

## Inverse Mold Design To Accommodate Manufacturing Warpage in Chemically Blown PMDI Foams

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**33<sup>rd</sup> Polymer Processing Society, Cancun, Mexico**

# (pMDI) Polyurethane Foams Are Challenging!

## Application Space

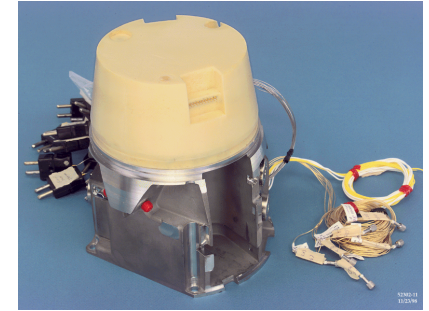
- PMDI is used as an **encapsulant** and as a **structural material** to mitigate against shock and vibration

## Problems Arise From a Short Pot Life and High Exothermicity

- Void Defects, Density Variations, **Residual Stress**
- **Short Term**: Meet Tight Geometric Specifications
- **Long Term**: Long term shape change/loss of component function

## The Current Design Approach Is Costly!

**Manufacturing and Acceptance through Trial and Error  
Modeling and Simulation Can Help!**



(Mike Gerding, UUR)



# The Challenge: Predicting How Manufacturing Conditions Impact Component Dimensional Stability

## A Typical Manufacturing-to-Lifetime Process

Mix Resin

Injection,  
foaming and  
initial curing  
at lower T

Run 030110-PMDI-4 60°C  
Free Rise  
Sandia National Labs

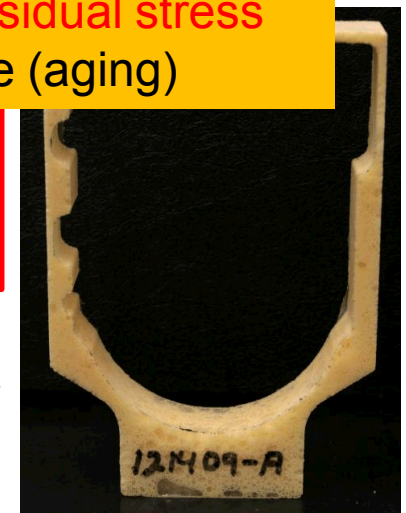
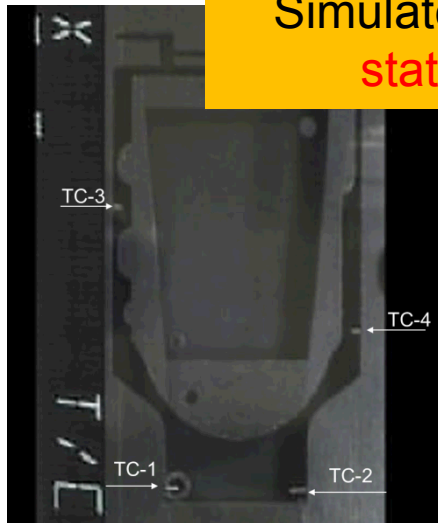
Secondary  
Cure at  
Higher  
Temperature

### Objective

Simulate the manufacturing process, develop the residual stress state, and predict component warpage over time (aging)

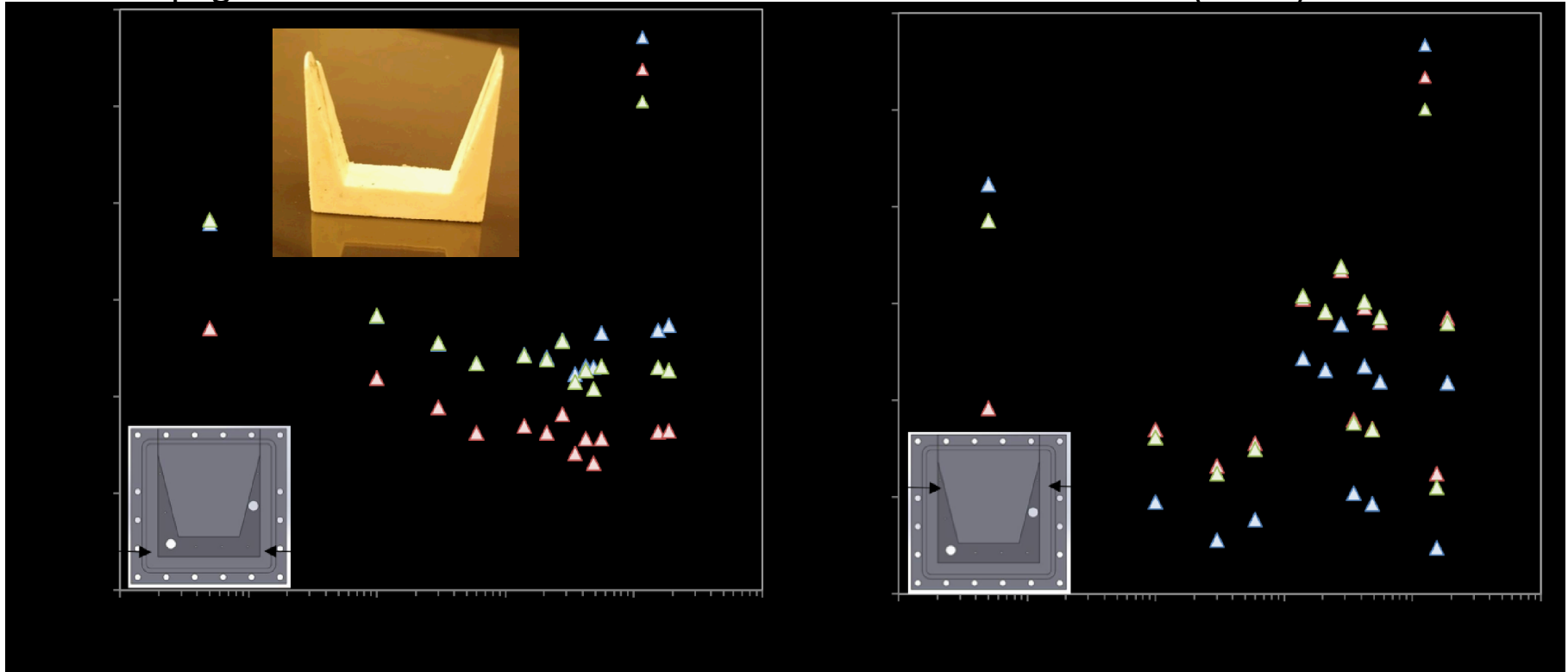
Remove  
from mold –  
predict cure  
and thermal  
stresses

Predict  
shape and  
size over  
years



# Experimental Motivation: Warpage and Aging Demonstration on the Sandia Staple

- PMDI 10 S packed to 12.5 PCF
- Cure Schedule: 30 C for 10 minutes, 4 hours at 120 C
- Warpage measurements with a Zeiss® Coordinate Measurement (CMM) Machine



Monotonic and Consistent Warpage Trend in Thick Regions

Non-Monotonic and Complex Warpage Trend On Thin Staple Arms

What are the key factors that make complex warpage behavior at the "staple arms"?



# Presentation Outline

1. Problem Definition and Customer Needs
2. "Cradle-to-Grave" Modeling and Simulation:
3. Parametric Studies on the Sandia Staple Mold from Resin Injection through 20 years of aging
4. Warpage Predictions on complex foam components
5. Summary and Key Findings

# MODEL SUMMARY

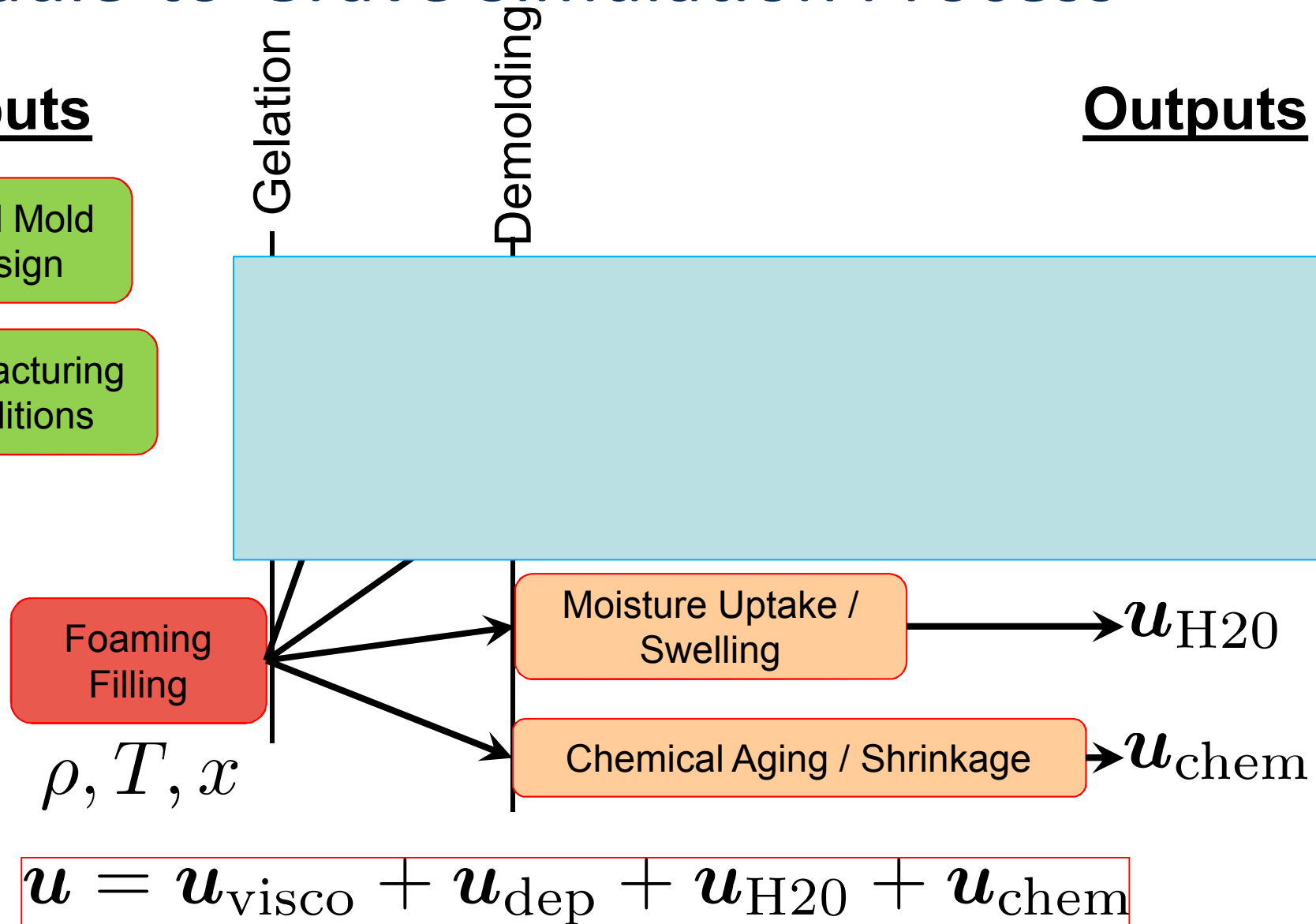
# Cradle-to-Grave Simulation Process

## Inputs

Initial Mold Design

Manufacturing Conditions

## Outputs



# Inverse Mold Design Process

## Inputs

Manufacturing  
Conditions

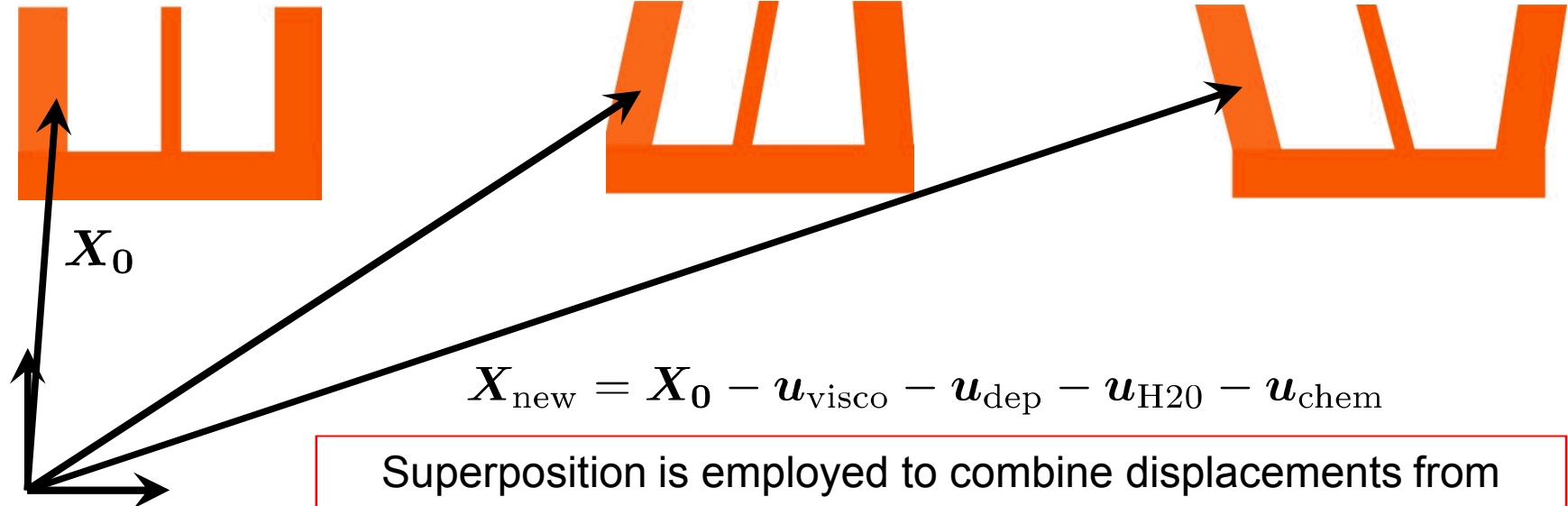
Initial Mold  
Design

Cradle-to-Grave  
Simulation

## Output

Final Mold  
Shape

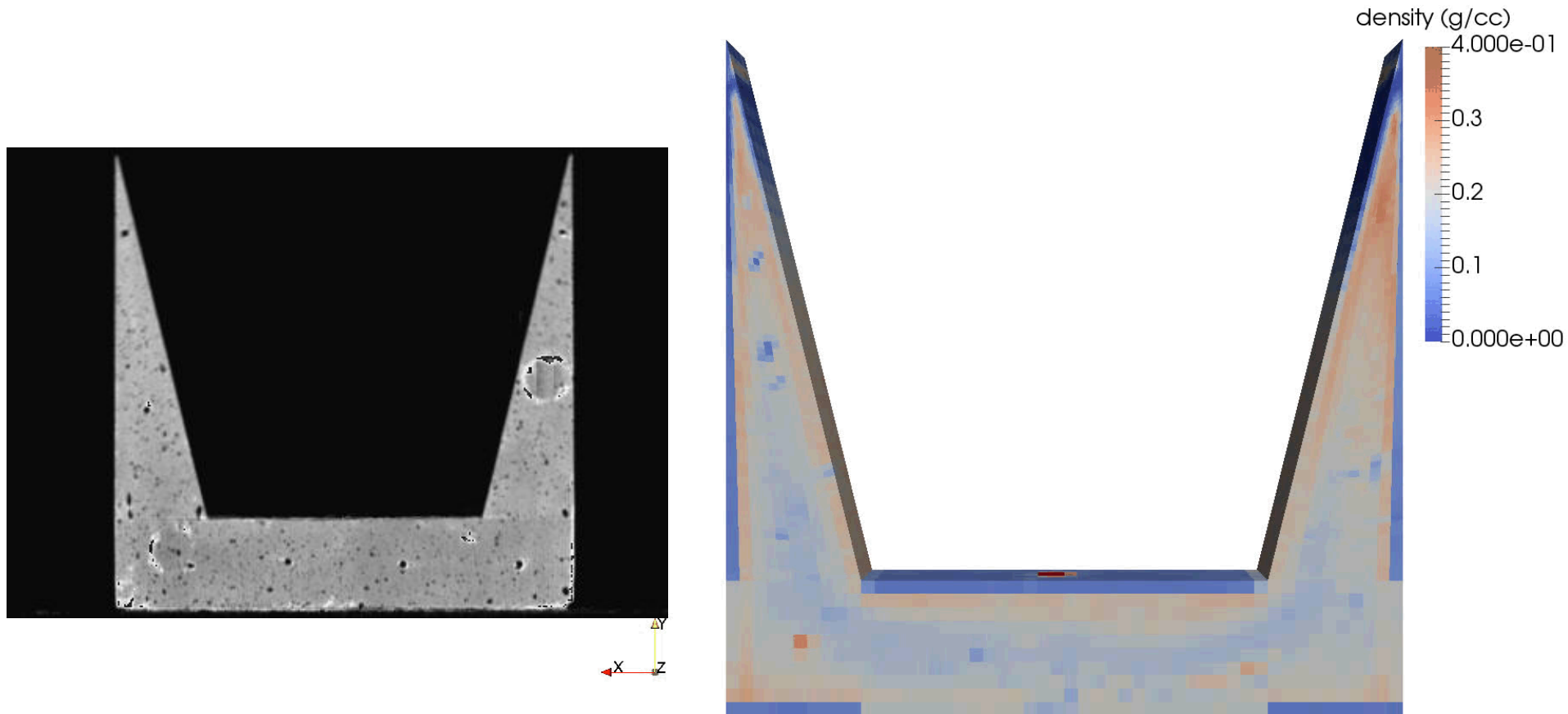
$$X[t] = X_0 + u_{\text{visco}} + u_{\text{dep}} + u_{\text{H2O}} + u_{\text{chem}}$$



Superposition is employed to combine displacements from different mechanisms and then to “inverse warp” the initial mold design

