

# Numerical Modeling of an Enclosed Cylinder

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## Abstract

Finite element models are regularly used in many disciplines to predict dynamic behavior of a structure under certain loads and subject to various boundary conditions, in particular when analytical models cannot be used due to geometric complexity. One such example is a structure with an entrained fluid cavity. To assist an experimental study of the acoustoelastic effect, numerical studies of an enclosed cylinder were performed to design the test hardware. With a system that demonstrates acoustoelastic coupling, it was then desired to make changes to decouple the structure from the fluid by making changes to either the fluid or the structure. In this paper, simulation is used to apply various changes and observe the effects on the structural response to choose an effective decoupling approach for the experimental study.

**Keywords:** acoustoelasticity, structural acoustics, finite element, direct simulation, design

## 1 Introduction

Acoustoelastic coupling can result in structural response different from the in-vacuo condition, even if structure is substantial and the internal fluid is air [1, 2]. For the coupling to be significant, the modes of the structure and internal fluid cavity need to be similar in both shape and frequency [3]. To support a recent test project in acoustoelasticity, a representative piece of hardware had to be designed with strong acoustoelastic coupling. Here, finite element (FE) simulation was utilized to design a shell structure with ovaling modes well aligned with internal acoustic modes. Then, simulations of the coupled structural-acoustic system were performed to assess how changes to the acoustic or structural component affect the structural response at frequencies near the coupled modes. Of particular interest was how to change the system to result in response similar to the in-vacuo structural response. Various changes were considered including adding mass to the structure, changing the fluid medium and adding damping to the internal fluid.

## 2 Designing an Acoustoelastic System Using Simulation

A research project was planned to study acoustoelastic response and methods to decouple the structural-acoustic system for modal testing [4]. To aid this project, a simulation-based study was conducted to design a simple shell with strong acoustoelastic coupling. Some of design objectives for the test hardware include 1) to have coupled ovaling modes below 4000 Hz to make modal testing easier, 2) to be less than 20 kg to be easy to handle, 3) to be inexpensive to build and easy to modify and 4) to be representative in size and wall thickness to structures of interest.

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<sup>1</sup> Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525

## 2.1 Parameterized Finite Element Shell Models

The geometry of the structure (a cylindrical shell) was chosen to match readily-available aluminum tubing from a metal supply catalog. A total of eight geometries were considered, shown in Table 1. To rapidly generate finite element models of the shell, a parameterized meshing script was developed which takes inputs of shell length, wall thickness and radius to build a 3D hex mesh. These models were small, around 150k degrees of freedom (DOF) so computing modes for several shell geometries was tractable in the FE software Sierra/SD [5].

## 2.2 Analytical Acoustic Modes Model

As the interior cavity of the shell is a cylindrical, hard-walled volume, an analytical model was used to estimate the acoustic modes. For each shell geometry described above, acoustic modes were estimated using the expression below for rigid walled cylindrical cavities:

$$f_{l,m,n} = \frac{c_0}{2} \sqrt{\left(\frac{\alpha_{mn}}{\pi a}\right)^2 + \left(\frac{l}{h}\right)^2}, \quad (1)$$

