SAND2017-11605C

# Dynamic Substructuring Applied to the Decoupling of Acoustic-Structure System

Ryan Schultz
Structural Dynamics Department
Sandia National Laboratories[1]
P.O. box 5800 – MS0557
Albuquerque, NM, 87185
rschult@sandia.gov

R. Benjamin Davis
College of Engineering
University of Georgia
ben.davis@uga.edu

**Abstract**
Dynamic substructuring allows analysts to combine component structural dynamics models into a system-level model. An analogous process can be used to subtract off a component model from the system model. Here, this decoupling approach is adapted for use with acoustic-structure systems. This has applications to model validation where structural models are correlated to test data. When acoustic cavities are present, coupling with the acoustic subsystem can confound the test response and inhibit the model validation effort. Here, the Transmission Simulator method of Component Mode Synthesis is applied to extract the structure-only response from a coupled structural-acoustic system using the acoustic cavity modes as the subtracted component. This approach is demonstrated on a simple plate-box system using component modes from analytical and numerical models.

**Keywords:** acoustoelasticity, component mode synthesis, structural acoustics, transmission simulator, substructuring

## 1 Introduction

Often, modal test results are used to correlate to or update finite element (FE) models of structures. Thus, it is important that the model matches the test conditions (and vice versa). One aspect seldom considered is the possibility that the air inside hollow structures could be modifying the structural modes measured in test. This modification of the structural response is due to coupling of the structure and acoustic modes, dubbed acoustoelasticity. For a system to exhibit acoustoelastic coupling, the modes of the hollow structure need to match closely to the modes of the internal air cavity in both shape and frequency [1]. When the modes are very similar in shape and close in frequency, the result is a split of the structural mode into two peaks, akin to a tuned absorber. While it is possible to prevent the acoustoelastic coupling in test by modifying the boundary conditions or adding damping to the air cavity, at times the problem of acoustoelastic coupling is not known until after the test [2]. As such, a method for decoupling the structure from the air is desired.

Component mode synthesis (CMS) has long been used to couple modal models of structures [3]. Similarly, related methods, such as the Transmission Simulator method, have been developed to remove components from system models. The objective of this work is to first apply CMS to couple a structural-acoustic plate-box system to get a coupled system with acoustoelastic coupling traits. Next, the Transmission Simulator method will be applied to the coupled system to decouple, that is, remove the acoustic component and result in a structure-only model. Comparisons will be made against direct numerical simulation of a finite element model of the plate-box system.

---

[1] Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525

## 2 Review of Component Mode Synthesis Methods

CMS is used to connect modal models of components to form a system model with accurate dynamics. Typically, the size of the component modal models is greatly reduced compared to the system models built with full degree of freedom (DOF) component models, bringing efficiency to system modeling. Additionally, this component assembly approach provides useful modularity, allowing for the insertion or removal of components. While many different CMS methods exist, here we consider the simple case of isolated component modes. That is, the component mode basis will be formed solely with the component normal modes without additional constraint or supplemental modes.

### 2.1 Coupling Dynamic Systems with Component Mode Synthesis

Consider two components, A and B, joined together to create a system, C. The CMS method used to couple $A + B = C$ is as follows [4]:

1. Obtain component modes and form the component shape, modal mass and modal stiffness matrices. Truncate or select the modes of interest. Modes can come from numerical simulation or experimental measurements.
$$\begin{array}{c}\text{Component A: } \omega_A, \Phi_A \\ \text{Component B: } \omega_B, \Phi_B\end{array} \tag{1}$$
2. Write constraint equations in matrix form which link the connected DOF in each component. This is typically called a boolean constraint matrix, $S$.
$$S\begin{Bmatrix} x_A \\ x_B \end{Bmatrix} = \{0\} \tag{2}$$
3. Cast this constraint matrix into modal space by multiplying by a block diagonal matrix of the component mode shape matrices.
$$\bar{S} = S\begin{bmatrix} \Phi_A & 0 \\ 0 & \Phi_B \end{bmatrix} \tag{3}$$
4. Generate a transformation matrix, $L$, which will connect the modal DOF of the components based on the modal constraint matrix. This is done by taking the null space of the modal constraint matrix.
$$L = null(\bar{S}) \tag{4}$$
5. Use the transformation matrix to form the coupled system modal mass and stiffness matrices using the diagonally stacked component modal mass and stiffness matrices.
$$\bar{M} = L^T \bar{M}_U L \quad , \quad \bar{K} = L^T \bar{K}_U L \tag{5}$$
6. Solve for the coupled system modes then transform back to physical space using the uncoupled component modes and the transformation matrix.
$$\Phi_C = \Phi_U L \bar{\Phi}_C \tag{6}$$