# Initial Calculations of the U-staple

- Uniform Gauge Pressure (12 psig), Density from the X-ray CT
- **1000 X Displacement for Visualization**



Depressurization consistently produces 1/10 or less the deformation compared with viscoelastic residual stress relaxation

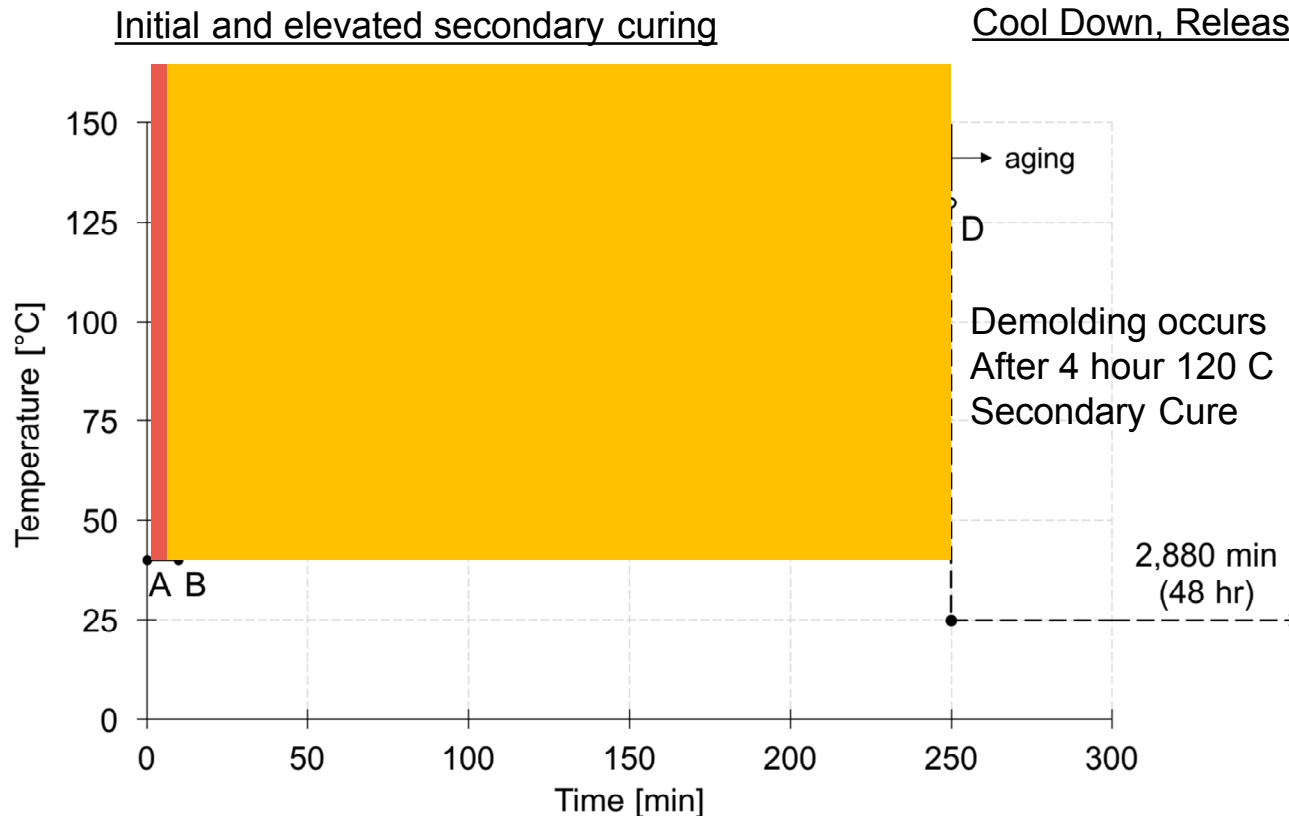
The role of residual stress and non-linear viscoelasticity in part warpage

# **RESIDUAL STRESS RELAXATION MANUFACTURING SENSITIVITY STUDY**



# The Sandia / AWE Staple Simulations

A benchmark problem to experimentally and computational investigate warpage quantitatively



## Boundary Conditions

### **Thermal:**

1. Essential BCs until The end of the 4 hour 120 C secondary cure
2. Convective cooling to room temperature thereafter

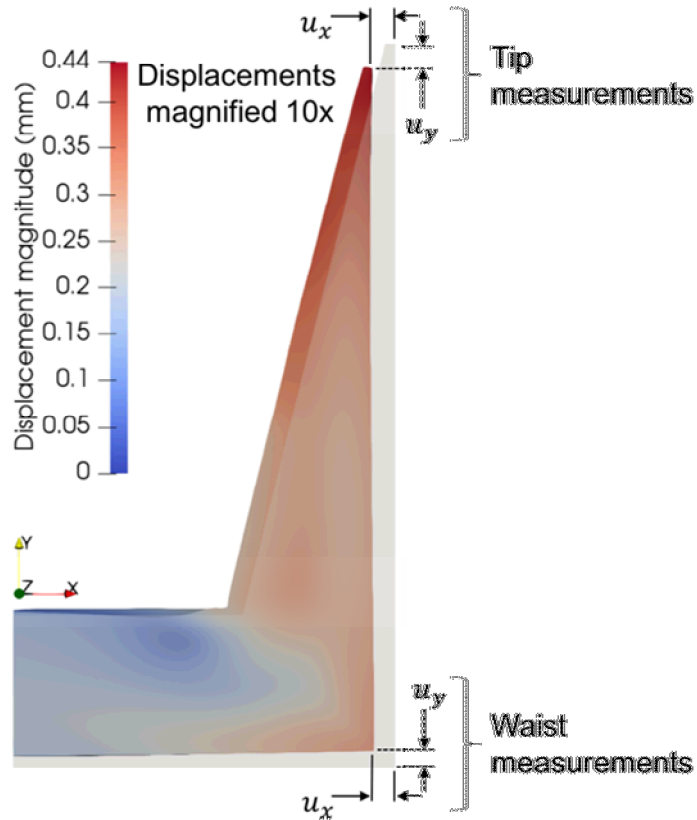
### **Mechanical:**

1. Essential (zero displacement) until demolding
2. Traction free post demolding

A KCNSC Cure Schedule for the PMDI10S Foam. Parametric Study Parameters During Cooldown

# Sandia Staple Displacement Definitions

## Displacements

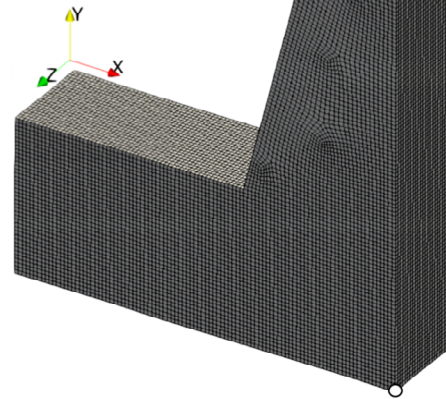


(A)

- Node 1 ( $x_1, y_1, z_1$ )
- Node 2 ( $x_2, y_2, z_2$ )

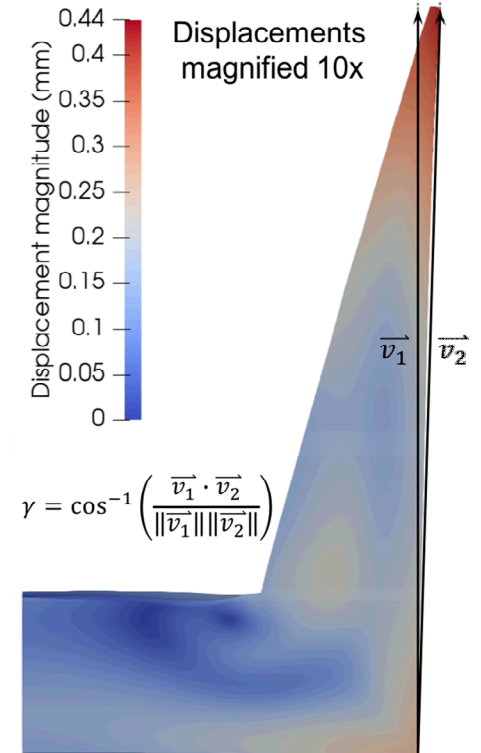
$$\vec{v}_1 = \begin{Bmatrix} 0 \\ y_0 \\ 0 \end{Bmatrix}$$

$$\vec{v}_2 = \begin{Bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{Bmatrix}$$

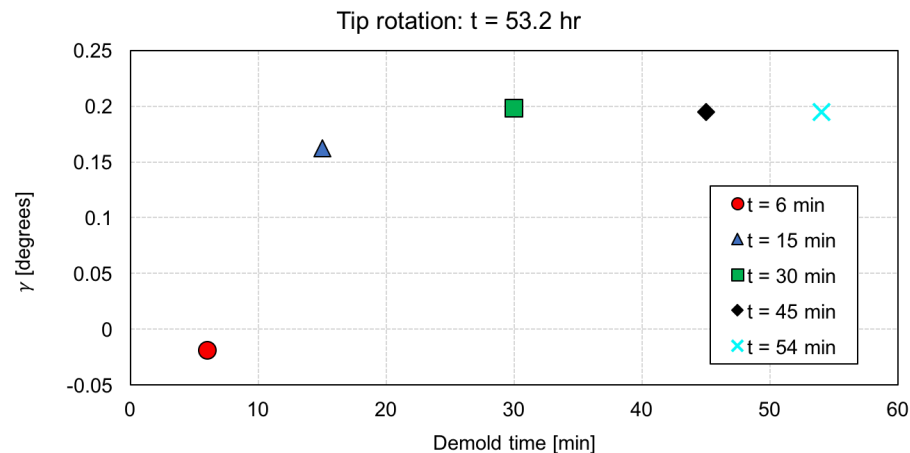
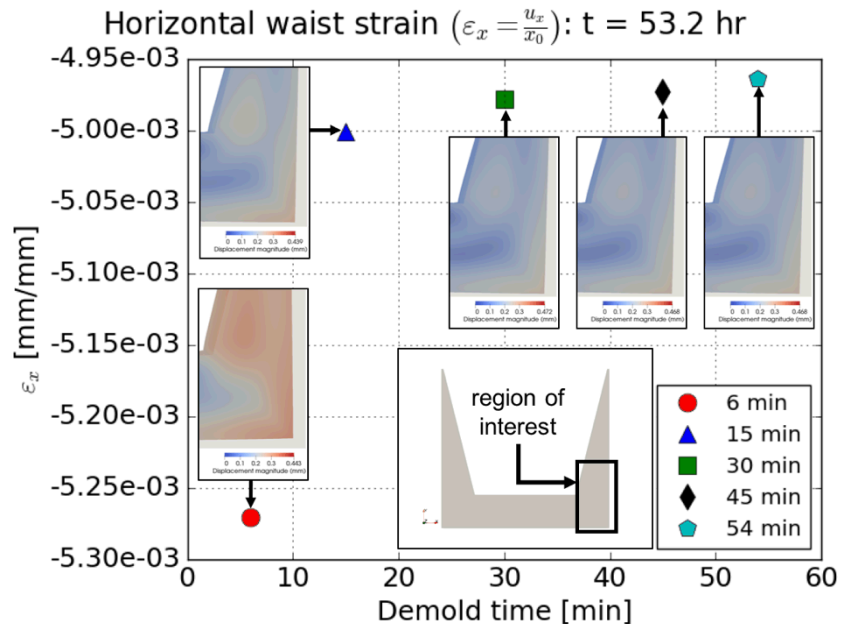
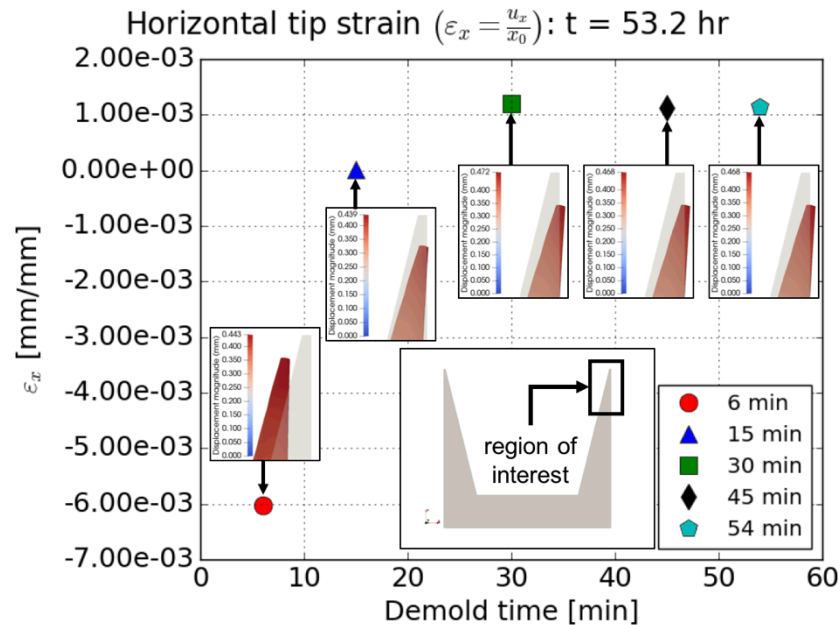


## Arm Angle

(B)

