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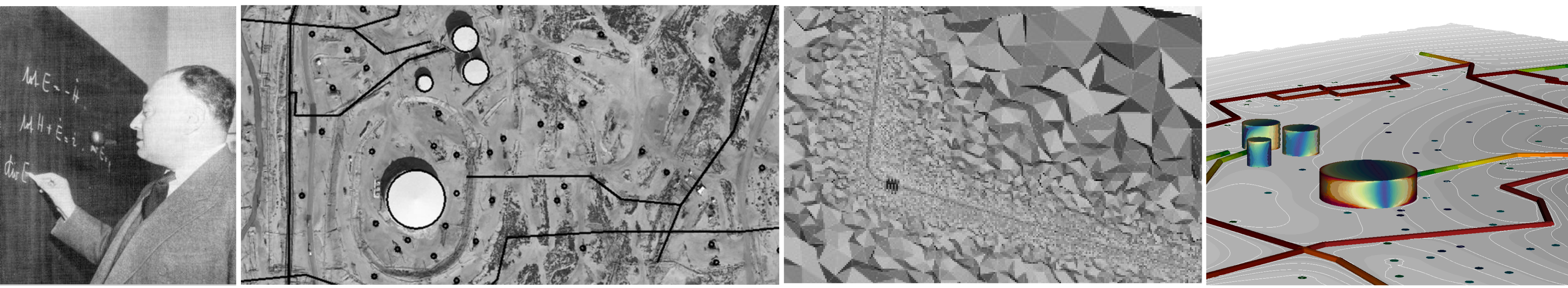

# Hierarchical Material Properties in Finite Element Analysis: The Oilfield Infrastructure Problem


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Built on LDRD Laboratory Directed Research and Development


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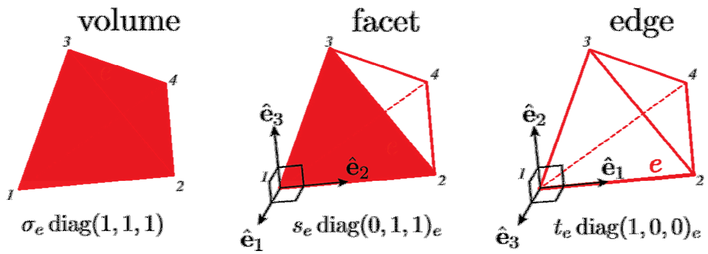
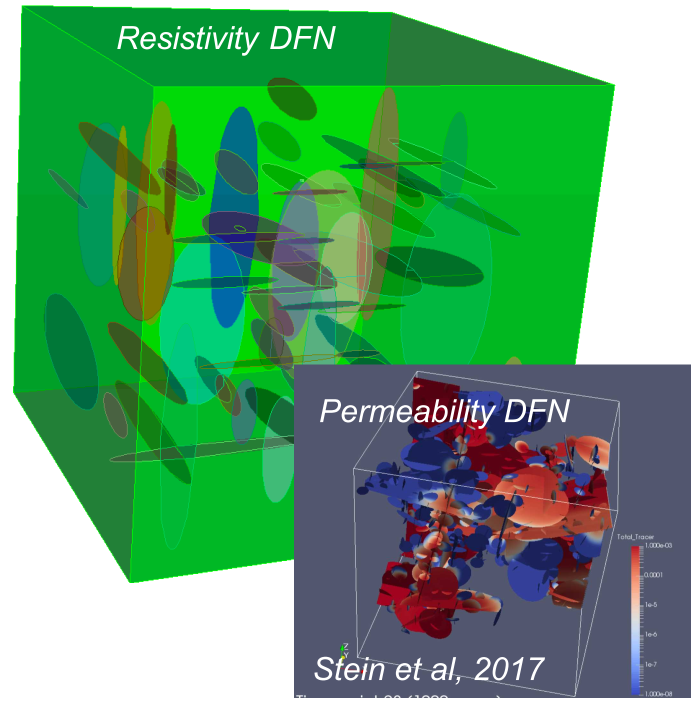
# The LDRD Team

Chester J Weiss (PI), SNL Geophysics Dept

Forward model discretization, fractional calculus and applications

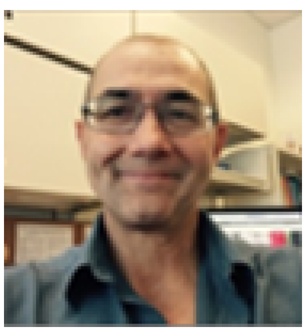



G Didem Beskardes (PostDoc),
SNL Geophysics Dept

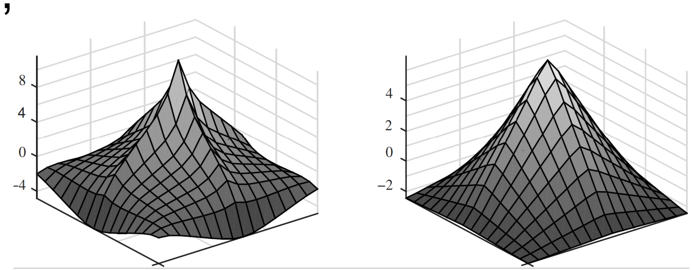
*started Aug 28, 2017*

Applications, discrete fracture networks and optimization



Bart van Bloemen Waanders (co-PI),
SNL Uncertainty Quantification Dept

High-performance computing, fractional calculus and optimization



# The pain of volumetric discretization

**Example problem:** discretization of steel casing in an oil well

0.2 m outer diameter, 0.025 m wall thickness, electrical conductivity 5e6 S/m

regular tet with edge length 0.025 m occupies a volume $(0.025\ \text{m})^3 / (6\sqrt{2}) = 1.84\text{e-}6\ \text{m}^3$

1 km of casing requires 7.4e6 tets

**Over a 1 km$^3$ Earth model discretized at, say 10 m, 7.4/(7.4 + 8.5)*100% = 46.5% of the tets are devoted to 0.0000014% of the mesh volume.**

This is **computationally explosive**, especially for realistic oilfield settings where there are 10s of km of steel casing + surface pipelines + storage tanks + electric cable + …

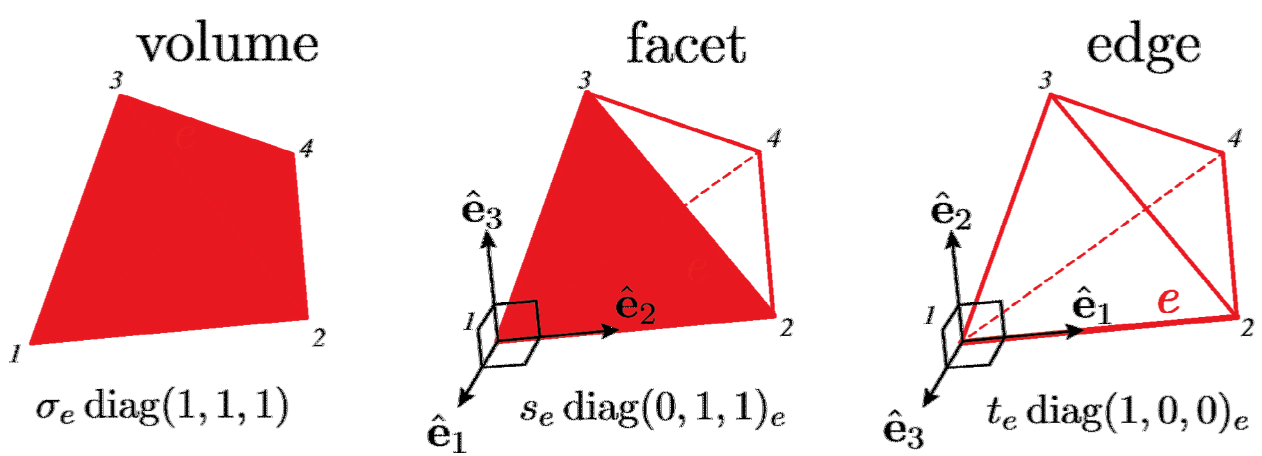
**Typical approaches to the problem are**

- specialized algorithms for parallel compute architectures (Commer et al., 2015, Hoversten et al., 2015, Um et al., 2015)
- Discretization of slightly "fatter" casing, whose large size reduces the element count with an acceptable reduction in accuracy (Haber et al., 2016; Weiss et al., 2016).

## A New Hope

Hanging the material properties on the tets, faces and edges of the unstructured tetrahedral mesh allows for thin conductors to be economically represented by facets and edges, rather than 100s of millions of tiny tets.

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$



# Hierarchical basis functions for material properties

This hierarchy of material distributions is made possible by using rank-2 tensor basis functions – an extension of the early work in 2D anisotropy by Weiss and Newman (Geophysics, 2002, 2003)

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) \; + \; \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) \; + \; \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$

$$\boldsymbol{\psi}_e^V(\boldsymbol{x}) = \mathrm{diag}(1,1,1) \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{volume } e \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\psi}_e^F(\boldsymbol{x}) = \mathrm{diag}(0,1,1)_e \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{facet } e \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{\psi}_e^E(\boldsymbol{x}) = \mathrm{diag}(1,0,0)_e \begin{cases} 1 & \text{if } \boldsymbol{x} \in \text{edge } e \\ 0 & \text{otherwise} \end{cases}$$

The tensor representation keeps the material properties local to the edges and facets in the Finite Element weak formulation / bilinear form.

## Assembly and solution of the linear system

Poisson Eq for electro/magnetostatics

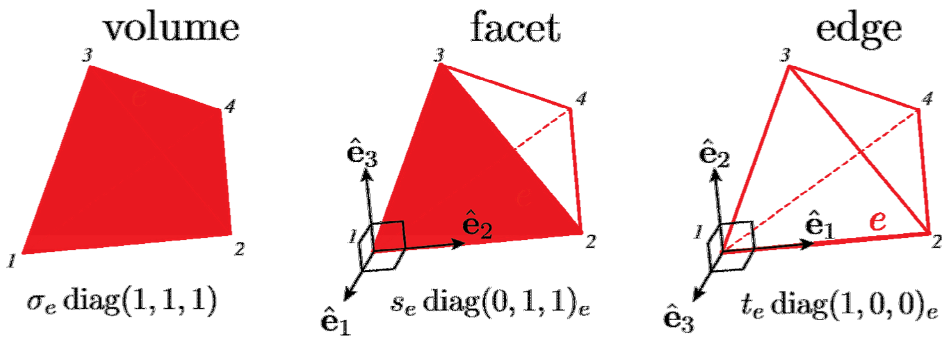
$$-\nabla \cdot (\boldsymbol{\sigma} \cdot \nabla u) = f \qquad \int_\Omega \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u)\ dx^3 = \int_\Omega v f\ dx^3$$

Sparse anisotropic conductivity collapses 3D gradients to 2D and 1D gradients…

$$\boldsymbol{\sigma} = \mathrm{diag}(0, \sigma, \sigma) \qquad \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) = \sigma \nabla_{23} v \cdot \nabla_{23} u$$

$$\boldsymbol{\sigma} = \mathrm{diag}(\sigma, 0, 0) \qquad \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u) = \sigma \nabla_{1} v \cdot \nabla_{1} u$$

…thus ensuring that the facet and edge material properties are local and not distributed over the tetrahedral volume.



# Assembly and solution of the linear system

Variational formulation: $\int_\Omega \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u)\ dx^3 = \int_\Omega vf\ dx^3$

Hierarchical model: $\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$

3D inner products collapse to 2D and 1D inner products

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_V} \sigma_e \int_{V_e} \nabla v \cdot \nabla u\ dx^3 = \sum_{e=1}^{N_V} \sigma_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^4 \mathbf{u}_e$$

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_F} s_e \int_{F_e} \nabla_{23} v \cdot \nabla_{23} u\ dx^2 = \sum_{e=1}^{N_F} s_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^3 \mathbf{u}_e$$

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_E} t_e \int_{E_e} \nabla_1 v \cdot \nabla_1 u\ dx = \sum_{e=1}^{N_E} t_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^2 \mathbf{u}_e$$

Global stiffness matrix is a sum of 3D, 2D and 1D element stiffness matrices.

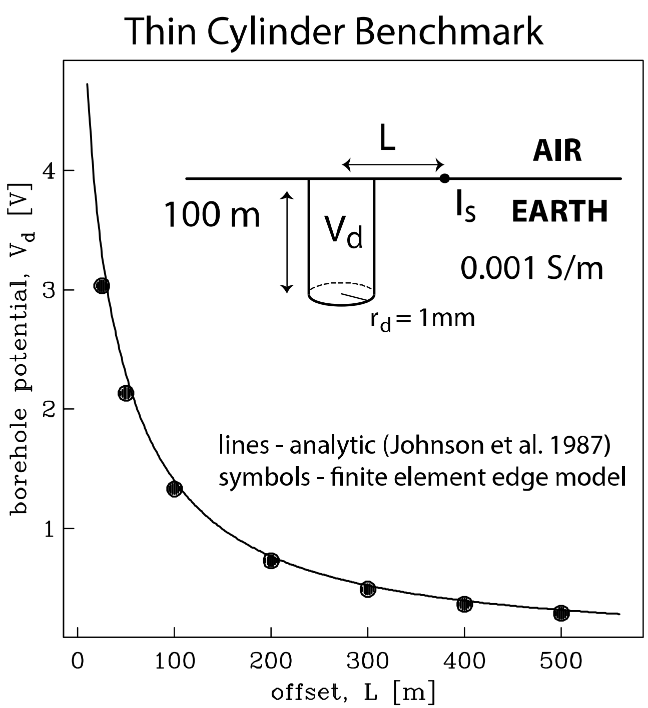
$$\mathbf{K}\mathbf{u} = \mathbf{b}$$

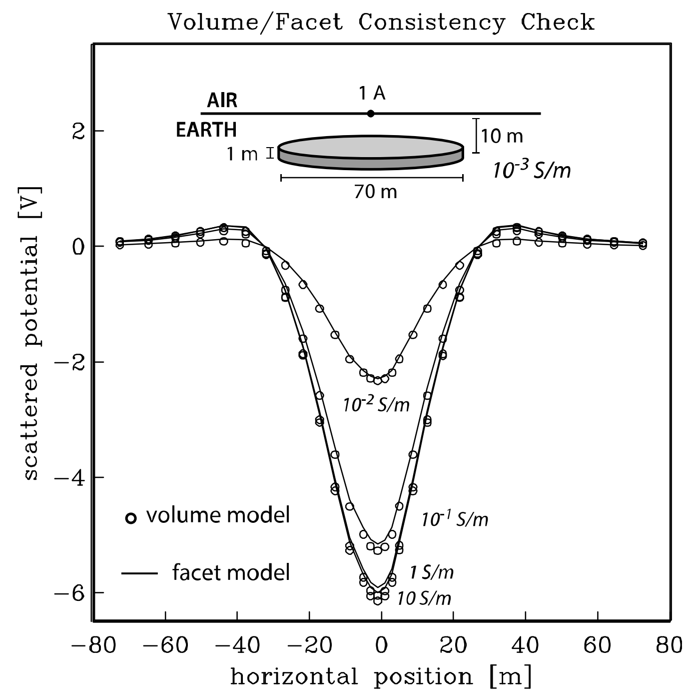
$$\mathbf{K} = \sum_{e=1}^{N_V} \sigma_e \mathbf{K}_e^4 + \sum_{e=1}^{N_F} s_e \mathbf{K}_e^3 + \sum_{e=1}^{N_E} t_e \mathbf{K}_e^2$$

Solve iteratively with Jacobi scaled conjugate gradients and on-the-fly matrix assembly (Weiss, 2001)

# Benchmarking with analytics

Benchmarking and internal consistency checks show that for thin conductors, the facet/edge representation achieves acceptable accuracy over a range of geometries and material properties.

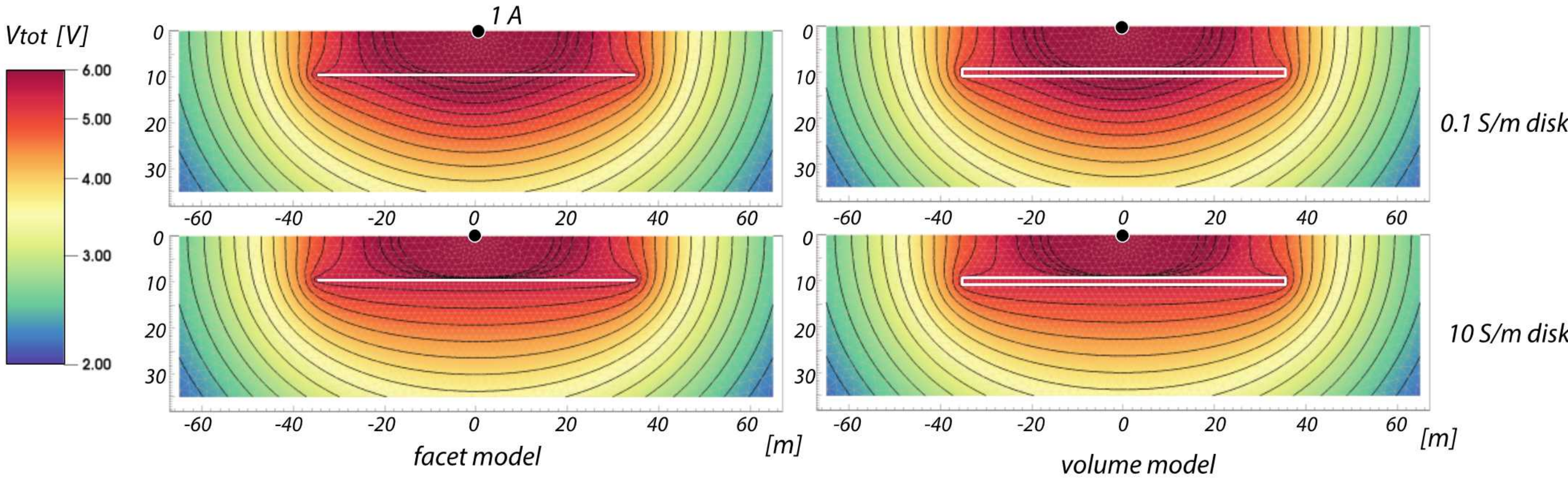


# Volume/facet consistency test

Visual inspection of thin disk results for facet elements (left) and many small tetrahedral elements (right).

