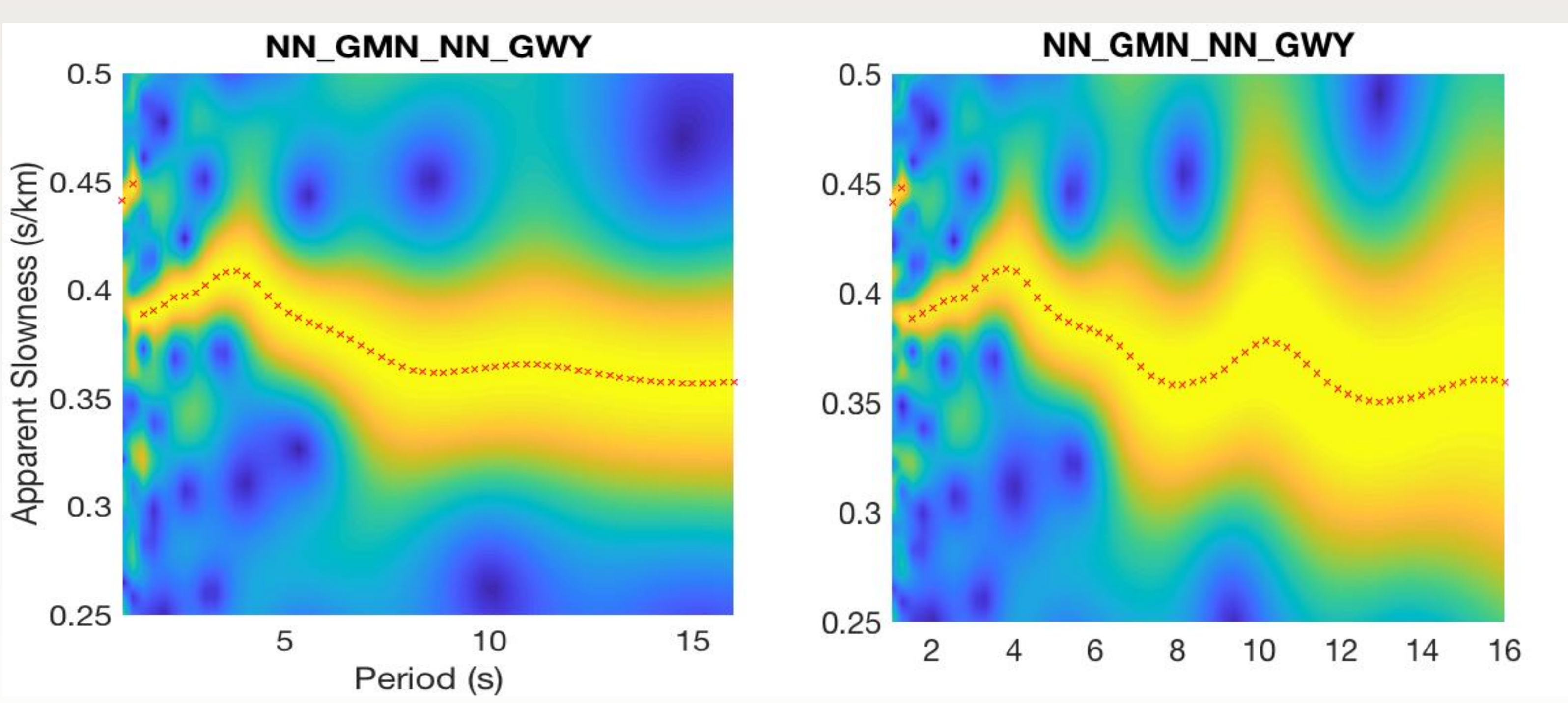
where $s(t)$ is the signal and $H(t, f_c \pm \Delta f)$ is a band-limited filter function with center frequency f_c. The signal is filtered and enveloped for a range of center frequencies and the result is plotted as a function of frequency and slowness.