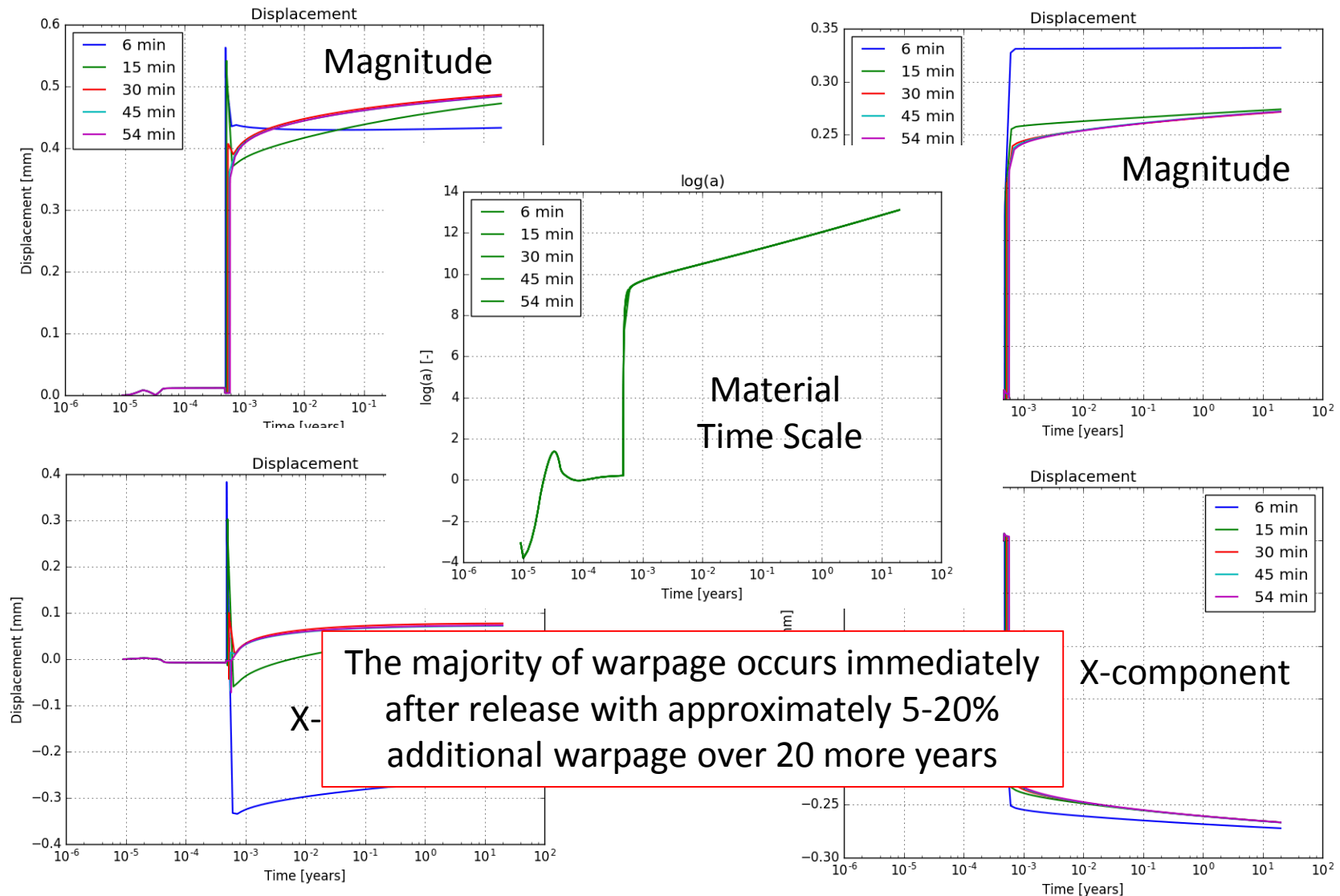


# Demolding Sensitivity Study

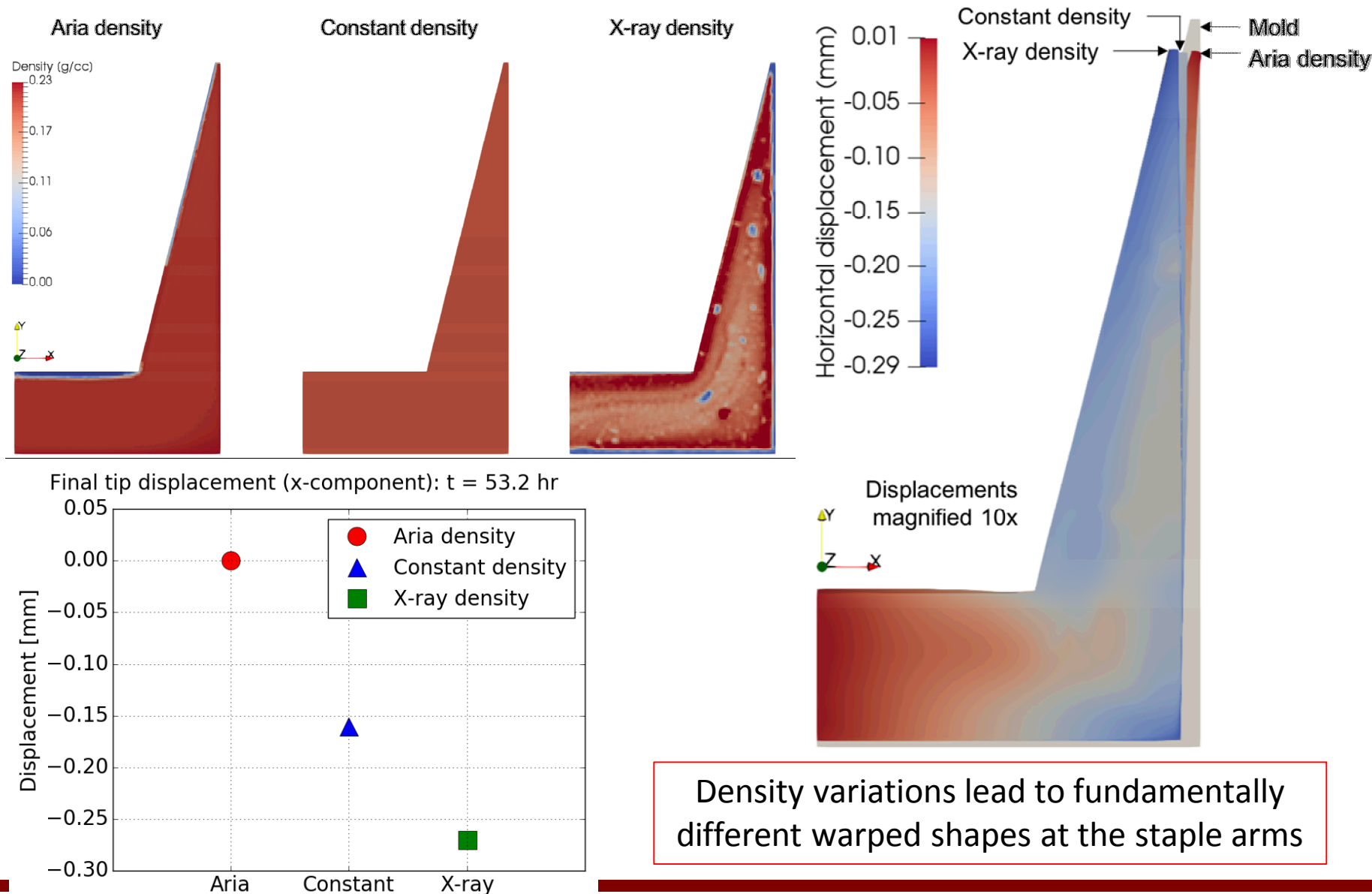


Demolding too hot (close to the glass transition) significantly changes the Warpage response

# Demolding Sensitivity Study: 20 Years of Traction Free / RT Physical Aging



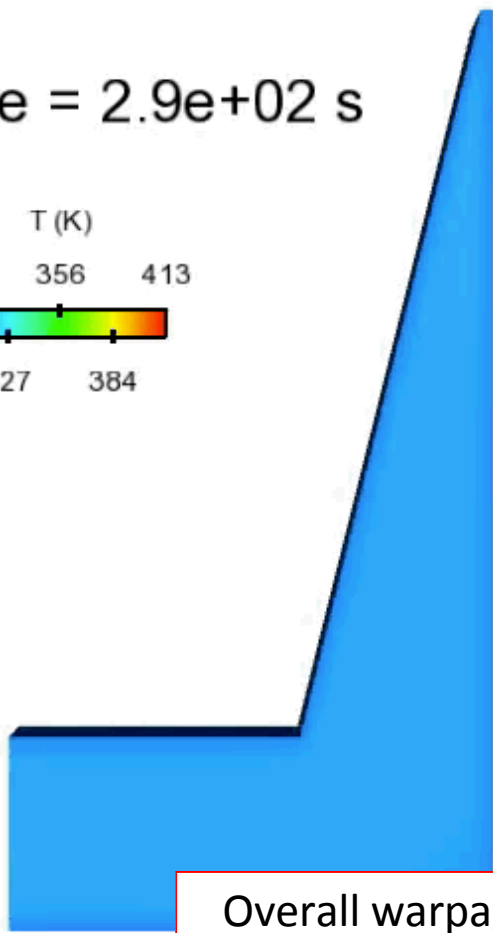
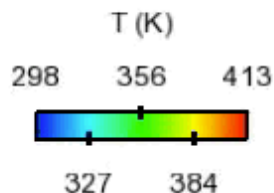
# Density Sensitivity Study



# Density Variation Time Histories

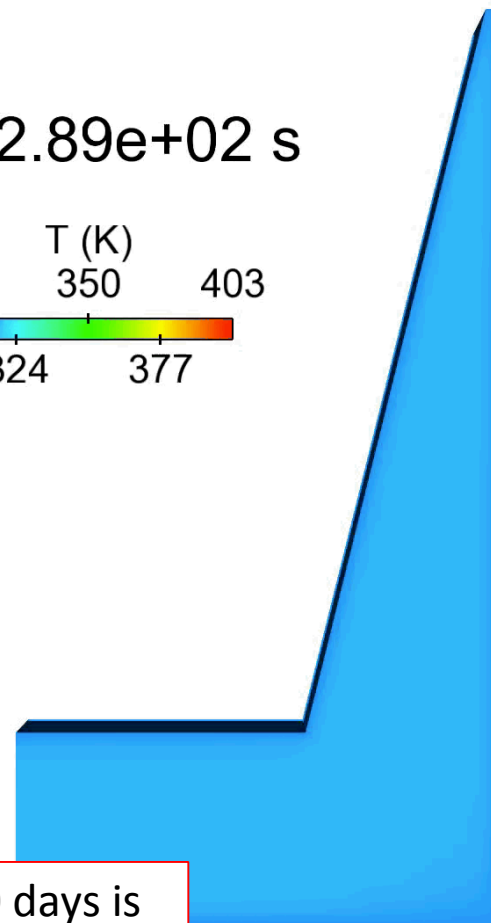
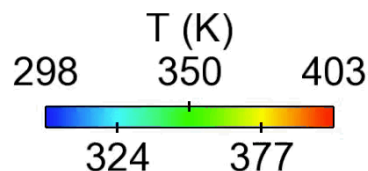
## Constant Density

Time =  $2.9 \times 10^2$  s



## X-Ray CT Density

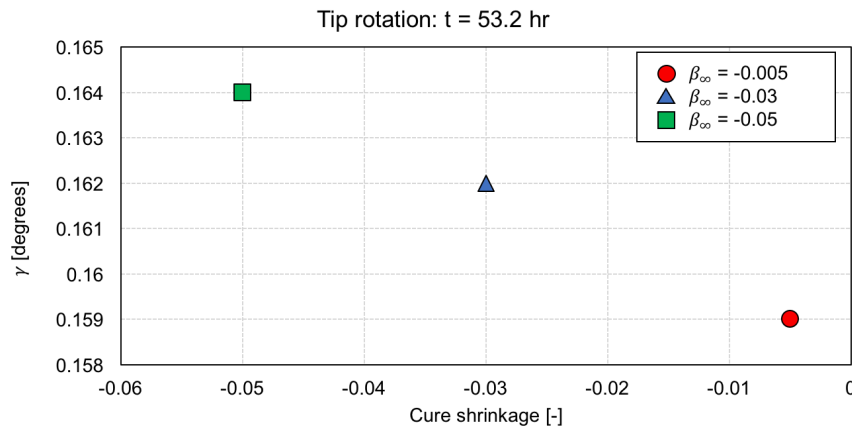
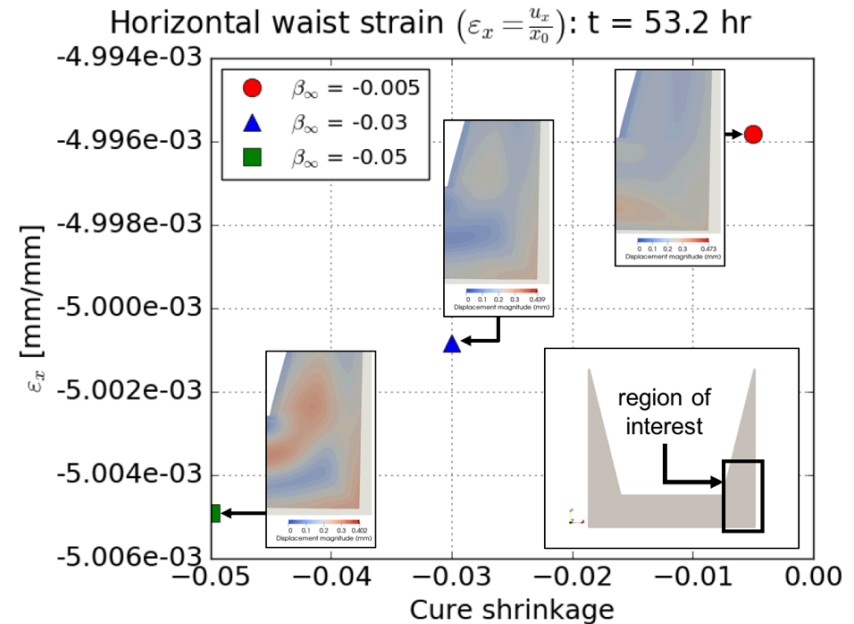
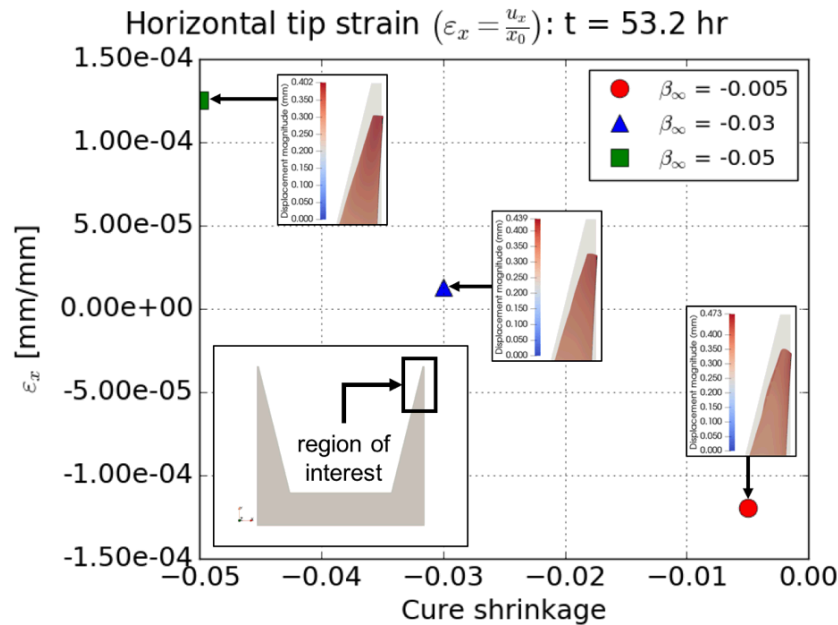
Time =  $2.89 \times 10^2$  s



Overall warpage behavior over the first 10 days is qualitatively similar between different densities



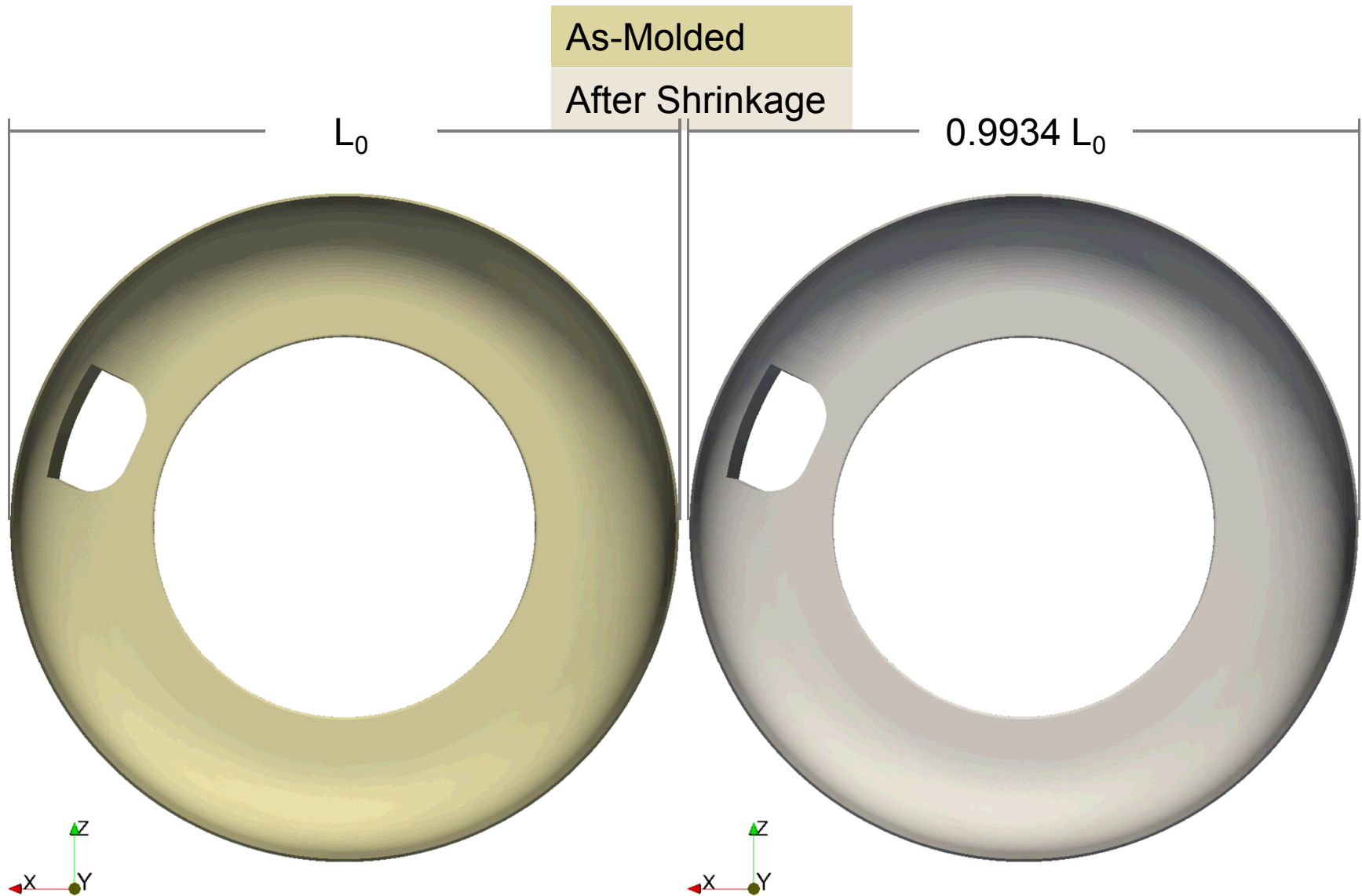
# Cure Shrinkage Sensitivity Study



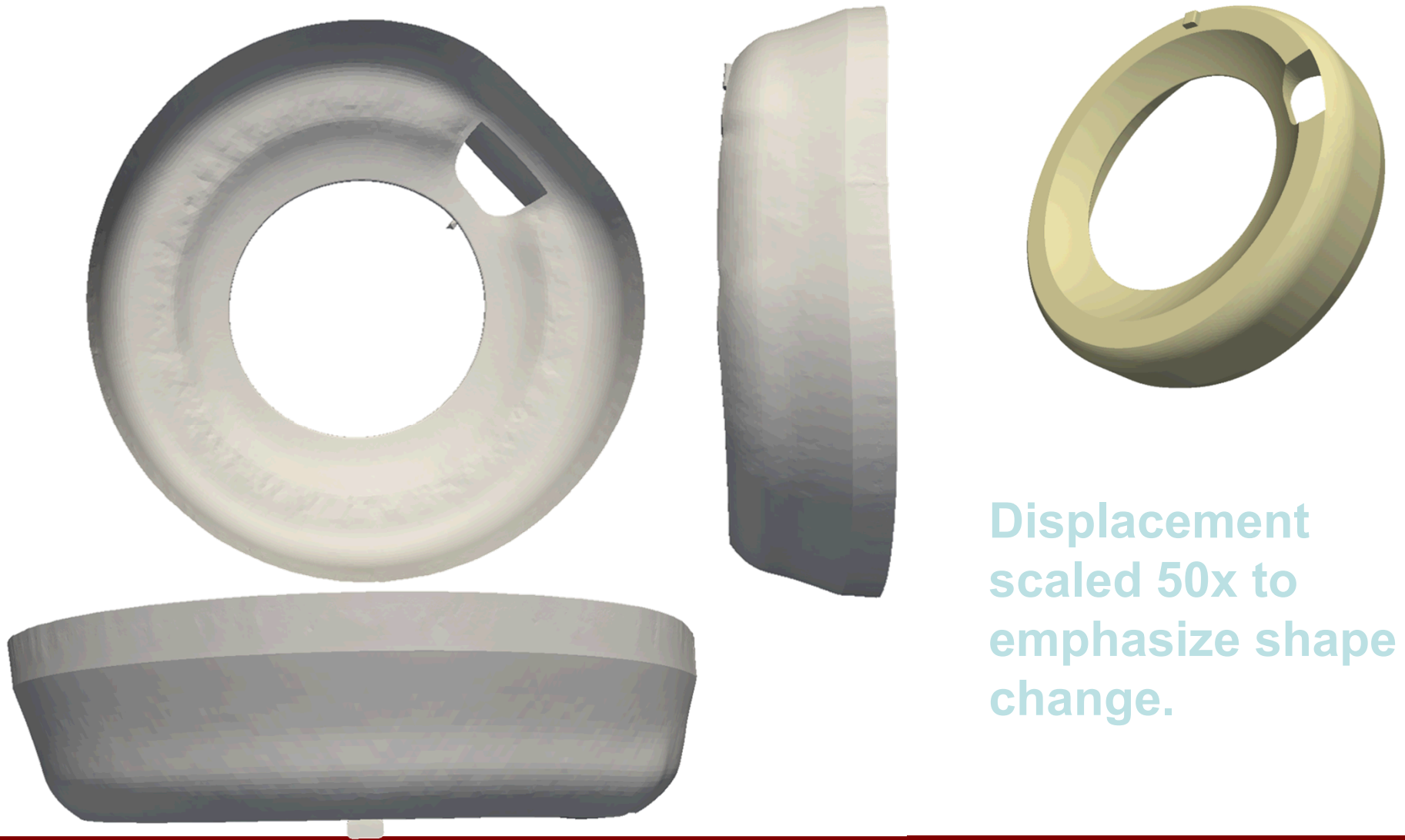
The effects of the cure volume change parameter on staple warpage are small.  
 → We don't need to accurately determine This parameter in the model

