National Nuclear Security Administration

Sandia National Laboratories

U.S. DEPARTMENT OF ENERGY

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Rate Games in Wireless Networks

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PURDUE ENGINEERING

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Introduction

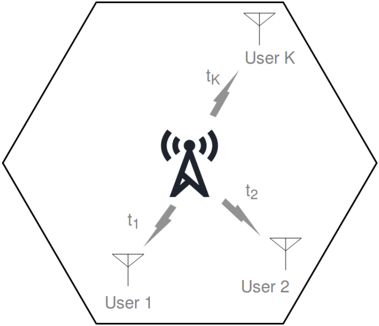
Game-theoretic models for wireless networks

Is absolute rate always an appropriate utility?

Example

Resource Scheduling in Wireless Networks

- BS seeks to maximize the sum-rate of K users
- Priority is given to the user with the best channel conditions (highest SNR)
- Users compete for highest **relative** rate



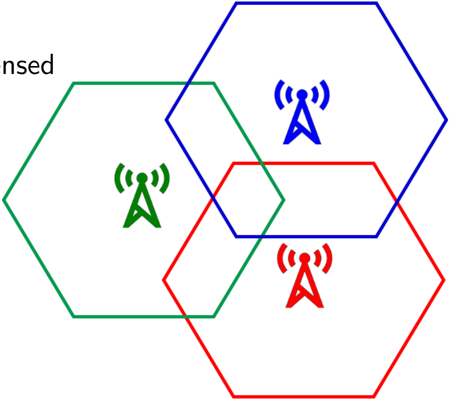
(https://www.darpa.mil/DDM _Gallery/sc-event-619-316.jpg, 2017)

Spectrum Collaboration Games

- The DARPA SC2
- Teams competes for the most reliable communication

Example

eNodeB interference in LTE Unlicensed



Problem Statement

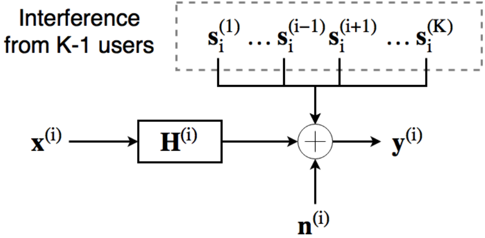
We model a game between K users in a wireless network

- Users compete for the highest rate
- User allocate a power budget between their channel inputs and interference towards other users

Game analysis

- Nash Equilibrium and stability analysis for $K = 2$
- We will show that when the game is symmetric, the sum SINR across all users goes to ∞ as K grows large.

Channel Model



Channel output for the ith user

$$\boldsymbol{y}^{(i)} = \boldsymbol{H}^{(i)}\boldsymbol{x}^{(i)} + \sum_{\substack{k=1 \\ k\neq i}}^{K} \boldsymbol{s}_i^{(k)} + \boldsymbol{n}^{(i)} \tag{1}$$

- Interference is independent of channel input
- For ease of analysis, $\boldsymbol{H}^{(i)}$ is known to all users

Optimization Problem

ith user's information rate

- Diagonalize $\boldsymbol{H}^{(i)}$ into N_i orthogonal channels
- Rate $R_i = \sum_{j=1}^{N_i} \log\left(1 + \lambda_j^{(i)}\right)$
- $\lambda_j^{(i)}$ is the ith user's SINR at the jth channel

The ith user solves the following optimization problem

$$\begin{aligned} \underset{\boldsymbol{x}^{(i)}, \boldsymbol{s}_k^{(i)}}{\text{maximize}} \quad & R_i - \max\{R_1, ..., R_{i-1}, R_{i+1}, ..., R_K\} \\ \text{subject to} \quad & \underbrace{\mathbb{E}[\boldsymbol{x}^{(i)\intercal}\boldsymbol{x}^{(i)}]}_{P_X^{(i)}\text{: sum channel input power}} + \underbrace{\sum_{\substack{k=1 \\ k \neq i}}^{K} \mathbb{E}[\boldsymbol{s}_k^{(i)\intercal}\boldsymbol{s}_k^{(i)}]}_{P_I^{(i)}\text{: sum interference input power}} \leq P_T^{(i)} \end{aligned} \quad (2)$$

Optimization Problem

We already know optimal solutions for inputs when $P_X^{(i)}$, $P_I^{(i)}$ fixed

- Waterfill $P_X^{(i)}$ over channel inputs
- $\mathbb{E}[\boldsymbol{s}_k^{(i)\intercal}\boldsymbol{s}_k^{(i)}] \propto P_X^{(k)}$ (Jorswieck & Boche)

$$\begin{aligned} \underset{P_X^{(i)},P_I^{(i)}}{\text{maximize}} \quad & R_i - \max\{R_1, ..., R_{i-1}, R_{i+1}, ..., R_K\} \\ \text{subject to} \quad & P_X^{(i)} + P_I^{(i)} \leq P_T^{(i)} \end{aligned} \tag{3}$$

