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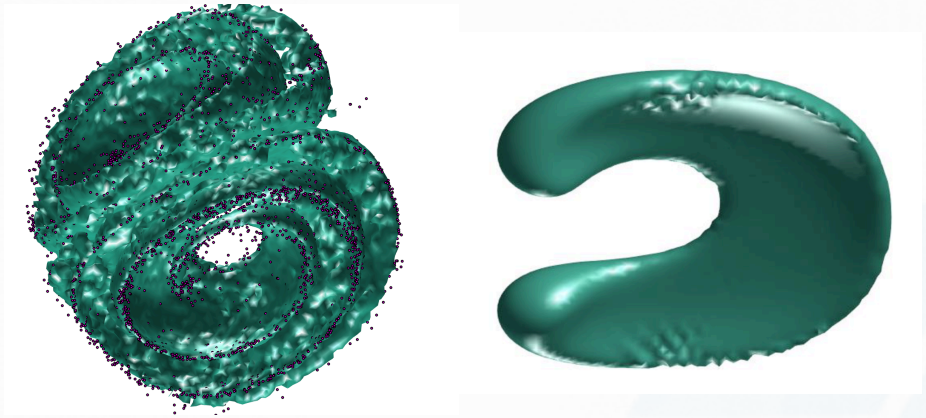

An Interpolative Particle Level Set Method for Interfacial Physics

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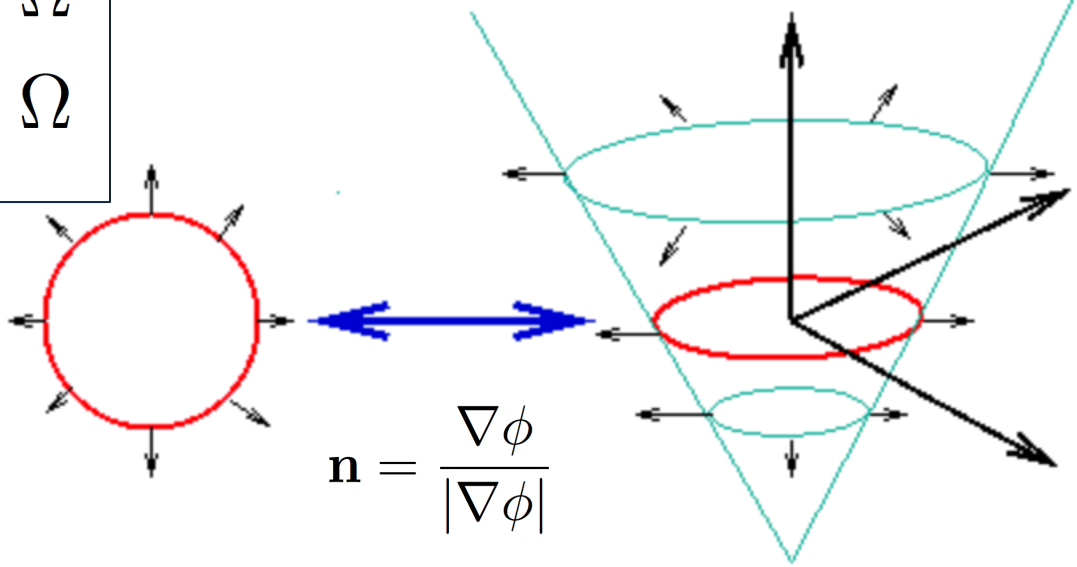
Interface Method Options

- Interface capturing (Eulerian, e.g. level set methods)
  - ✓ Natural merging and pinch-off
  - ✓ Normal vector and curvature calculations
  - x Mass conservation problems
  - x Limited by grid size
- Interface tracking (Lagrangian particle methods)
  - ✓ Conservative by design
  - ✓ Excellent at resolving fine scale dynamics
  - x No connectivity/difficult to define normal vector/curvature
  - x Needs reseeding under distorted velocity conditions

Level set (signed distance) method

$$\phi(\mathbf{x},t) > 0 \quad \text{for} \quad \mathbf{x} \in \Omega$$
$$\phi(\mathbf{x},t) \leq 0 \quad \text{for} \quad \mathbf{x} \notin \Omega$$