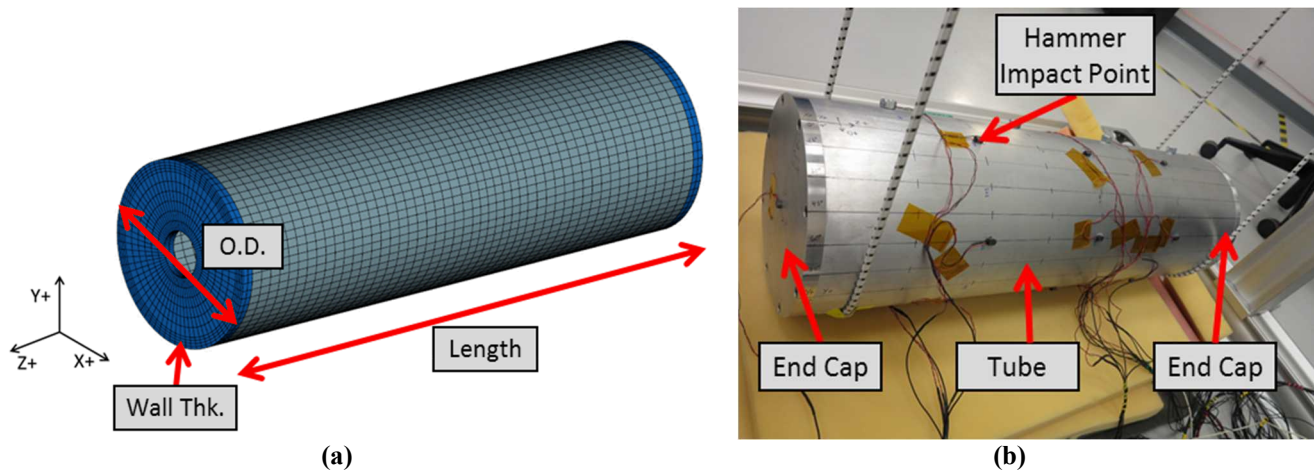
where  $c_0$  is the sound speed,  $a$  is the radius,  $h$  is the length,  $\alpha_{mn}$  is the  $n$ -th zero of  $m$ -order Bessel function of the first kind, and  $l, m, n$  are the acoustic mode orders in the length, circumference, and radius, respectively. Only radial mode orders of  $n = 1$  were considered here, so the acoustic mode orders are listed in Table 1 as  $(m, l)$ . An acoustic mode with order  $(2,1)$  has two node lines around the circumference and one down the length and is the same shape as a  $(2,1)$  ovaling mode in the shell.

## 2.3 Final Design Selection

Table 1 shows the various shell geometries considered along with the FE-predicted shell mode frequencies and the analytically-predicted acoustic mode frequencies for compatible modes. Modes with shell and acoustic modes within five percent are highlighted. Of the various geometries, a shell with length 24 inches (61 cm), outer diameter 8 inches (20.3 cm) and 0.5 inch (1.2 cm) wall thickness proved to be the best match of acoustic modes to structural modes. Since both the  $(2,1)$  and  $(3,0)$  mode frequencies were within five percent, this geometry was selected. The actual hardware was designed with removable end caps, one of which has a hole in the end to allow for microphones and acoustic damping material to be added to the cavity. Fig. 1 shows the FE mesh and final hardware design.

**Table 1:** Shell geometries considered for the design, with shell and acoustic mode frequencies for various ovaling modes. Possible acoustoelastic modes are highlighted.