The choice of component modes affects the system modes; enough of the right component modes are needed to properly span the space. Several of the CMS approaches supplement or alter the free component normal modes with additional modes to enrich the basis. In particular, the Transmission Simulator method uses an additional component to provide more realistic connection boundary conditions on the components. An interesting aspect of the Transmission Simulator method is that the Transmission Simulator component is removed from the system via a subtraction operation as described in the next section.

### 2.2 Decoupling Dynamic Systems with the Transmission Simulator Method

To remove the Transmission Simulator component from the system assembly, a procedure similar to the coupling process is used; however here there is a subtraction of component modes. This decoupling of a component from a system can notionally be considered as: C - B = A, with B being be the Transmission Simulator and A being the desired, extracted component.

The process for removing a Transmission Simulator component from a coupled system is as follows [4, 5]:

1. Obtain system and Transmission Simulator component modes and form the shape, modal mass and modal stiffness matrices. Truncate or select the modes of interest.
$$\begin{array}{c}\text{Coupled System: } \omega_C, \Phi_C \\ \text{Transmission Simulator Component: } \omega_B, \Phi_B\end{array} \tag{7}$$

2. Create the modal space constraint matrix using a softened form of the constraint equations which uses a pseudo-inverse of the Transmission Simulator component modes. $\Phi_B^+$ is the pseudo-inverse of the B component shape matrix.

$$\bar{S} = [\Phi_B^+][\Phi_C \quad -\Phi_B] \tag{8}$$

3. Generate a transformation matrix, $L$, which will connected the modal DOF of the components based on the modal constraint matrix. This is done by taking the null space of the modal constraint matrix.

$$L = null(\bar{S}) \tag{9}$$

4. Use the transformation matrix to form the uncoupled system modal mass and stiffness matrices using the diagonally stacked system and component modal mass and stiffness matrices. This time, however, there is a subtraction of the $B$ component modal matrices:

$$\bar{M}_U = \begin{bmatrix} \bar{M}_C & 0 \\ 0 & -\bar{M}_B \end{bmatrix} \;,\; \bar{K}_U = \begin{bmatrix} \bar{K}_C & 0 \\ 0 & -\bar{K}_B \end{bmatrix} \tag{10}$$

$$\bar{M} = L^T \bar{M}_U L \quad , \quad \bar{K} = L^T \bar{K}_U L \tag{11}$$

5. Solve for the decoupled system modes then transform back to physical space using the uncoupled component modes and the transformation matrix.

$$\Phi_A = \Phi_U L \bar{\Phi}_A \tag{12}$$

## 3 Application of Component Mode Synthesis to Couple and Decouple an Acoustoelastic System

Typically, the Transmission Simulator is used to provide a better set of basis modes for the A component. For this acoustoelastic system decoupling problem, we propose to use the same method but treat the Transmission Simulator component, B, as the entrained air in a hollow structure, A, for which we have coupled modes from system C. More concisely, we want to achieve A using modes of C and B. In this work, an example acoustoelastic system is created to explore the use of CMS and Transmission Simulator approaches to the coupling and decoupling of acoustoelastic systems. Below is a diagram of the coupling of components to form an acoustoelastic system and the decoupling of an acoustoelastic system to leave an uncoupled component.