- 5th order HJ-WENO scheme for the gradient operator
- 3nd order TVD RK for the time derivative

Re-initialization equation

$$\frac{\partial\phi}{\partial\tau} + S(\phi_0)(|\nabla\phi| - 1) = 0$$

$$S(\phi_0) = \frac{\phi_0}{\sqrt{\phi_0^2 + (\Delta x)^2}}.$$

Hybrid particle-level set method

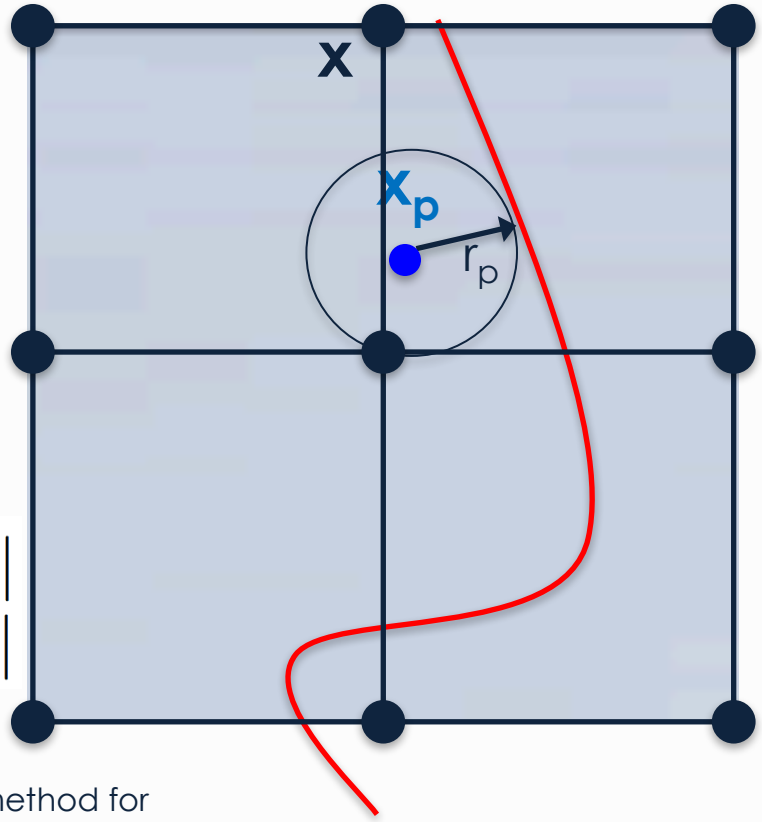
- Particles are placed near the interface and initialized with a sign and distance from the interface
- This information is used to update the level set field

$$\phi_p(\mathbf{x}) = s_p(r_p \pm |\mathbf{x} - \mathbf{x}_p|)$$

$$\phi^+(\mathbf{x}) = \max_{p \in E^+}(\phi_p, \phi)$$

$$\phi^-(\mathbf{x}) = \min_{p \in E^-}(\phi_p, \phi)$$

$$\phi(\mathbf{x}) = \begin{cases} \phi^+(\mathbf{x}) & \text{if} \quad |\phi^+(\mathbf{x})| \leq |\phi^-(\mathbf{x})| \\ \phi^-(\mathbf{x}) & \text{if} \quad |\phi^+(\mathbf{x})| > |\phi^-(\mathbf{x})| \end{cases}$$