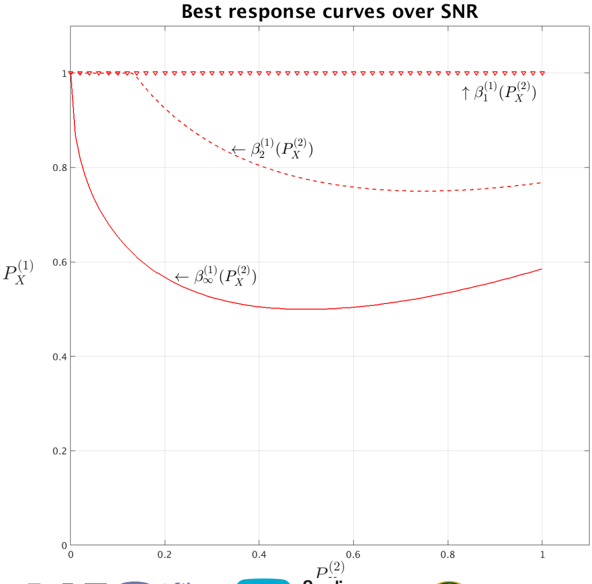
Optimization Problem

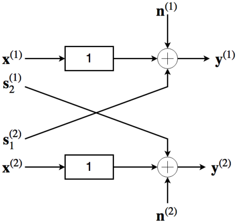
We define the ith user's best response as

$$\beta_{SNR}^{(i)}(P_X^{(-i)}) = \arg \max_{P_X^{(i)} \leq P_T^{(i)}} \{R_i - \max[R_1, ..., R_{i-1}, R_{i+1}, ..., R_K]\}.$$

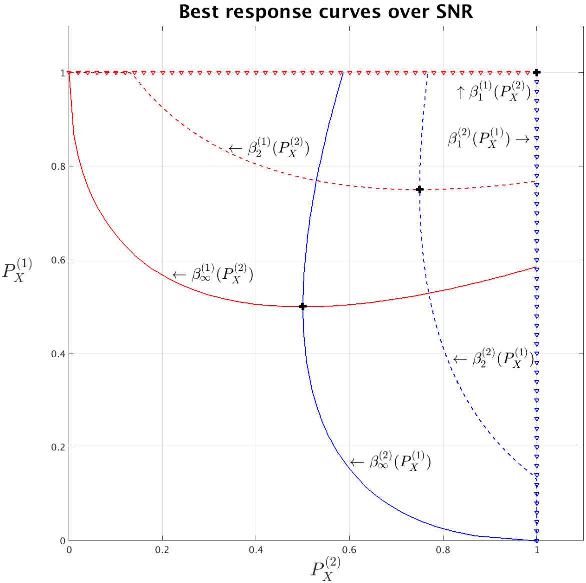
- The ith user's utility is a function of the other users' sum channel input power
- It will be useful to compare the best response of the ith user with the SNR averaged over N_i channels.

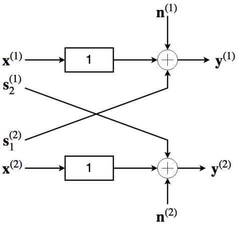
Two user game $(K = 2)$





Two user game $(K = 2)$





Two user game $(K = 2)$

Theorem

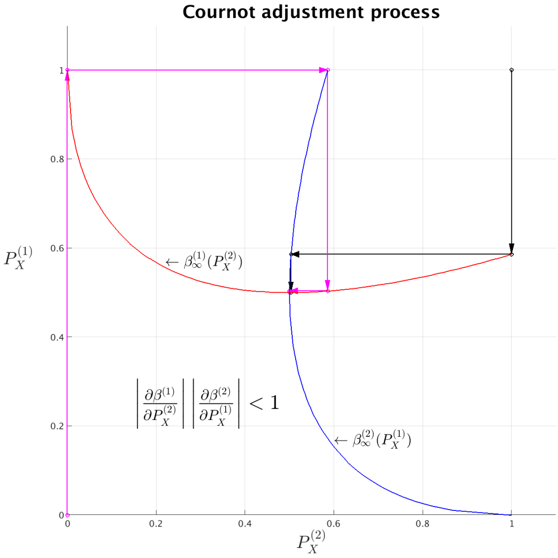
For $K = 2$, the unique Nash Equilibrium occurs when the ith user's sum channel input power is

$$P_X^{(i)*} = \frac{N_i(N_j\sigma_n^2 + P_T^{(i)})}{2N_iN_j\sigma_n^2 + N_iP_T^{(i)} + N_jP_T^{(j)}} \ , i \neq j. \qquad (4)$$

Proof: $P_X^{(i)*}$ is the only fixed point in the best response curves. ■

How "good" of a prediction is the Nash Equilibrium?

Asymptotic Stability of Solution



Two user game $(K = 2)$

Consider the symmetric case

- $P_T^{(1)} = P_T^{(2)} = P_T$
- $N_1 = N_2 = N$

At high SNR $(\sigma_n^2 \to 0)$, $P_X^{(1)} = P_X^{(2)} = 1/2$

- SINR $= 1 << $ Optimal
- We can do better

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Symmetric K user game

We make the game symmetric such that each user has identical channel structure and power budget

Theorem

In a symmetric K user game at high SNR, the ith user's sum interference input power at the Nash Equilibrium converges s.t.

$$P_I^{(i)*} \xrightarrow{K\to\infty} 0. \tag{5}$$

Proof: At high SNR, the ith user's utility is concave over a convex power constraint. We can verify that $P_X^{(i)*} = (K-1)/K$ satisfies the KKT conditions of the ith user's convex optimization problem. ∎