# **WARPAGE PREDICTIONS IN 10 PCF PMDI STRUCTURAL COMPONENTS 48 HOURS AFTER RESIN POOR**

# Exemplar Part With Featured Regions

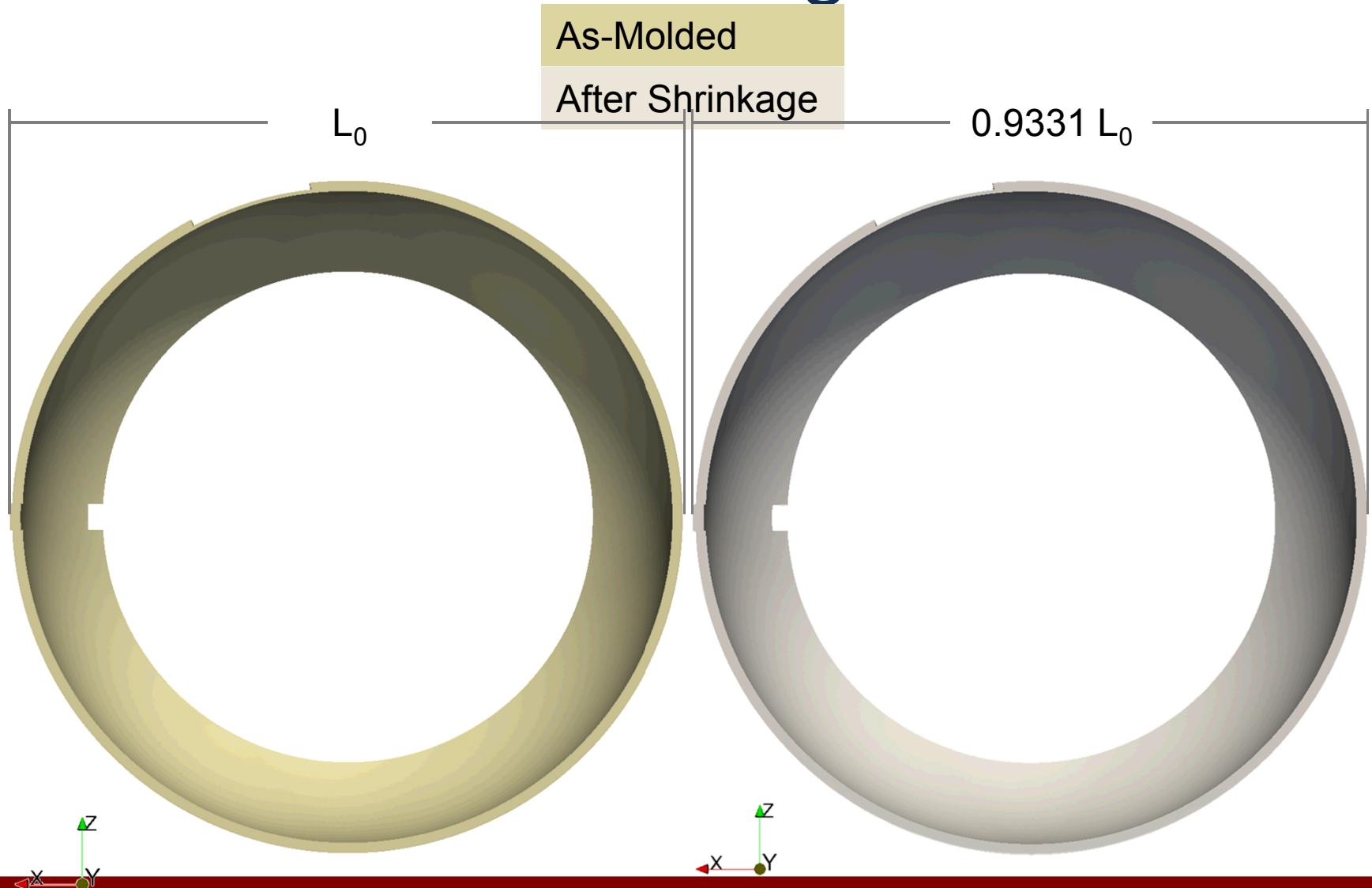


# Warpage accentuated near holes and slender regions

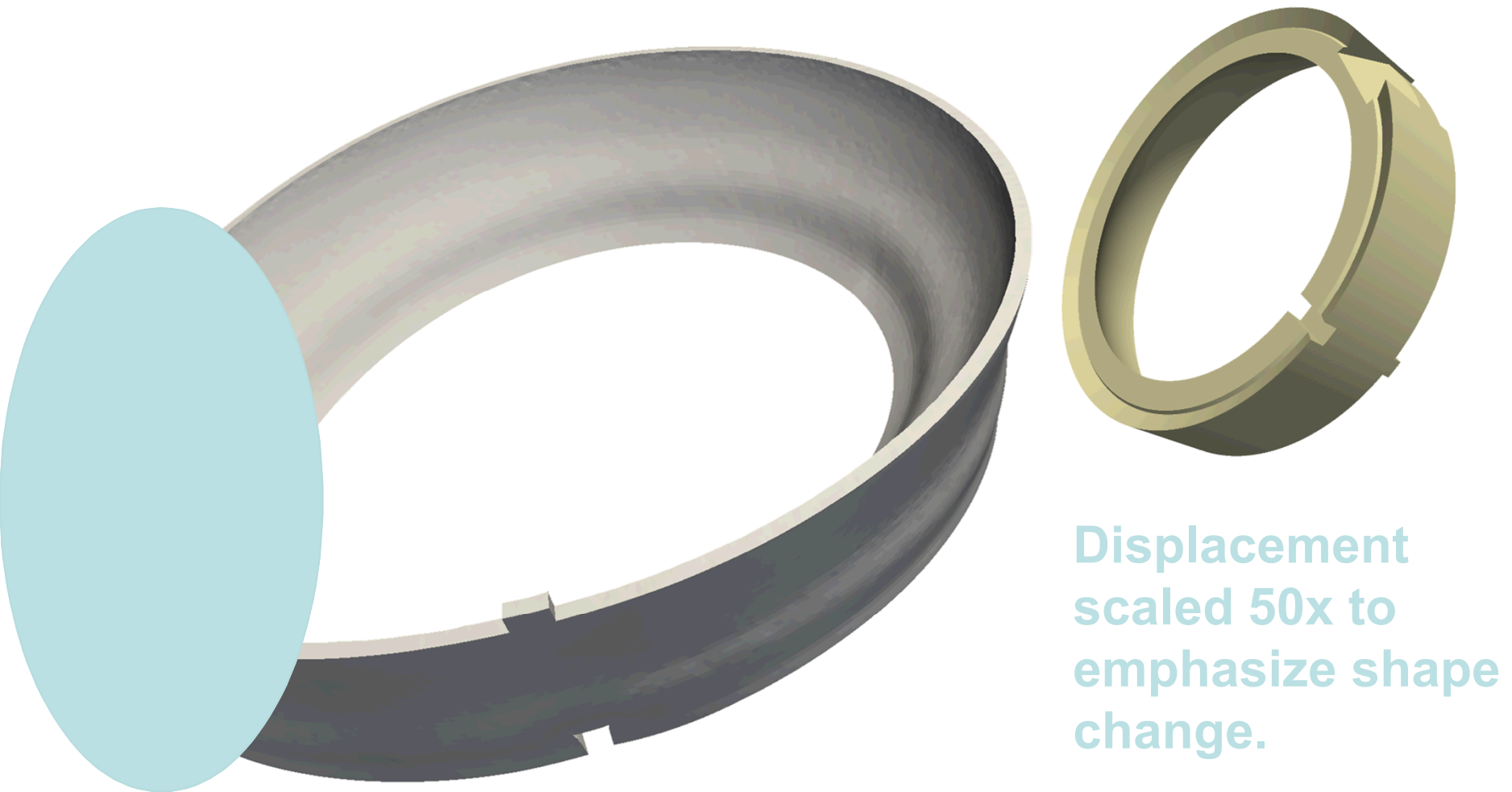


Displacement  
scaled 50x to  
emphasize shape  
change.

# Exemplar Part With Notched Features and Thin and Thick Region Transitions



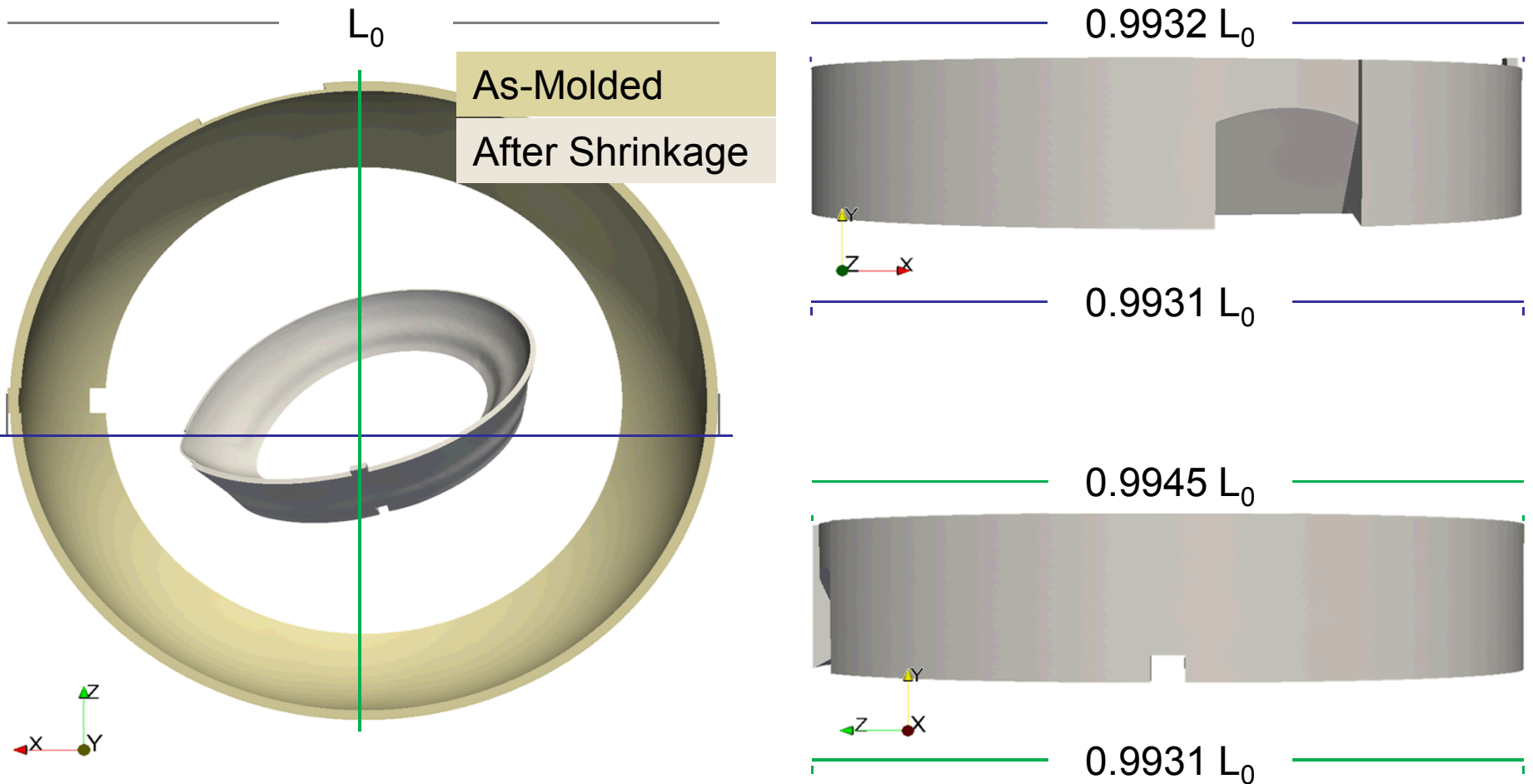
# Shrinkage Accentuated Near Thin-Thick Region Transition