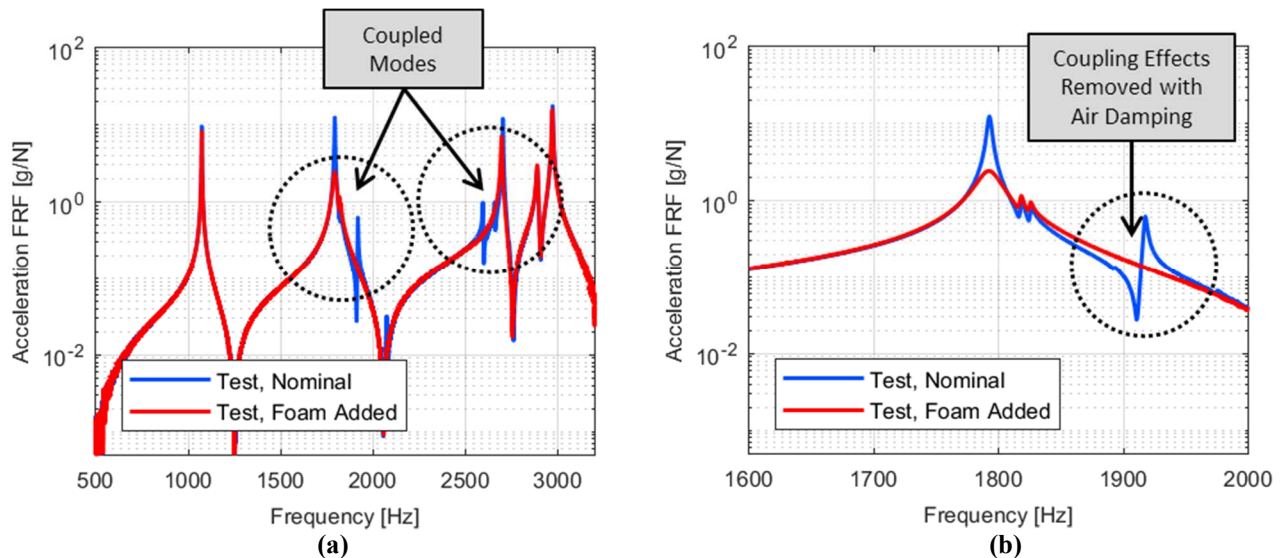
O.D. [in.]	Wall [in.]	Length [in.]	Shell 2,0	Shell 2,1	Shell 3,0	Shell 3,1	Shell 4,0	Shell 4,1	Acs. 2,0	Acs. 2,1	Acs. 3,0	Acs. 3,1	Acs. 4,0	Acs. 4,1
6	0.50	10	2574	5316	5229	6868	9347	-	2622	2708	3611	3673	4574	4623
6	0.25	12	1525	3777	2512	3605	4530	5176	2384	2449	3282	3330	4158	4196
6	0.50	12	2264	4359	5085	6182	-	-	2622	2682	3611	3654	4574	4608
8	0.25	24	650	1586	1304	1620	2432	2596	1748	1771	2407	2424	3049	3062
8	0.50	20	1177	2292	2728	3196	5046	5381	1873	1903	2579	2601	3267	3284
8	0.50	24	1077	1857	2690	2988	-	-	1873	1894	2579	2594	3267	3279
10	0.25	24	618	1704	873	1348	1559	1780	1380	1408	1900	1921	2407	2424
10	0.50	24	824	1815	1730	2140	3191	3454	1457	1484	2006	2026	2541	2556



**Fig. 1** FE mesh of the shell structure **(a)** and final shell design in free-free test setup **(b)**

## 2.4 Example Test Data

An example modal hammer test frequency response function (FRF) is shown in Fig. 2. Here, a modal hammer impacts radially at the location shown in Fig. 1 and the response is measured by accelerometers at several locations. Included in this figure are drive point FRFs for two test configurations: the nominal, coupled structure and the structure with foam added to the cavity to damp out the acoustic modes, a technique previously demonstrated on a different coupled structure [2]. The two coupled modes can be seen around 1890 Hz and 2600 Hz, evidenced by an additional peak in the FRF which is removed when the air is damped with foam. A more detailed summary of the experimental examination of this hardware is found in [4]. If the structure and acoustic modes were very close in frequency, a split in the mode near 1800 Hz would be expected. As the coupling instead looks like an additional peak around 1890 Hz with little shift in frequency of the main peak, the hardware does not have perfectly aligned structure and acoustic modes.



**Fig. 2** Experimental FRFs for the designed acoustoelastic system showing the effect on coupling of adding air damping, broadband **(a)** and zoomed in at the (2,1) mode **(b)**

## 3 Finite Element Simulations to Explore Changes to the Acoustoelastic System

To support the experimental study, a coupled structural-acoustic FE model was developed to explore how changes to the structure or acoustic components would affect the coupled system FRFs. This provides a simulation-based approach to evaluate decoupling techniques which could be used in modal tests of a structure. In this section, direct frequency response simulations near the first coupled mode are used to demonstrate the effect on structural FRFs of changes to air damping, sound speed of the air, and added mass to the shell. These mimic the system perturbations which could be employed in a test of an acoustoelastic system and signify which changes are most impactful to shift or eliminate the acoustoelastic coupling effects.

### 3.1 Coupled Finite Element Fluid Mesh

An acoustic finite element mesh was built contiguous with the shell mesh at the wetted surface. A mesh size of around 1 cm was used in the acoustic mesh to give at least six elements per wavelength up to 4000 Hz. Direct frequency response simulation was used because it is the most accurate method for simulation of structural-acoustic systems with significant acoustic absorption. To mimic the test results shown above, a broadband unit input force was applied at the hammer impact location and the acceleration response output was computed at that drive point.