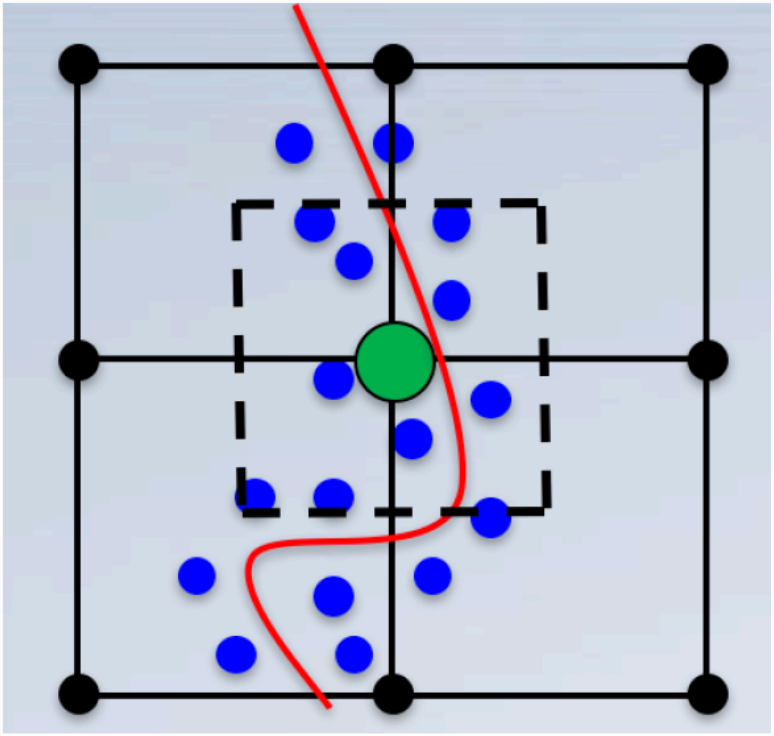
D. Enright, R. Fedkiw, J. Ferziger, I. Mitchell, A hybrid particle level set method for improved interface capturing, Journal of Computational Physics 183 (1) (2002) 83–116.

Interpolative Particle Level Set Method

- Particles are placed near the interface and intialized with a signed distance from the interface (equivalent to the level set value)
- Particles are used as a form of Lagrangian refinement around the interface
- We use (bi/tri) linear interpolation to update the 'coarse' level set field on the grid using the 'fine' level set field at particle locations

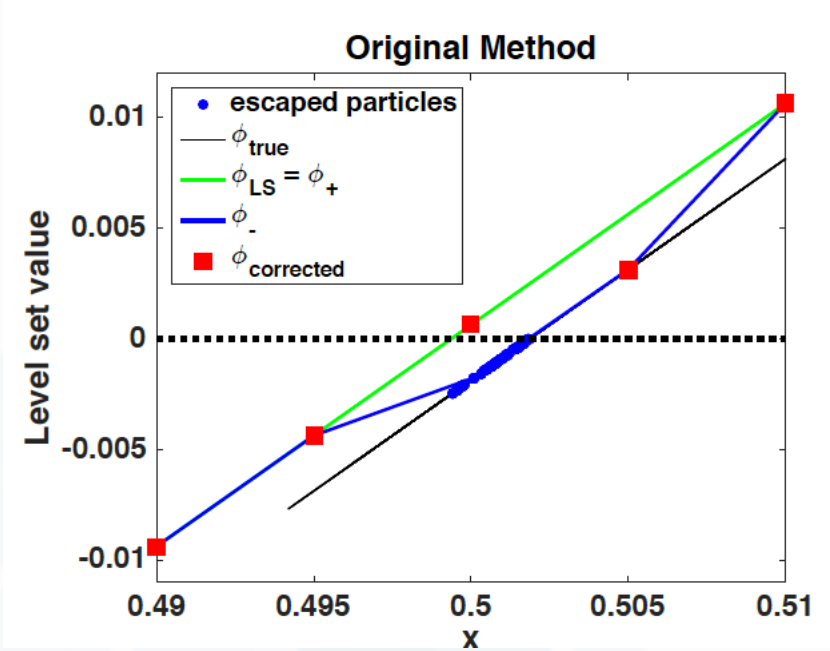
$$\phi(\mathbf{x}) \approx a_0 + a_1 x + a_2 y + a_3 xy$$