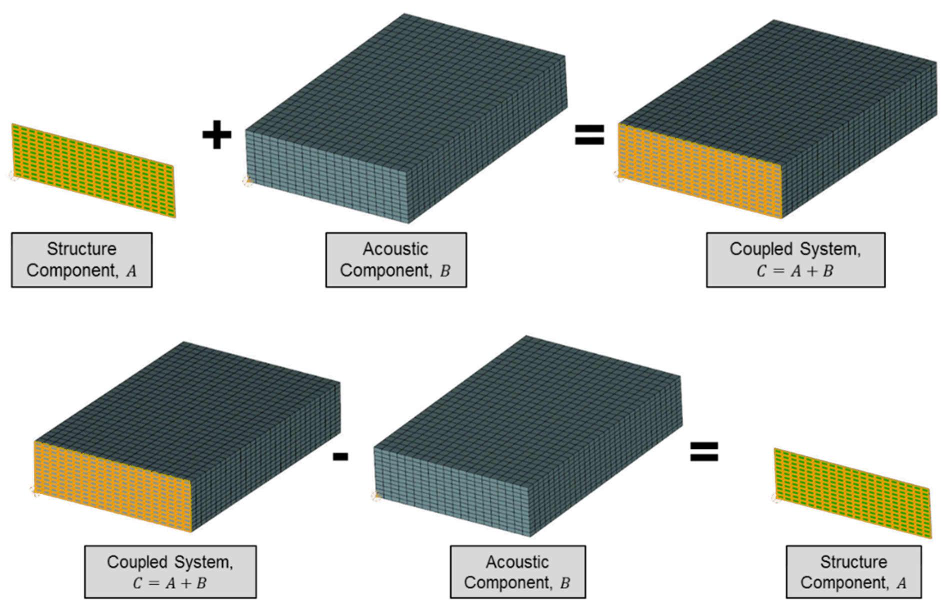



**Fig. 1** Coupling of the structural-acoustic plate-box system (top) and decoupling of the system (bottom)

Ideally, the decoupling process would be attempted using coupled system modes from a numerical simulation. However, obtaining coupled structural-acoustic system modes from FE packages can be tricky. Coupled modes from a FE simulation was tried here but the results were not trustworthy and seemed to indicate an issue with scaling of the mode shapes coming out of the FE package. As such, for this work the decoupling process is trialed using coupled system modes obtained by coupling together structure and acoustic components with the CMS process described above. This is not especially satisfying since the notional math is simply: (A + B) - B = A, however until a robust method for obtaining useful coupled structural-acoustic system modes is found, this is an option.

### 3.1 Acoustoelastic Systems

Acoustoelastic systems are comprised of a structure entraining an acoustic fluid volume and have structural modes with similar frequencies and shapes to the hollow cavity acoustic modes. This mode similarity allows for a coupling of the acoustic and structural components which changes the response of the structure. Depending on mode shape similarity and the proximity of the natural frequencies, the coupling effect can be large or small [6].

### 3.2 Example System: Plate-Box

A simple acoustoelastic system is a box of air with an elastic plate on one side, an example used in the acoustoelasticity literature [7]. A finite element (FE) model of such a system was developed with dimensions to align the first mode of a 4 [mm] thick simply supported aluminum plate with the first mode of a rigid-walled, air-filled box. A diagram showing the mesh and dimensions is shown below. The modulus of elasticity of the plate was tuned to closely align the structure and acoustic mode frequencies. The mesh is a regular grid with 31 nodes in X, 13 nodes in Y, and 21 nodes in Z, providing 273 nodes on the wetted surface.

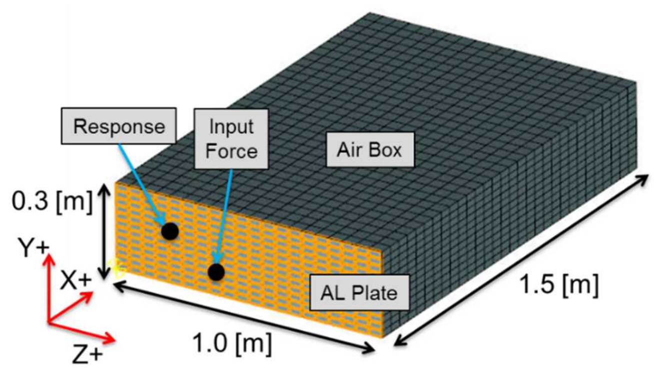



**Fig. 2** Dimensions of the plate-box system and locations of the input and response points

### 3.3 Structure Component Modes

The plate-only (*in vacuo*) modes were computed using Sierra/SD, part of the Sierra Mechanics finite element package [8]. Plate-only shapes and frequencies are shown below alongside the rigid-walled acoustic modes.

