SAND2017-13087C

# Stochastic Partial Differential Equation Solver for Hydroacoustic Modeling: Improvements to Paracousti sound propagation solver

Leiph Preston (lpresto@sandia.gov), Geophysics Dept., Sandia National Laboratories, Albuquerque, NM


## Abstract

Marine hydrokinetic (MHK) devices offer a clean, renewable alternative energy source for the future. Responsible utilization of MHK devices, however, requires that the effects of acoustic noise produced by these devices on marine life and marine-related human activities be well understood. Paracousti is a 3-D full waveform acoustic modeling suite that can accurately propagate MHK noise signals in the complex bathymetry found in the near-shore to open ocean environment and considers real properties of the seabed, water column, and air-surface interface. However, this is a deterministic simulation that assumes the environment and source are exactly known. In reality, environmental and source characteristics are often only known in a statistical sense. Thus, to fully characterize the expected noise levels within the marine environment, this uncertainty in environmental and source factors should be incorporated into the acoustic simulations. One method is to use Monte Carlo (MC) techniques where simulation results from a large number of deterministic solutions are aggregated to provide statistical properties of the output signal. However, MC methods can be computationally prohibitive since they can require tens of thousands or more simulations to build up an accurate representation of those statistical properties. An alternative method, using the technique of stochastic partial differential equations (SPDE), allows computation of the statistical properties of output signals at a small fraction of the computational cost of MC. We are developing a SPDE solver for the 3-D acoustic wave propagation problem called Paracousti-UQ to help regulators and operators assess the statistical properties of environmental noise produced by MHK devices. In this presentation, we present the SPDE method and compare statistical distributions of simulated acoustic signals in simple models to MC simulations to show the accuracy and efficiency of the SPDE method.


## Method

We start with the linear, coupled, first-order set of partial differential equations for velocity and pressure in an acoustic media and expand both the dependent and independent variables in terms of chaos polynomial bases, which are functions of random variables only, e.g.

$$p(\mathbf{x},t) = \sum_{j=0}^{N} P_j(\mathbf{x},t)\Phi_j(\theta)$$

for pressure. We similarly expand the 3 components of velocity, the source terms, bulk modulus, and buoyancy giving coefficients

$$V_{x,i}, V_{y,i}, V_{z,i}; S_{x,i}, S_{y,i}, S_{z,i}, M_i; K_i; R_i$$

Plugging these expansions into the partial differential system and projecting onto the k$^{th}$ basis function yields

$$\frac{\partial V_{x,k}}{\partial t} = -\sum_{i=0}^{N}\sum_{j=0}^{N_r} e_{ijk} R_j \frac{\partial P_i}{\partial x} + \sum_{i=0}^{N_{S_x}}\sum_{j=0}^{N_r} e_{ijk} R_j S_{x,i}$$

$$\frac{\partial V_{y,k}}{\partial t} = -\sum_{i=0}^{N}\sum_{j=0}^{N_r} e_{ijk} R_j \frac{\partial P_i}{\partial y} + \sum_{i=0}^{N_{S_y}}\sum_{j=0}^{N_r} e_{ijk} R_j S_{y,i}$$

$$\frac{\partial V_{z,k}}{\partial t} = -\sum_{i=0}^{N}\sum_{j=0}^{N_r} e_{ijk} R_j \frac{\partial P_i}{\partial z} + \sum_{i=0}^{N_{S_z}}\sum_{j=0}^{N_r} e_{ijk} R_j S_{z,i}$$

$$\frac{\partial P_k}{\partial t} = -\sum_{i=0}^{N}\sum_{j=0}^{N_k} e_{ijk} K_j \left[\frac{\partial V_{x,i}}{\partial x} + \frac{\partial V_{y,i}}{\partial y} + \frac{\partial V_{z,i}}{\partial z}\right] + \frac{\partial M_k}{\partial t}$$

where

$$e_{ijk} = \langle \Phi_i \Phi_j \Phi_k \rangle$$

is the Galerkin multiplication tensor.