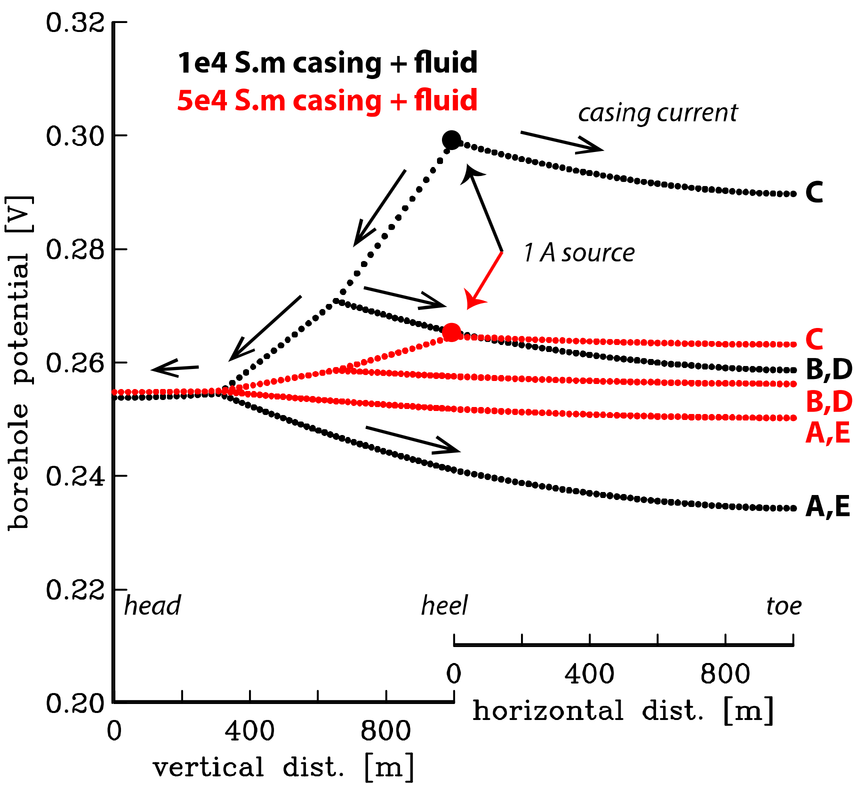
Shown is a cross section of electric potential through the disk and surrounding geology for a weak conductor (top) and strong conductor (bottow). Background conductivity is 0.001 S/m.

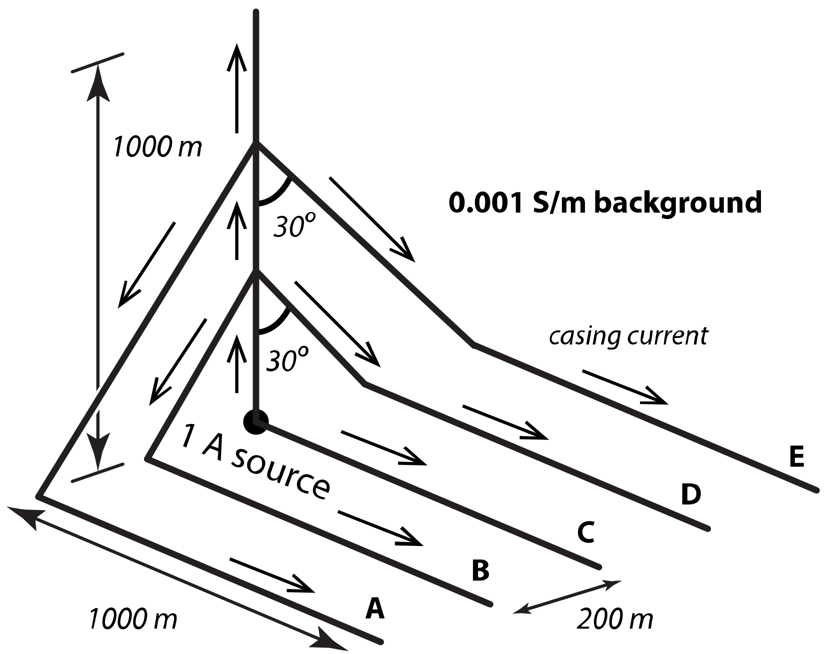


With the mathematical framework in place along with favorable benchmarking results, we're now emboldened to investigate oilfield problems.

EXAMPLE PROBLEM: an idealized multi-lateral (edge elements for casing)

What is the effect of casing conductivity on the casing potential?



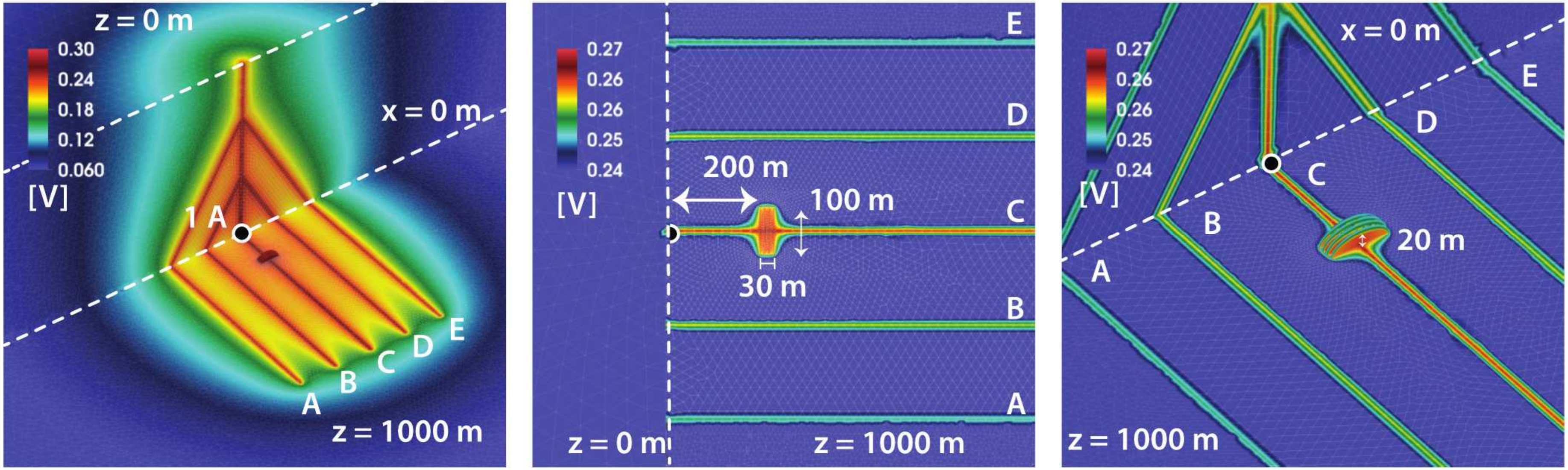



With the mathematical framework in place along with favorable benchmarking results, we're now emboldened to investigate oilfield problems.

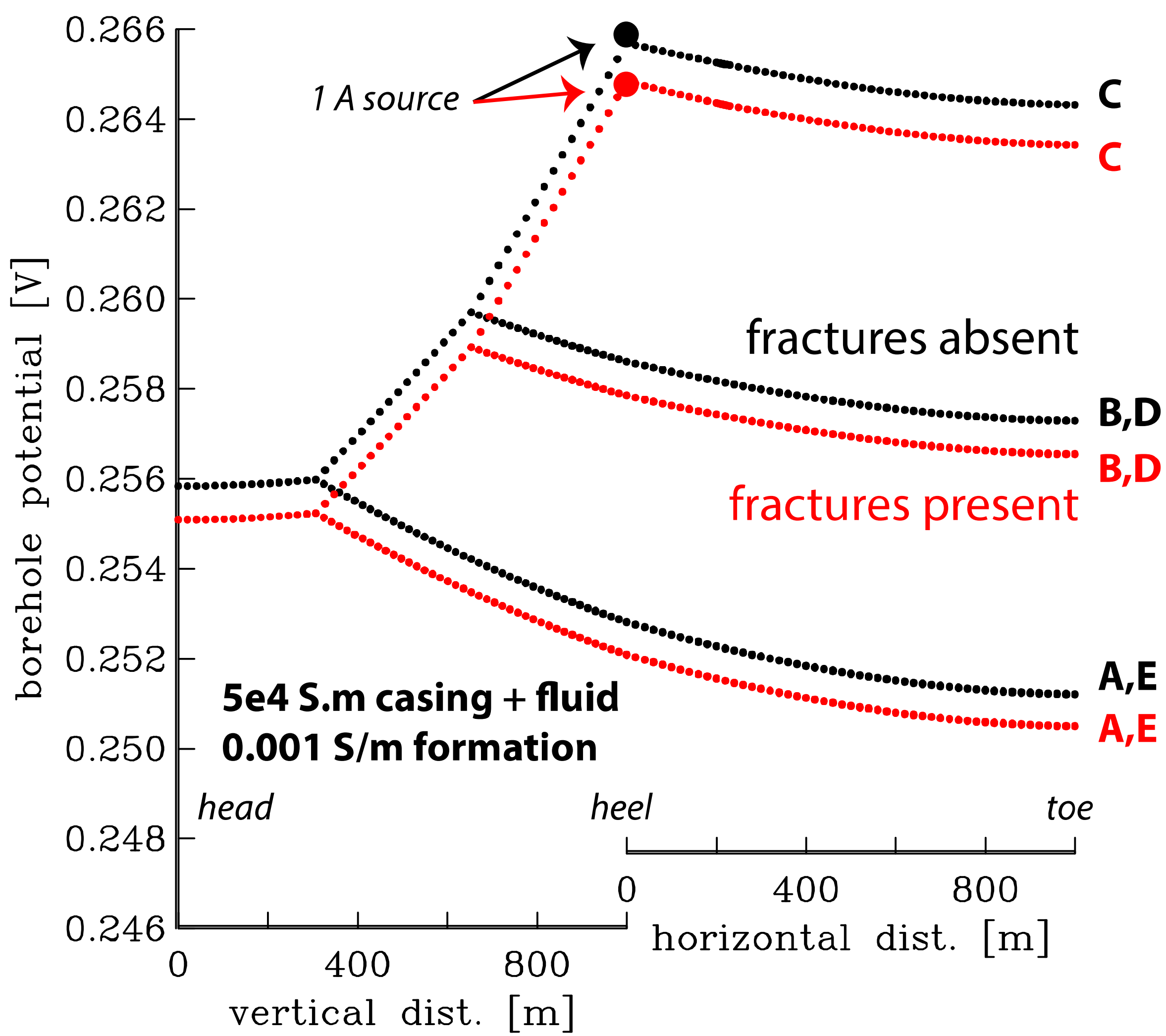
EXAMPLE PROBLEM: an idealized multi-lateral, now with a fracture.

What effect does a conductive fracture have on the casing system?

casing: edge elements, fracture: facet elements

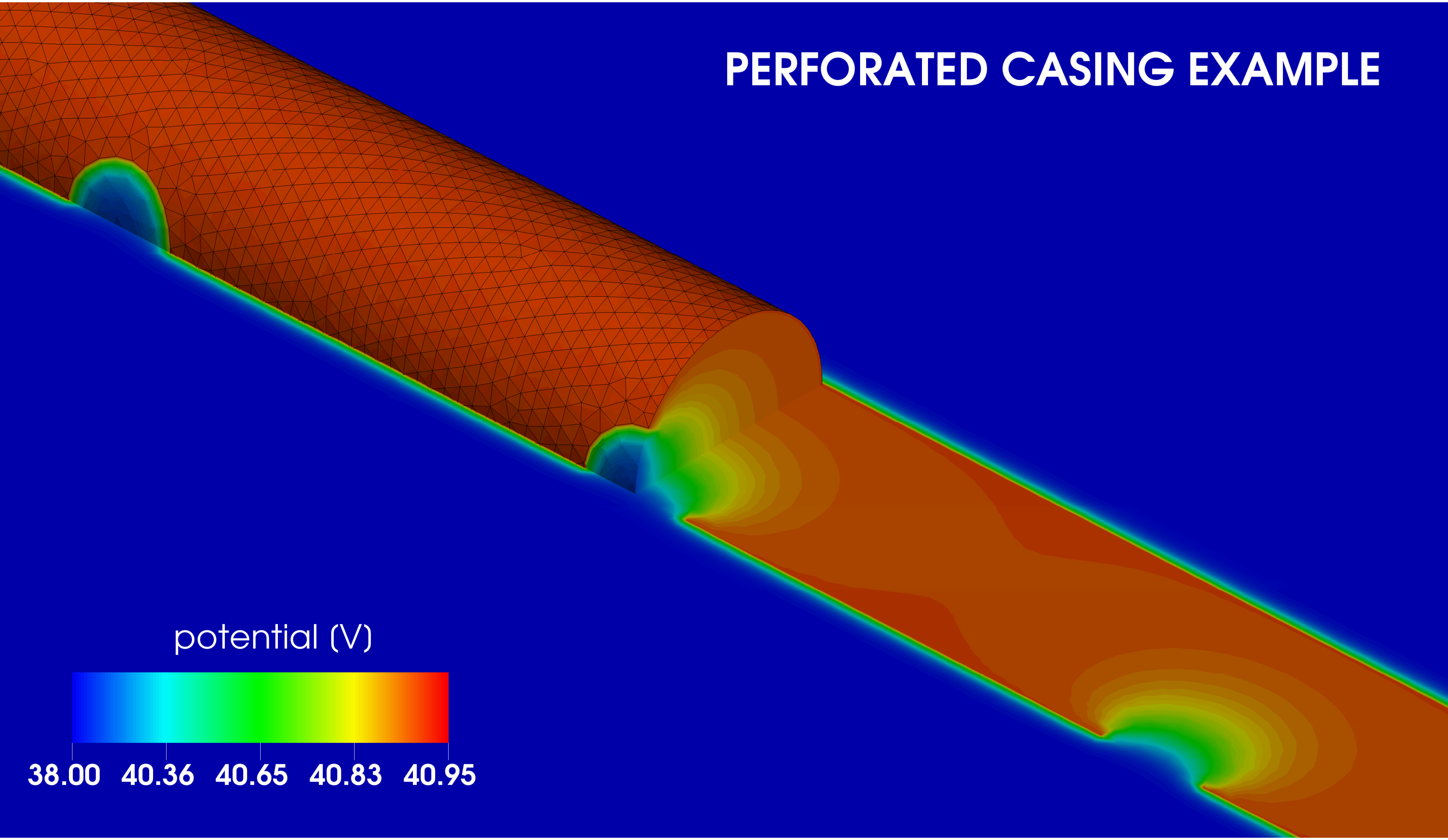


# 4D time-lapse response of casing response with conductive fractures introduced into the system



## Proof of Concept Example 2: Perf design studies

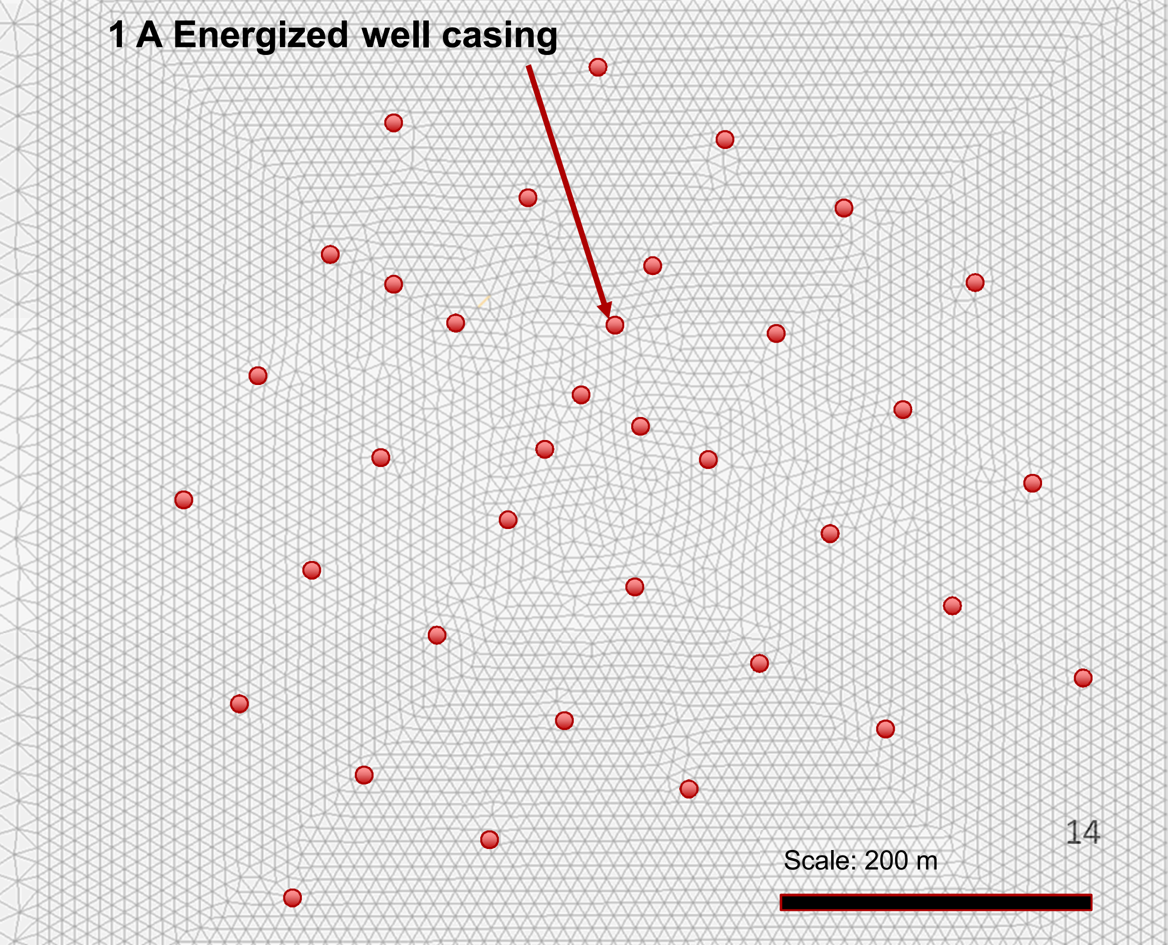
Over scales of a few meters, model the borehole cylinder explicitly with facet conductivities.