### 3.4 Acoustic Component Modes

The acoustic component modes were first computed with rigid-wall boundary conditions for the purposes of visualizing the acoustic pressure mode shapes, shown below alongside the plate modes. In the figure below, it can be seen that the first plate and acoustic modes are compatible; the shapes and frequencies are similar. No other mode pair in the frequency range shows the same similarity, thus we expect only one coupled mode split peak in the response.

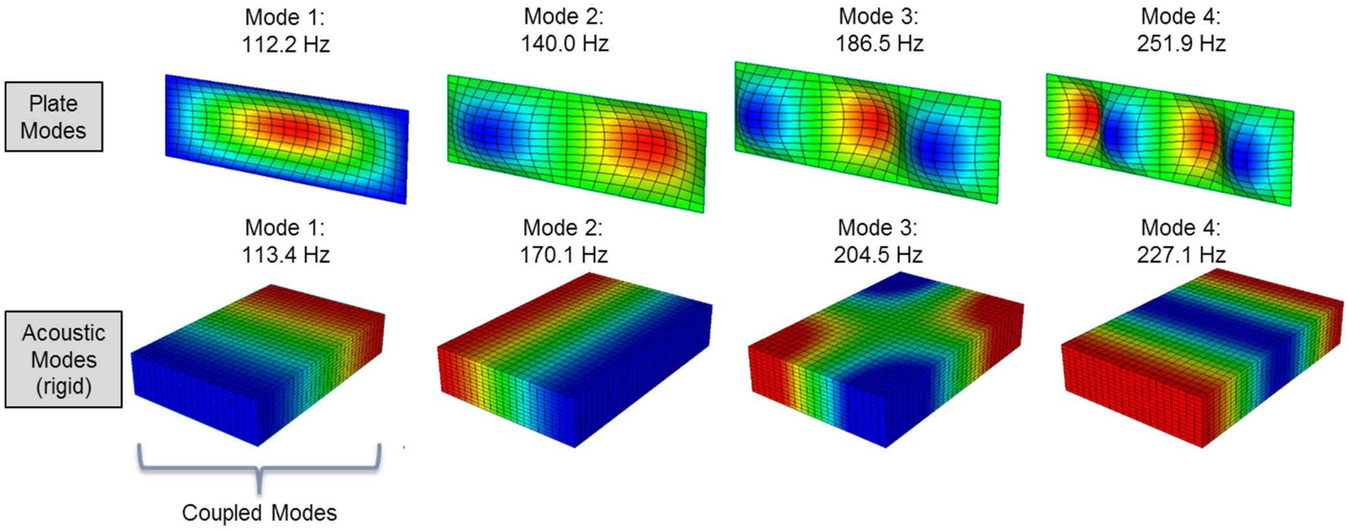



**Fig. 3** Component modes of the plate (top) and rigid-walled box (bottom)

Rigid-walled boundary conditions cannot be used in this CMS process because there is zero acoustic particle velocity at the plate-box wetted surface; a condition that will result in unusable acoustic mode shapes in the $B$ component basis. Thus, a different set of acoustic modes was needed. Using a pressure release boundary condition at the wetted surface and rigid walls elsewhere provides modes with non-zero particle velocity at the wetted surface. However, computing acoustic modes with any type of absorbing boundary condition can be challenging using most finite element packages. For this simple geometry and idealistic boundary conditions, the acoustic modes can be computed analytically as [9]:

$$p(x,y,z,n_x,n_y,n_z) = A_{scale}\, sin\left(\frac{(2n_x+1)\pi x)}{2L_x}\right) co\, s\left(\frac{n_y \pi y}{L_y}\right) co\, s\left(\frac{n_z \pi z}{L_z}\right) \quad (13)$$

$$f_{n_x,n_y,n_z} = \frac{c_0}{2\pi}\sqrt{\left(\frac{(2n_x+1)\pi}{2L_x}\right)^2 + \left(\frac{n_y \pi}{L_y}\right)^2 + \left(\frac{n_z \pi}{L_z}\right)^2}, \quad (14)$$

where $p(x,y,z,n_x,n_y,n_z)$ is the pressure mode shape as a function of $x,y,z$ space and $n_x,n_y,n_z$ mode orders. The box dimensions are $L_x$ by $L_y$ by $L_z$. The shape scaling, $A_{scale}$ is 1.0 to unit scale the shapes. $f_{n_x,n_y,n_z}$ is the mode natural frequency and $c_0$ is the acoustic sound speed. The first mode of the box is order (1,0,0), meaning it is a quarter sine in X and constant in Y and Z.