This system of equations is N times larger than the original system, where N is the maximum polynomial order for the chaos expansion. Additionally, several sums appear that are not present in the original system. However, many of the Galerkin tensor terms are trivially zero, and with a proper choice of basis functions, the extra terms can be minimized.

## Choice of Basis Functions

| PDF | chaos polynomial | Support | weight function |
|---|---|---|---|
| Gaussian | Hermite | $(-\infty,+\infty)$ | $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ |
| Gamma | Laguerre | $[0,+\infty)$ | $\frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}$ |
| Uniform | Legendre | $[a,b]$ | $\frac{1}{b-a}$ |

The table lists some of the chaos polynomials and their respective weight functions. For example, when one has a Gaussian process, the optimal chaos polynomials are Hermite polynomials of the random variables. When one uses the optimal polynomial, a minimum number of terms are needed to expand a given term. For example, if one has a Gaussian process with a given mean and variance, only two terms (maximum first order Hermite polynomials) are needed to exactly express that term. Even with perfectly Gaussian independent input, the dependent variables may not be perfectly Gaussian, so more expansion terms are typically needed for dependent variables than independent variables.

## Implementation

The stochastic PDE system is discretized using a 4$^{th}$ order spatial and second order temporal finite-difference scheme on a standard staggered grid.

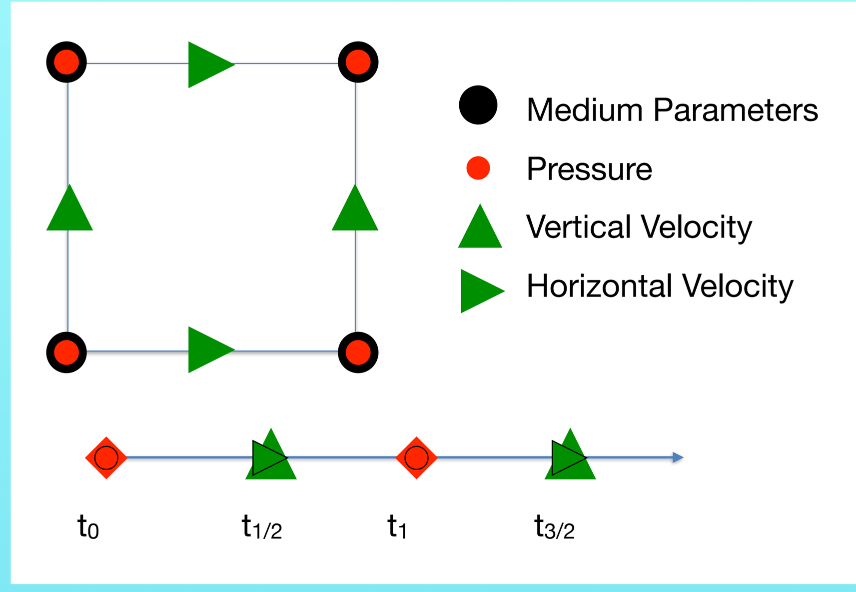


Figure 1: Unit cell (top) and time axis (bottom) for the staggered finite-difference scheme.

## Zeroth Order Validation

The stochastic PDE system should reduce to the deterministic solution using Paracousti when all independent variables only have a zeroth order expansion (deterministic). I verify this using an homogeneous whole space.