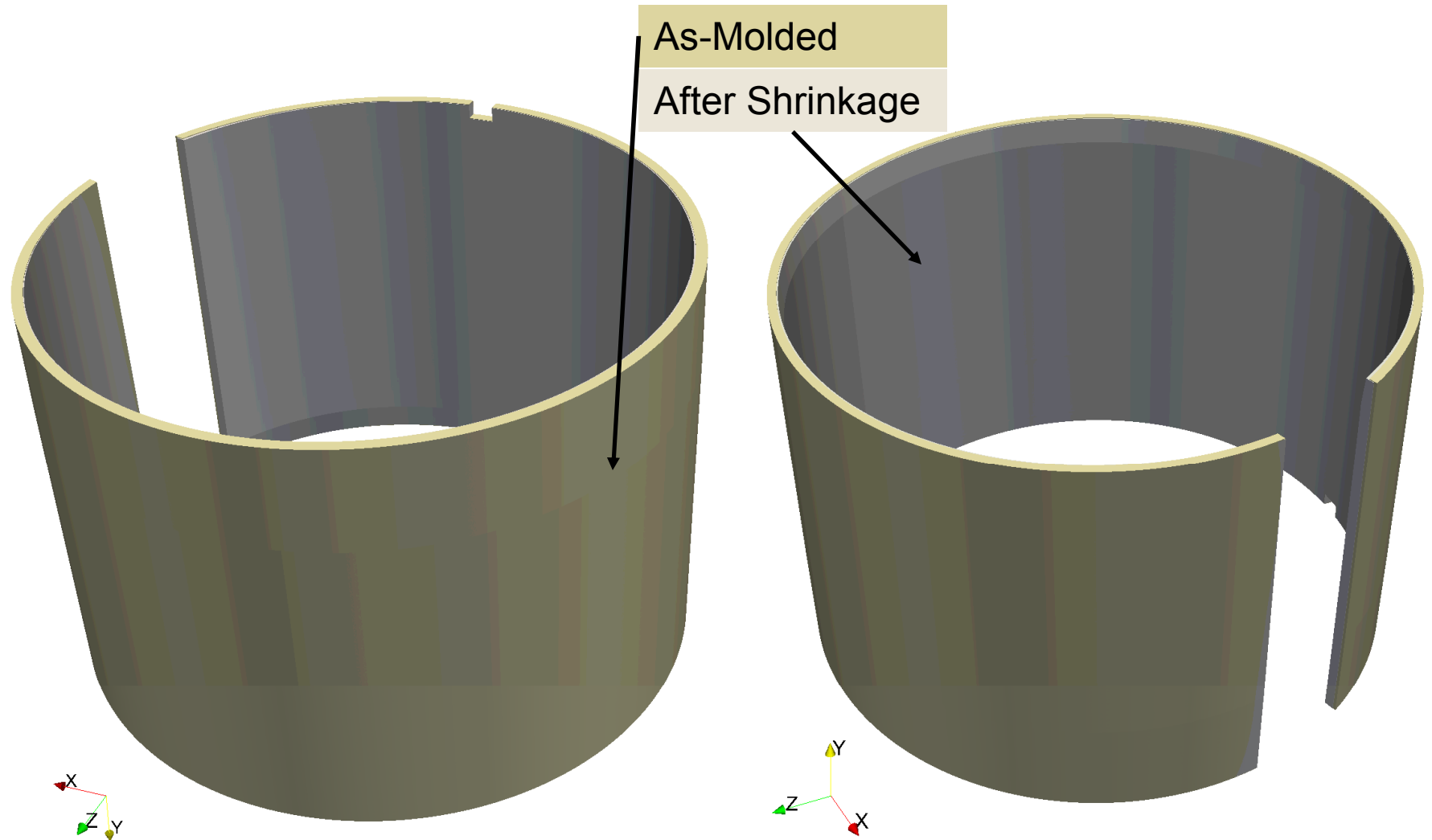
Notch Remains Largely Unwarped



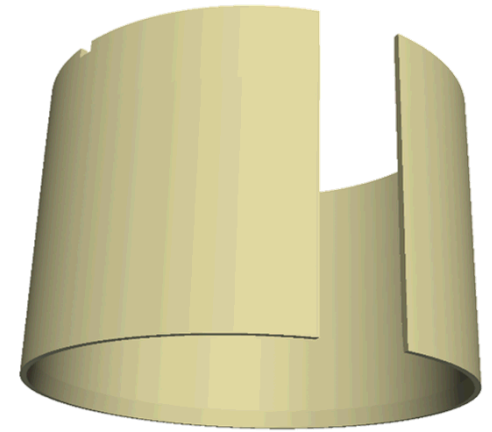
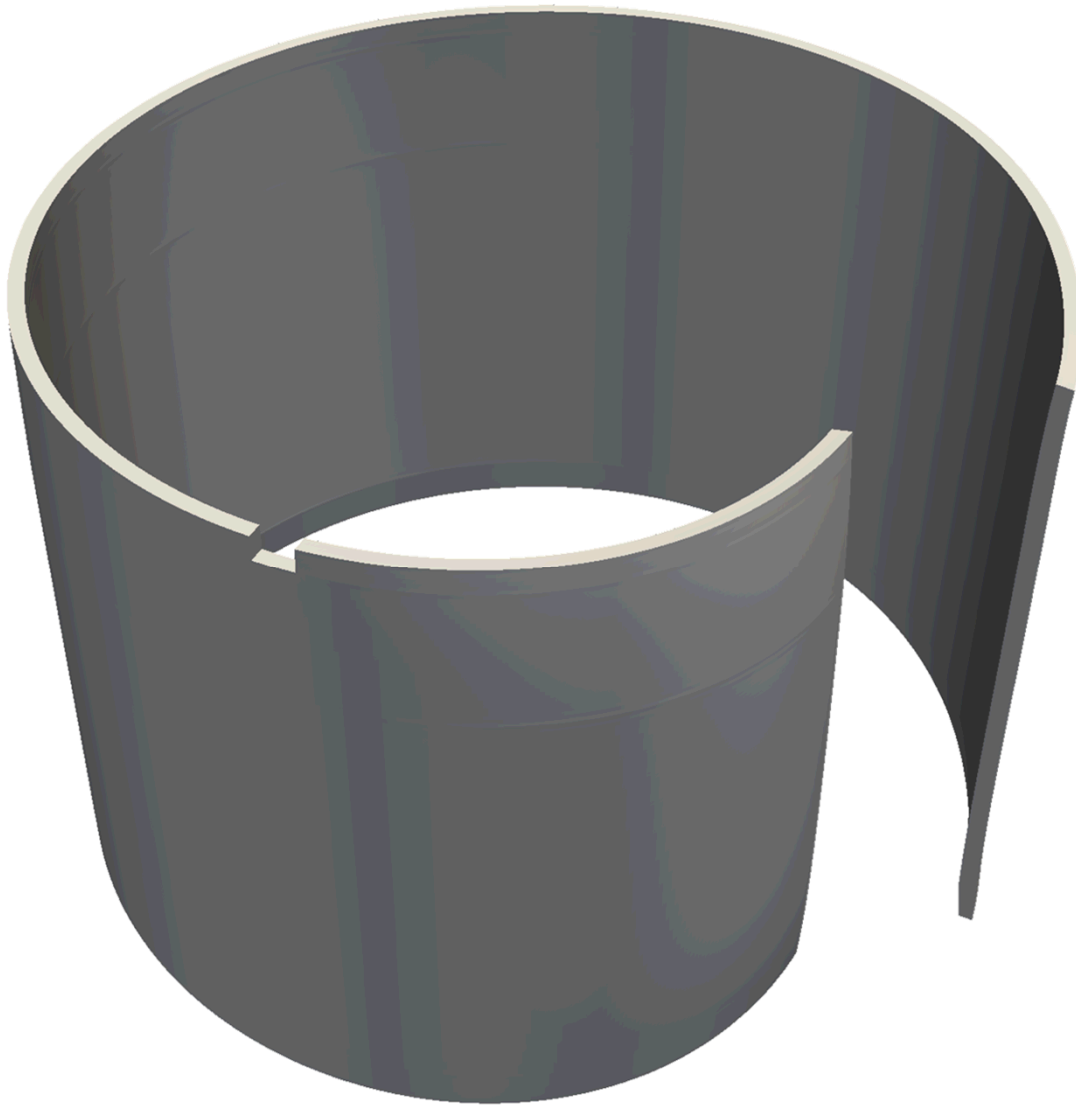
# Top and Bottom Shrink Differently



Different Top/Bottom Warpage Suggests The Component Will Not Simply Shrink in Volume Uniformly



As-Molded Exterior vs. After-Shrinkage Interior



This design is particularly susceptible to bending given the integrated error around the specimen circumference

Displacement  
scaled 20x to  
emphasize shape  
change.

# Key Findings and Conclusions

Current **inverse mold design** simulations were **successfully developed and deployed** on a number of support components

- Demolding hot results in more warpage in the present study compared with cooling further into the glass
- Density variation strongly influences shape change in slender regions and should be reduced
- Warpage predictions were in reasonably good agreement with KCNSC CMM measurements experiments (not shown here)

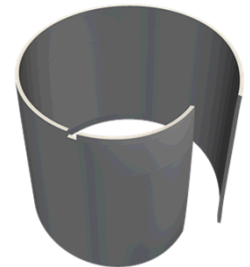
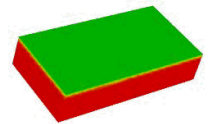
Kevin Long, [knlong@sandia.gov](mailto:knlong@sandia.gov)

# QUESTIONS?

# Improving Component Manufacturing Yields and In-Service Reliability:

Time = 5.0050

1. Filling support to reliably fill complex molds
  - Reduce defects and density variation
2. Post-manufacturing support to improve product acceptance yields
  - Change manufacturing conditions AND inverse mold design to reduce out-of-tolerance component warpage
3. Long-term assessment of dimensional stability of foamed components to support surveillance efforts



## Objective

Simulate the manufacturing process, develop the residual stress state, and predict component warpage over time (aging)



# Non-Linear Curing Viscoelastic Solid Modeling

## Balance Laws and Solution Fields:

- Mass + Momentum (*Displacements*) ← Lagrangian FEM
- Species Balance (*Chemical Reaction Extent*) ← ALE FEM
- Energy (*Temperature*) ← ALE FEM

## Solid State Non-Linear Viscoelastic (NLVE) Model Initial Conditions

- Initialize *temperature, foam density, and reaction extent* from simulation stage 1
- Directly initialize the stress-free reaction and temperature (expansion free)
- Assume the NLVE viscous stresses are initially zero

## Stress prediction based on the universal curing model developed at SNL

DB Adolf and RS Chambers, "A thermodynamically consistent, nonlinear viscoelastic approach for modelling thermosets during cure," *J. Rheology*, 2007.

Cauchy Stress: SNL Non-linear Viscoelastic Curing Model (Adolf & Chambers 2007)

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}[\log \underline{\underline{U}}, T, x, \text{histories}]$$

Logarithmic Strain

Temperature

Extent of matrix cure

### Material and Laboratory Time Relation

$$dt^* = \frac{dt}{a[t]}$$

$$\log a = -\hat{C}_1 \left( \frac{N}{\hat{C}_2 + N} \right)$$

### Density Scaling

$$\psi[\rho_0] = \left( \frac{\rho_0}{\rho_{ref0}} \right)^p \psi[\rho_{ref0}] \quad \text{Free Energy}$$

$$\underline{\underline{\sigma}}[\rho_0] = \left( \frac{\rho_0}{\rho_{ref0}} \right)^p \underline{\underline{\sigma}}[\rho_{ref0}] \quad \text{Cauchy Stress}$$

# Curing NLVE Model Continued

Relaxation behavior and mechanical properties depend on the *temperature, extent of cure, and histories of deformation*

## Material Time Dependencies

Thermal

Pressure

$$\begin{aligned}
 N = & \left\{ \left[ T(t) - T_{ref} \right] - \int_0^t ds \, f_1(t^* - s^*) \frac{dT}{ds}(s) \right\} + C_3 \left\{ I_1(t)_{ref} - \int_0^t ds \, f_1(t^* - s^*) \frac{dI_1}{ds}(s) \right\} \\
 & + C_4 \left\{ \int_0^t \int_0^t ds \, du \, f(t^* - s^*, t^* - u^*) \frac{d\varepsilon_{dev}(s)}{ds} : \frac{d\varepsilon_{dev}(u)}{du} \right\} + C_5(x(t)) \left\{ \left[ x(t) - x_{ref} \right] - \int_0^t ds \, f_1(t^* - s^*) \frac{dx}{ds}(s) \right\}
 \end{aligned}$$

Shear Deformation
Matrix Cure

## Glass Transition Evolution

$$T_{ref}(x) = T_{ref} - \frac{[C_3\beta_\infty + C_5(x(t))](x(t) - x_{ref})}{(1 + C_3\alpha_\infty)}$$

$$C_5(x(t)) \equiv C_{5a} + C_{5b} x$$

## Shear Modulus

$$G_g(T) = G_{gref} + \frac{\partial G_g}{\partial T}(T - T_{ref}) + \frac{\partial G_g}{\partial x}(x - x_{ref})$$

$$G_\infty(T) = \left\{ G_{ref} + \frac{\partial G_\infty}{\partial T}(T - T_{ref}) \right\} \left[ \frac{x^m - x_g^m}{x_{ref}^m - x_g^m} \right]^n$$

# Simple Macroscale Depressurization Model

Macroscale Volumetric Kinematic Split:  
Mechanical, Thermal, Depressurization

$$\mathbf{F}^{\text{vol}} = \lambda^v \mathbf{1} = \mathbf{F}_M^{\text{vol}} \mathbf{F}_\theta \mathbf{F}_d = \lambda_M^v \lambda_\theta \lambda_d \mathbf{1}$$

Total Rate of Volume Deformation

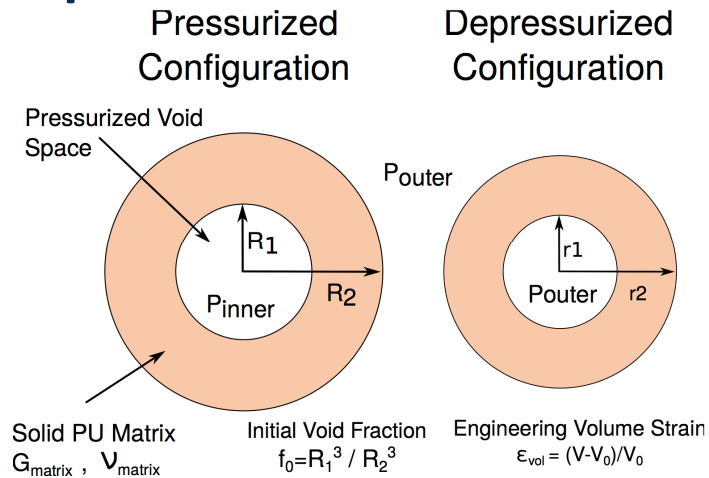
$$\mathbf{D}^{\text{vol}} = \frac{\dot{\lambda}^v}{\lambda^v} \mathbf{1} = \left( \frac{\dot{\lambda}_M^v}{\lambda_M^v} + \frac{\dot{\lambda}_\theta}{\lambda_\theta} + \frac{\dot{\lambda}_d}{\lambda_d} \right) \mathbf{1} = \mathbf{D}_M^{\text{vol}} + \mathbf{D}_\theta + \mathbf{D}_d$$

Rate of Isotropic Thermal Deformation

$$\mathbf{D}^\theta = \alpha_L \dot{T} \mathbf{1}$$

Hypoelastic Constitutive Law on the  
Mechanical Component Component of the  
Deformation Gradient

$$\dot{\sigma} = \frac{K \text{tr} \dot{\epsilon}_M}{3} \mathbf{1} + 2G \text{dev} \dot{\epsilon}_M$$



Outer Boundary Radial Displacement  
Normalized by its Reference Radius

$$\frac{U_2}{R_2} = \frac{(G(P_1 f_0 - P_2)/3 + K f_0 (P_1 - P_2)/4)}{(GK(1 - f_0))}$$

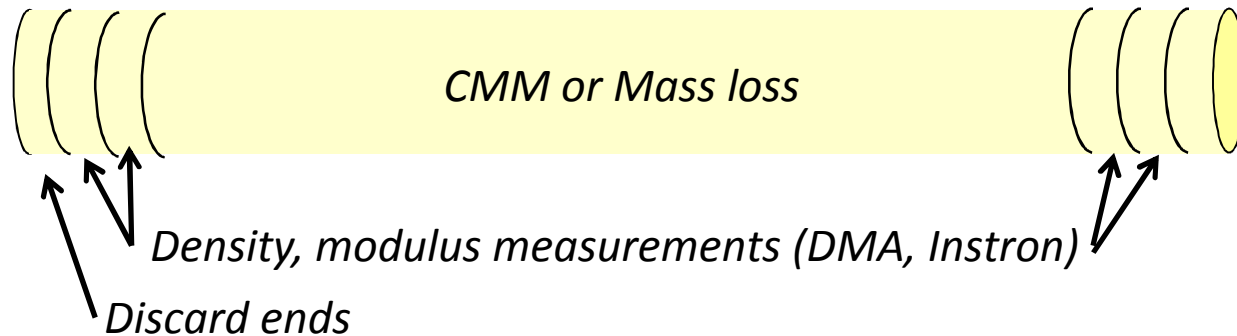
$$\frac{\dot{U}_2}{R_2} = \frac{f_0(4/3G + K)}{(4GK(1 - f_0))} \frac{d}{dt} (P_1 - P_2)$$

Rate of the Depressurization Volume  
Deformation at the Macroscale

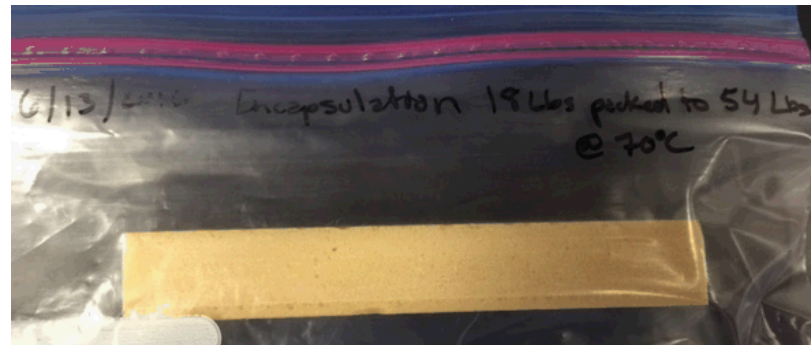
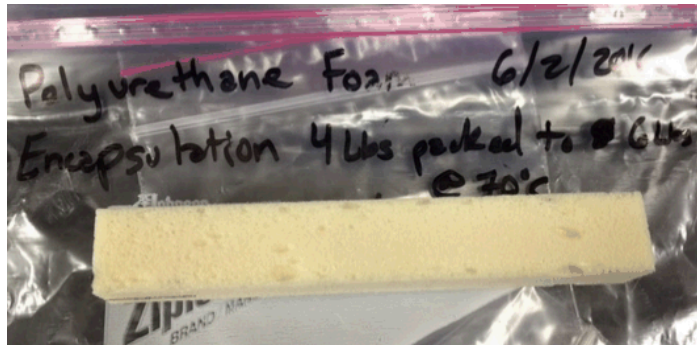
$$\mathbf{D}_d = \frac{\dot{U}_2}{R_2 + U_2} \mathbf{1} = \frac{\dot{U}_2/R_2}{1 + U_2/R_2} \mathbf{1}$$

See Sokolnikoff, Timoshenko for the elasticity solution

# Shrinkage and Mass Loss in Foam Cylinders



- $\frac{3}{4}$ "X8" and 1"X8" cylinders (with skin) and machined to square cross-sections (without skins)
- Density from 6 to 54 pcf at different over packings



# Shrinkage and Mass Loss in Foam Cylinder

## Manufacturing Process

- PMDI foam injected at 40 °C, overpacked to various densities
- After 15 mins, cured in oven at 120 °C for 4 hrs

## Dimensional Change via a Xyce Coordinate Measurement Machine (CMM)

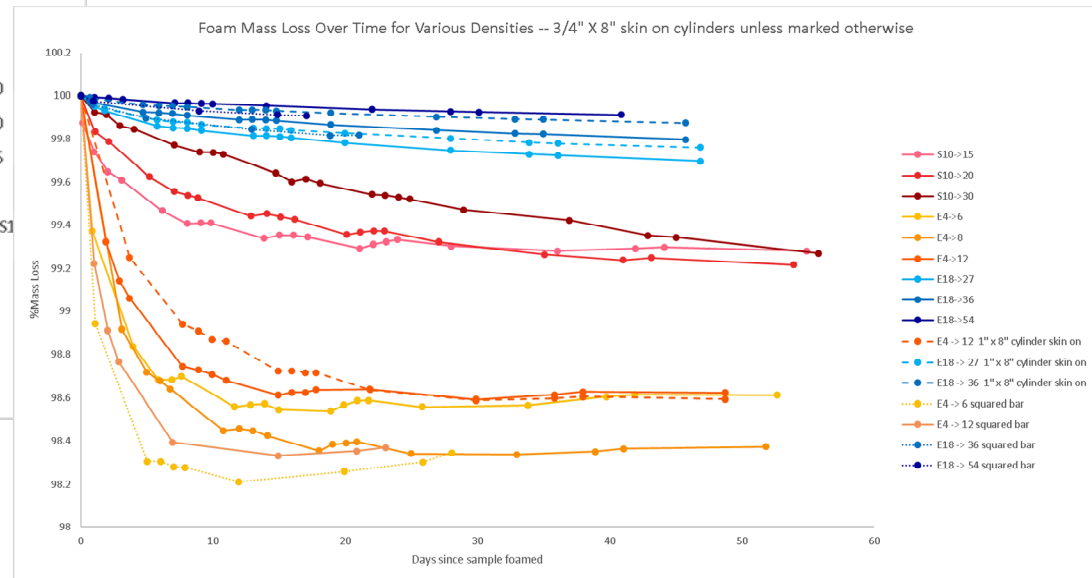
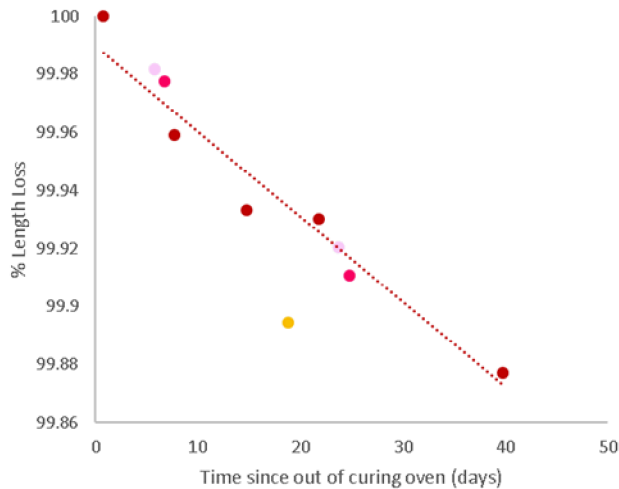
## Mass Changes Measured on a Milligram Scale at Room Temperature