2b. The wavelet transform

The generalized wavelet transform,

$$W[s(t,C)] = \int_{t_0}^{T} s(t-\tau)\phi(t,C) d\tau$$

decomposes a signal into its constituent wavelet coefficients by convolving the signal with a real- or complex-valued function, ϕ. Each wavelet is scaled by C, which controls the center frequency of the wavelet.



Two methods to estimate surface wave dispersion. Left: results of applying a series of Gaussian filters to a vertical-component autocorrelogram formed from station pair GMN/GWY of the UNR network. The narrow band signals are then enveloped via the Hilbert transform and normalized. The peak power for each period, as identified by the red dots correspond to the group velocity, U, for the Rayleigh wave. Right: similar to the left panel, except we use a wavelet transform to decompose the signal. To produce these results, we use the first-order, complex-valued Slepian wavelet.

3. The multi-wavelet transform

3a. Theory

The integration kernels in the MWT are Slepian wavelets, which are a family of even-odd wavelet pairs that are time-frequency optimized (Lilly and Park, 1995). They are formed by solving the eigenvalue problem

$$A\underline{\phi} = -\lambda\underline{\phi}$$

where

$$A_{mn} = \frac{\sin[2\pi(f_c + f_w)\Delta t(m-n)]}{\pi(m-n)} - \frac{\sin[2\pi(f_c - f_w)\Delta t(m-n)]}{\pi(m-n)}$$

Solving the eigenvalue problem gives $k = 1,2,\ldots N$ eigenvectors, which are normalized and form the complex valued wavelet pairs

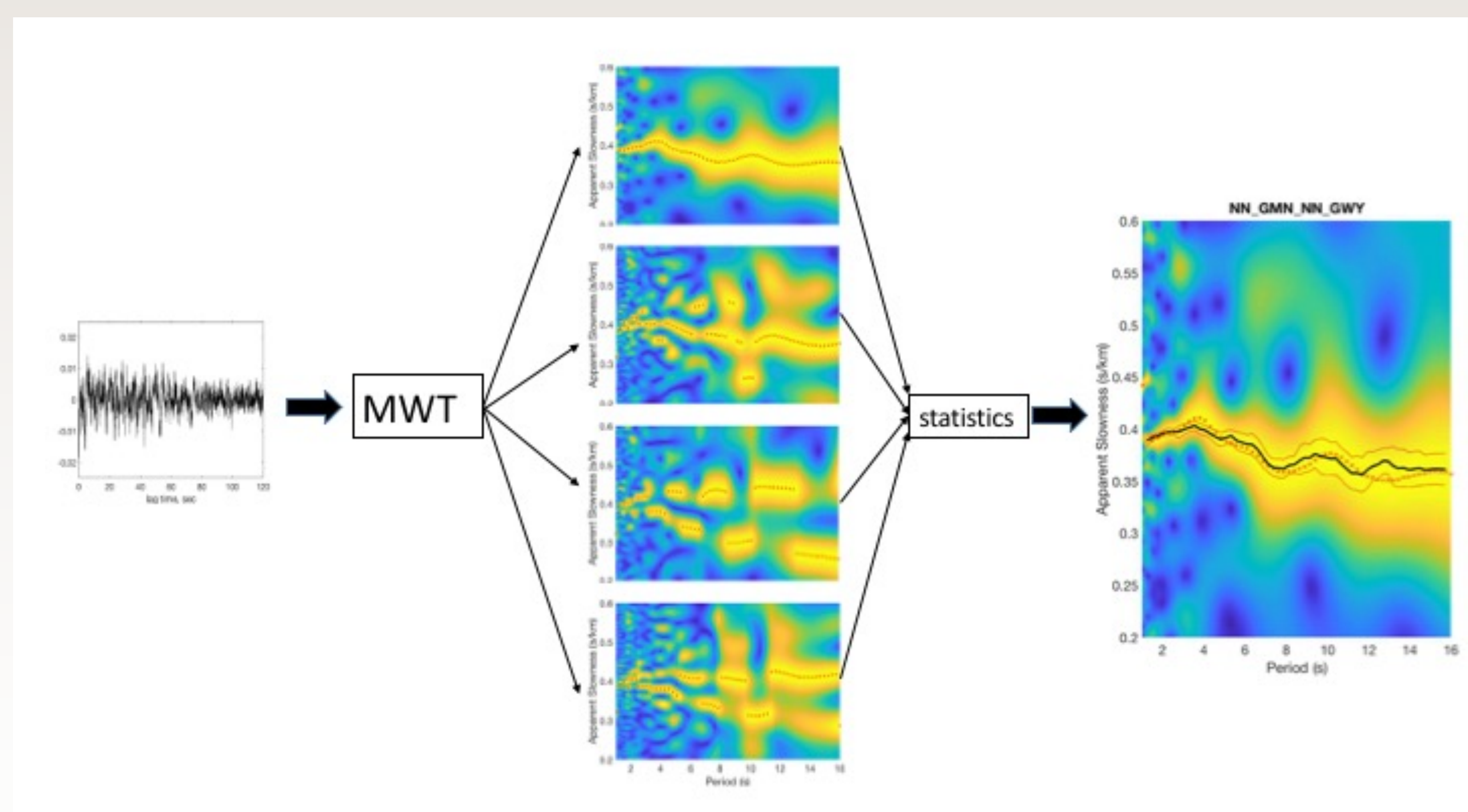
$$\phi^{\{k\}}(t,C) = \phi_e^{\{k\}}(t,C) + \mathrm{i}\phi_o^{\{k\}}(t,C)$$