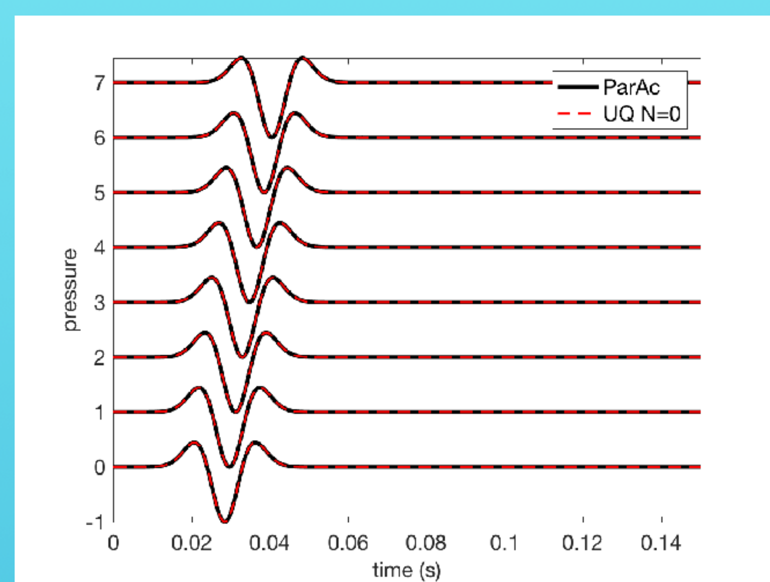


Figure 2: Comparison of pressure traces from deterministic Paracousti (black) and Paracousti-UQ (red dashed) with all maximum chaos polynomial orders set to zero.

## Monte Carlo Comparisons

Up to 800,000 Monte Carlo simulations were completed for each validation test of the algorithm. The two figures below show the distribution for bulk modulus from the Monte Carlo simulations and the convergence of the maximum of the peak in time of the standard deviation versus number of Monte Carlo simulations.

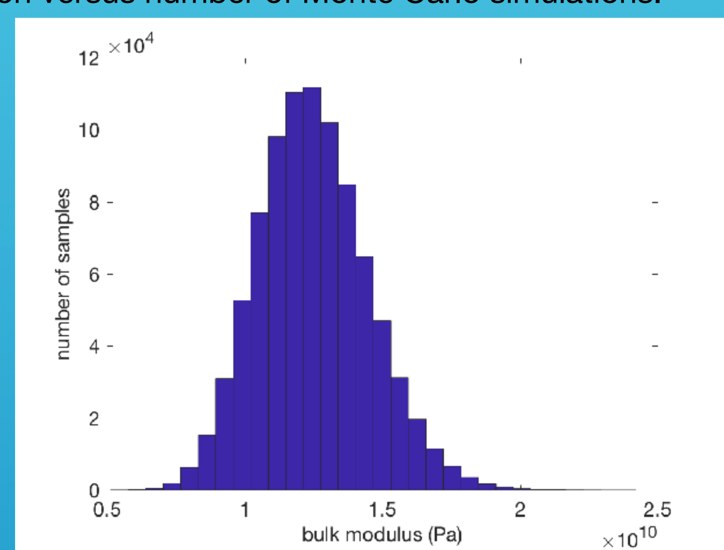


Figure 3: Distribution of bulk modulus used in Monte Carlo simulations.

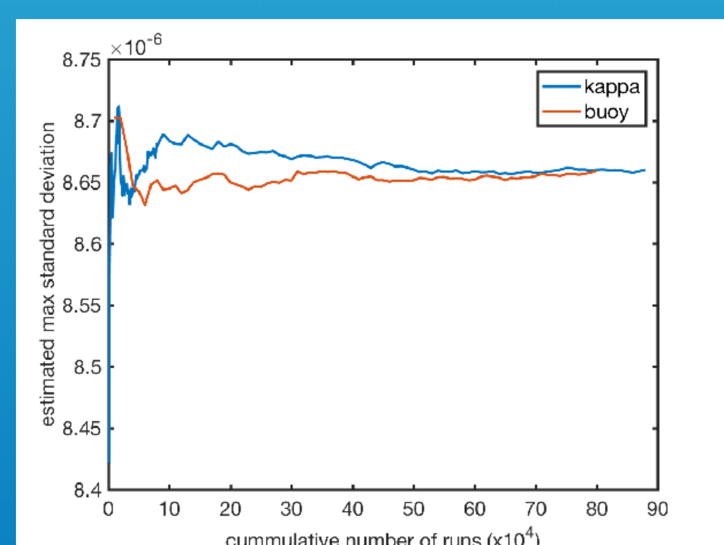


Figure 4: Estimated maximum of standard deviation in pressure at the nearest receiver as a function of cumulative number of Monte Carlo simulations for bulk modulus and buoyancy distributions.