Linear shrinkage seems less sensitive to density than mass loss in our limited data to date



**CMM measures  
dimensional changes**

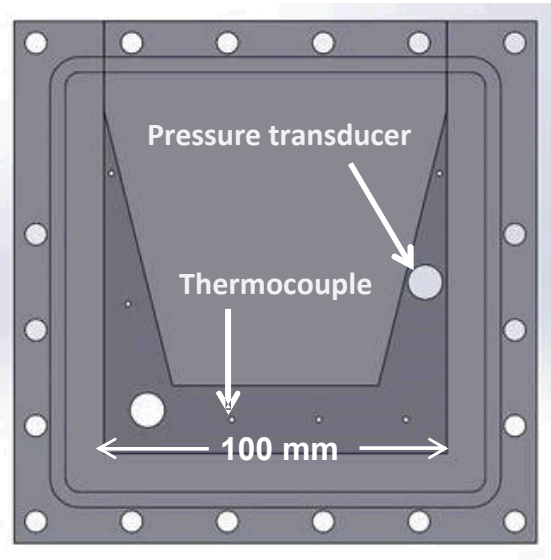
Summary S10 and E4 (w/o 6/29 data)



# Shrinkage in a Relevant Geometry

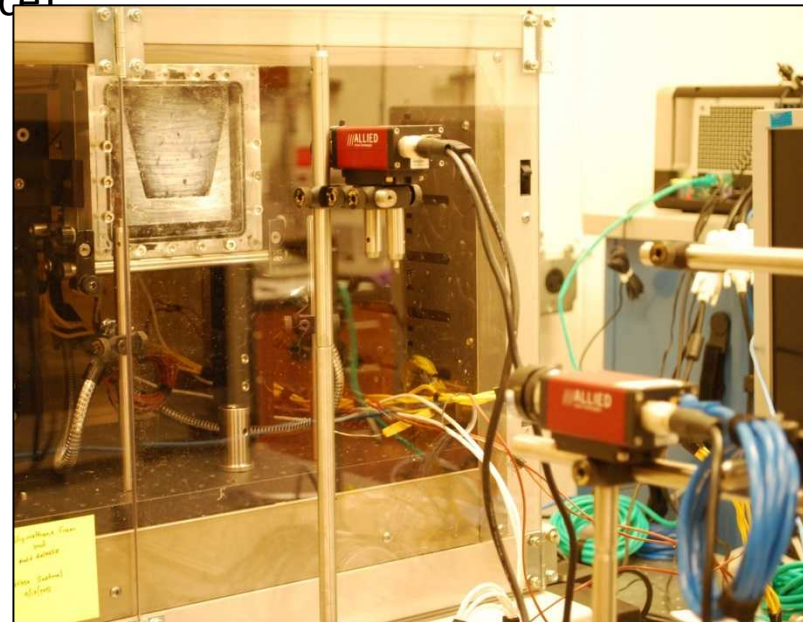
**Goal:** Quantify warpage over months to provide model validation data and physical insight

- Geometry involves both thin and bulky regions
- Initially, filling conditions approximate in-house cure schedule
  - **PMDI S10** foam injected at **40 °C**, overpacked to **12.5 lb/ft<sup>3</sup>**
  - After 15 mins, cured in oven at **121 °C for 4 hrs**
  - Two separate filling orientations “C” and “U”
- Coordinate Measurement Machine (CMM by Xzyce)



Ports for thermocouples and pressure transducers to record parameters during foaming.

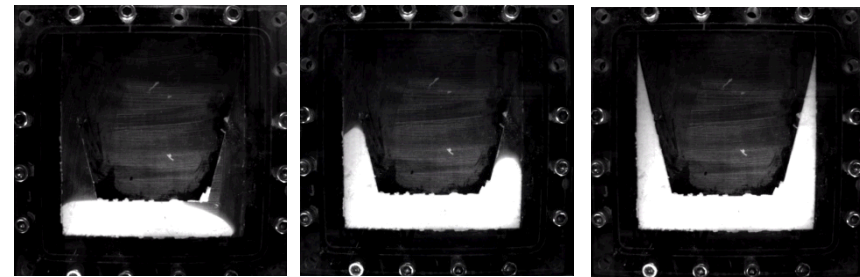
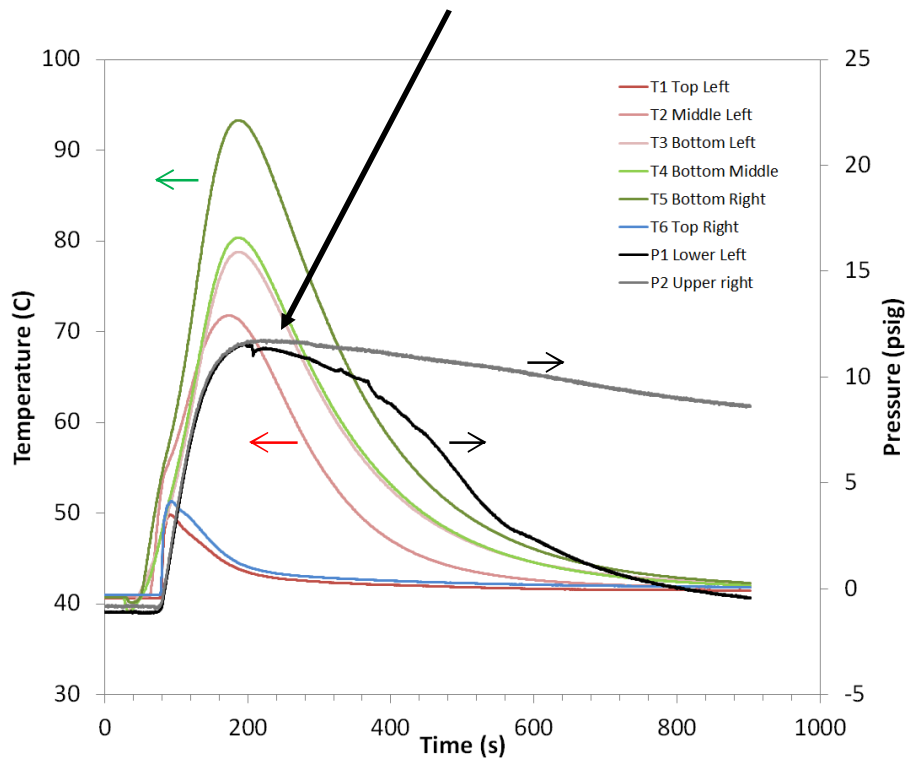
Fill filmed using cameras, transparent oven door



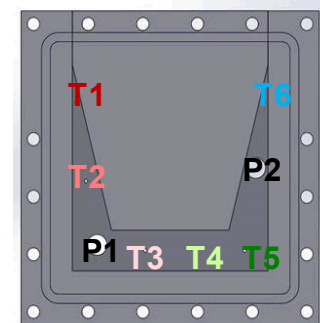
# Foaming U-shaped staple mold

- Over many repeats, temperature, pressure, and flow profile are remarkably repeatable
- Imperfectly symmetric fill common
- Pressure rises as foam expands, relaxes at lower corner and stays positive at P2

Gauge Pressure Estimate of ~12 psig



Some slight asymmetry due to  
bias of initial injection



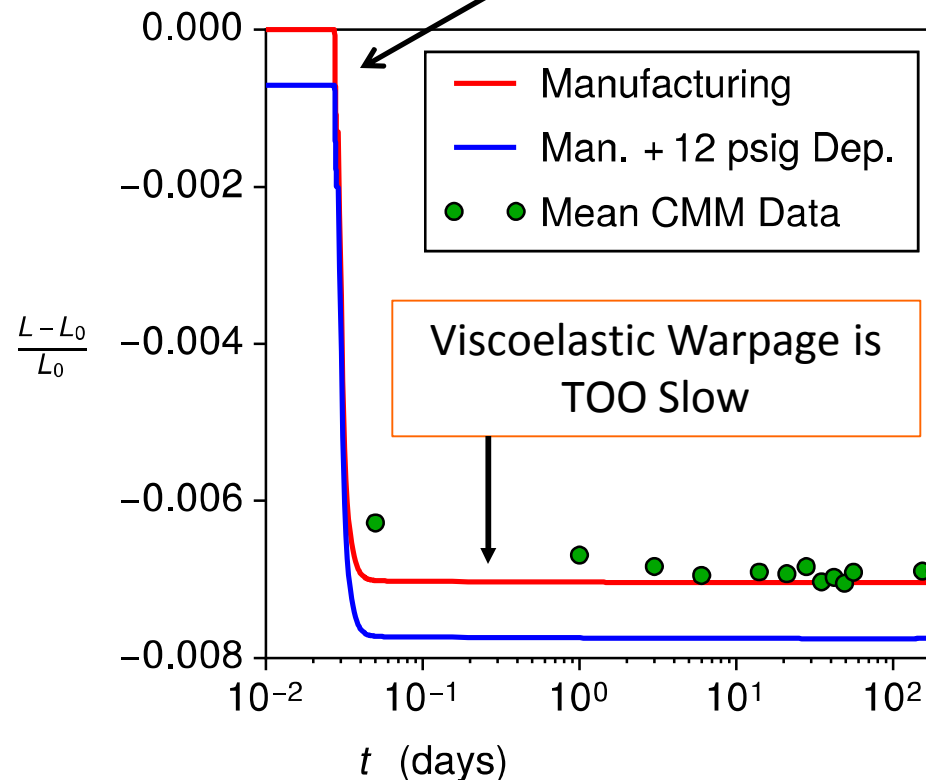


# Our First Hypothesis:

## Viscoelastic Relaxation of Residual Stress

Manufacturing → Cure Shrinkage +  
Boundary Conditions → Residual  
Stress Generation

Elastic Unload +  
Thermal-Contraction



Model captures the  
elastic unload and cool-  
down reasonably well,  
but it misses the long  
term aging response

The Viscoelastic Time  
Scale is TOO Long (by a  
few orders of magnitude)

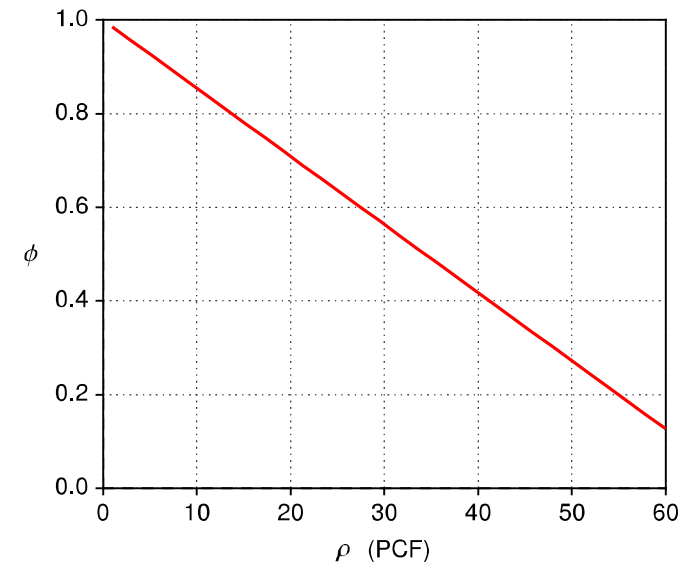
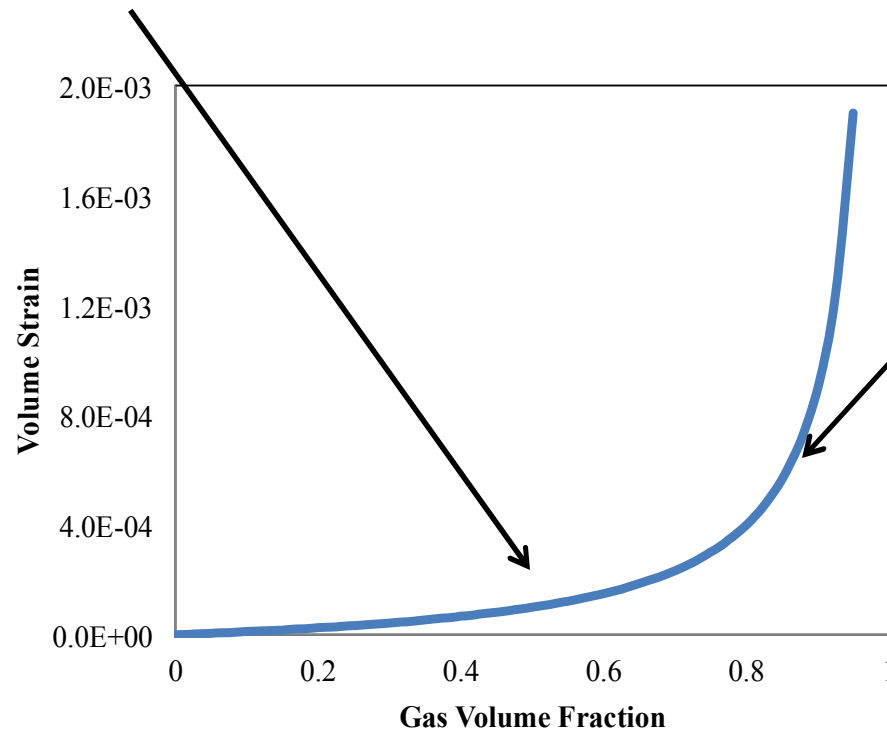


# What about Bubble Depressurization?

Originally, we focussed on higher density foams: 40, 50, 55 PCF

→ Depressurization was insignificant

Below 20 PCF, it matters!

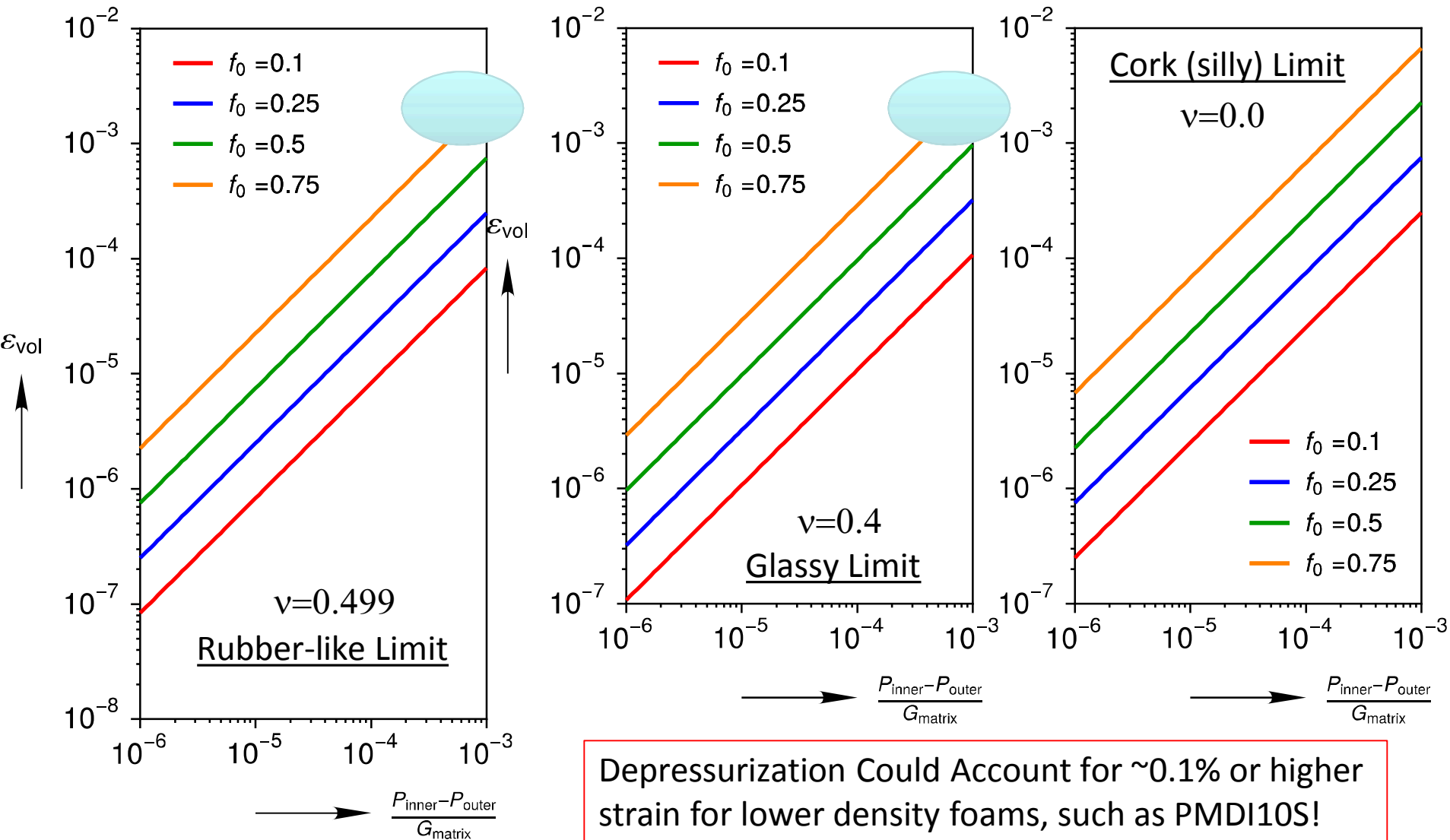


Bubble depressurization matters especially for smaller density (assuming a fixed gauge pressure...)