At the wetted surface, the plate DOF are constrained to the acoustic DOF in terms of displacements, not pressures. Thus, the pressure mode shapes had to be converted to a displacement-like quantity. Acoustic particle velocity is proportional to the gradient of pressure [10]:

$$\vec{u} \propto \vec{\nabla} p \quad (15)$$

This gradient was computed numerically by generating the pressure shape in three dimensions and then computing the gradient of that grid of shape values. The acoustic modes need to be mass normalized to be useful in a CMS process so a modal mass was determined using a volume integral of the square of the acoustic pressure gradient mode shape:

$$m_j = \rho_0 \int_V \vec{\nabla}\Phi_{\mathrm{p,j}} \cdot \vec{\nabla}\Phi_{\mathrm{p,j}}\, dV \quad (16)$$

### 3.5 Variations of Component Modes

The acoustic modes were computed at the fine mesh grid shown above. However, this is an impractically large number of acoustic DOF so subsets of these modes were also considered. Next, only the acoustic DOF at the wetted surface were retained in both the coupled system (C) and acoustic component (B) modes. Finally, the acoustic DOF were removed entirely from the C coupled system DOF, leaving only the plate DOF to be constrained to the B component acoustic DOF. Here, there are no B component DOF in the C coupled modes, and so the pseudo-inverse of the partitioned C shapes used to form the modal constraint matrix have to be replaced by the A DOF in the C shapes. This issue may cause some of the errors in the extracted B component modes observed in Fig. 8b.

### 3.6 Numerical Simulations of Uncoupled and Coupled Systems

Direct frequency response simulations were performed using the plate-only and plate and box coupled system model using Sierra/SD. These frequency response function (FRF) curves will be the basis of comparison for the response predictions made using the CMS coupled and decoupled systems.

A node was chosen for the input DOF and another for the output DOF, shown below. Then, a unit force linear spectrum was input over a broad frequency range and the response extracted at the output DOF. As this is a direct simulation, proportional damping was used. 0.5% damping was desired for the plate and 1.0% for the air. The proportional damping terms were determined to achieve nearly flat damping in the range of 50-500 Hz, but there is some overshoot in the damping at high frequencies.

## 4 Results

### 4.1 Direct Simulation for In-vacuo Structural Component and Coupled System Response

FRFs from direct simulation using Sierra/SD for the plate-only and the coupled plate-box system are shown below. As expected, there is strong evidence of acoustoelastic coupling near the first mode of the plate. As these shapes are compatible and the frequencies are very close, there is a dramatic split peak observed in the coupled system FRF. The CMS coupled system predictions will be compared against the coupled system FRF from this simulation and the CMS decoupled plate predictions will be compared against the plate FRF.