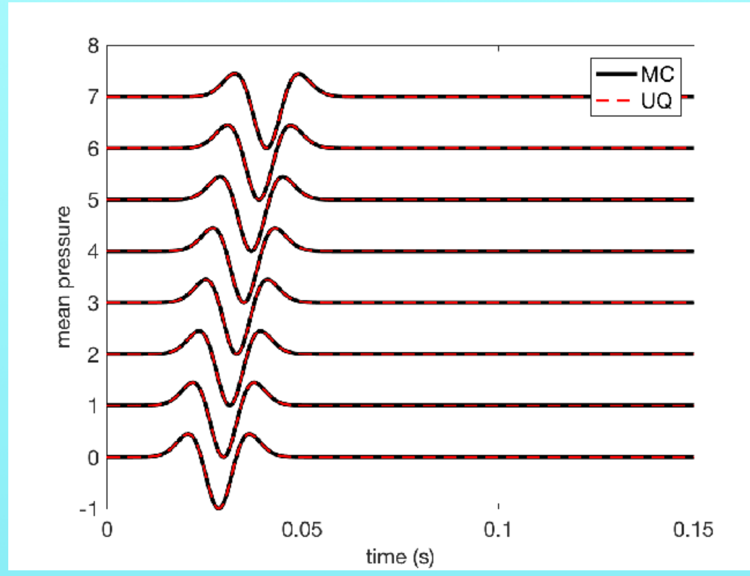


Figure 5: Comparison of mean pressure traces between MC and Paracousti-UQ for bulk modulus distribution.

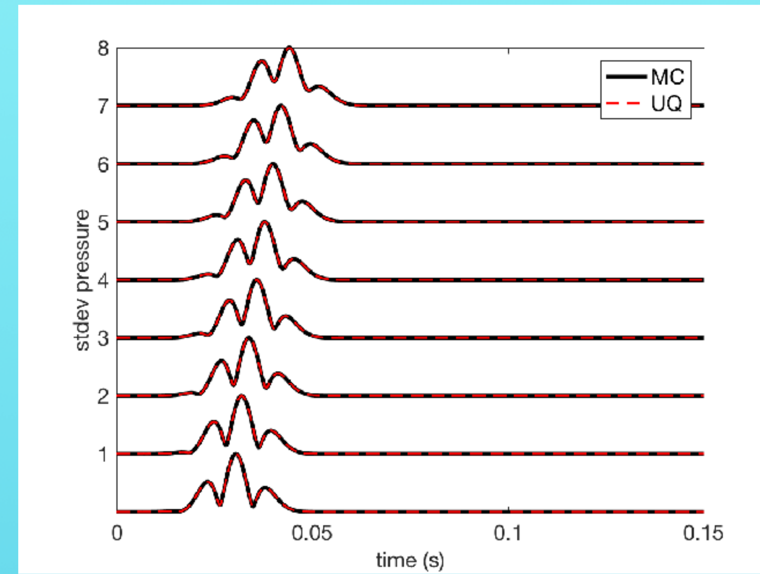


Figure 6: Comparison of standard deviation pressure traces between MC and Paracousti-UQ for bulk modulus distribution.

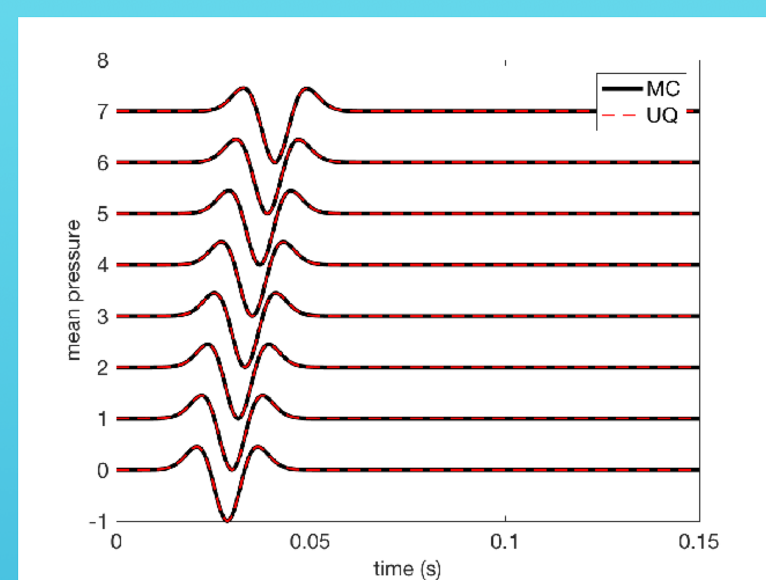


Figure 7: Comparison of mean pressure traces between MC and Paracousti-UQ for buoyancy distribution.