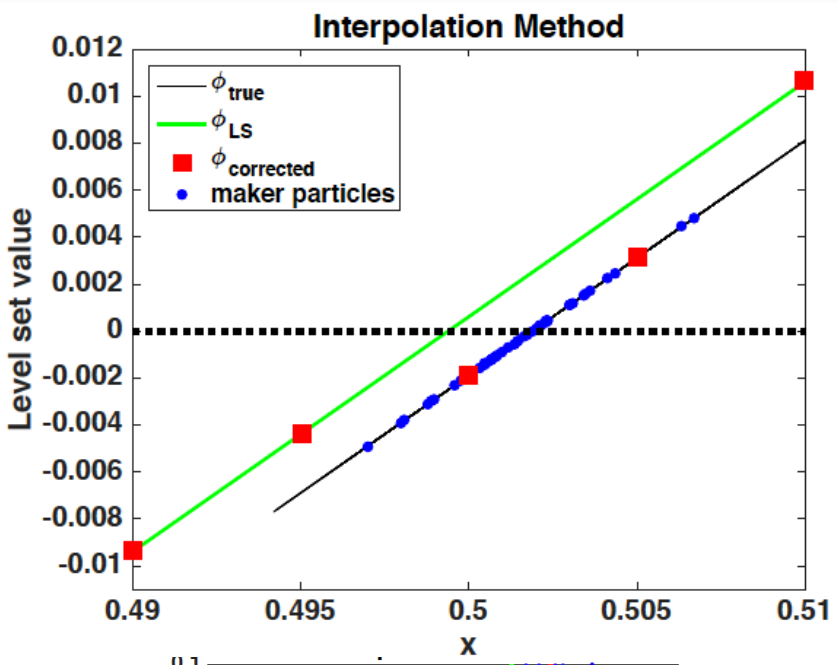
$$\begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_1 & y_2 & x_1 y_2 \\ 1 & x_2 & y_1 & x_2 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} f(Q_{11}) \\ f(Q_{12}) \\ f(Q_{21}) \\ f(Q_{22}) \end{bmatrix}$$

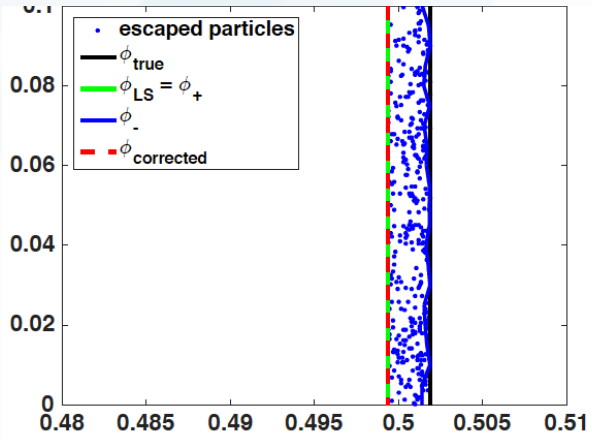


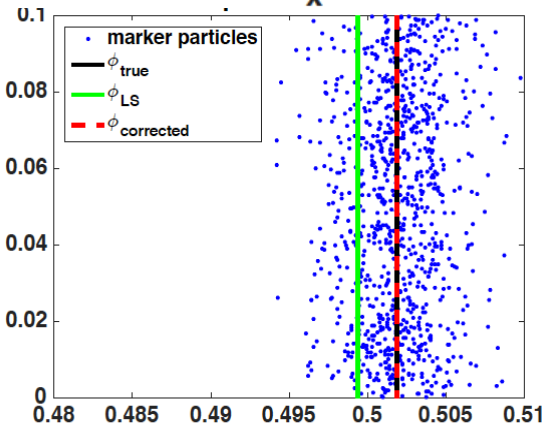
Erickson, Morris, Poliakoff, Templeton, "An interpolative particle level set method," in preparation.

Particle Level Set Method[1] versus Interpolative PLS[2]









[1] Enright, Fedkiw, Ferziger, Mitchell, "A hybrid particle level set method for improved interface capturing," J. Comp. Phys. (2002).
[2] Erickson, Morris, Poliakoff, Templeton, "An interpolative particle level set method," submitted to JCP.