# Consider Simple (Analytic) Depressurization Strains

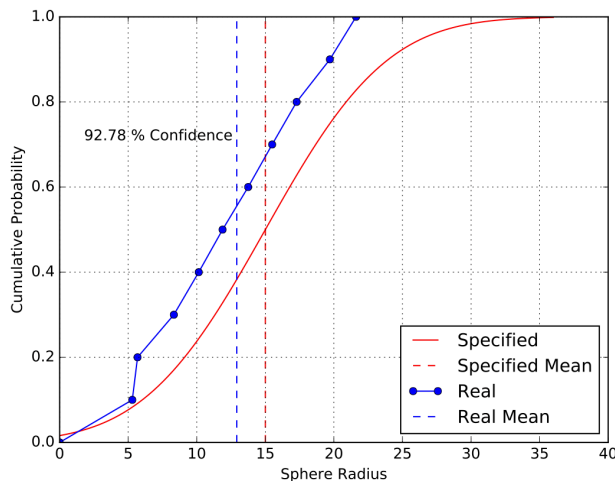
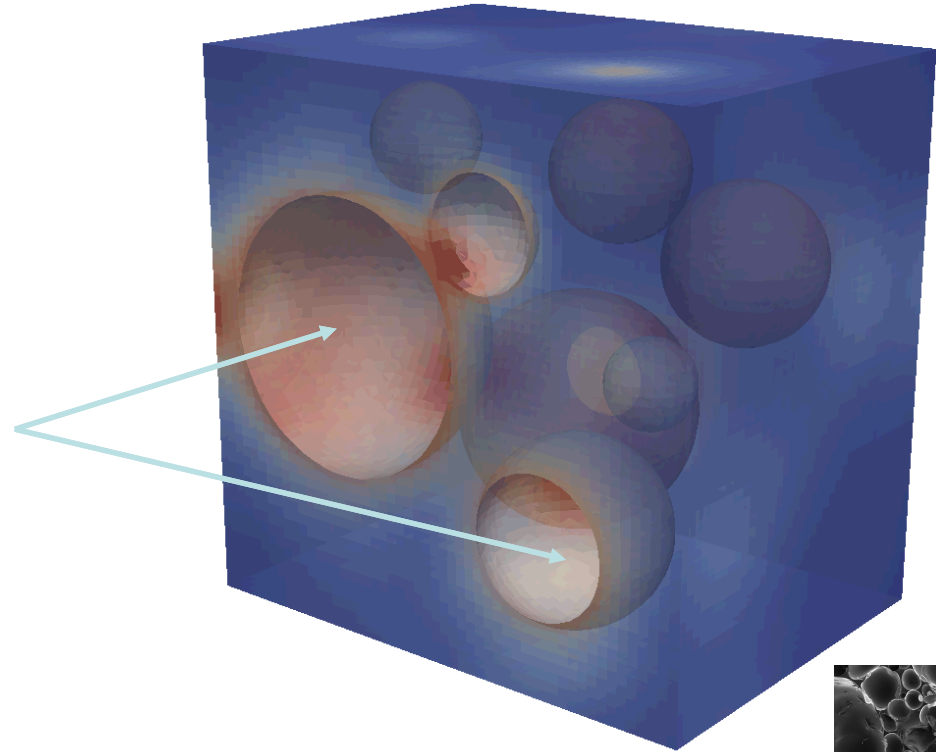
- Imagine:
  - The foam is of uniform density (no voids, no skin)
  - The foam is initially in a state of uniform gauge pressure
  
- Ignoring the dynamics associated with  $\text{CO}_2$  leaving the component, we can estimate the (isotropic) shrinkage strain a foamed component would experience as a function of:
  - Gauge pressure normalized by the matrix shear modulus (glassy shear modulus of the foam)
  - Poisson Ratio of the Matrix Phase
  - Porosity (Void Volume Fraction)

# Consider Simple (Analytic) Depressurization Strains



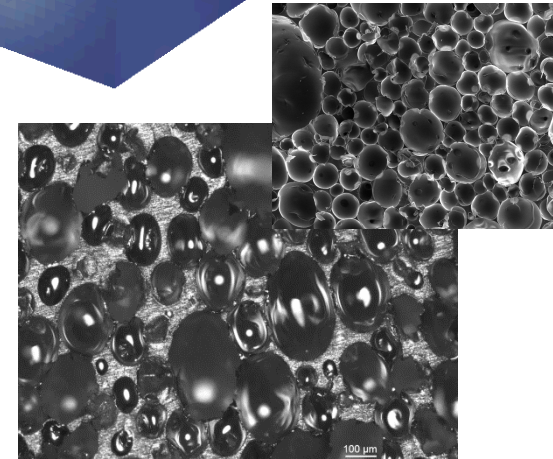
# Micromechanics Validation of the Analytic Model

- Prescribed porosity
- Depressurization from 20 psig applied to pores

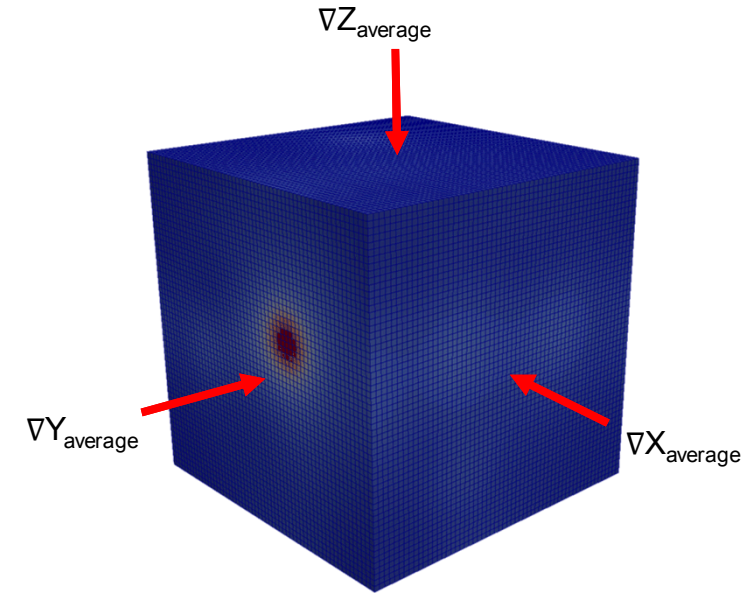
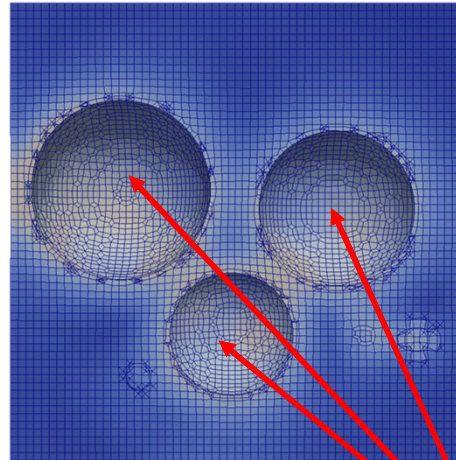
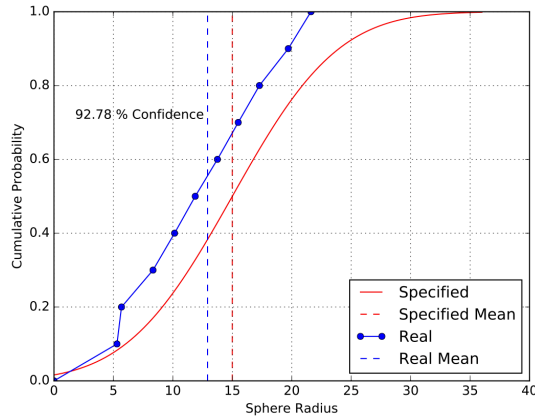


Different size distributions

- Gaussian, Uniform



# Micromechanics Validation of the Analytic Model



Pore geometries generated for multiple porosities (Gaussian distribution)



Porous cubes created and meshed

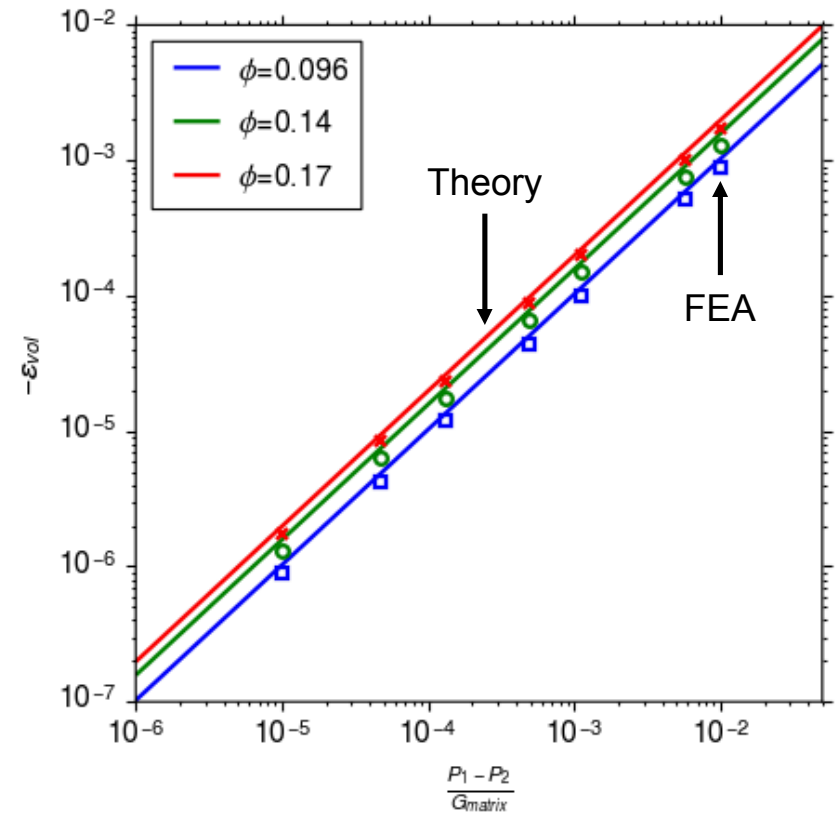
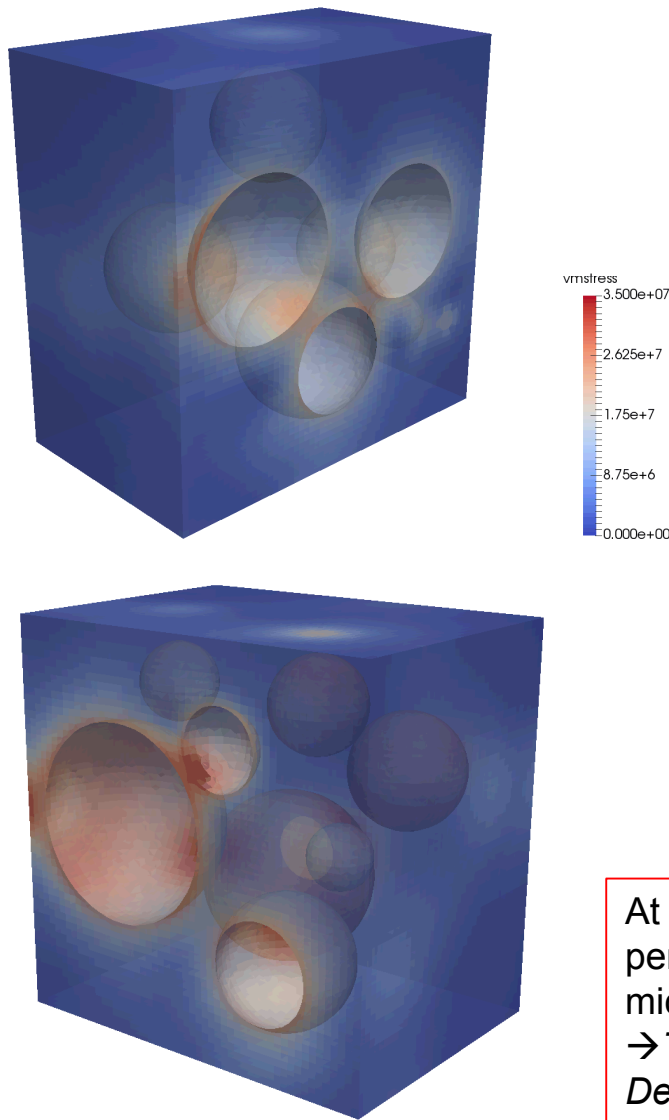


Decreasing gauge pressure applied to pore surfaces



Volume strains calculated

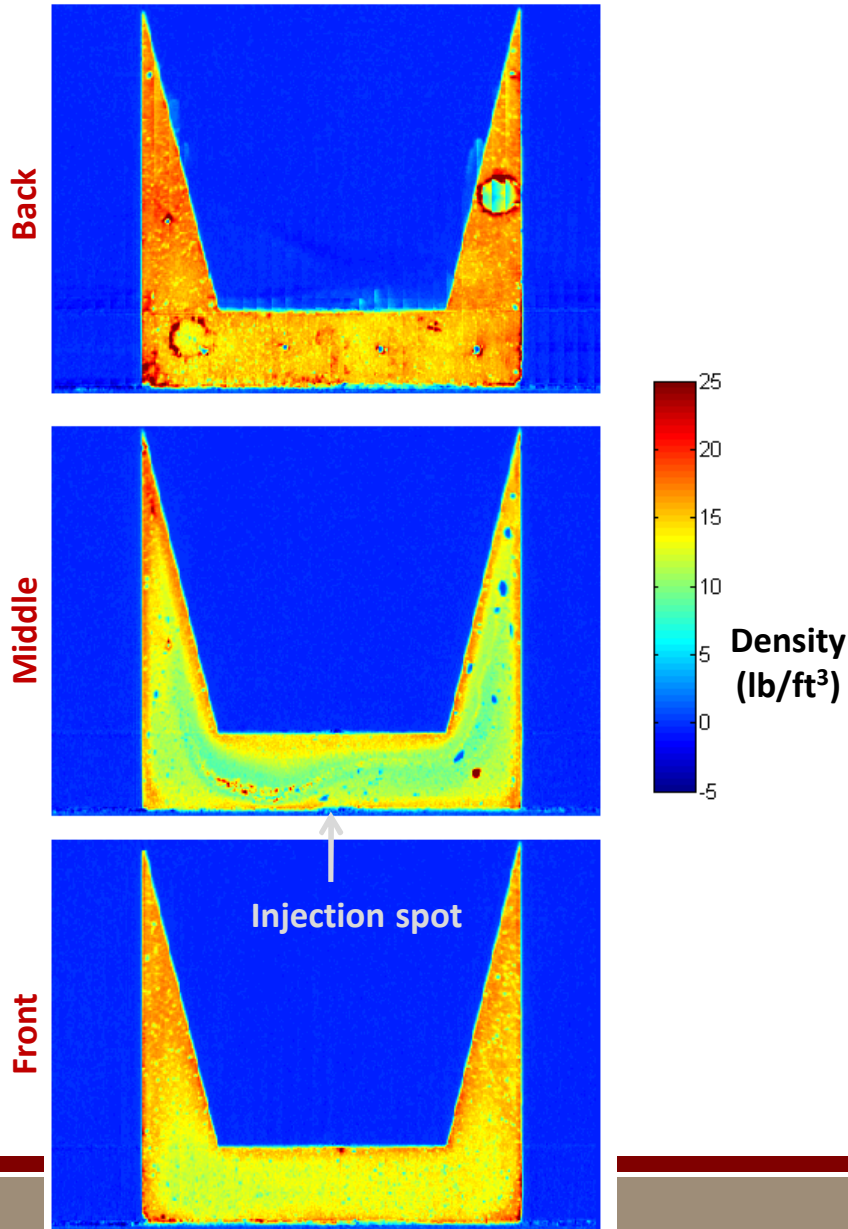
# Micromechanics Validation of the Analytic Model



At low porosities, where the analytic model is less likely to perform well, we have good agreement between the micromechanics FEA and Analytic Shell Model  
→ *The Analytic Model Reasonable Well Represents Deformation Due to Depressurization for Isotropic Foams*

# Density is important! Let's measure it with X-ray CT

U-Shaped staple



A skin is apparent ( $25 \text{ lb/ft}^3$ ), whereas the interior density is as low as  $7 \text{ lb/ft}^3$

Large voids-- primarily in the arms of the staple.