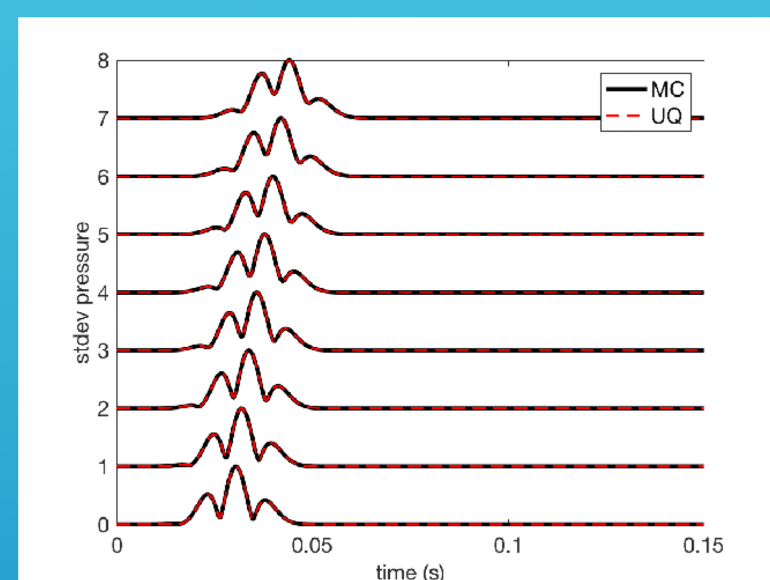


Figure 8: Comparison of standard deviation pressure traces between MC and Paracousti-UQ for buoyancy distribution.

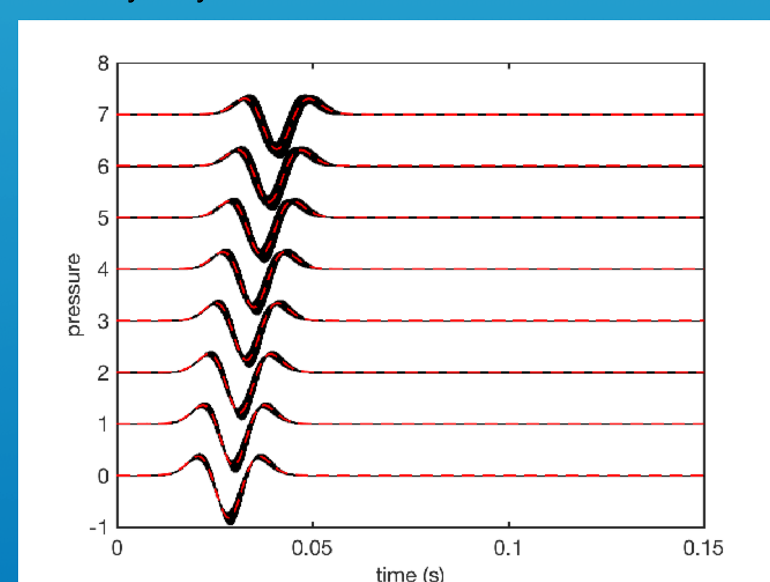


Figure 9: Mean (red dash) and one standard deviation (black) as a function of time for pressure traces for buoyancy distribution.