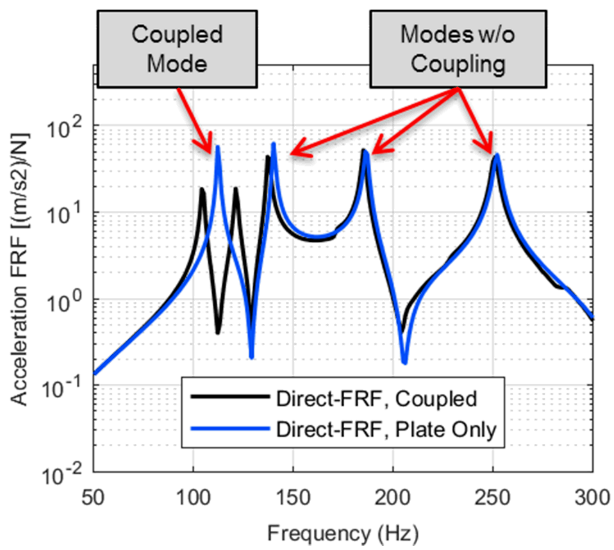
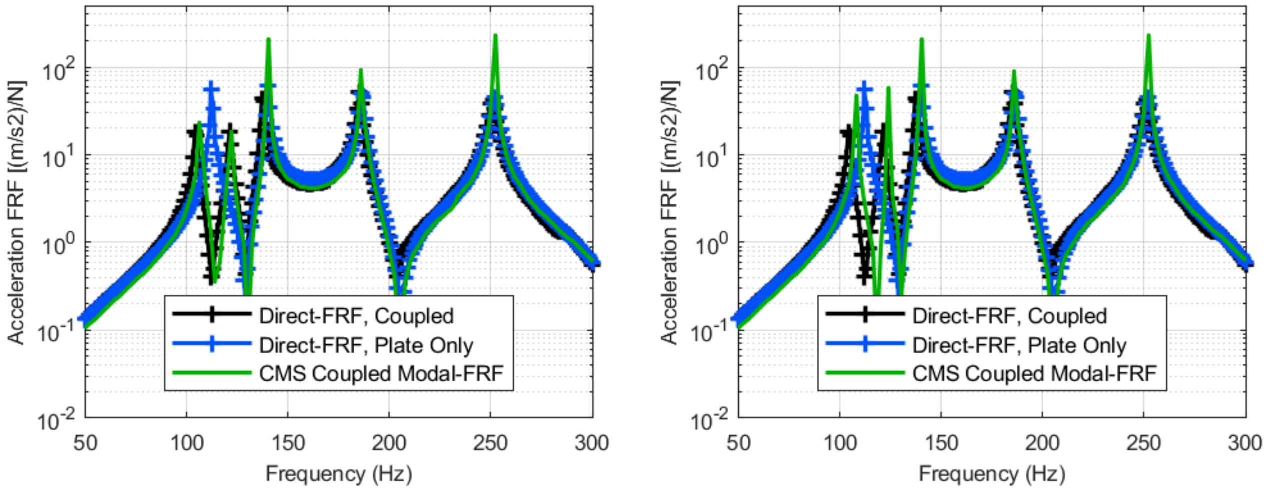



**Fig. 4** FRFs from direct simulation of the plate component and the coupled plate-box system

### 4.2 Coupled Structural-Acoustic System Response

The FRFs for the CMS coupled system are compared with the coupled system response predicted with direct simulation in the plot below. The CMS-coupled system agrees very well with the direct FRF simulation, capturing the coupled modes effect well. There is some error in the peaks at the higher frequency modes, but this may be due to the damping models being different; in the CMS mode-based predictions, the CMS system FRF is computed with modal damping whereas the direct FRF simulations use proportional damping.

As expected for CMS problems, the number of modes in the components affects the results, as seen in Fig. 5 (b). Introducing more acoustic modes into component B improves accuracy.




**(a)** **(b)**

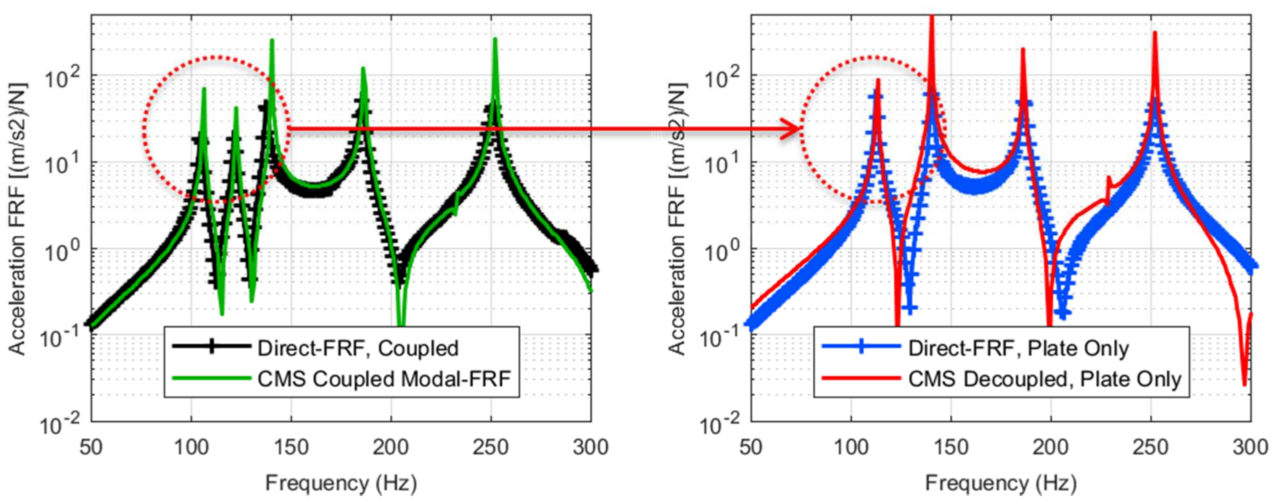
**Fig. 5** FRFs of the CMS-coupled plate-box system compared with the direct simulation using many modes (50 structure, 100 acoustic) in the plate and box basis **(a)** and fewer modes (30 structure, 10 acoustic) in the basis **(b)**