Coalescence seen in other thin PMDI samples

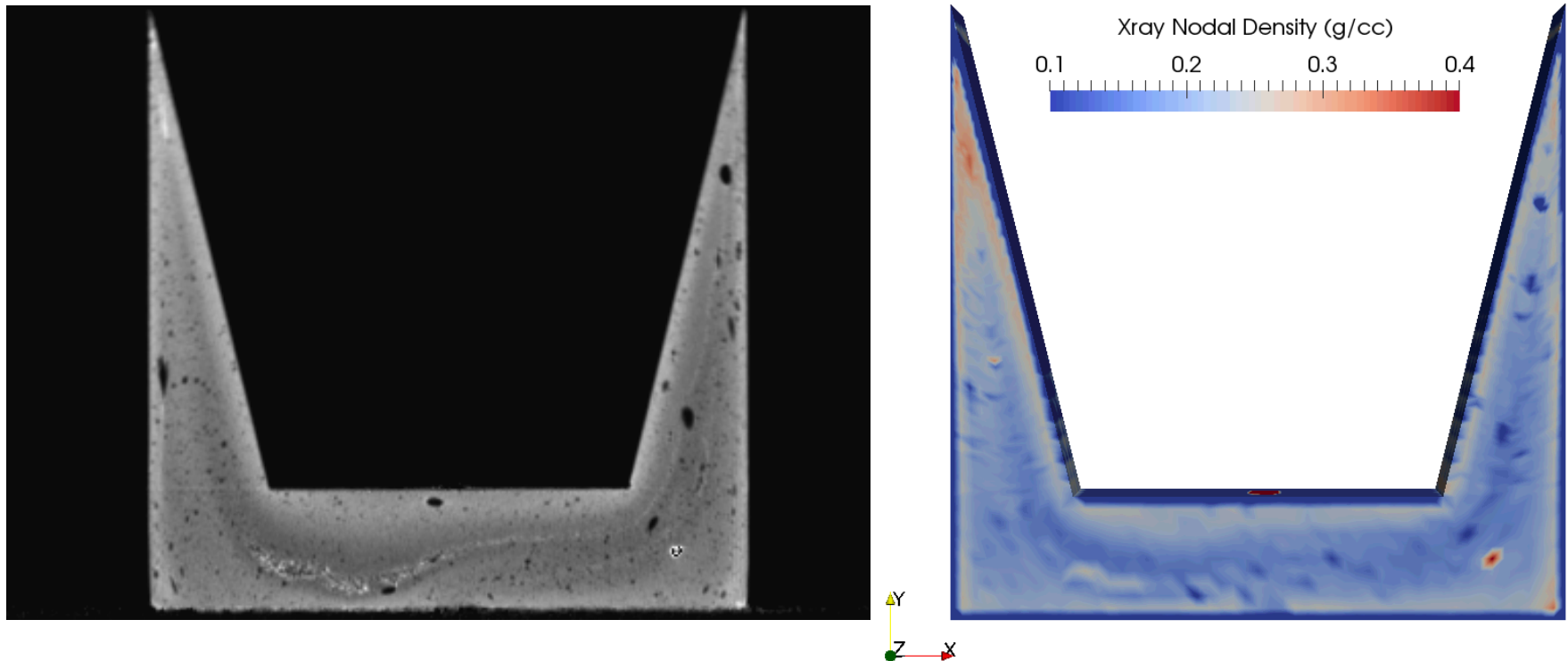
Larger numbers of voids in C-shaped staple (more shear overall)

Large bubbles could be source of pressure decay not predicted by model

Focus on bottom portion for shrinkage measurements

# Initial Calculations of the U-staple

- Non-Uniform Density from X-ray CT
- Uniform Depressurization (12 PSIG)



Mid-plane Cross-section showing the raw data (X-ray CT) and the Interpolated Nodal Data (Field Input to the Simulation)



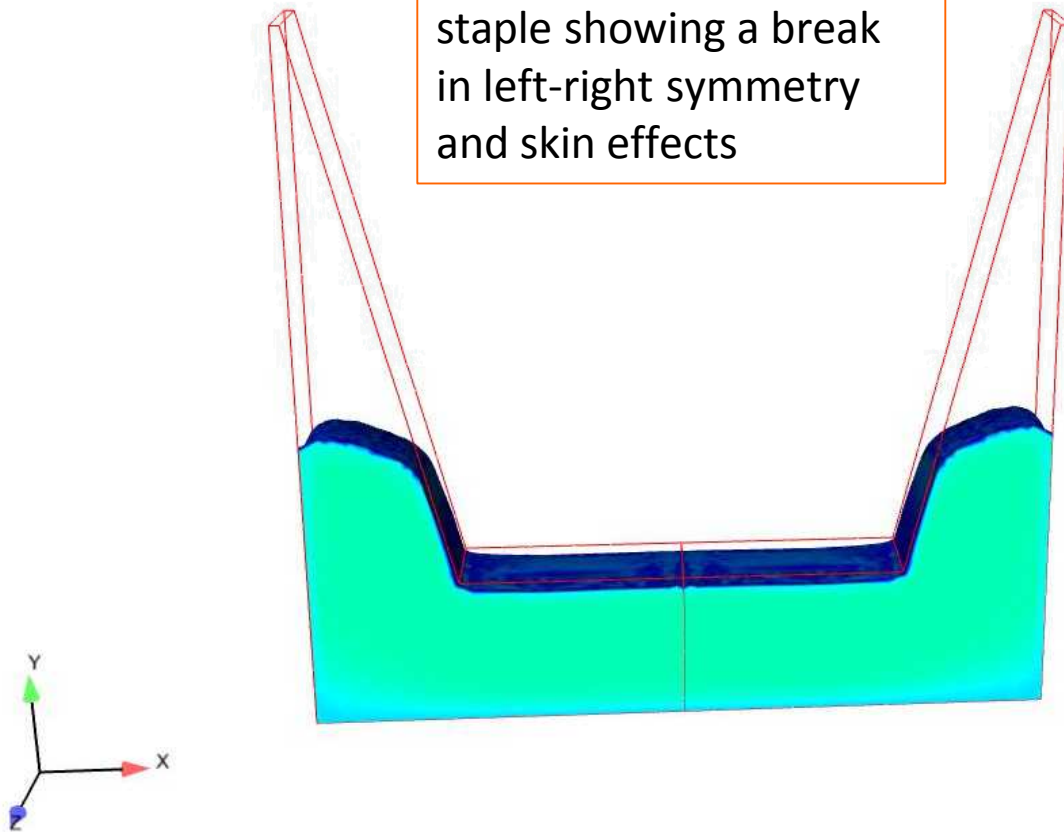
# Conclusions

- Viscoelasticity and Residual Stress relaxation properly account for the majority of the warpage due to manufacturing
  - The Time Scale for Continued warpage is too slow from this mechanism compared with our experimental data
- Bubble depressurization results in warpage over the right time scale (according to the literature), but the magnitude of warpage is a strong function of the bubble gauge pressure
  - How do we measure this?
  - Are the moduli of the PU matrix phase a function of the CO<sub>2</sub> concentration?
- Is the CMM machine the most robust method for monitoring warpage in soft materials?

# Future Efforts: Mold Filling Simulations That Better Predict Density and Gauge Pressure

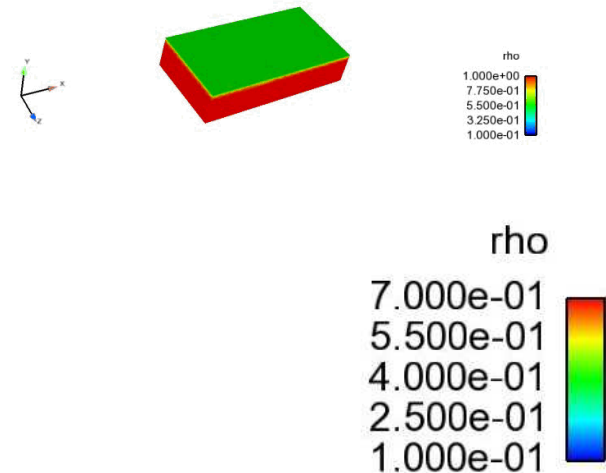
Time = 46.56

Example of filling the U-staple showing a break in left-right symmetry and skin effects

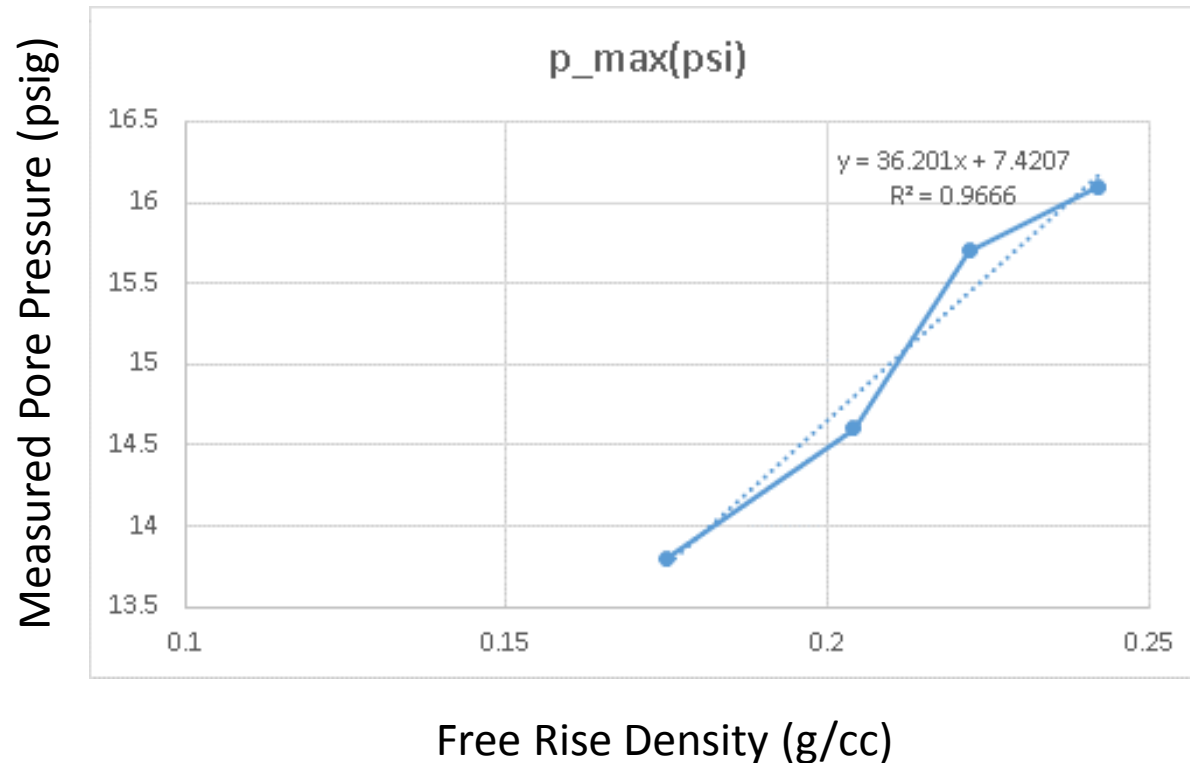


Quarter Symmetry Filling Example

Time = 5.0050



# Determine How Gauge Pressure Depends on the Free Rise Density and Overpacking



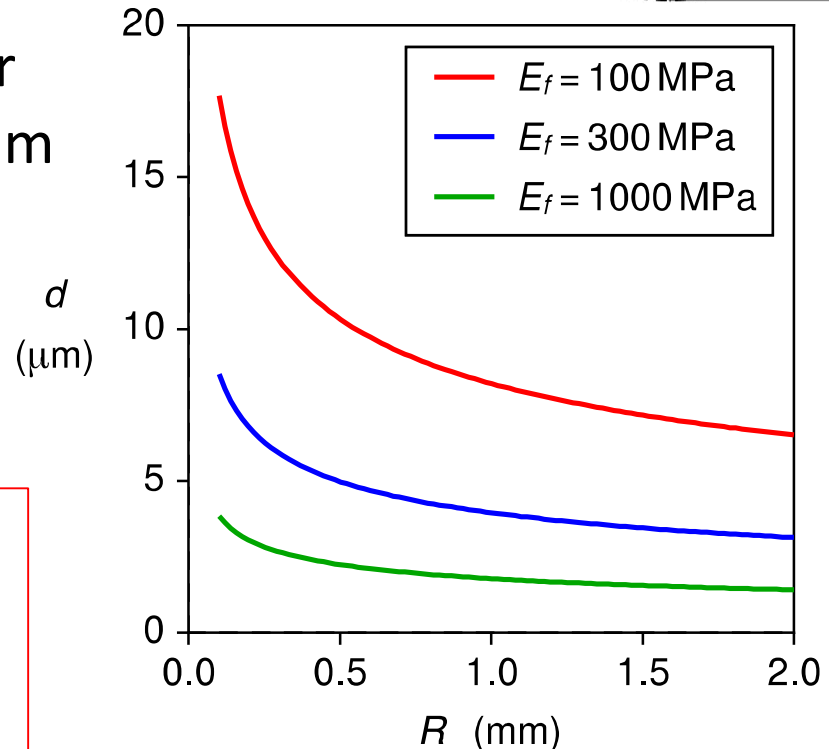
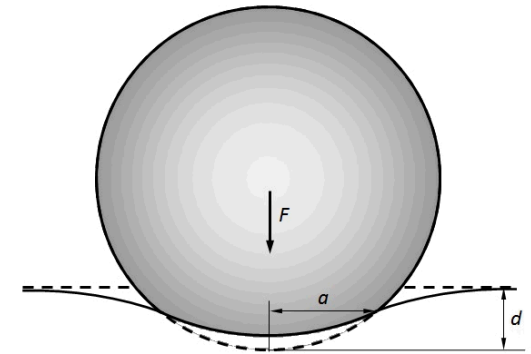
# How Much Displacement Does the CMM Probe Cause During Contact?

Hertz Contact Solution<sup>1</sup>

- 100 mN Force Probe Force
- ASSUME the metallic sphere is much stiffer than the foam
- Foam Young's Modulus of Approximately 300 MPa for glassy 10 PCF PMDI10S Foam

$$d = \left( \frac{9F^2}{16E_*^2 R} \right)^{1/3}$$
$$\frac{1}{E_*} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \approx \frac{1 - \nu_2^2}{E_2}$$

Depending on the foam density (Young's Modulus), the apparent "strain" for the Staple 10 cm span due to probe indentation ranges from 4E-5 to 1.6E-4...These Are HUGE

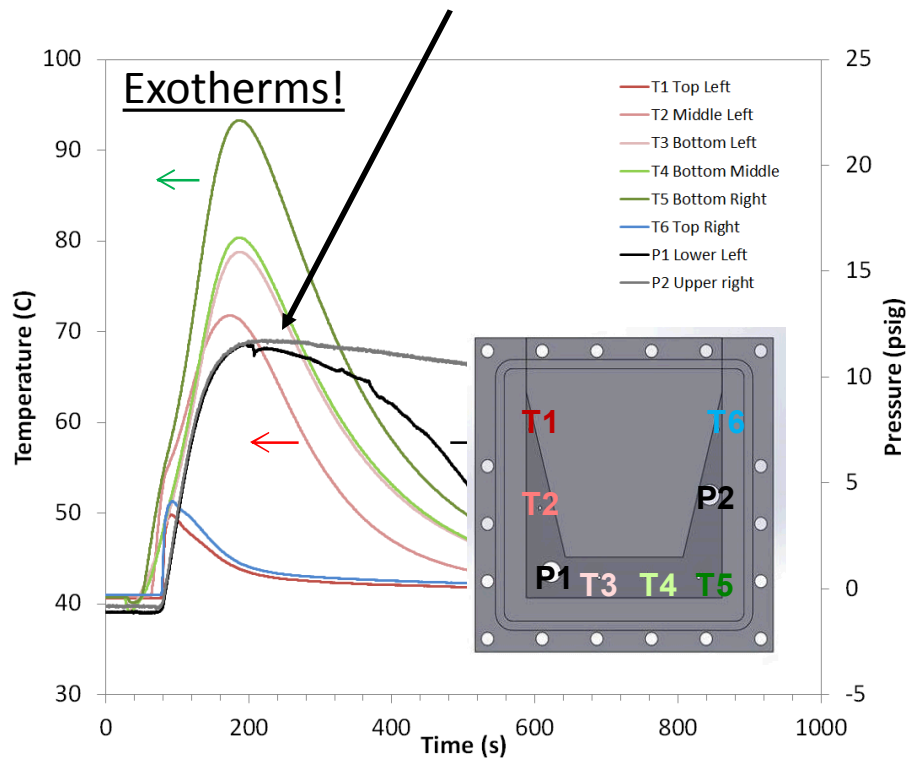


<sup>1</sup>Wikipedia: Contact Mechanics

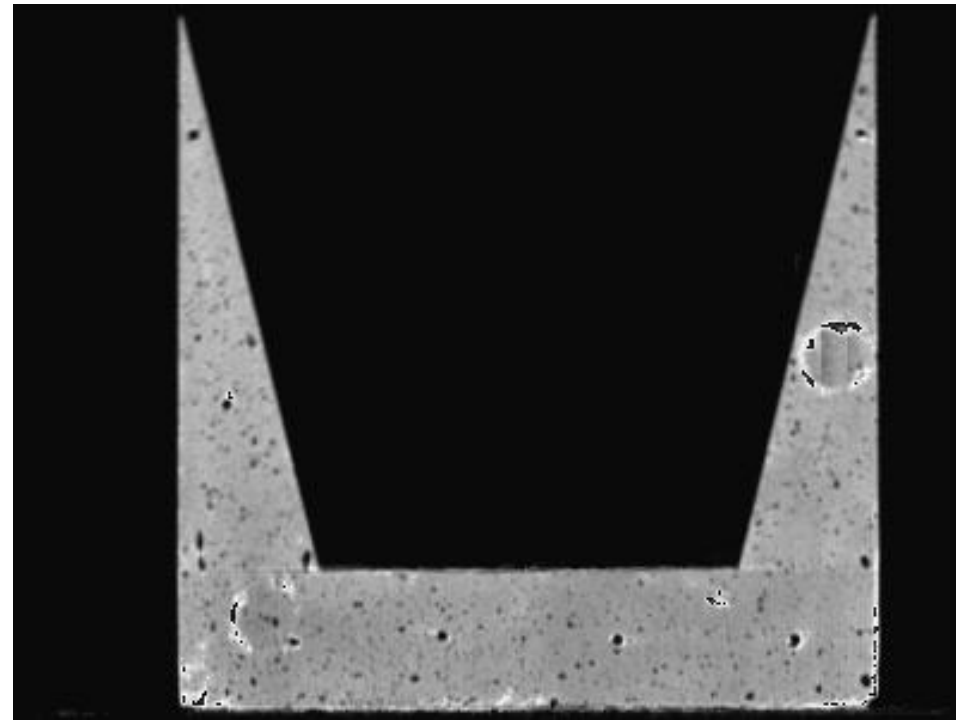
# Foaming U-shaped staple mold

- Over many repeats, temperature, pressure, and flow profile are remarkably repeatable
- Imperfectly symmetric fill common
- Pressure rises as foam expands, relaxes at lower corner and stays positive at P2

Gauge Pressure Estimate of ~12 psig



**Significant Density Variation Exists from Near Free-Rise Conditions AND Injection Process**



# Chemical and Physical Processes from Manufacture to Component Aging