### 3.2 Coupled Boundary Element Fluid Mesh

A separate approach was also considered to model the interior fluid using a lumped parameter boundary element approach [6]. In this approach, the structural mesh is used as the basis for the boundary mesh and the force to displacement transfer function due to the acoustic pressure field is computed and transformed to modal space. The interior and exterior acoustics can be captured with this approach which can lead to reduced meshing and solution time when compared to the finite element fluid mesh approach. However, losses in the fluid are not captured so that an investigation of the interior cavity damping cannot be completed. Thus, only the coupled FE model was used to simulate changes to the system.

### 3.3 Simulating Changes to the Air Damping

As shown in the experimental results, increasing the damping in the air can change the acoustoelastic coupling effect. This was achieved in the model by changing the proportional damping terms of the air volume from a nominal level of 0.1% to 0.5, 1.0 and 2.0%, over the range 1000 to 4000 Hz. It is not immediately obvious how to correlate a damping ratio to the amount of foam or absorbing material needed in a test. However, the trends, shown in Fig. 3 (a), can also be observed experimentally [2]. Direct frequency response simulation appears to capture well the effects of adding damping to the air component.

### 3.4 Simulating Changes to the Air Sound Speed

As an alternative method for changing the acoustoelastic coupling, the fluid medium or temperature could be changed such that the sound speed of the air is increased or decreased. To see how a change in sound speed affects the drive point FRF, the air properties were changed from nominal, room temperature sound speed of 343 m/s to cold (0 deg. C), 331 m/s and hot (40 deg. C), 355 m/s, temperatures. Fig. 3 (b) shows that changing sound speed does shift the acoustic mode frequency up and down as expected. Interestingly, for the 355 m/s case the acoustic mode is shifted further away from the structural mode, but the coupling effect remains largely intact. For the 331 m/s case, the modes are moved closer together and the split-peak effect is stronger, causing a shift down in frequency for the structure-dominant first peak.

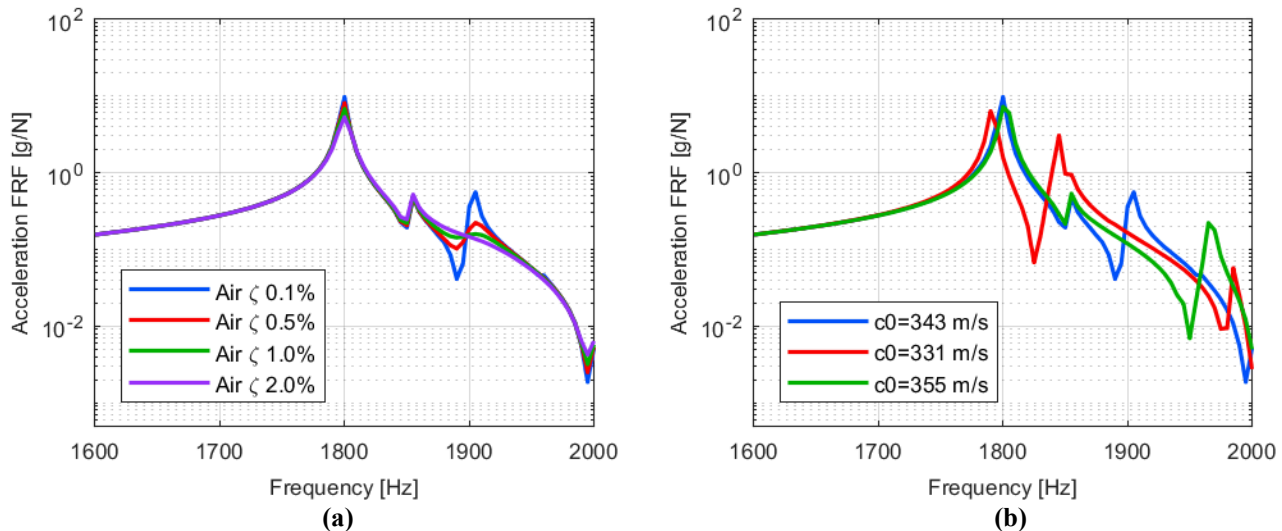
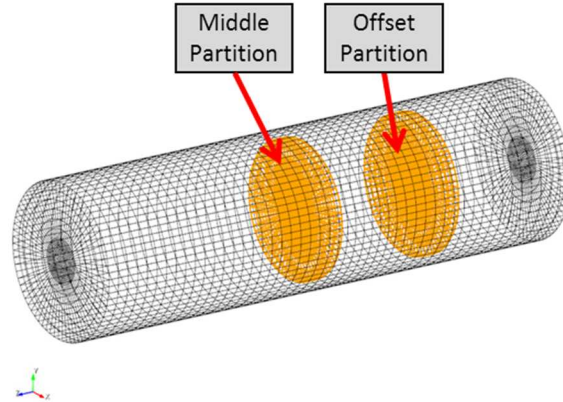