Particle Level Set Method[1] versus Interpolative PLS[2]

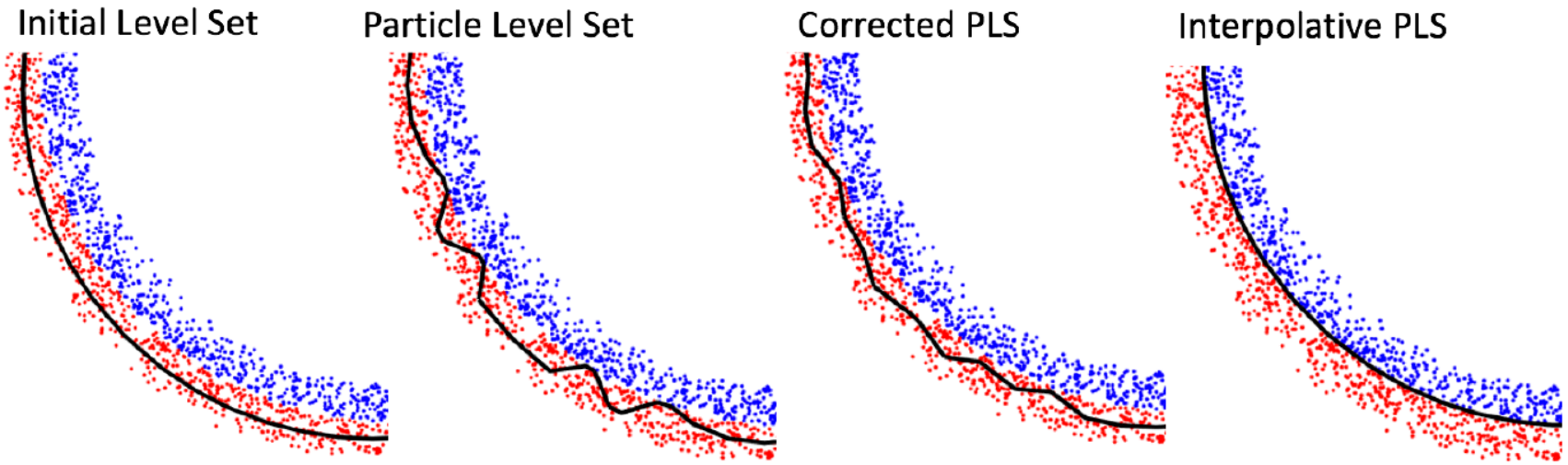
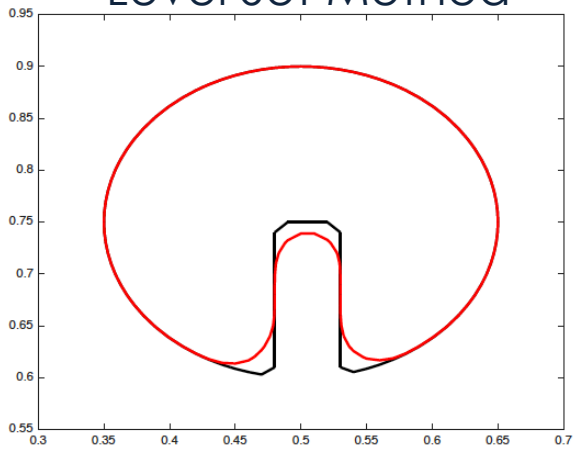


Figure 5: A close up view of the circle test for the original particle level set method versus and the corrected PLS versus our interpolative particle level set method. The original and corrected particle level set methods have a more irregular interface after the correction step due to the min/max function in Equation 16, while our interpolative method is able to fully correct the level set function in a smooth fashion.

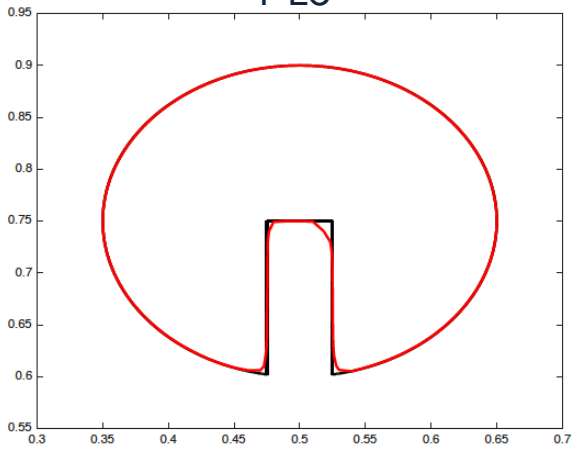
[1] Enright, Fedkiw, Ferziger, Mitchell, "A hybrid particle level set method for improved interface capturing," J. Comp. Phys. (2002).
[2] Erickson, Morris, Poliakoff, Templeton, "An interpolative particle level set method," submitted to JCP.