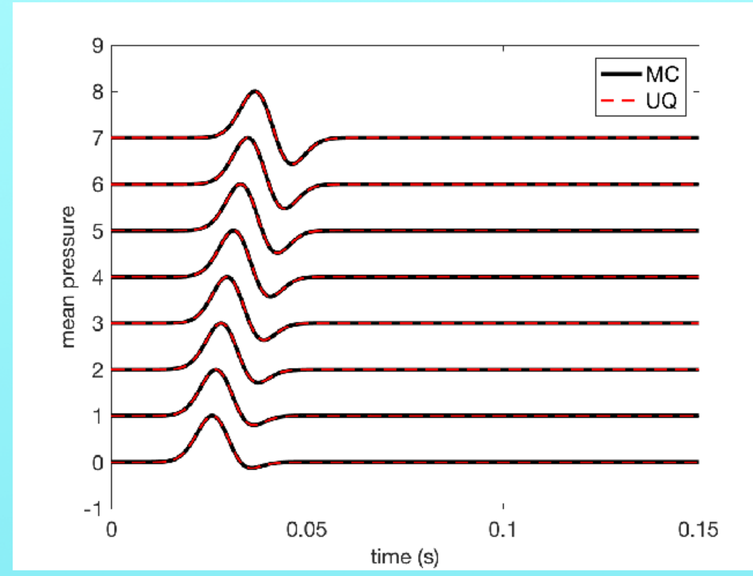


Figure 10: Comparison of the mean pressure traces between Monte Carlo and Paracousti-UQ simulations for a buoyancy distribution with a force source.

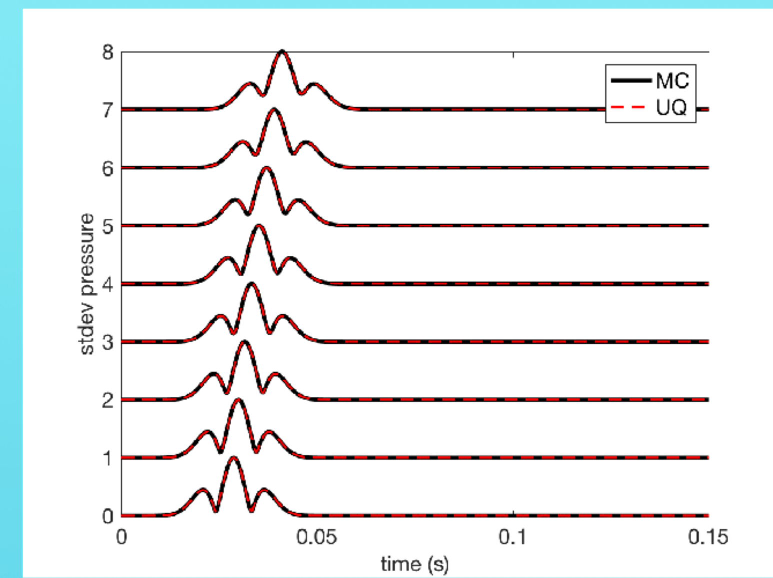


Figure 11: Comparison of the standard deviation of pressure traces between Monte Carlo and Paracousti-UQ simulations for a buoyancy distribution with a force source.

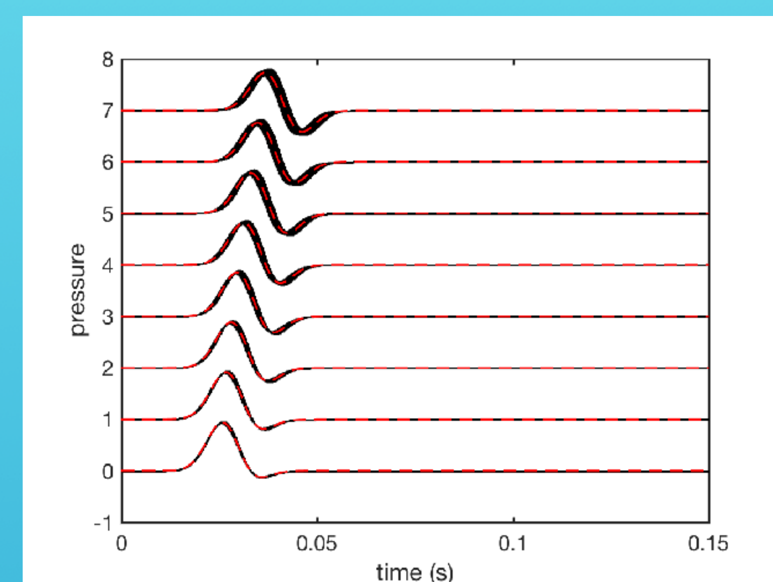


Figure 12: Mean (red dash) and one standard deviation region (black) for pressure traces from a force source.