# Real World Example 1: Shallow, heavy oil reservoir.

10 m node spacing, 16 km of casing, 36 vertical wells: 1628 edges
10 m node spacing on air/earth interface over oilfield
1.4M tets, 238k nodes, 10 x 10 x 5 km domain

Casing model:
20 cm OD
2.5 cm wall thickness
5e6 S/m conductivity
t_e = 5e4 S.m



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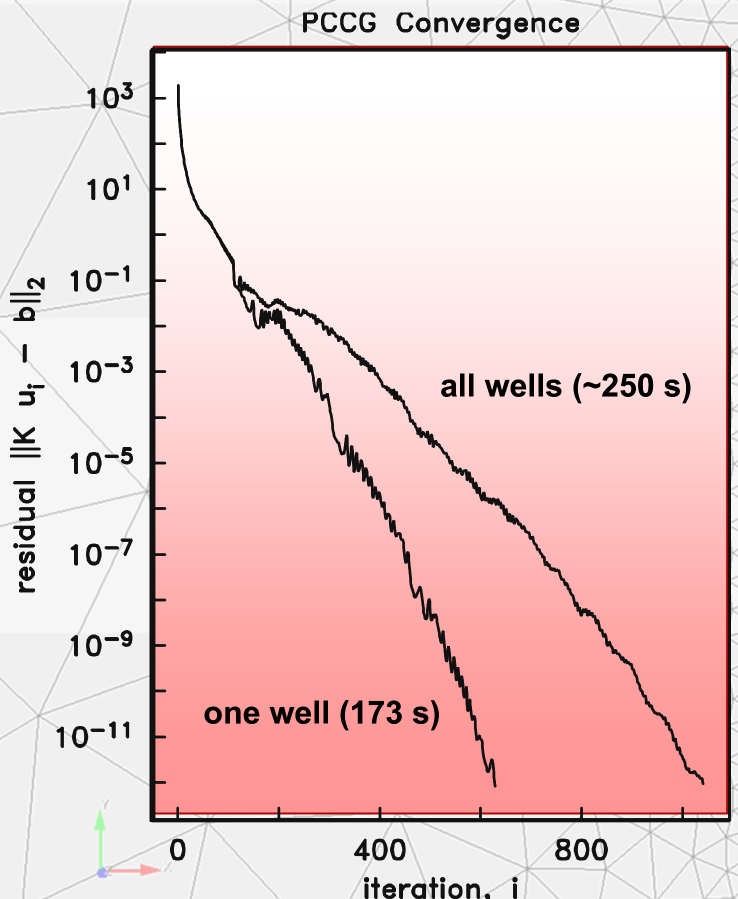
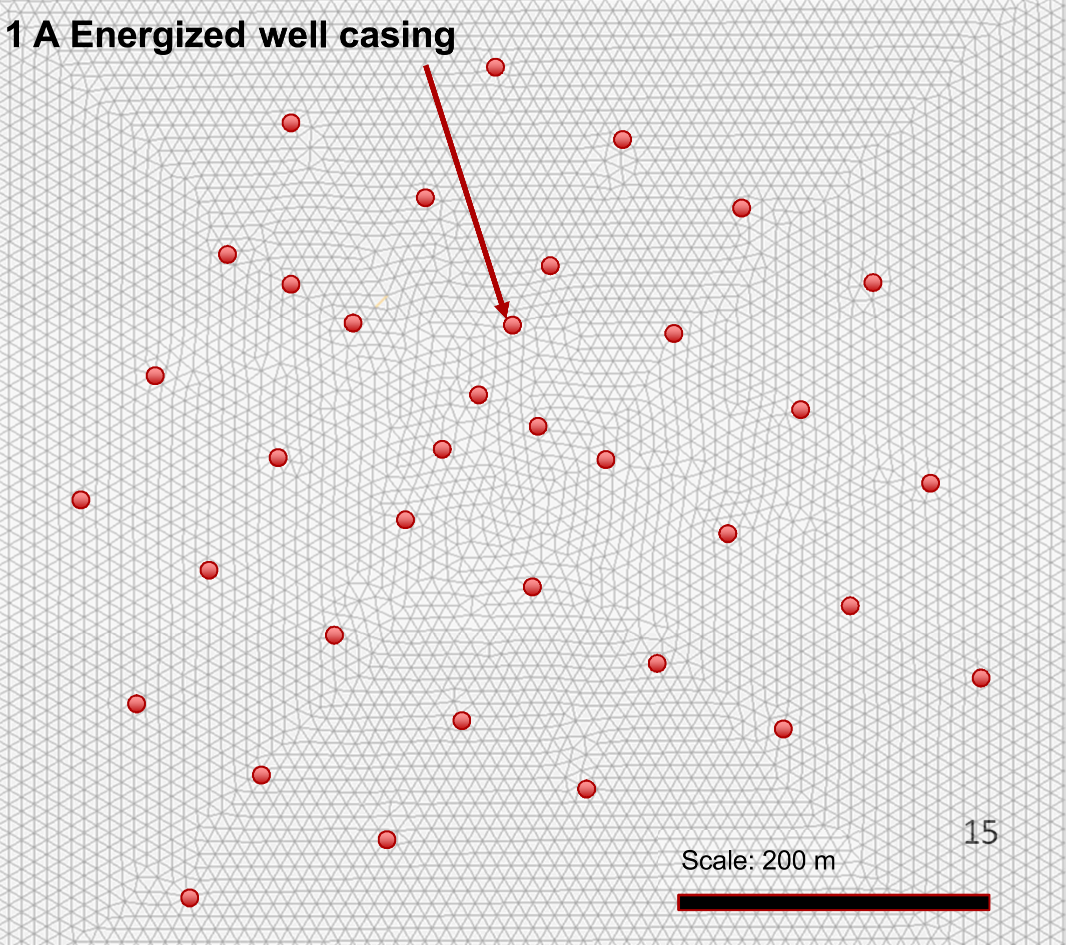
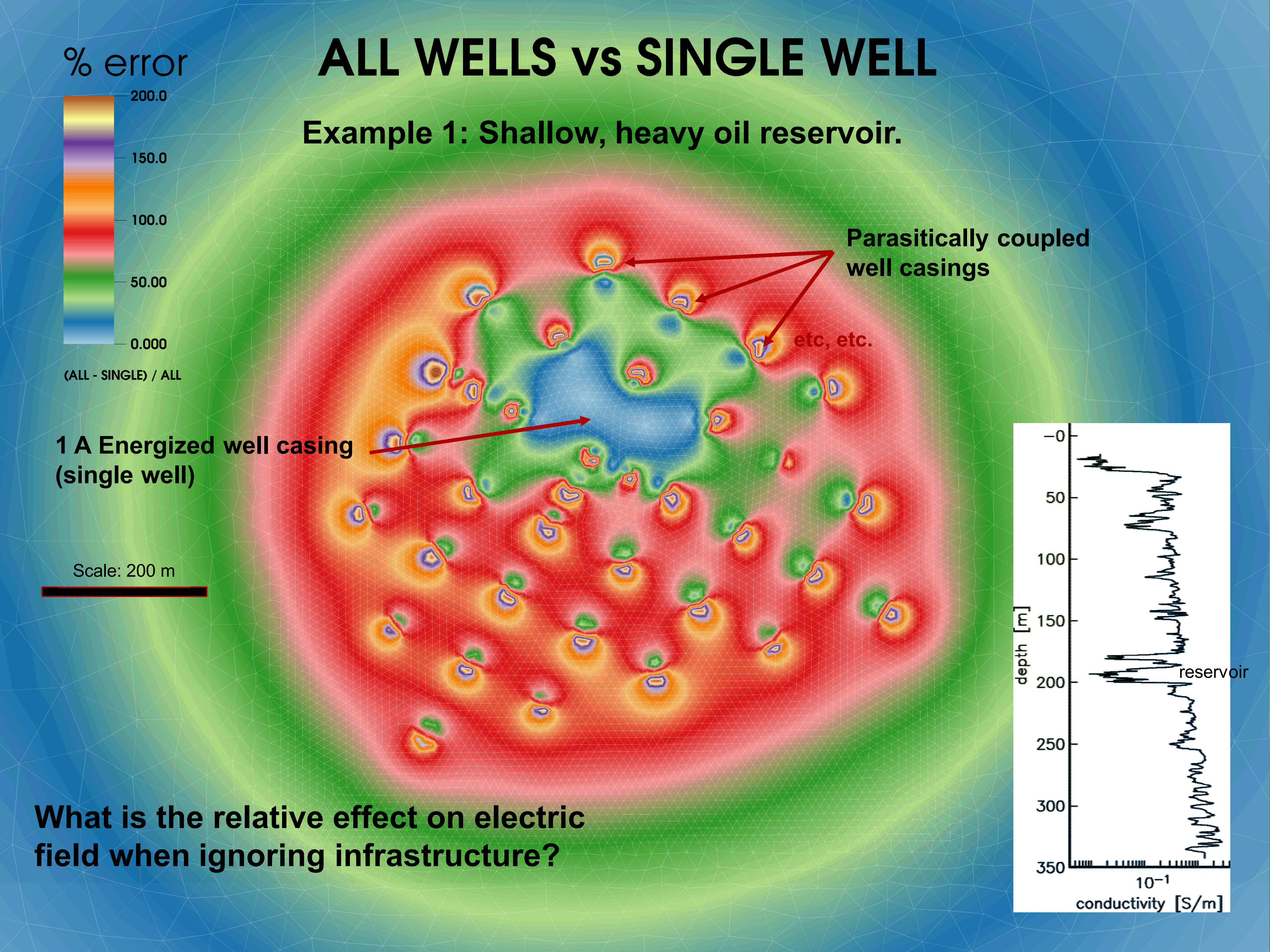
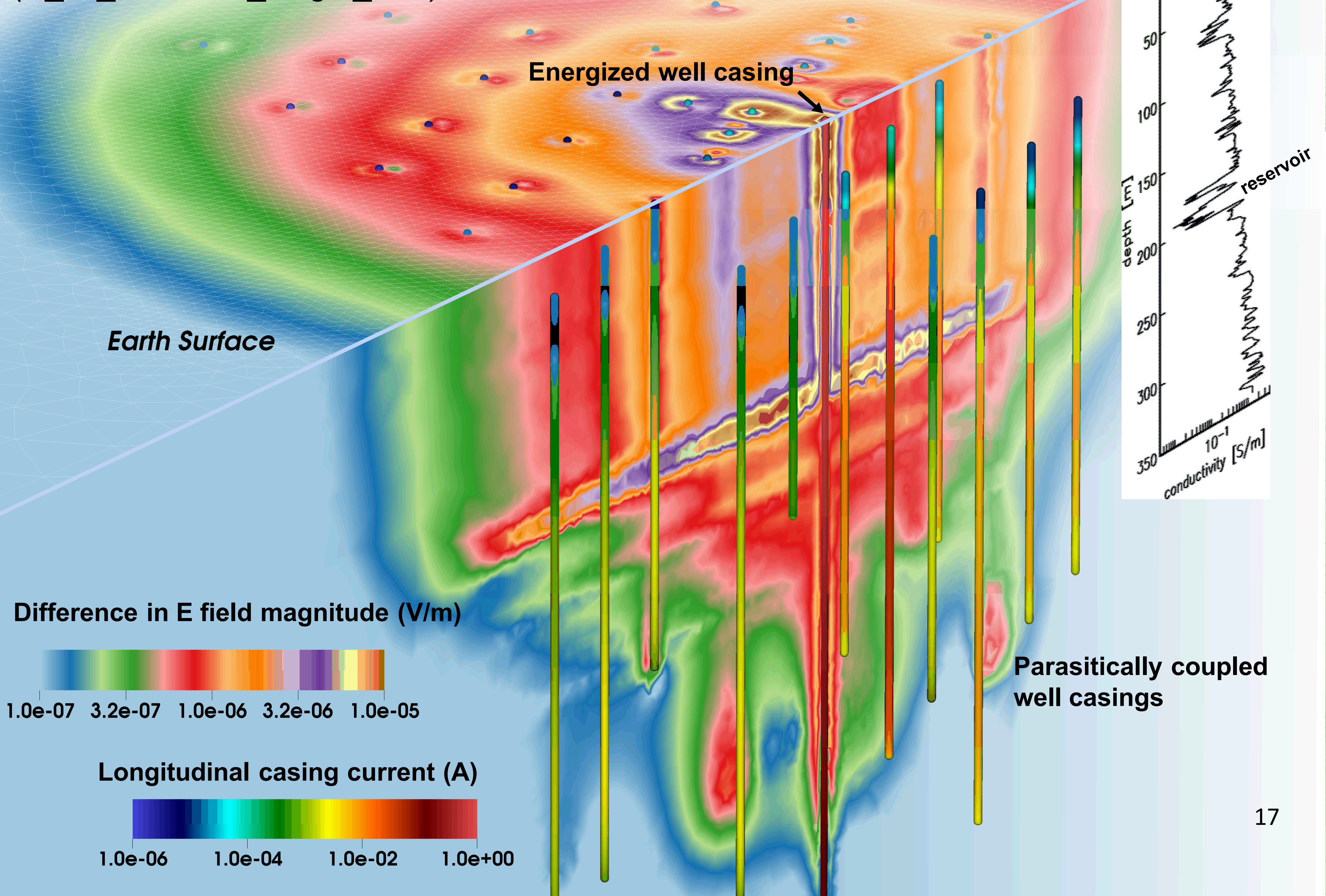



ALL WELLS vs SINGLE WELL
Example 1: Shallow, heavy oil reservoir.
% error
200.0
150.0
100.0
50.00
0.000
(ALL - SINGLE) / ALL
Parasitically coupled well casings
etc, etc.
1 A Energized well casing (single well)
Scale: 200 m
What is the relative effect on electric field when ignoring infrastructure?
0
50
100
150
200
250
300
350
depth [m]
reservoir
$10^{-1}$
conductivity [S/m]

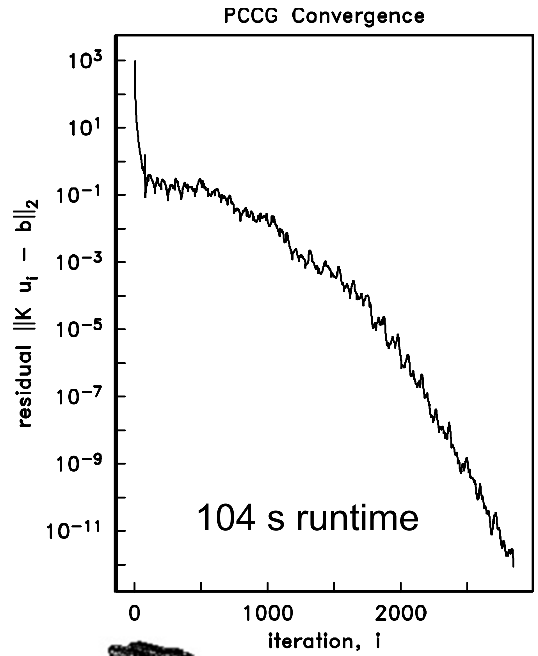
# Effect of full clutter model on Electric fields