Particle Level Set Method[1] versus Interpolative PLS[2]

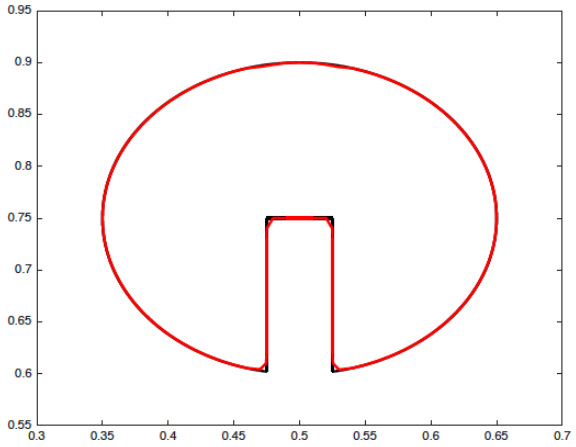
Level Set Method

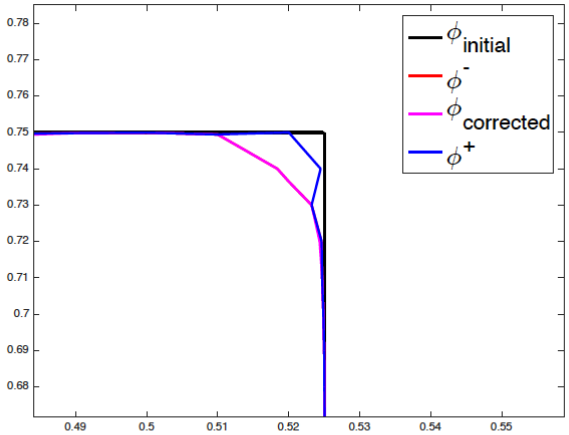


PLS



IPLS





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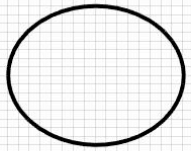
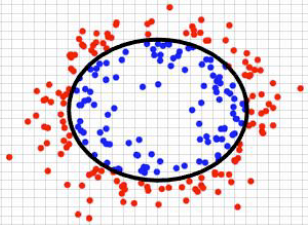
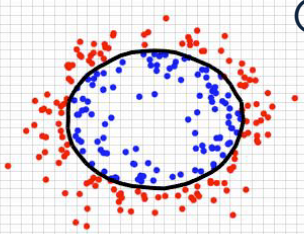
Circle in a vortex flow test for resolving thin filaments (shearing)