### 4.3 Decoupled Structural-Acoustic System Response

Predictions of response for the decoupled plate response are shown below for various sets of basis modes and degrees of freedom in the coupled system modes.

#### 4.3.1. All Acoustic DOF in Coupled System & Subtracted Component

Fig. 6 shows the FRF for the coupled and then decoupled system using all the plate DOF and all the acoustic DOF in components C and B. Of course, it is unrealistic to obtain this DOF count using experimentally-derived structure and acoustic shapes. In the next subsections, cases with a reduced DOF count are considered.




**(a)** **(b)**

**Fig. 6** FRFs of the CMS-coupled plate **(a)** which is then decoupled using the Transmission Simulator method **(b)**

#### 4.3.2. Reduced Count Acoustic DOF in Coupled System & Subtracted Component

Next, shown in Fig. 7a, only the acoustic DOF on the wetted surface in components C and B are retained in the basis modes. This is a drastic reduction in the DOF count, from 8463 acoustic DOF to 273 acoustic DOF. The decoupling procedure is again successful at removing the coupled modes peak at mode 1. Good results were obtained using 30 basis modes for component C and 20 modes for B.

Entirely removing the acoustic DOF from the C component modes was also considered, Fig. 7b. Here, only the 273 displacement DOF were retained in the C modes and then the 273 acoustic DOF were retained in the B modes. This is a more realistic configuration as typically only structural response measurements (modes) would be made on a coupled system.

Then, the acoustic modes for the B component could be obtained from a model. It was found that 20 C component modes and 5 B component modes provide good results. Curiously, increasing the number of modes in both components to 40 in C and 30 in B introduced inaccuracies. Typically in CMS problems there is monotonic convergence with increasing modes, which indicates that this process on this type of system is not robust.