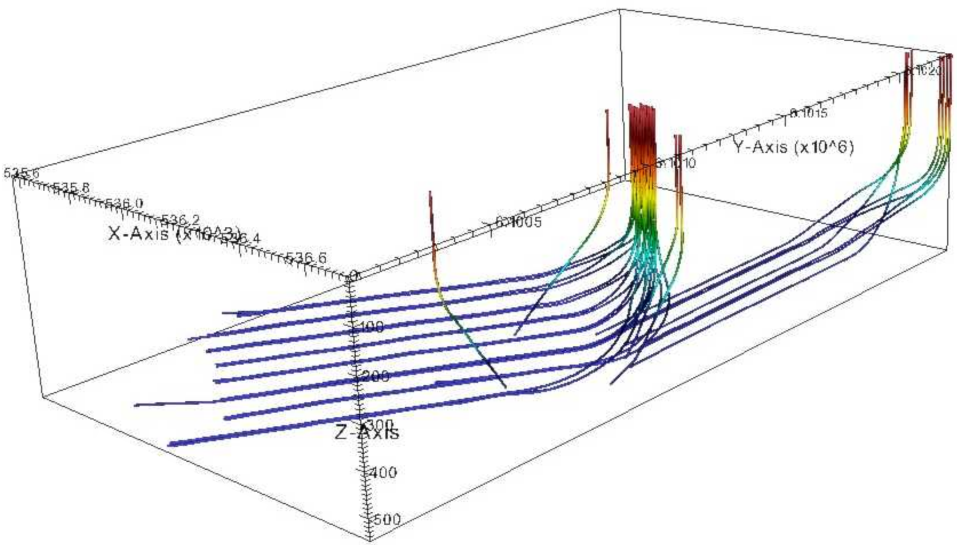
*(E_all_wells – E_single_well)*

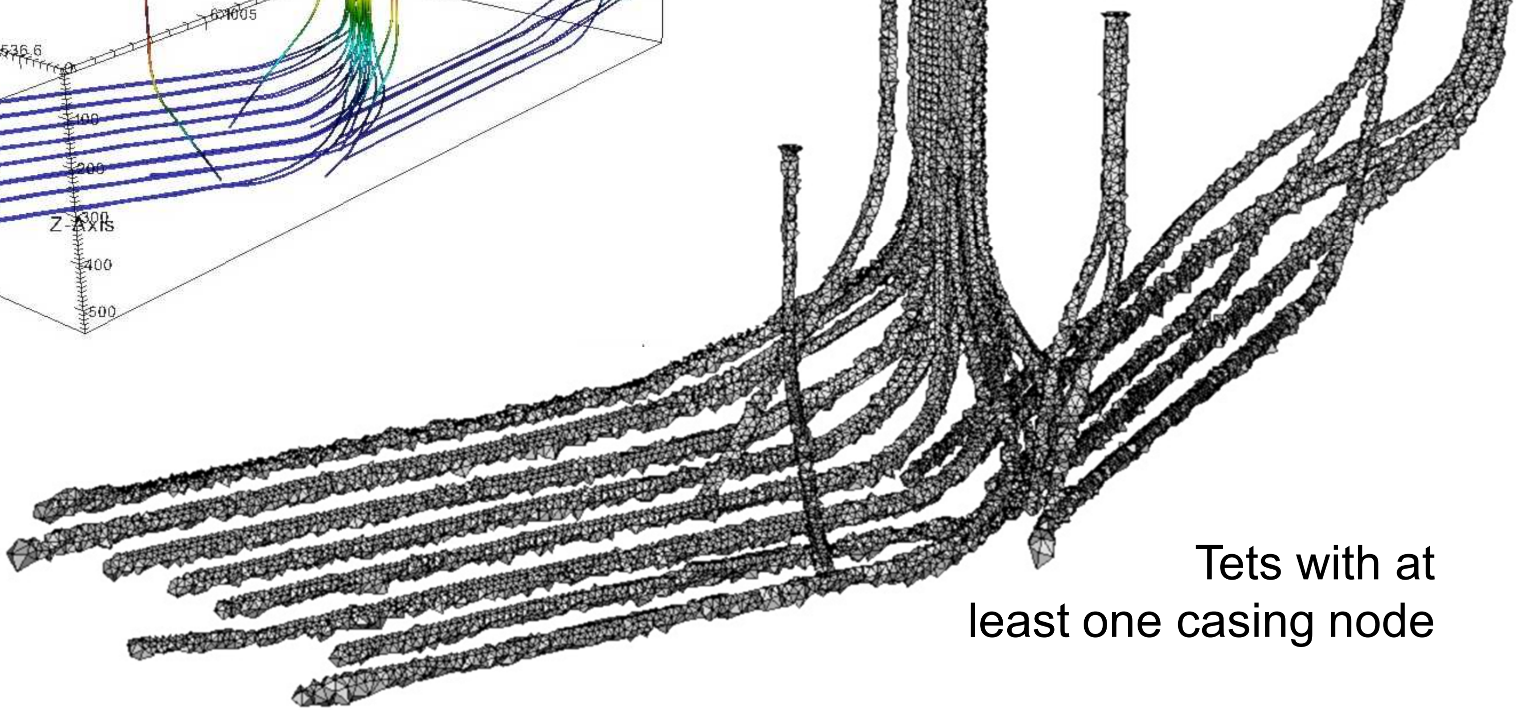


# Real World Example 2: SAGD multilateral

**20 m node spacing, 45 km of casing, 31 wells: 2313 edges**
**50 m node spacing on air/earth interface over oilfield**
**332k tets, 60k nodes, 10 x 10 x 5 km domain**

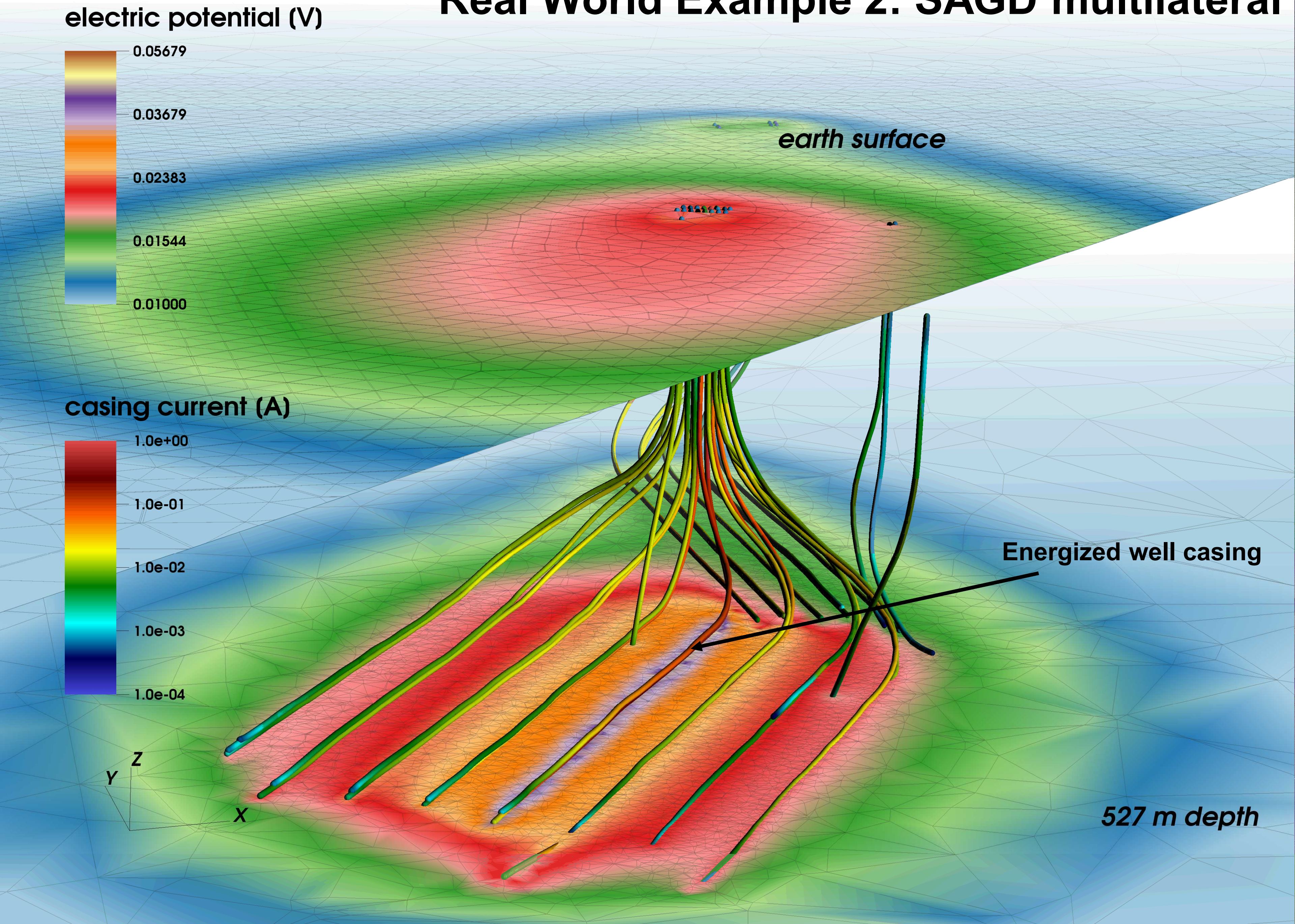


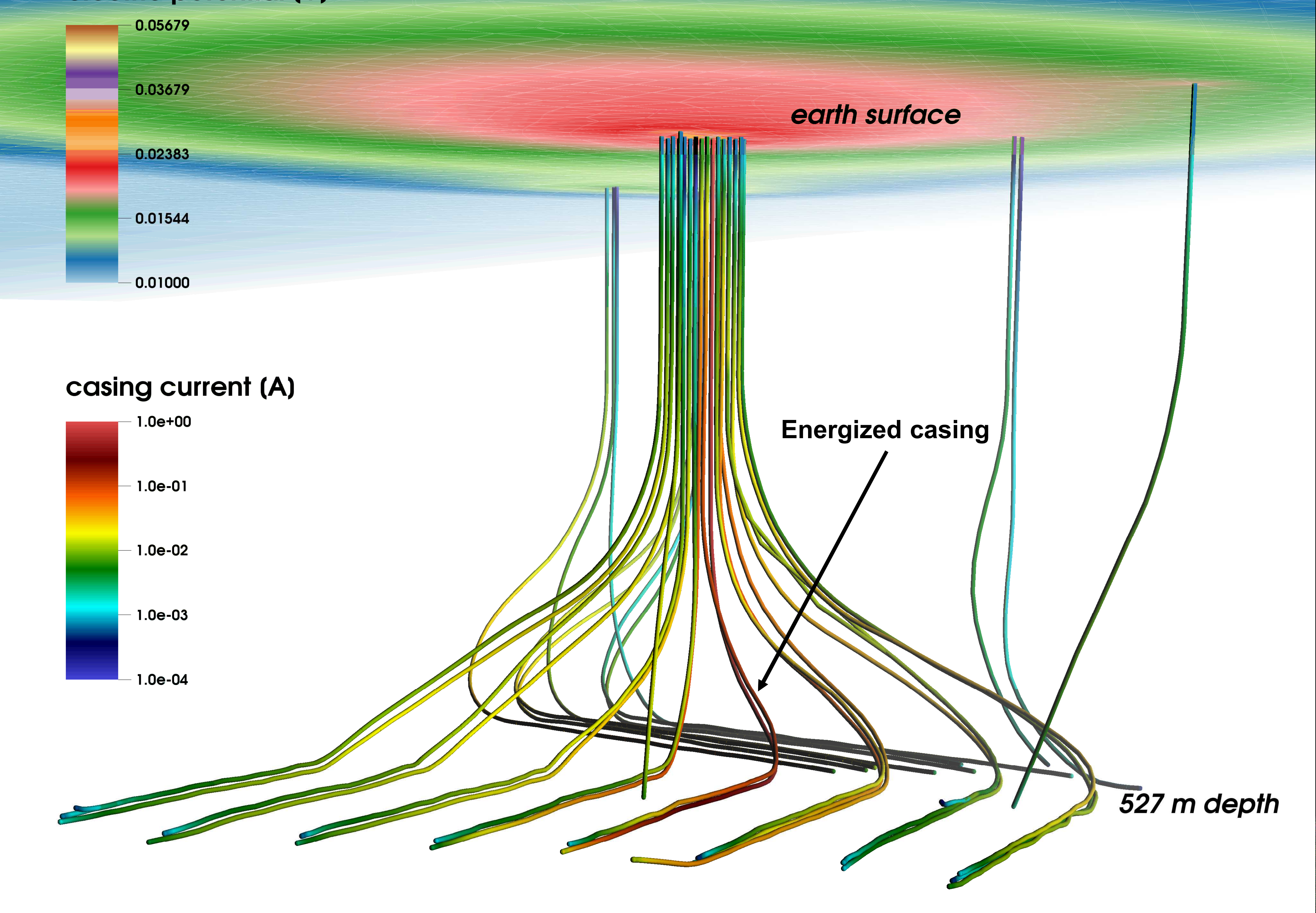



Tets with at least one casing node

# Real World Example 2: SAGD multilateral



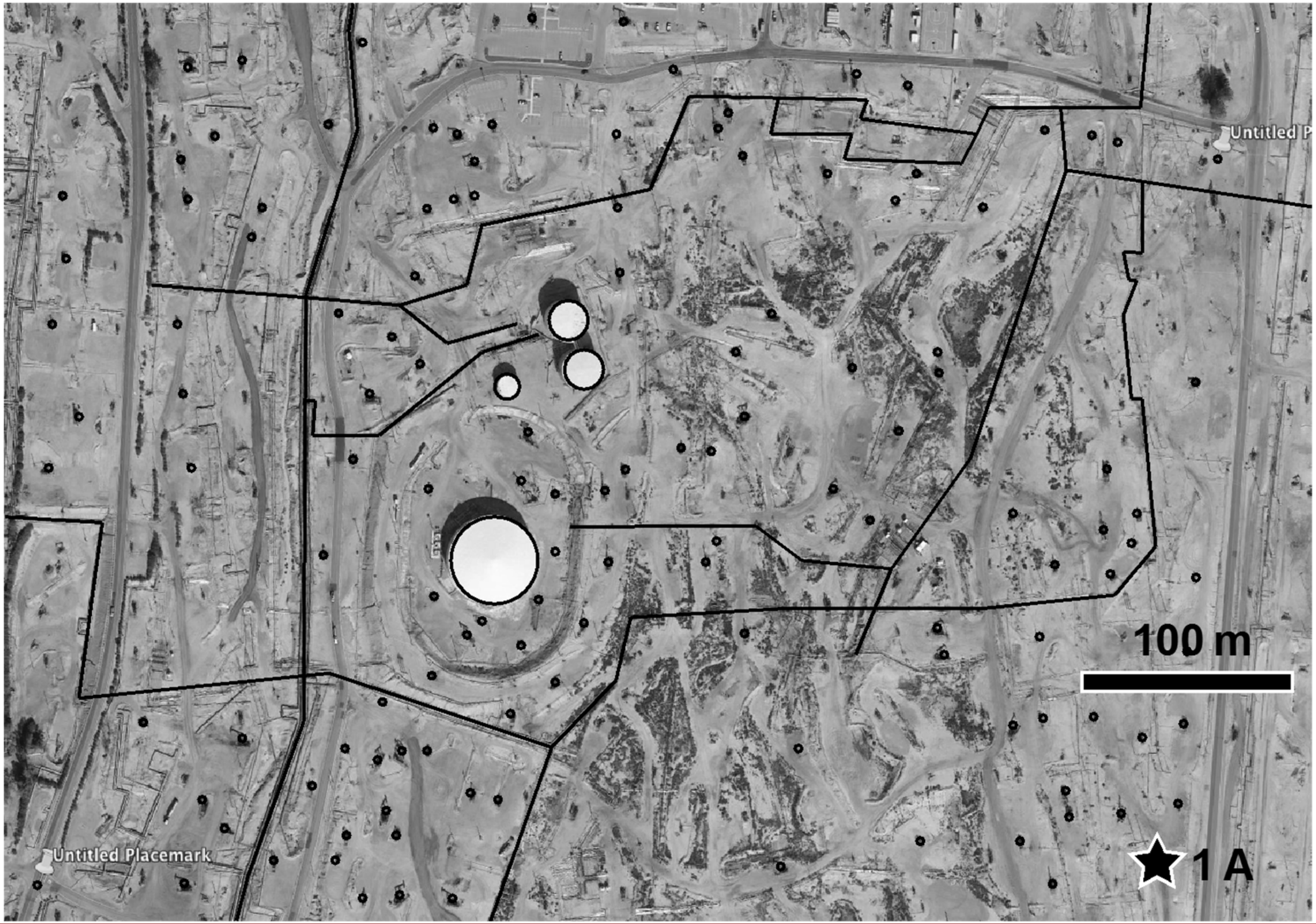
# Real World Example 2: SAGD multilateral