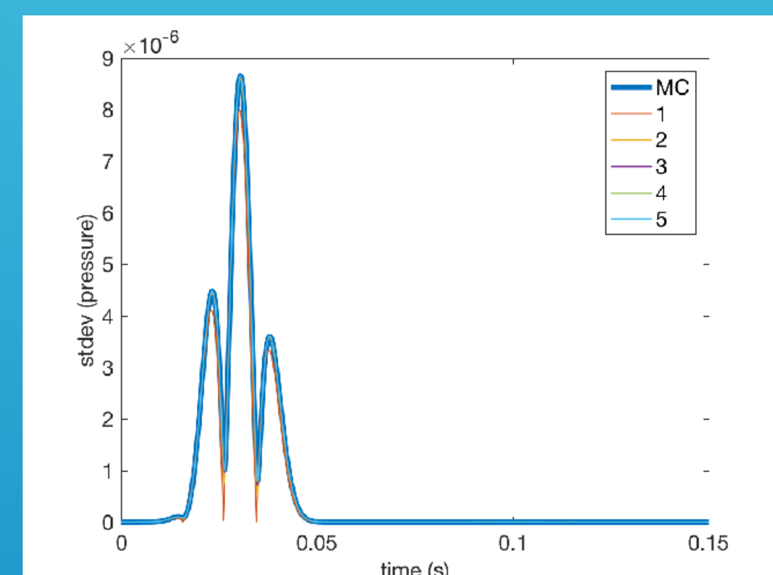


Figure 16: Convergence of Paracousti-UQ based on maximum polynomial order for standard deviation of pressure at the nearest receiver.

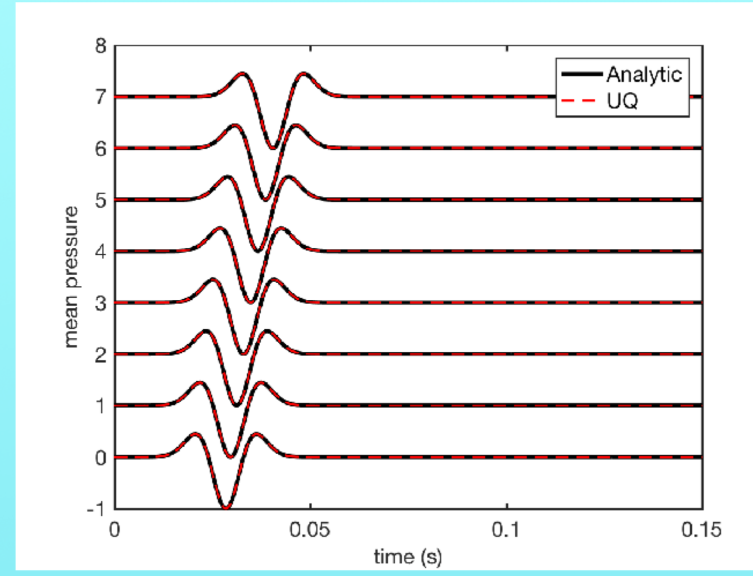


Figure 13: Comparison of the mean pressure traces between the analytic solution and Paracousti-UQ for the explosive source time function distribution.