Fig. 3 Simulated drive point FRFs demonstrating the effects of adding air damping (a) and changing the air sound speed (b)

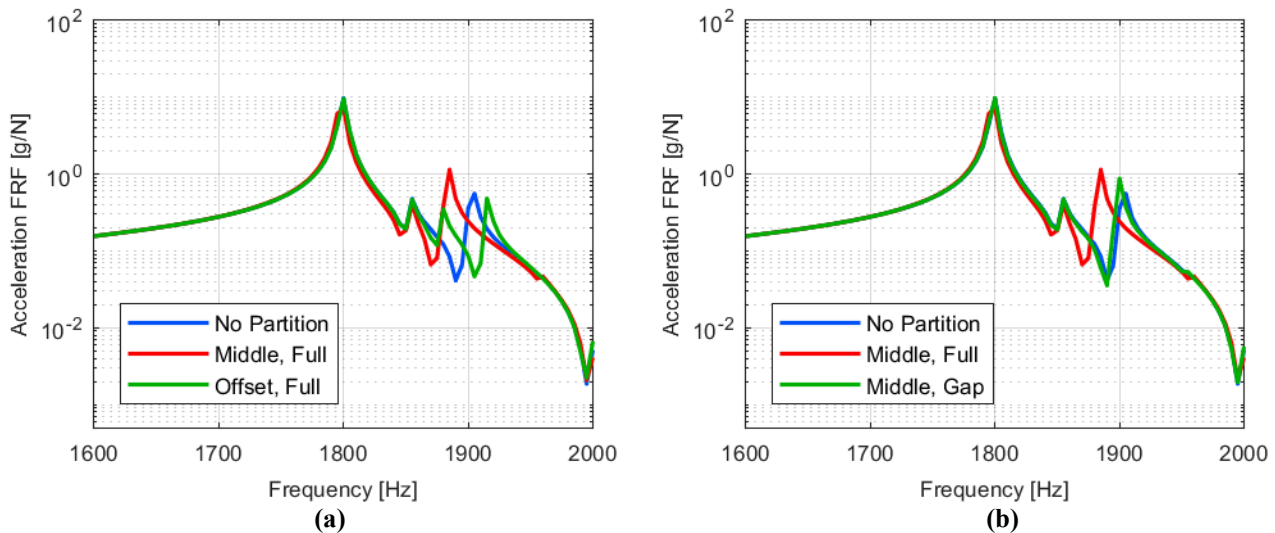
### 3.5 Simulating Changes to the Geometry of the Air Cavity – Adding a Partition

The acoustic modes can also be altered by disrupting the air cavity with a non-structural wall or partition. Intuitively, the location of the partition relative to the acoustic mode shape will determine how effective the partition is at changing the

acoustoelastic coupling. Two partition locations are considered and shown below in Fig. 4. These were employed by removing a disc of elements from the acoustic cavity from the mesh, thereby creating reflective boundary at the partition location. As the (2,1) mode is of interest here, the middle and offset partition locations should affect the acoustic modes and thereby the acoustoelastic coupling, in different ways. Fig. 5 (a) shows that the partitions do affect the coupled modes and the offset location seems to introduce an additional peak in the response. This is likely due to an additional acoustic mode with shifted frequency due to the different cavity lengths. Additionally, it would be difficult to completely seal off a partition to the shell wall in practice, so a partition with a circumferential gap was also simulated. Fig. 5 (b) shows that if the partition is not well sealed, the FRF is basically the same as if the partition is not there at all.