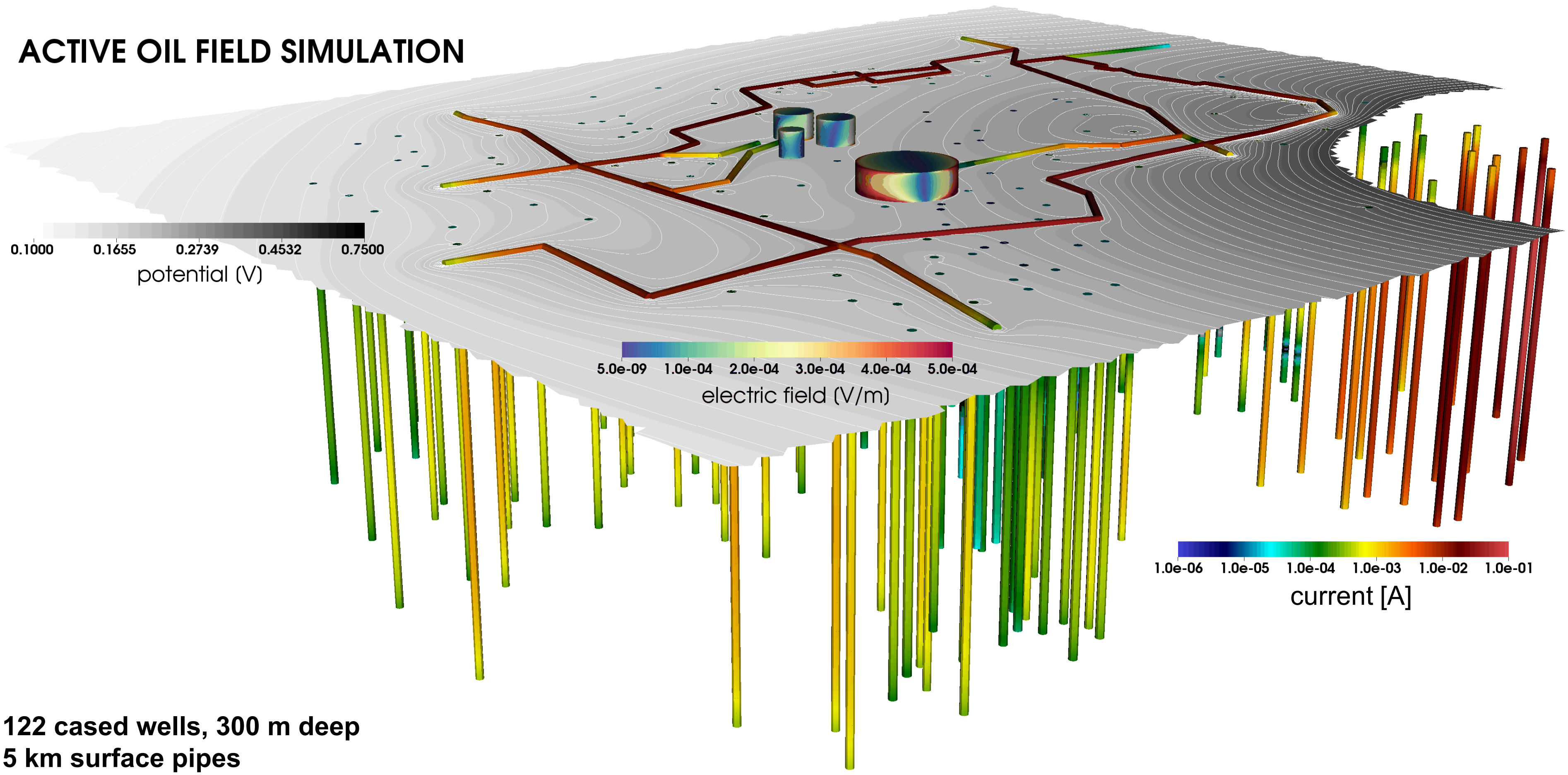
# Real World Example 3: Casing + surface infrastructure

Kern River Formation Site

0.7 $km^2$ area + 122 wells + ~2 km surface pipes



# Real World Example 3: Casing + surface infrastructure




**122 cased wells, 300 m deep**
**5 km surface pipes**
**~35 km pipeline/casing modeled at 10 m grid spacing: 3500 elements**
**Traditional FEM requires ~7e6 elements per km of pipline/casing.**
**HFEM decreases computational burden by ~4 orders of magnitude in this example (10 min vs 2 mo, estimated runtime)**

## Conclusions and comments

Hierarchical material properties in finite element analysis offers a computationally economical way for modeling sharp, volumetrically insignificant regions, with elevated material property values (e.g. conductivity in electrostatics)

The reduction is computational burden over volumetric discretization can reach several orders of magnitude, thus leading to “real time” solutions and evaluation of problems previously believed intractable.

For the electrostatic problem, hierarchical FE solutions compare favorably with independent analytic solutions and are internally consistent with solutions from volume discretizations.

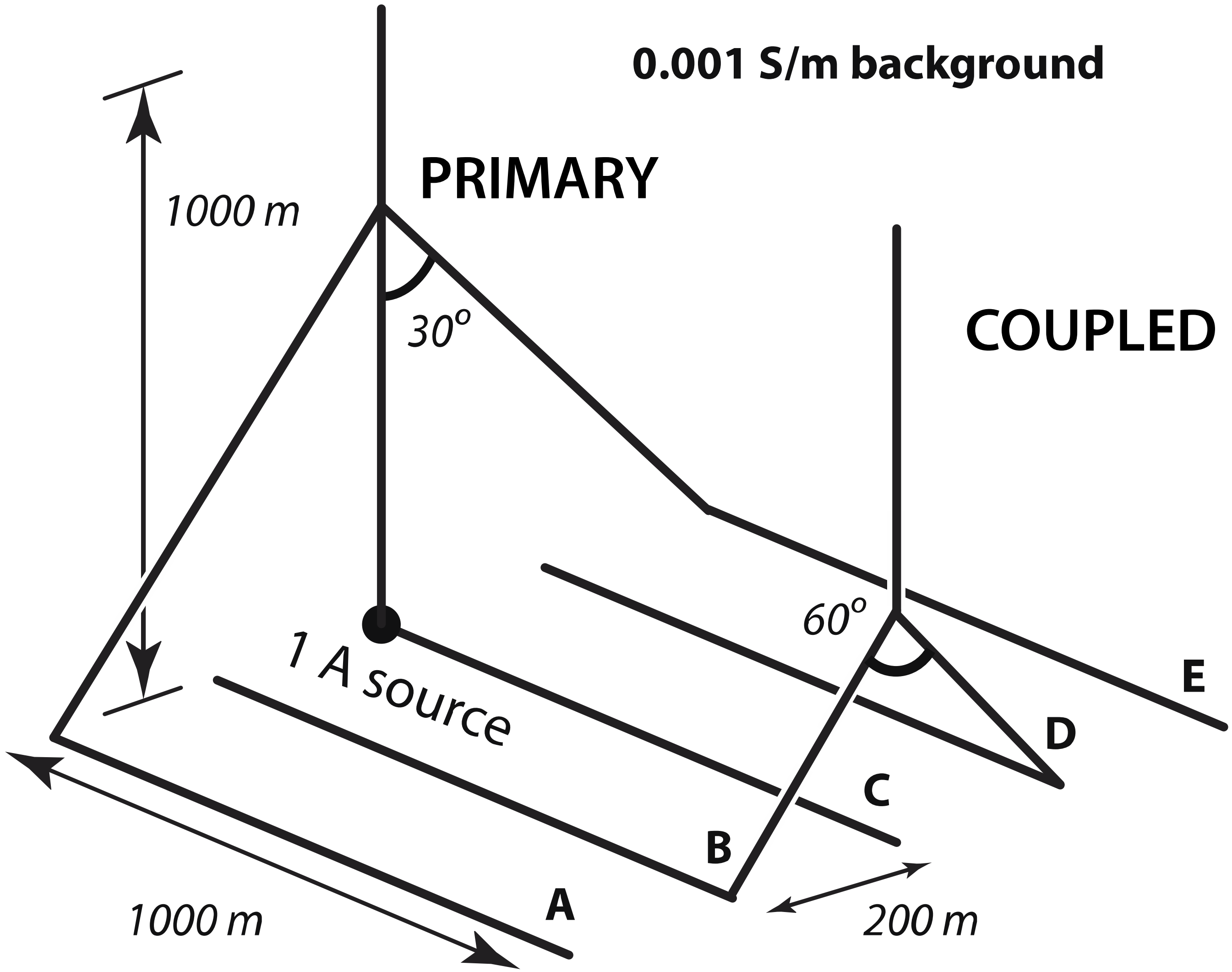
Hierarchical FE method has been applied to various “real world” oilfield examples with complex infrastructure.  Although solution times are fast (10s of seconds to a minute or so), mesh generation and metadata management issues are more acute.

**Acknowledgements**
Sandia National Laboratories LDRD Program.

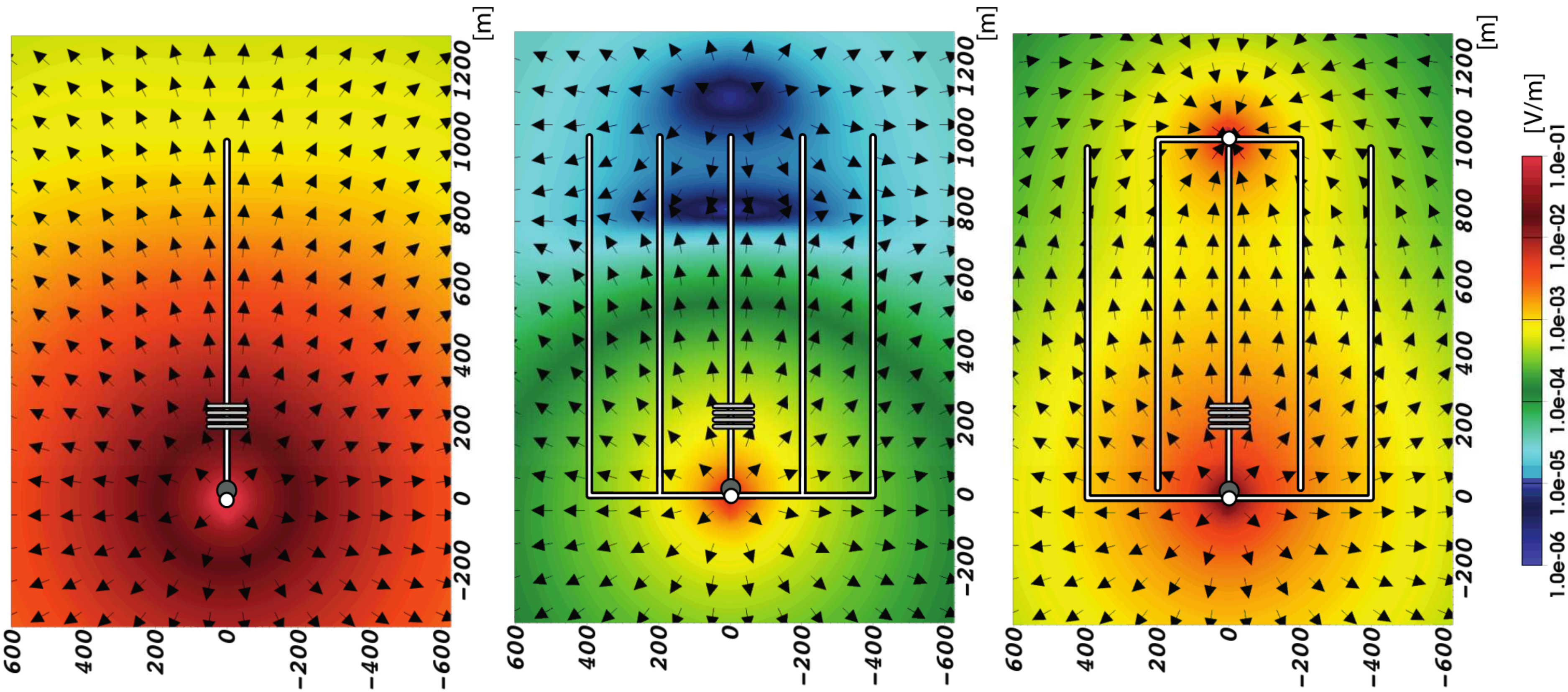
Sandia National Laboratories

BACKUP SLIDES

What happens when some of the horizontals aren't in direct electrical contact? I.e. passive coupling between all the steel in the ground.

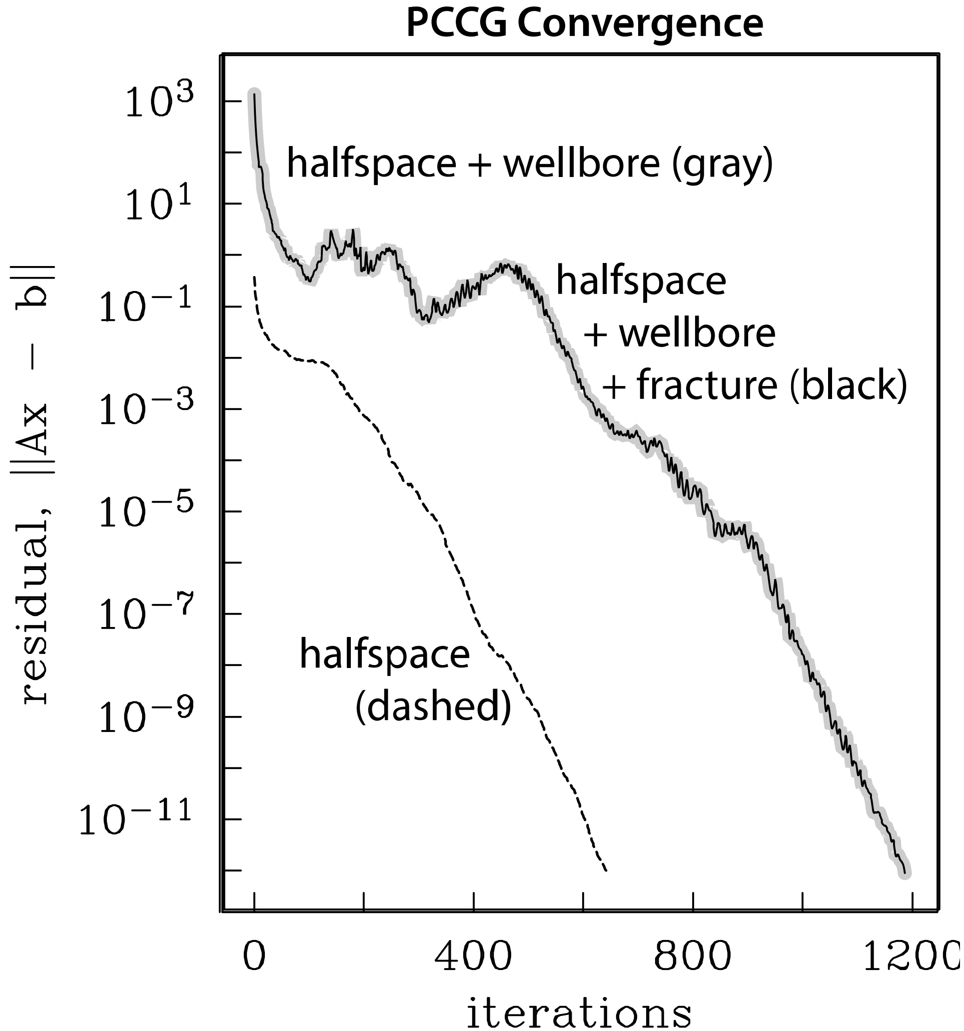


What does the time-lapse surface electric field look like for different casing configurations? Better get all the steel in your model!

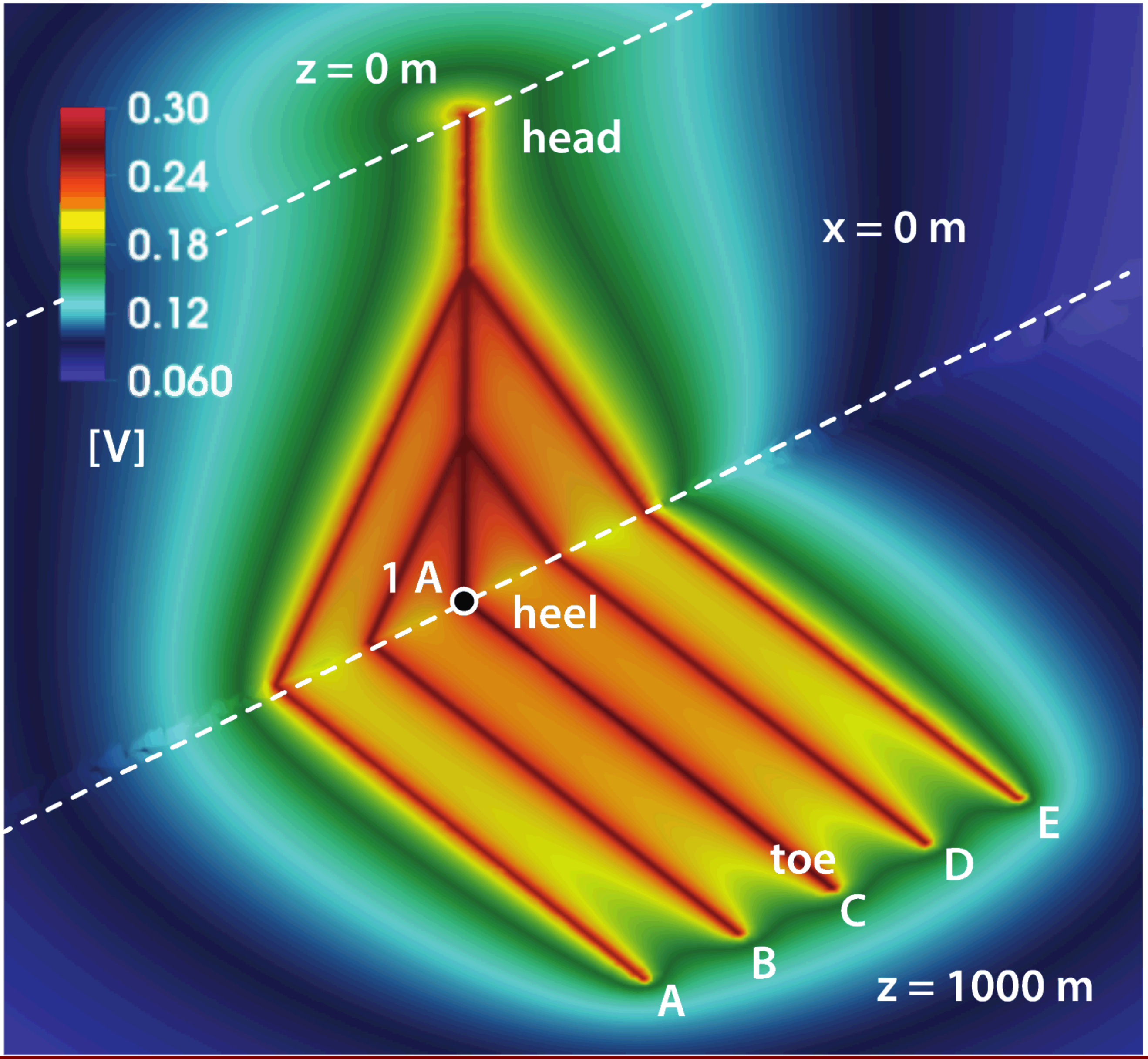


# Effect of hierarchical elements on PCCG convergence

Adding steel to the model is known to increase the runtime. But still… the runtimes are on the order of a few minutes, rather than hours.