Finally, the multiwavelet transform is given by

$$W^{\{k\}}[s(t,f)] = \int_{t=T/2}^{t+T/2} s(t-\tau)\phi_e^{\{k\}}(t,C) + \mathrm{i}\int_{t=T/2}^{t+T/2} s(t-\tau)\phi_e^{\{k\}}(t,C)\, d\tau$$

3b. Implementation

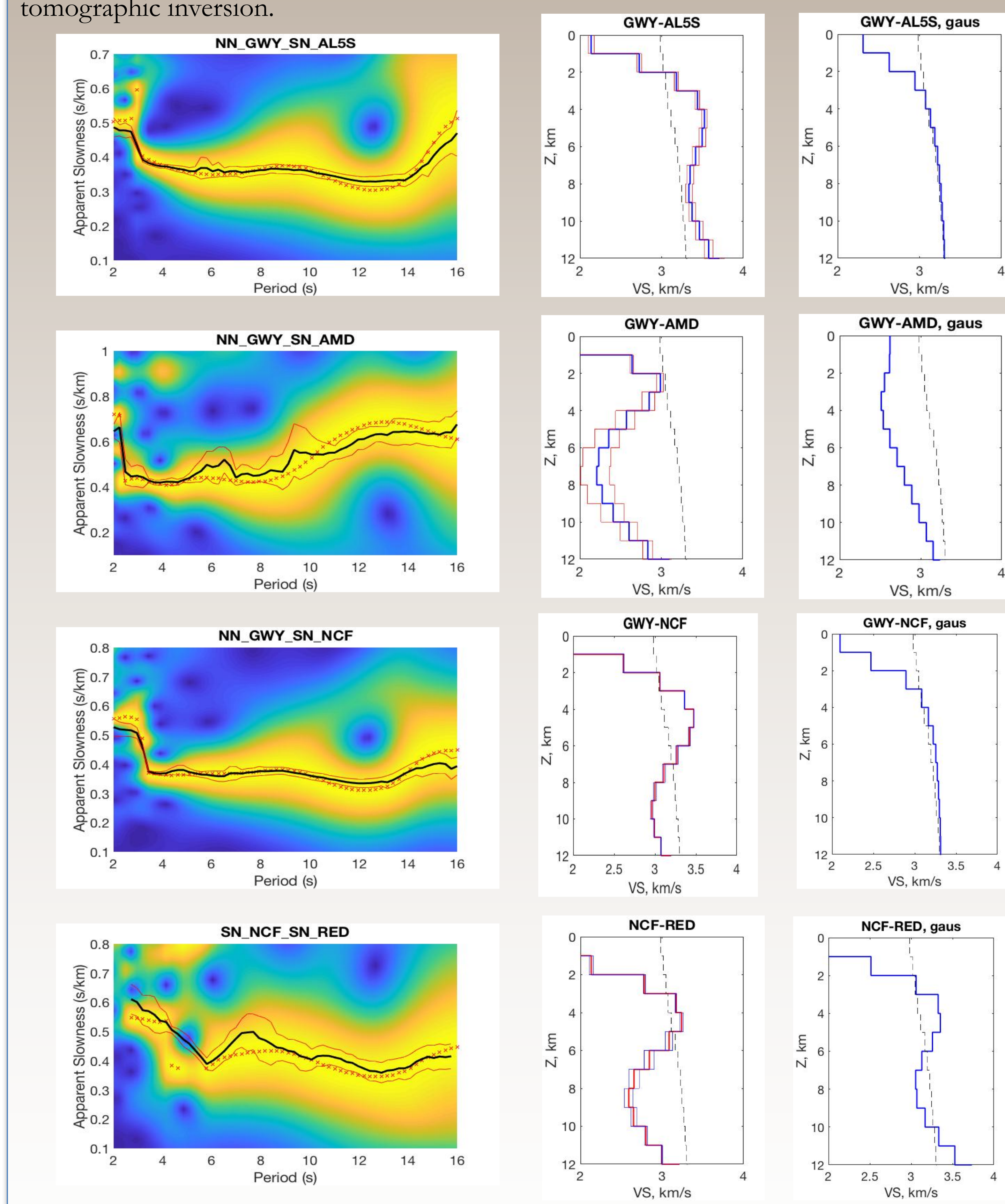
Each k^{th} complex wavelet is orthogonal to all other complex wavelets. Therefore, the wavelet decomposition $W^{\{k\}}[w(t,f)]$ for $k = 1,2,\ldots N$ yields N mutually orthogonal estimates of the signal's time-frequency composition. Because we use the wavelet decompositions to estimate group velocity of surface waves, we can use the N estimates to form uncertainty estimates of group and phase velocity using standard statistical methods (e.g. mean, standard deviation, etc).