Level set method



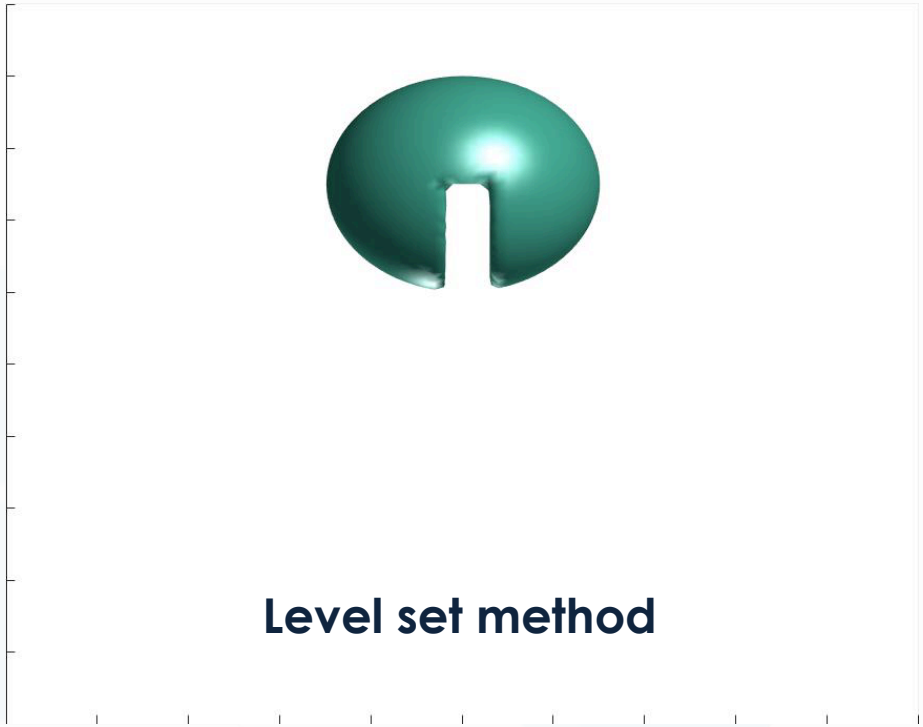
Original particle level set method



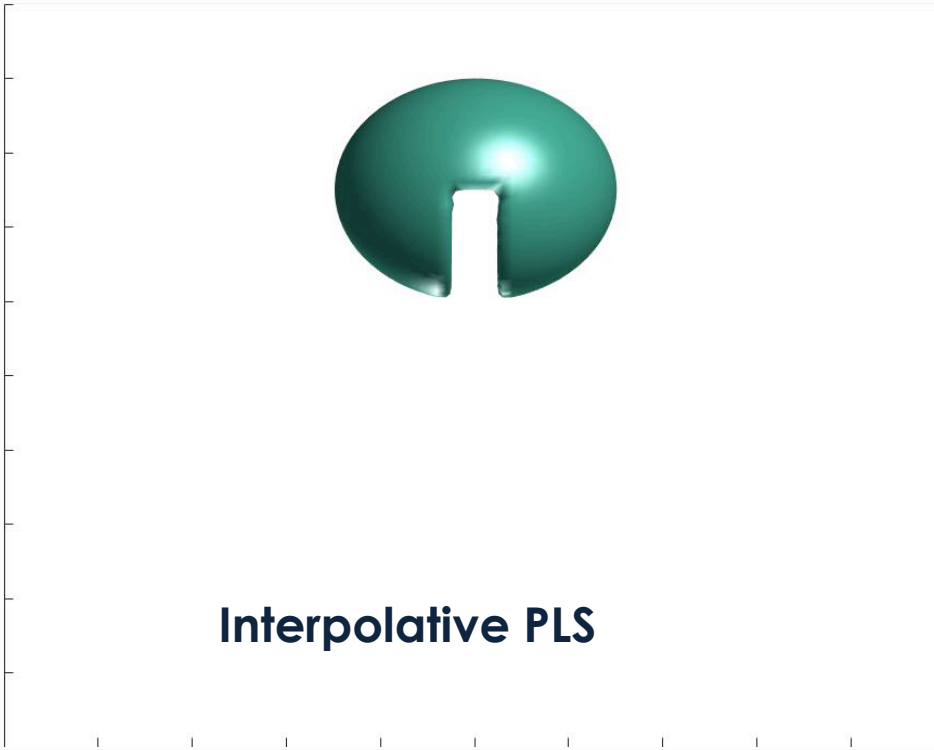
Interpolative PLS

Test for the method's ability to resolve thin filaments. (80 x 80 grid) Interpolative PLS is better able to capture the interface below the grid resolution