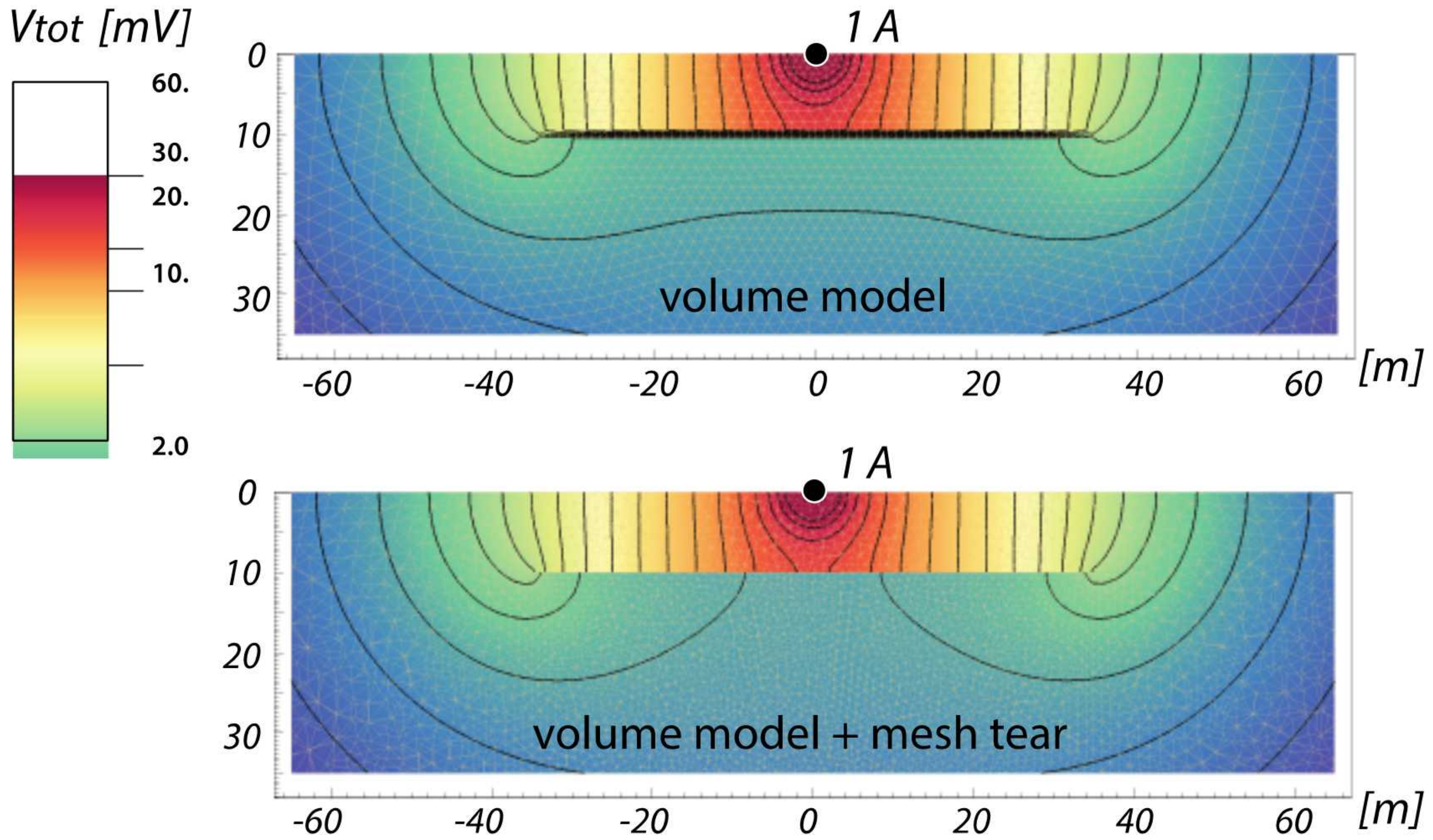


# 3D rendering of the multi-lateral system



## What about thin resistive elements?

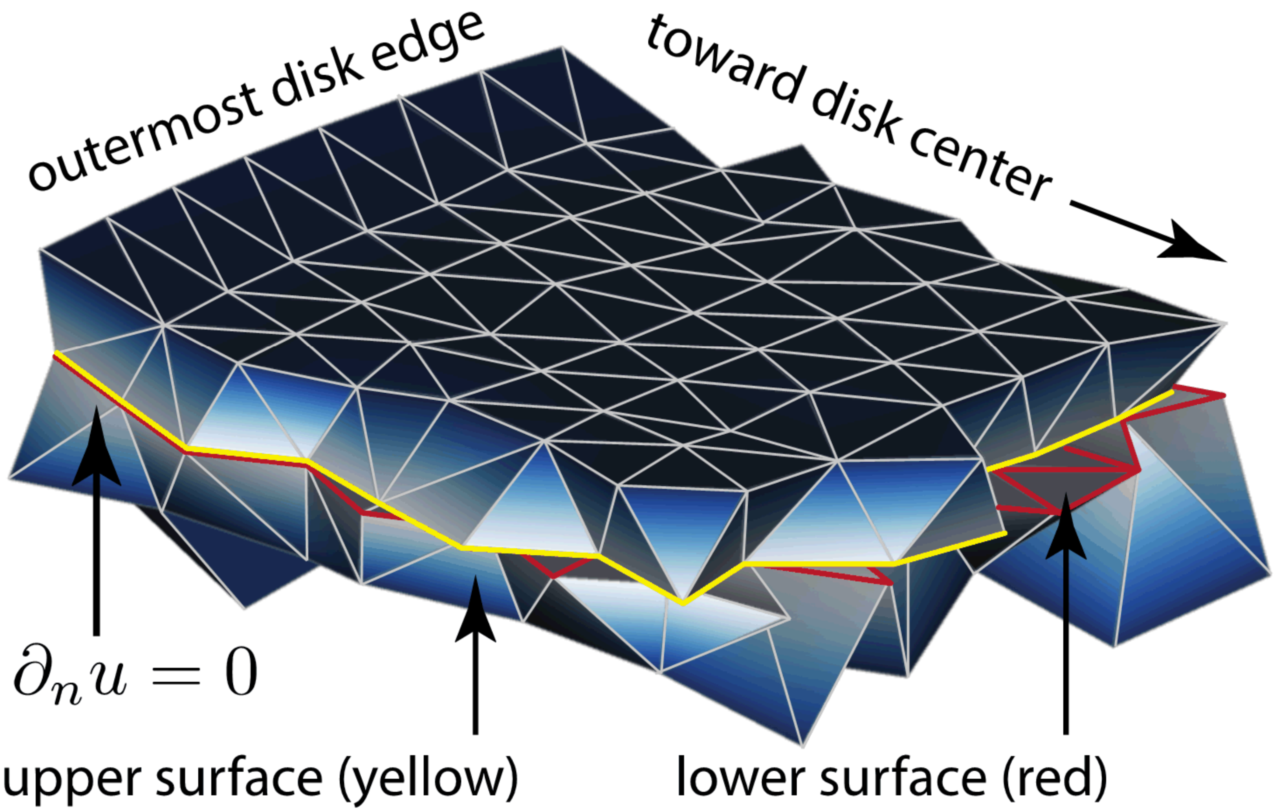
Because of the continuity condition, thin resistors require a little more intervention.  Specifically, we require that the infinitely thin resistor be multi-valued on the disk surface, with either side of the surface subject to a Neumann boundary condition.  This is called a “tear” in the FE literature.



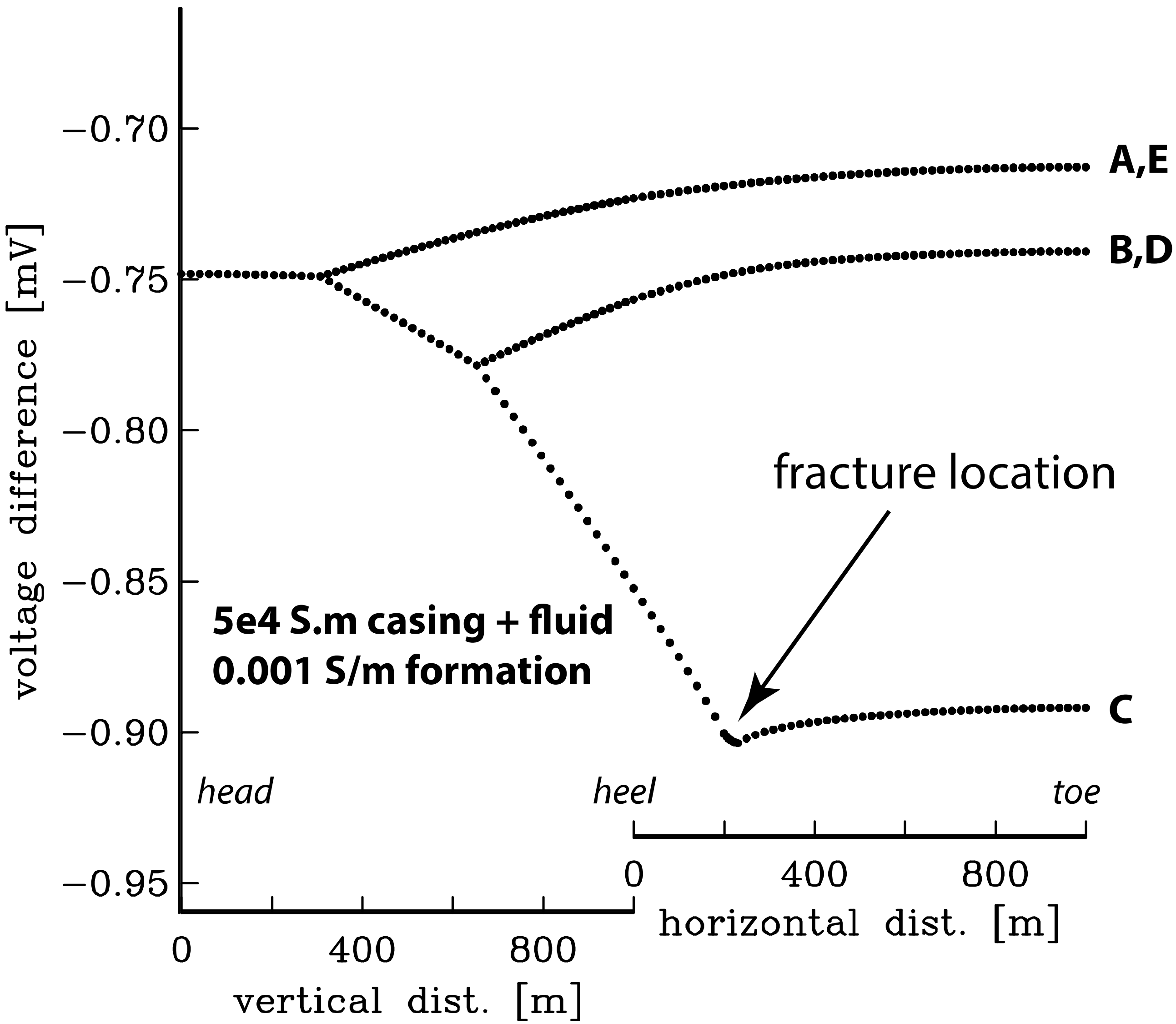
# Mesh modification for perfect resistors

The ”tear” representing the thin resistor is doubly discretized, with one set of nodes corresponding to tets on one side of the tear, and second set for tets on the other side.  Still, the surface is infinitely thin and we avoid extreme discretization of a thin, but finite thickness “slab” filled with millions of tiny tets.

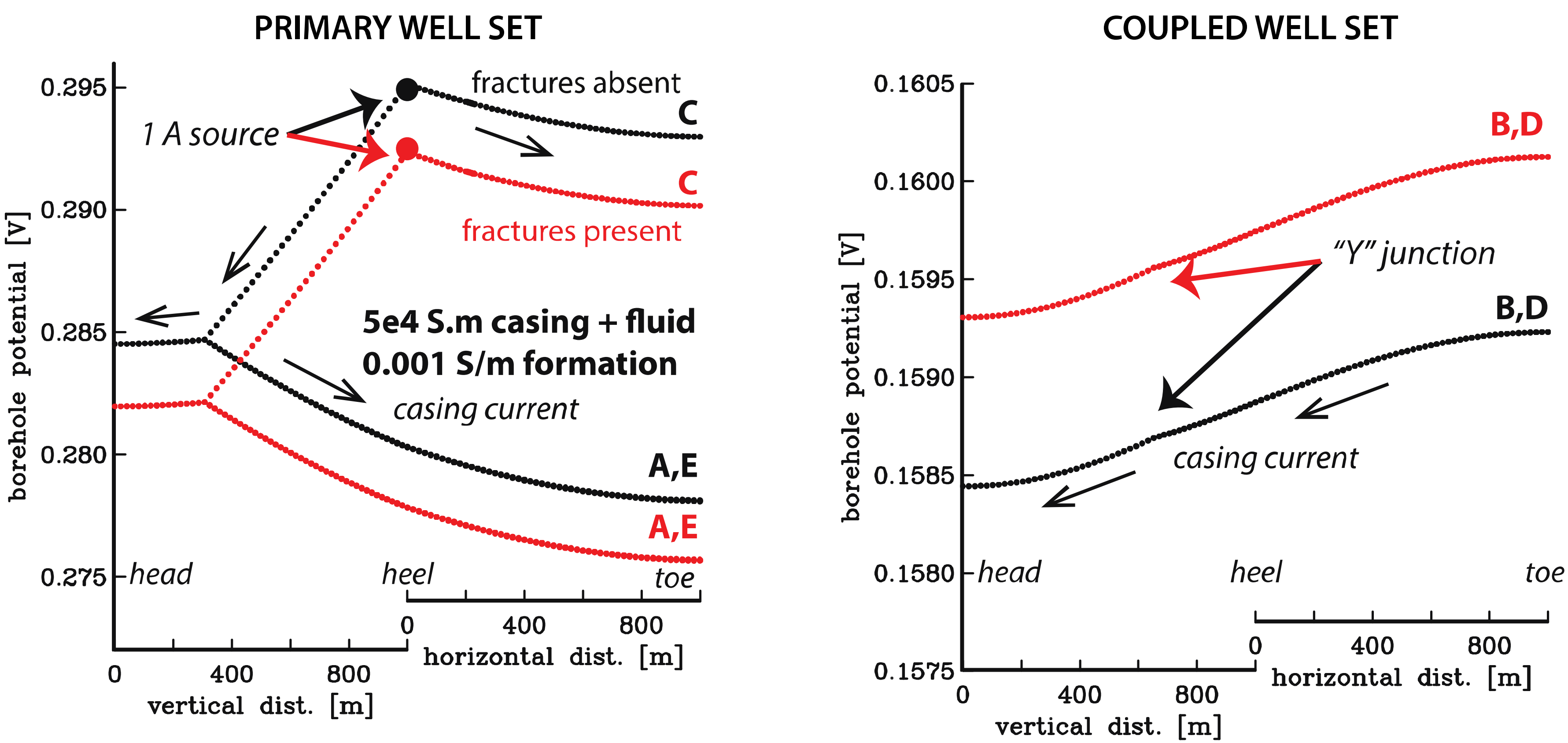
## Mesh Details for Tear Model



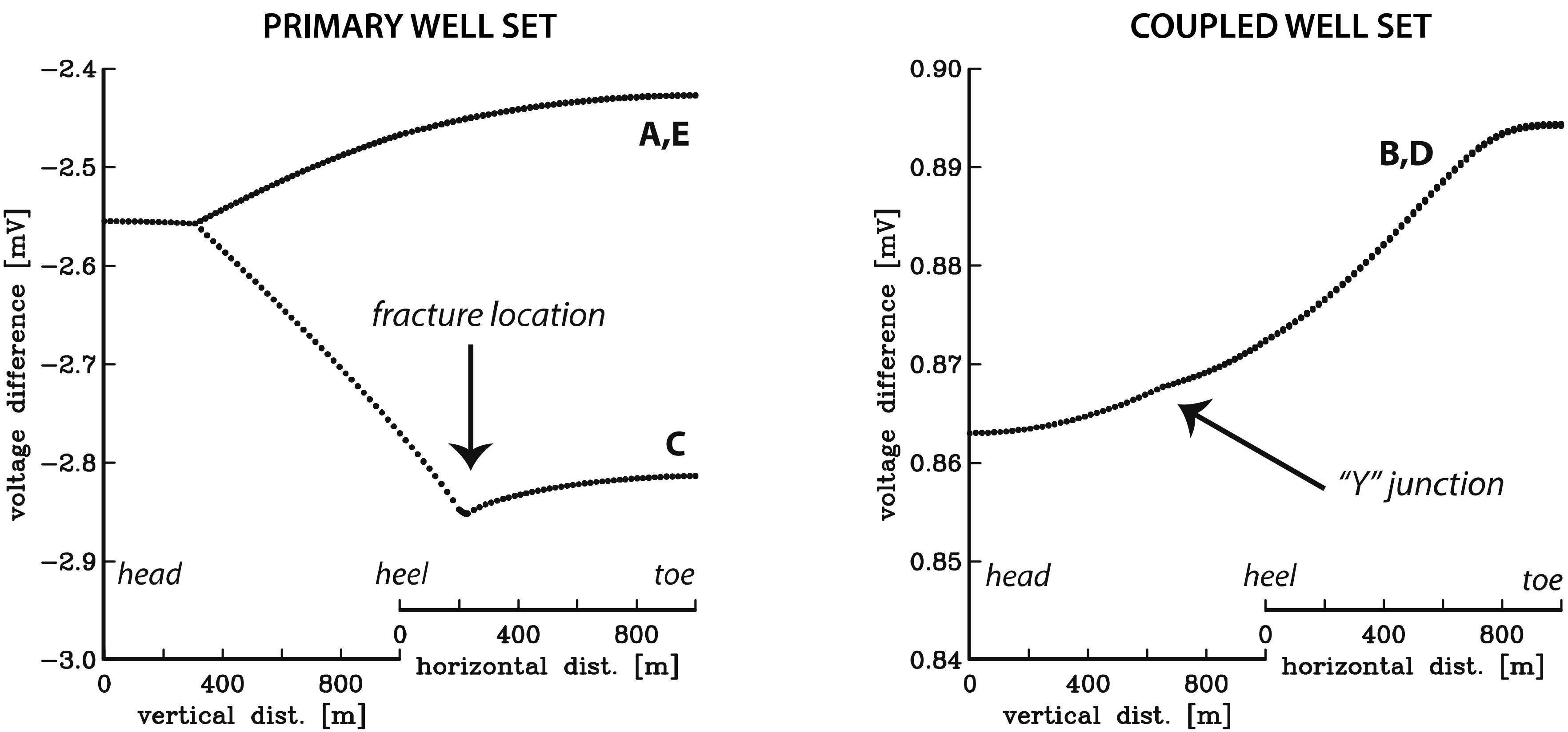
# 4D time-lapse differences of casing potential due to introduced fractures.