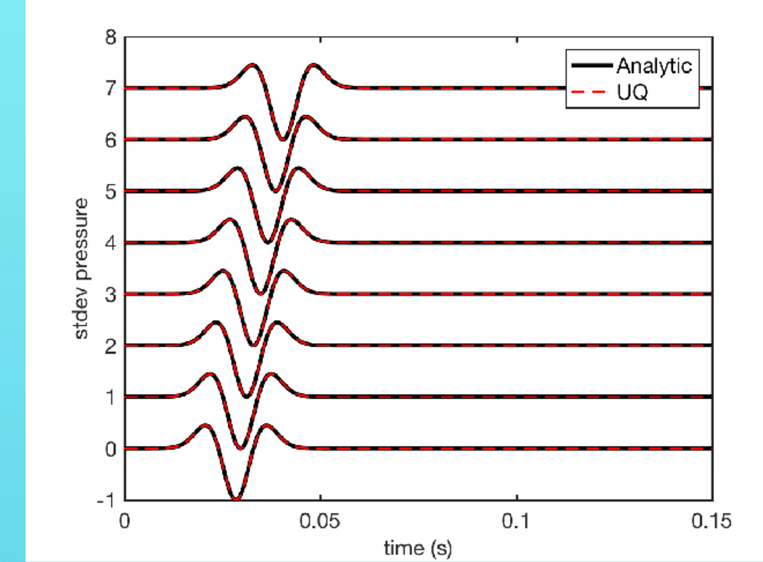


Figure 14: Comparison of the standard deviation pressure traces between the analytic solution and Paracousti-UQ for the explosive source time function distribution.

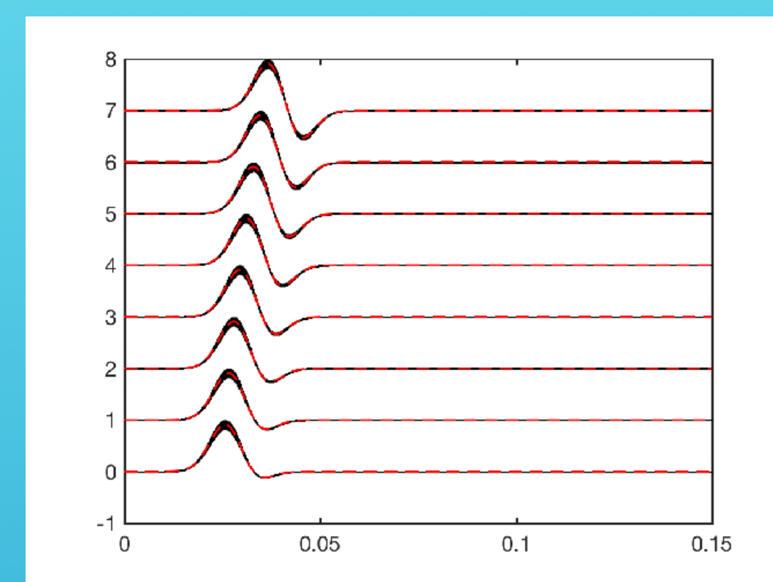


Figure 15: Mean (red dash) and one standard deviation region (black) from pressure traces from a force source distribution.

## Validation Tests

Multiple validation tests were performed by varying which independent parameters were deterministic and uncertain: bulk modulus, buoyancy, or source amplitude. Gamma distributions with mean and standard deviation of 1.25e10±2e9 Pa; 5e-4±8e-5 m$^3$/kg; and 1±0.1 J (N) for bulk modulus, buoyancy and explosive (force) source amplitude distributions, respectively. The models were spatially homogeneous, approximately equivalent to a medium with acoustic wave speed 2500±200 m/s and density of 2000 kg/m$^3$. Figures 5-6 show mean and standard deviation for bulk modulus distribution; Figures 7-9: buoyancy distribution for explosive source; Figures 10-12: buoyancy distribution for force source; Figure 13-14: explosion source amplitude distribution; Figure 15: Force source amplitude distribution.

## Discussion/Conclusions

- We validated the stochastic acoustic solver Paracousti-UQ against Monte Carlo simulations
- No more than 21 equivalent deterministic simulations were needed to obtain the stochastic PDE solution that matched the Monte Carlo simulations with <0.5% error.
- At a minimum, in these simple models, the stochastic solution is over 1,000 times faster than Monte Carlo to obtain equivalent accuracy for the standard deviation
- Next steps are to investigate more complex uncertain media


## Acknowledgements

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia LLC, a wholly owned subsidiary of Honeywell International Inc. for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.