3D Slotted disk: Level set versus IPLS



Level set method

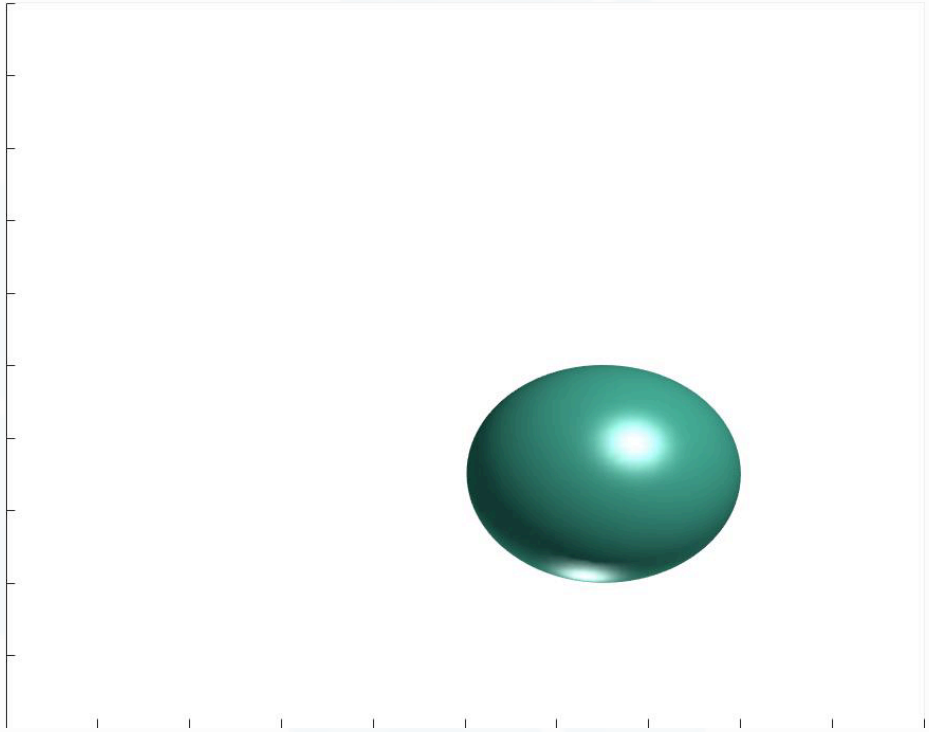


Interpolative PLS

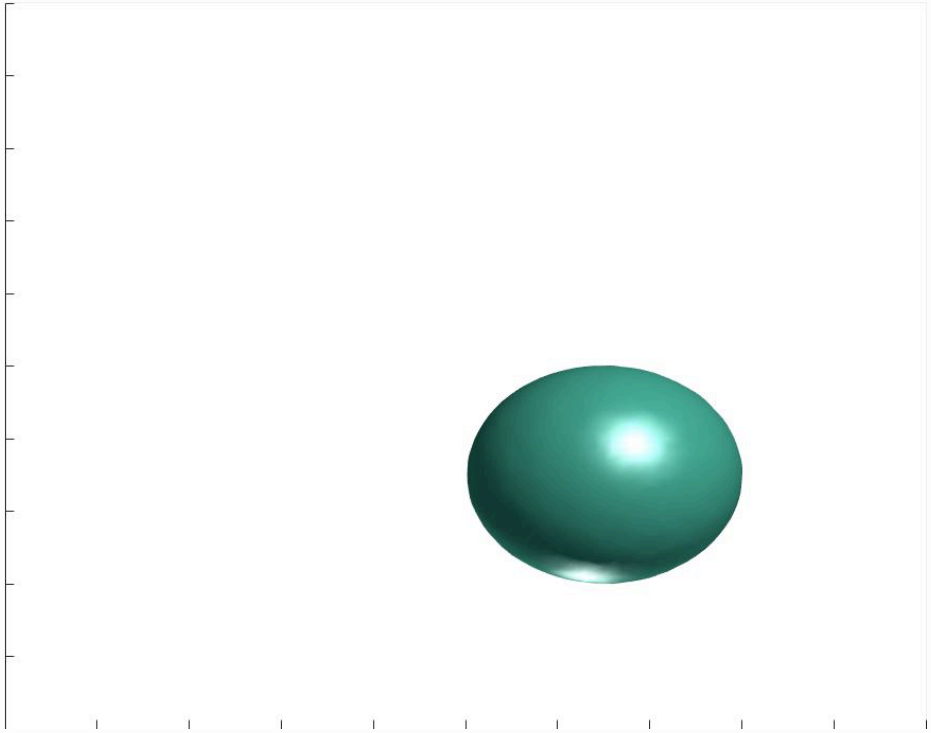
Test for the method's ability to limit the effects of numerical diffusion (100 x 100 x 100 grid)

3D vortex flow: Level set versus IPLS

Level set method



Interpolative PLS



Test for the method's ability to resolve thin filaments(100 x 100 x 100 grid)