What happens when some of the horizontals aren't in direct electrical contact? I.e. passive coupling between all the steel in the ground.

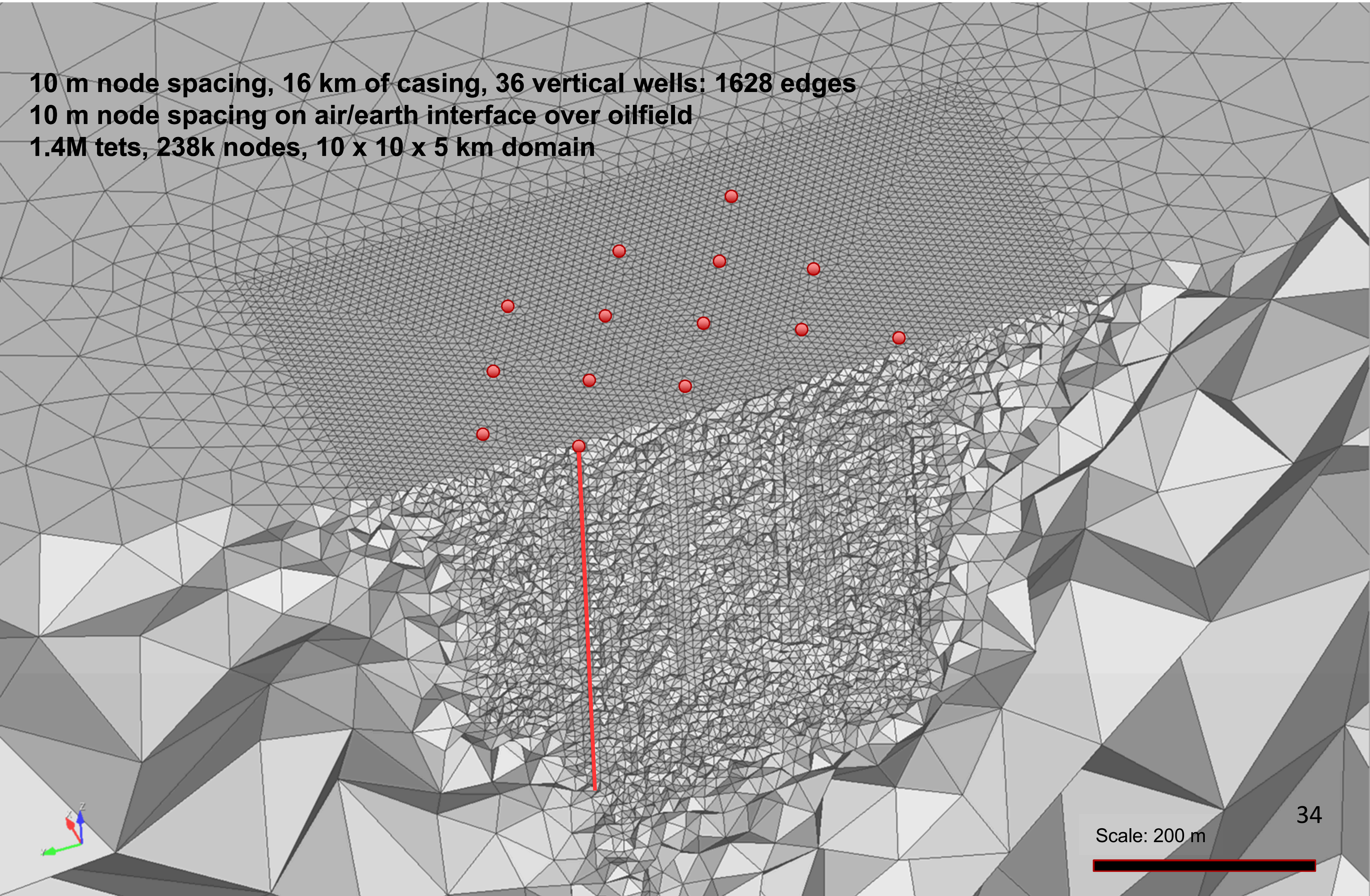


# 4D differences of casing potential for passively coupled casings.



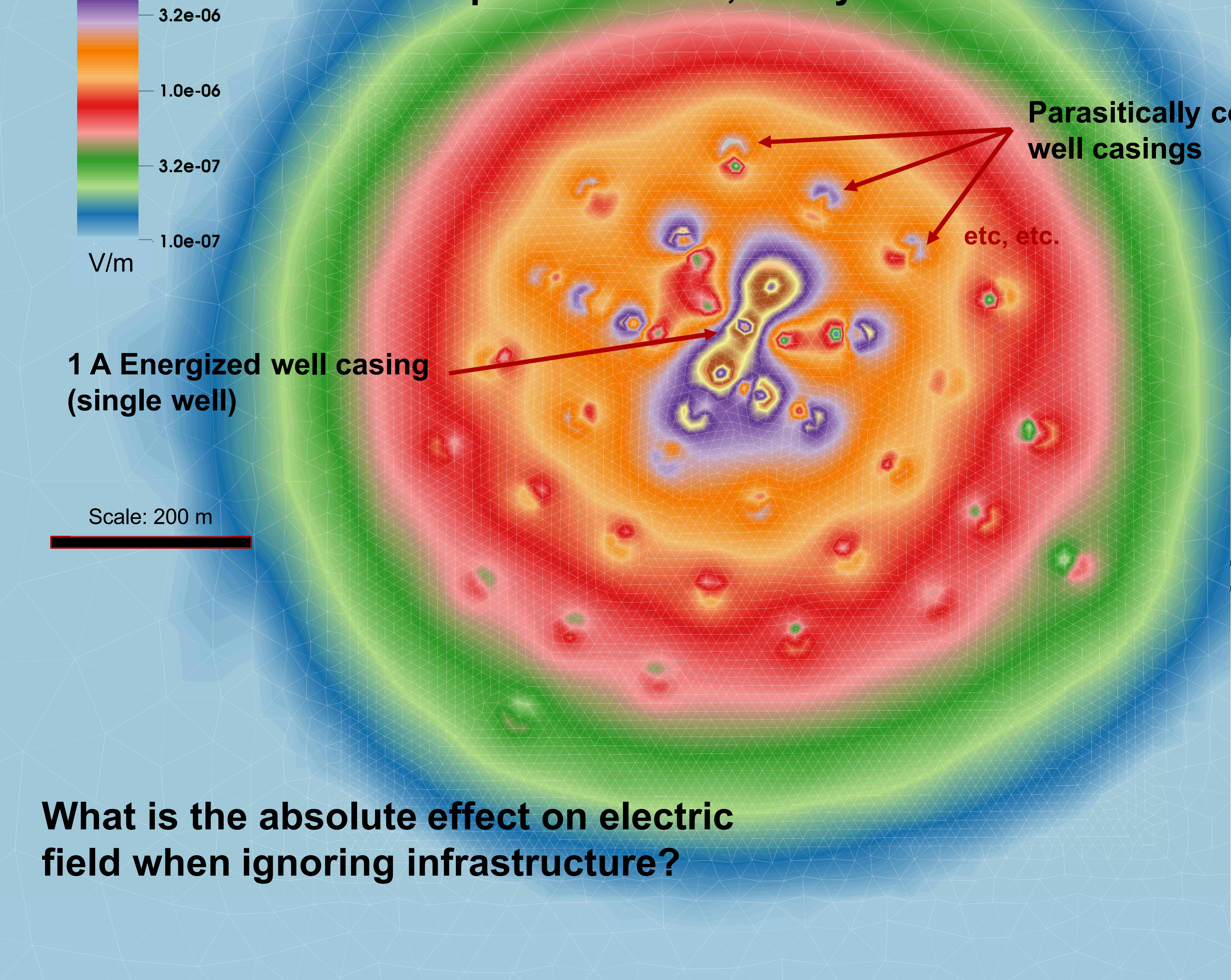
# Real World Example 1: Shallow, heavy oil reservoir.

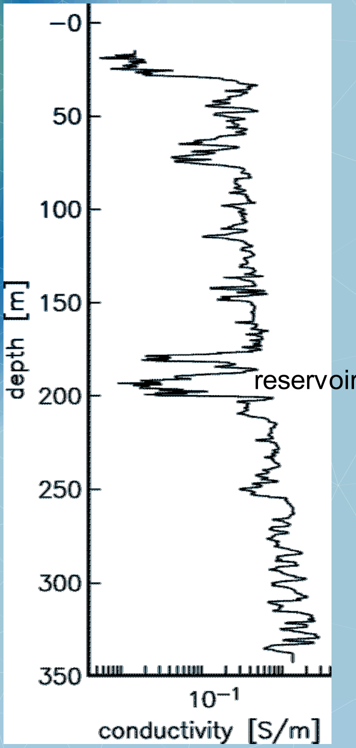
10 m node spacing, 16 km of casing, 36 vertical wells: 1628 edges
10 m node spacing on air/earth interface over oilfield
1.4M tets, 238k nodes, 10 x 10 x 5 km domain