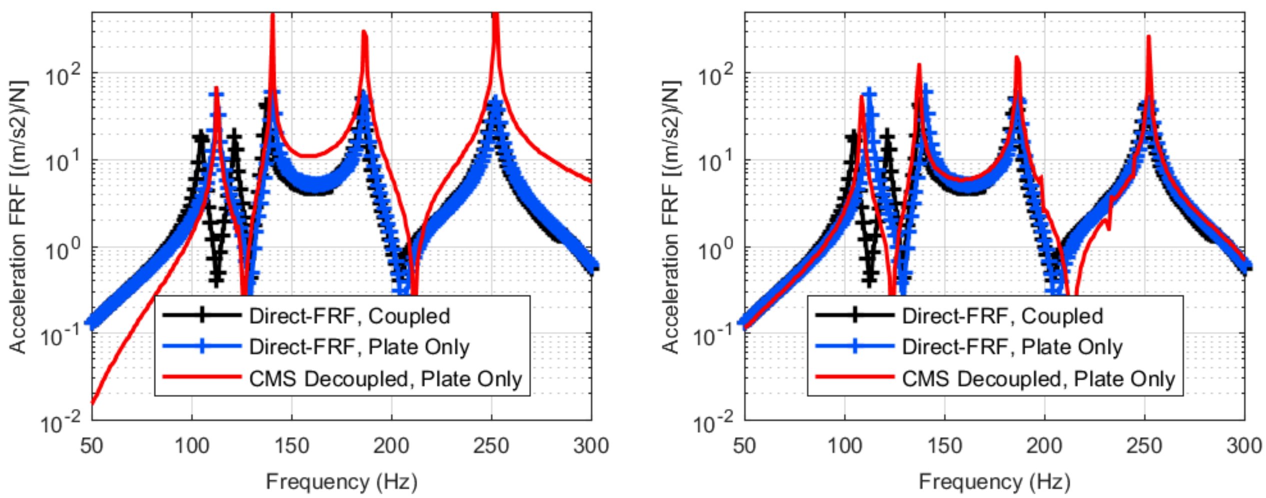



**Fig. 7** FRFs of the decoupled CMS system using **(a)** only DOFs on the wetted surface (no interior acoustic DOFs in B) and **(b)** using only displacement DOFs in component C (no acoustic DOFs in C)

## 5 Conclusions and Future Work

Traditional CMS methods were applied to a new type of system modeling, acoustoelastic systems. CMS was effective to couple together a structure and acoustic model, and the resulting coupled system response matched predictions from a coupled system direction frequency response simulation. As expected, the number and type of modes used in the component basis affects the accuracy of results. Next, the Transmission Simulator method was used to decouple a coupled system, subtracting off the acoustic component from an acoustoelastic system leaving the *in vacuo* structural response. Frequency response comparisons of a plate extracted from the coupled system compare reasonably well with response from direct simulation of a plate, but errors do exist and the results were quite sensitive to the choice of modes in the basis. Additionally, as is common for subtraction CMS problems several of the de-coupled system modes from the Transmission Simulator process are erroneous, either highly complex or degenerate with duplicate frequencies and near zero amplitude.

Further study is needed to determine, in general, what type of acoustic modes are required to sufficiently span the necessary space for this class of problems. Other acoustic domain boundary conditions at the wetted surface will be considered, for example impedance boundary conditions that lie between pressure release and rigid boundaries. Future work will focus on how to apply this technique to the ultimate goal: removing acoustic coupling effects from structural response measurements. This will require better understanding of how few structure and acoustic DOF are needed to provide component modes. Using simulation to determine coupled system modes would also be useful to validate these methods, provided the coupled system mode results are reliable.

## References

[1] E. H. Dowell, G. F. Gorman III and D. A. Smith, "Acoustoelasticity: General Theory, Acoustic Natural Modes and Forced Response to Sinusoidal Excitation, Including Comparisons with Experiment," *Journal of Sound and Vibration,* vol. 52, no. 4, pp. 519-542, 1977.

[2] R. Schultz and B. R. Pacini, "Mitigation of Structural-acoustic Mode Coupling in a Modal Test of a Hollow Structure," in *IMAC-XXXV International Modal Analysis Conference*, Garden Grove, CA, 2017.

[3] D. de Klerk, D. J. Rixen and S. N. Voormeeren, "General Framework for Dynamic Substructuring: History, Review,and Classification of Techniques," *AIAA Journal,* vol. 46, no. 5, pp. 1169-1180, May 2008.

[4] D. R. Roettgen, "Experimental Dynamic Substructuring Using Nonlinear Modal Joint Models," Doctoral Dissertation, University of Wisconsin - Madison, Madison, WI, 2016.

[5] R. L. Mayes and M. Arviso, "Design studies for the Transmission Simulator Method of Experimental Dynamic Substructuring," in *International Seminar on Modal Analysis (ISMA2010)*, Lueven, Belgium, 2010.

[6] S. A. Hambrick, S. H. Sung and D. J. Nefske, Engineering vibroacoustic analysis: methods and applications, Wiley, 2016.

[7] A. J. Pretlove, "Free Vibrations of a Rectangular Panel Backed by a Closed Rectangular Cavity," *Journal of Sound and Vibration,* vol. 2, no. 3, pp. 197-209, 1965.

[8] Sierra Structural Dyanmics Development Team, "Sierra Structural Dynamics - User's Notes," Sandia National Laboratories SAND Report, 2015.

[9] D. T. Blackstock, Fundamentals of Physical Acoustics, Austin, TX: John Wiley & Sons, 2000.

[10] A. D. Pierce, Acoustics, An Introduction to its Physical Principles and Applications, 1981: Acoustical Society of America, 1994.