The figure above illustrates the use of the MWT to estimate surface wave group velocity and uncertainty. The signal (left panel) is decomposed into N estimates of its wavelet coefficients, and plotted as function of slowness and period (middle panel). The peak power for each period is identified for each estimate of the wavelet decomposition (indicated by the the red dots). For each period, there are N estimates of slowness, which can be used to compute statistics (right panel). The right panel shows the mean slowness as estimated by the MWT(black line) as well as the 95% confidence interval (red line). For comparison, we overlaid the dispersion curve as estimated by a conventional Gaussian filter bank (red dots).

4. Application: surface wave tomography

We demonstrate the potential utility of our approach by applying it to four ambient noise correlograms collected by the University of Nevada, Reno seismic network which is near the Nevada National Security Site (NNSS). Using the MWT, we estimated the group and phase velocity of the Rayleigh wave for each station pair. We then invert the velocity estimates to recover the 1D shear wave velocity model using Computer Programs in Seismology (Herrman and Ammon, 2002). We propagate the uncertainty of surface wave velocity estimation into the tomographic inversion by performing the inversion for the mean estimated dispersion curve and then once again for the upper and lower bounds (+/- one standard deviation) of the dispersion curve. For comparison, we also estimated the dispersion curve using a Gaussian filter bank, and performed the same 1D tomographic inversion.



Dispersion curves as estimated by the MWT (black line) and their corresponding 95% confidence intervals (red lines). For comparison, we've overlain the dispersion curves estimated using a Gaussian filter bank (red dots).

Left column: 1D velocity estimates obtained by inverting the dispersion curves as estimated from the MWT. The heavy read line is the velocity estimated from the mean dispersion curve and the thin red lines show the velocity estimates from the upper and lower confidence intervals. Right column: shear wave velocities estimated from the dispersion curves obtained using a Gaussian filter bank. The dashed black line shows the starting shear-wave velocity model. For all cases, the model had 13 layers, 1km thick, and all layers were damped equally.

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