# ALL WELLS vs SINGLE WELL

## Example 1: Shallow, heavy oil reservoir.



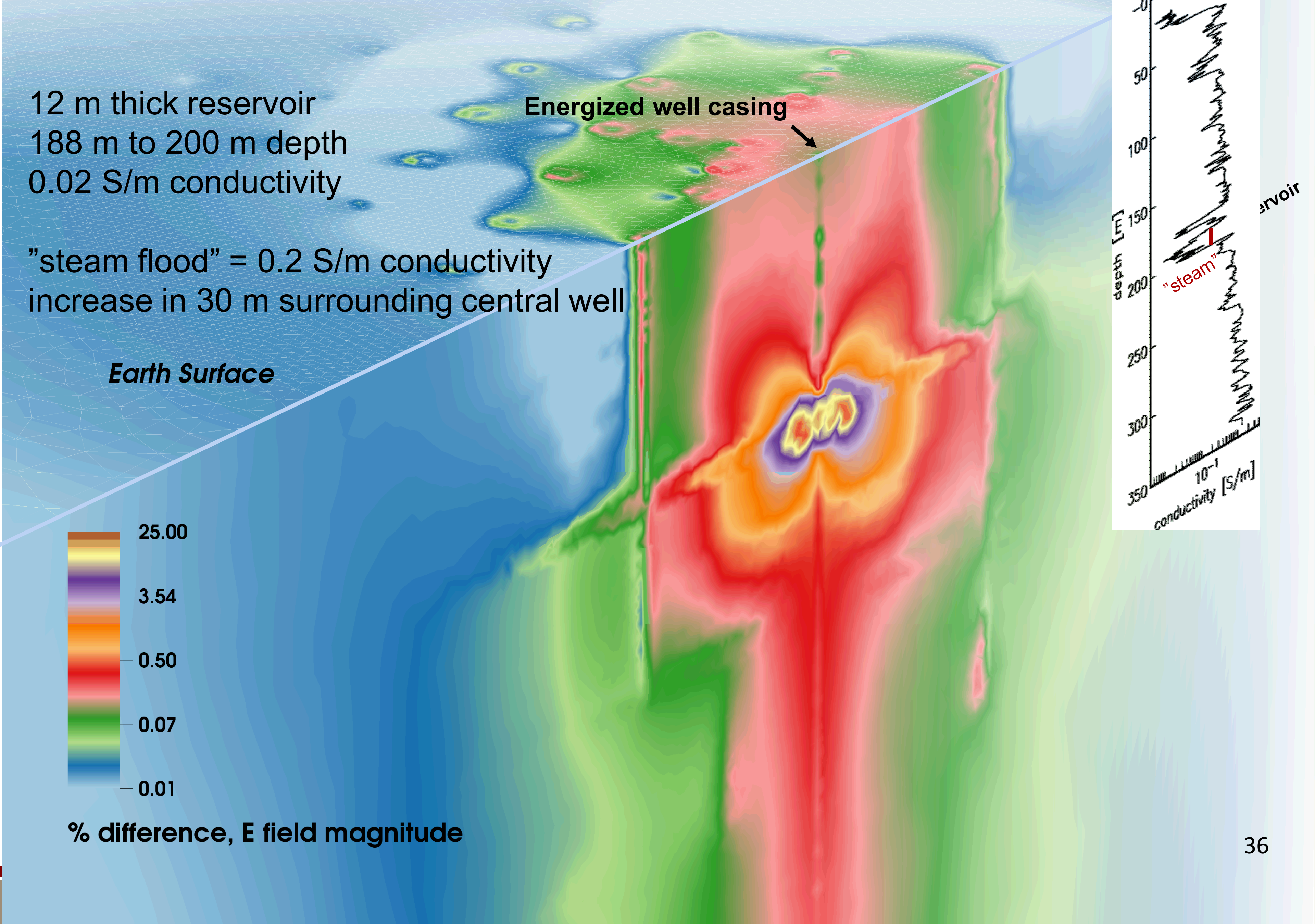




**What is the absolute effect on electric field when ignoring infrastructure?**

# Full clutter + steam injection: Electric fields

12 m thick reservoir
188 m to 200 m depth
0.02 S/m conductivity

”steam flood” = 0.2 S/m conductivity
increase in 30 m surrounding central well



# ALL WELLS vs SINGLE WELL

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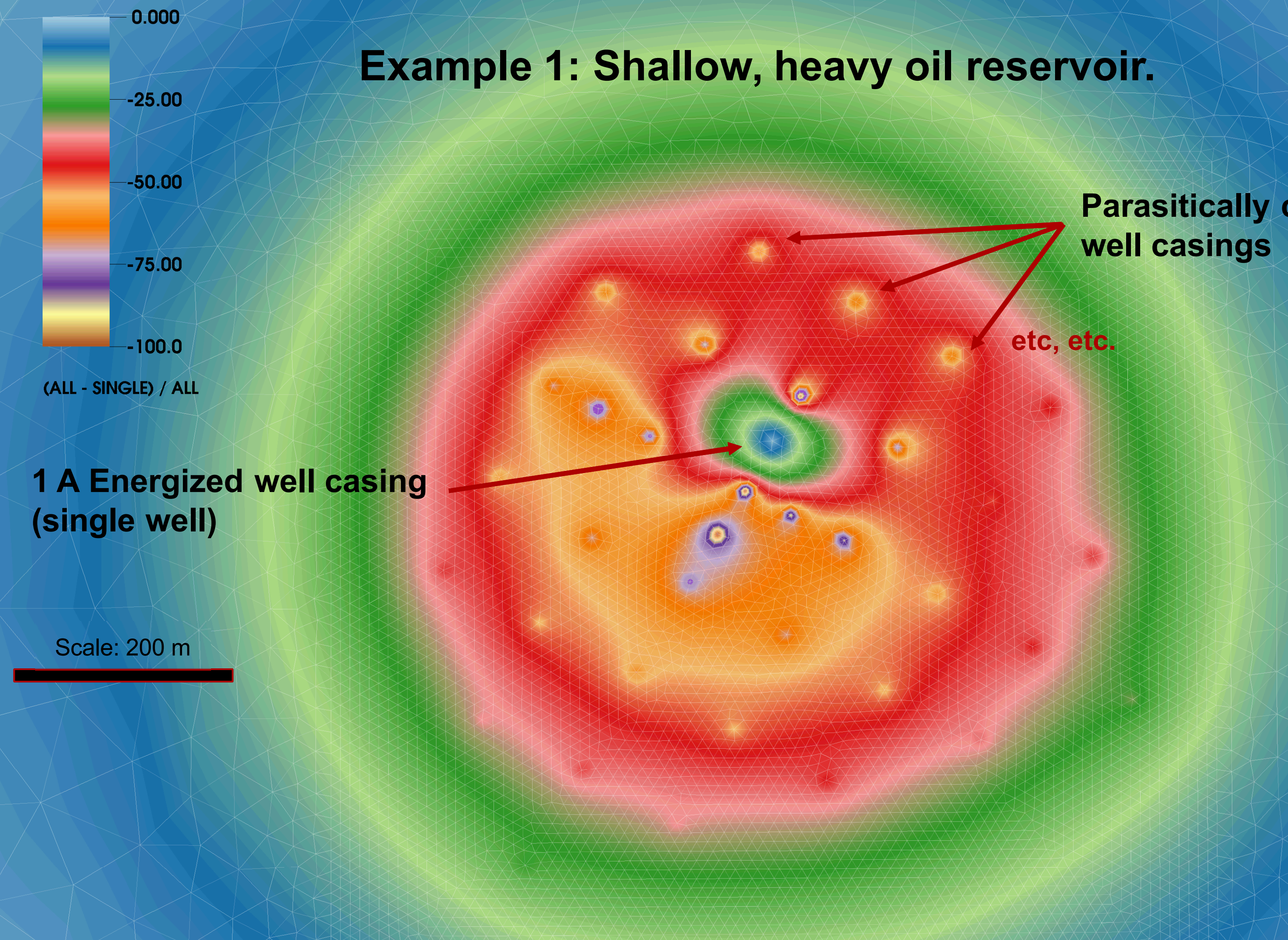
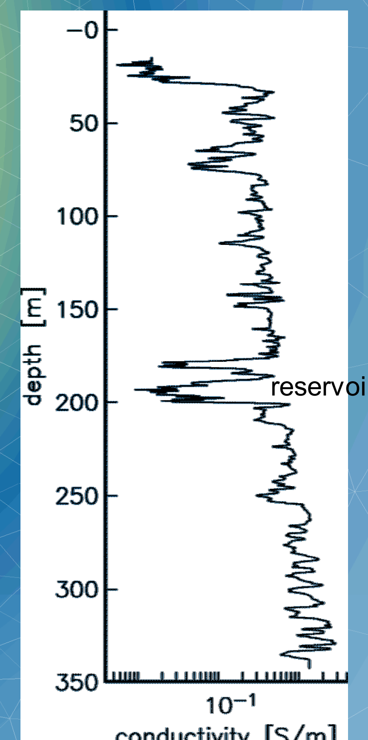
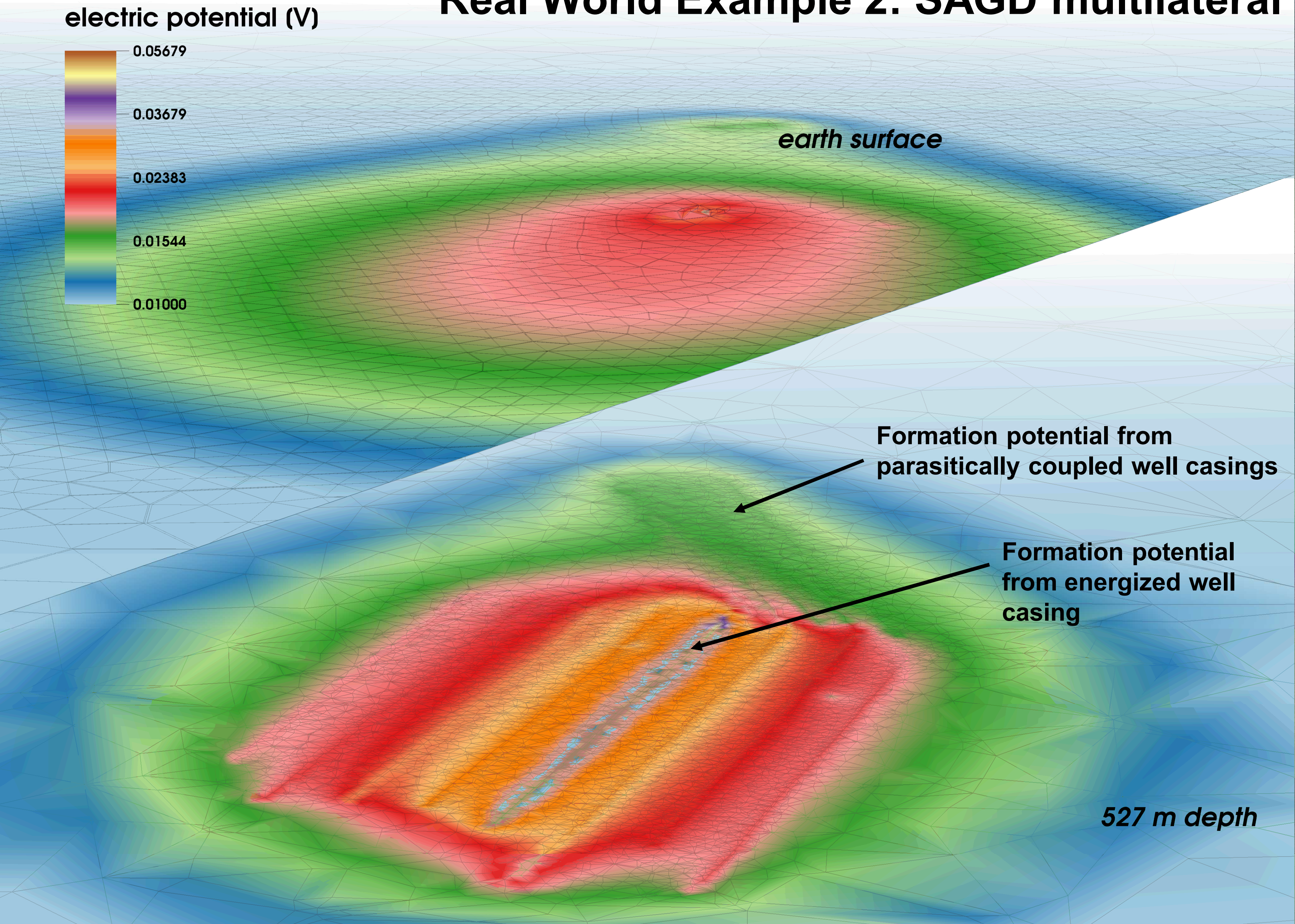


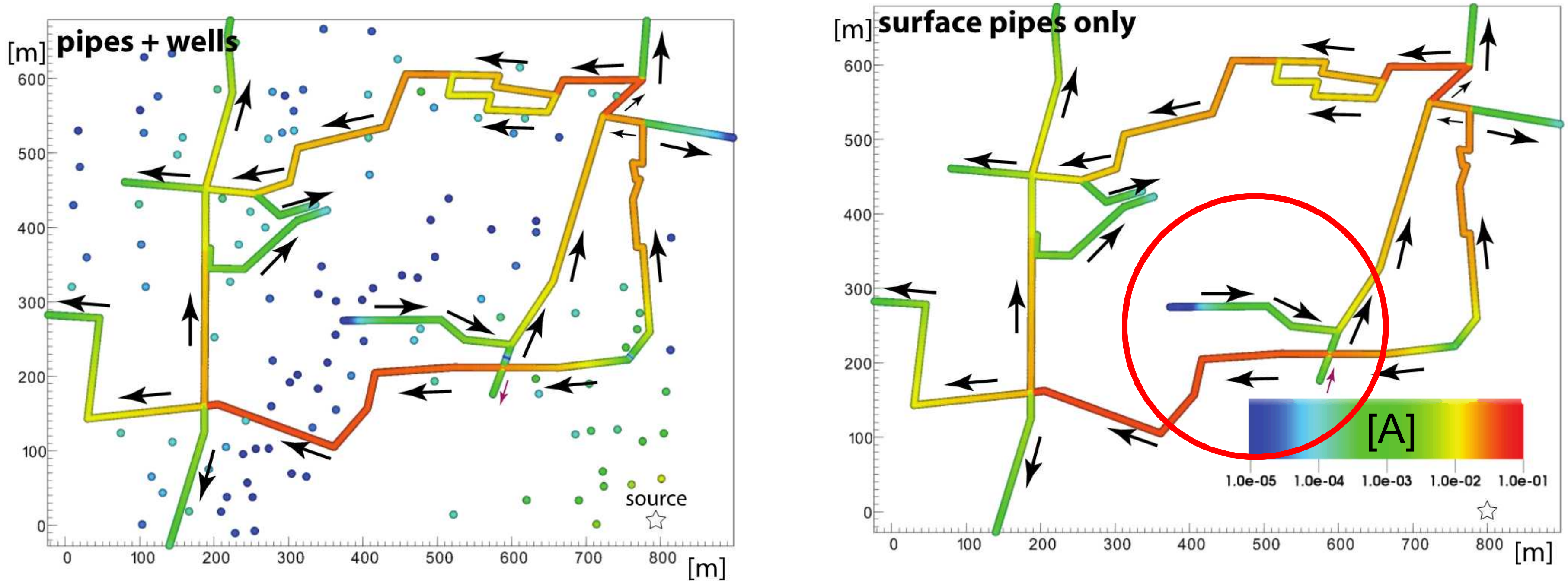

**What is the relative effect on electrostatic potential when ignoring infrastructure?**

# Real World Example 2: SAGD multilateral



# Real World Example 3: Casing + surface infrastructure

Longitudinal current (colors + arrows) for full casing/pipeline coupling (left) and pipeline alone (right).

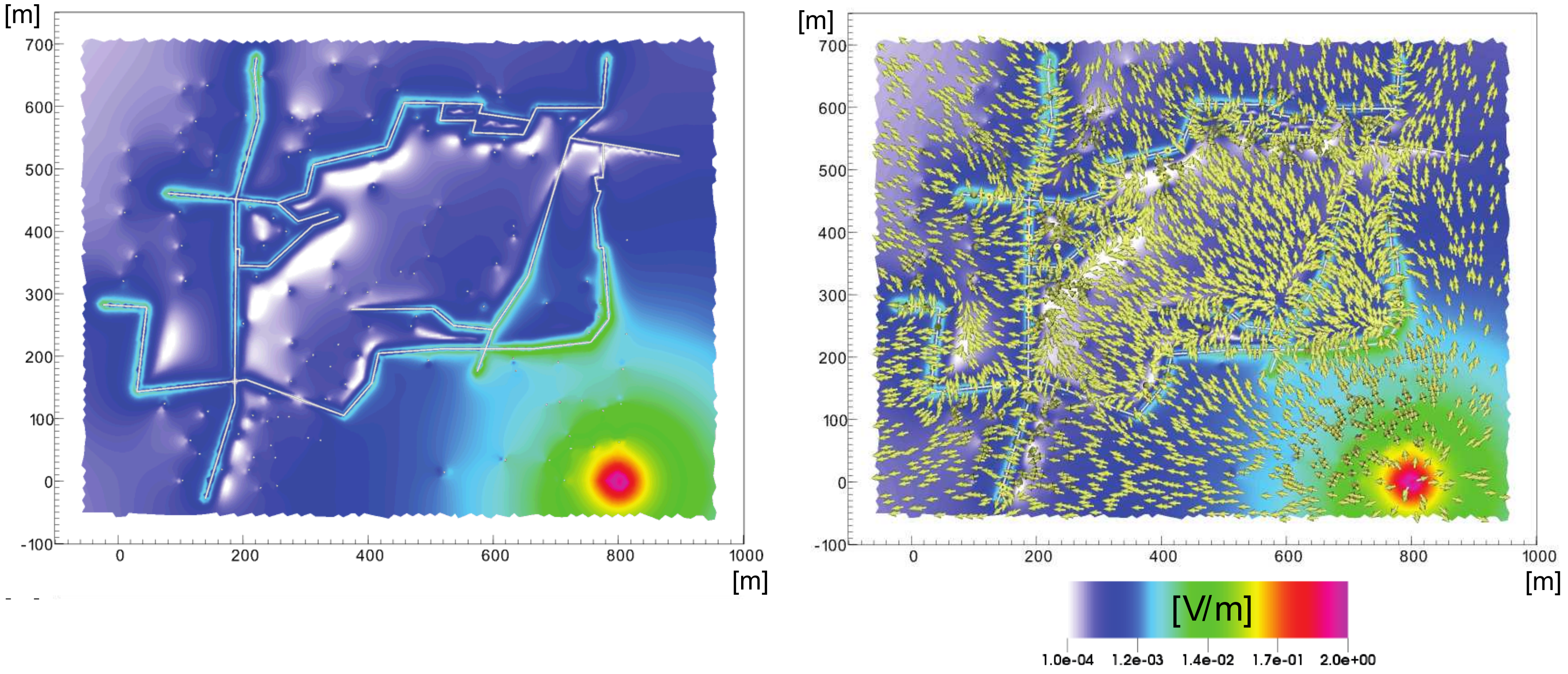





Weiss and Wilson (2017)

# Real World Example 3: Casing + surface infrastructure

Surface electric field for fully coupled model. Amplitude only (left), amplitude + direction (right)



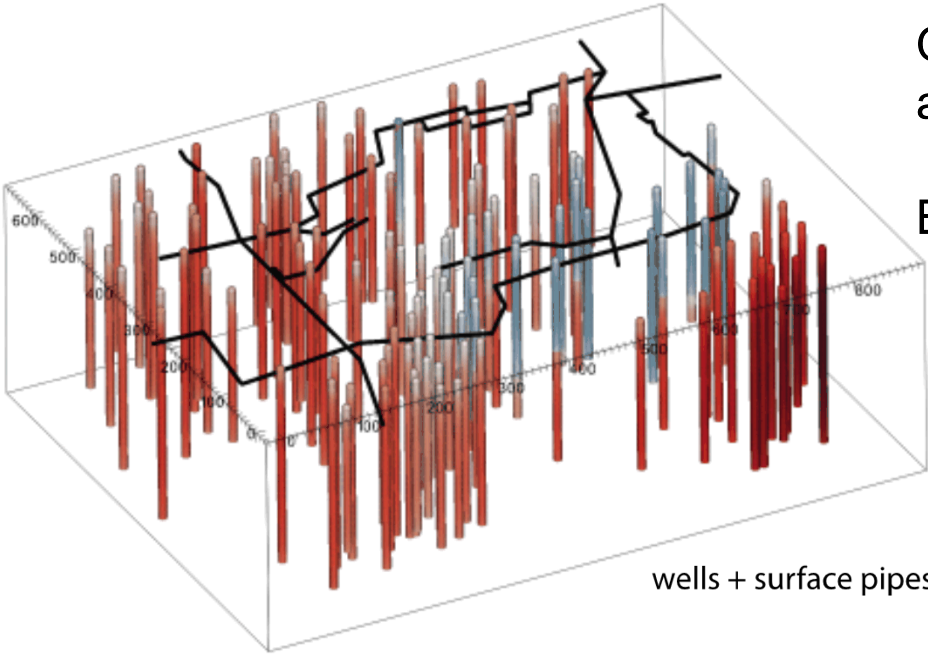



Weiss and Wilson (2017)

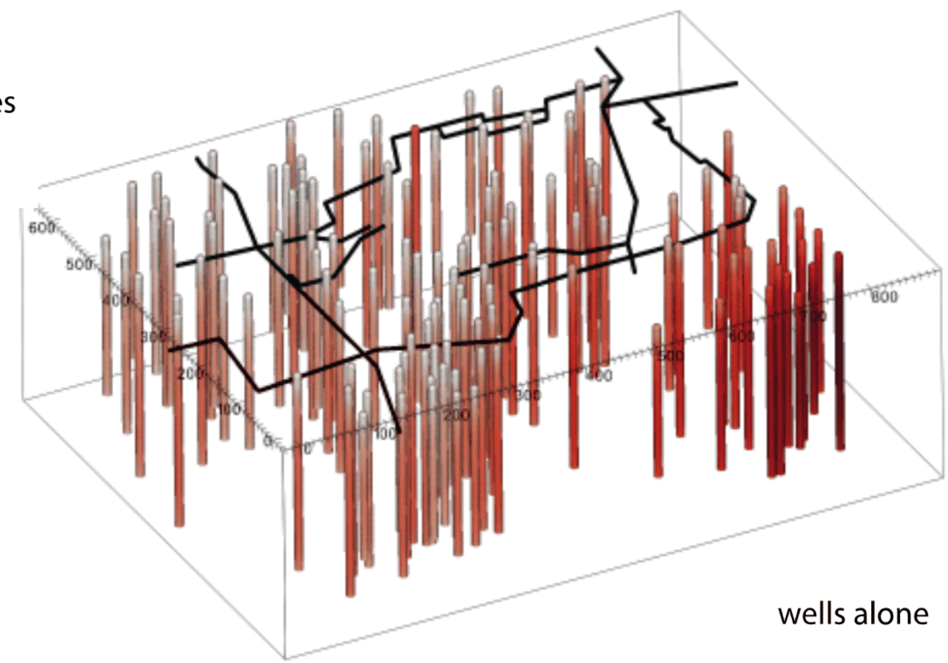
# Kern River Formation Site

QUESTION: How does pipeline coupling affect casing current direction?

Blue, upward current; red, downward.




wells + surface pipes




wells alone