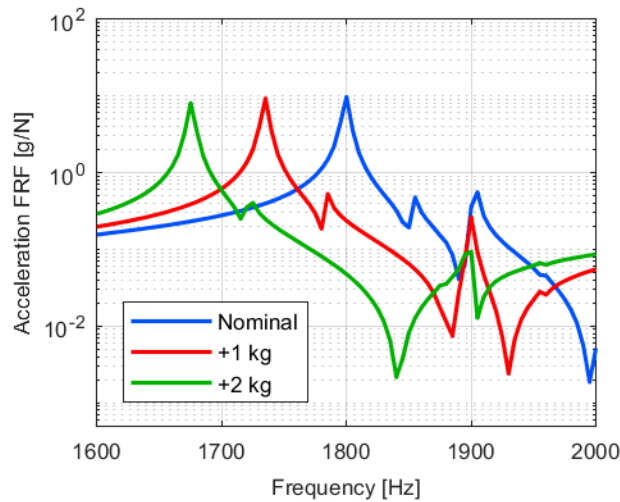
**Fig. 4** Reflective partition locations in the acoustic mesh



**Fig. 5** Effects of partition axial location (a) and partition sealing to the shell wall (b)

### 3.6 Simulating Changes to the Structural Mass

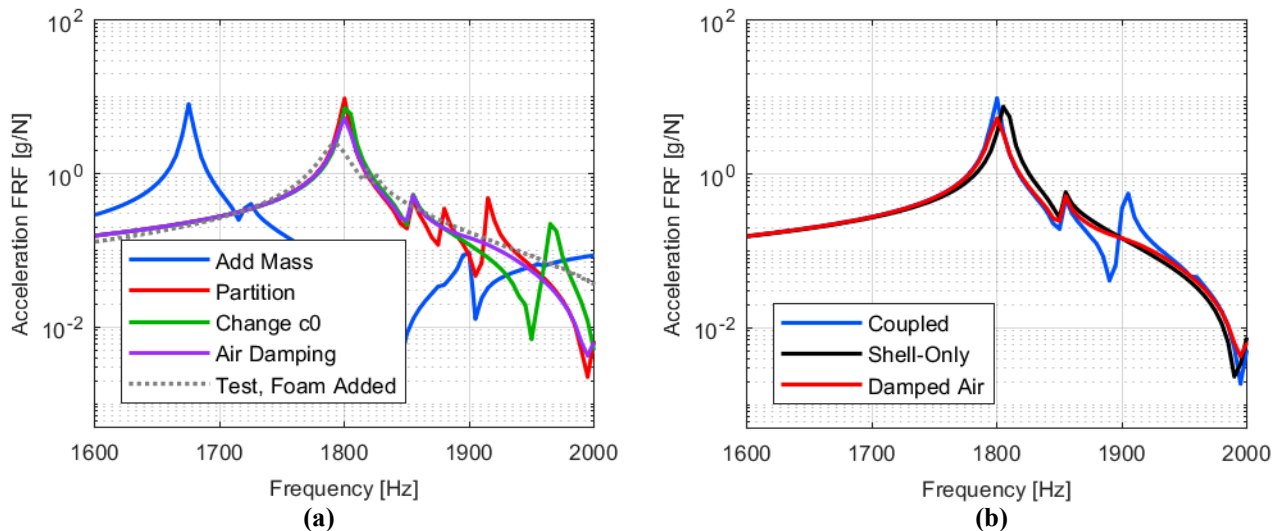
The shell structure could also be modified to change the frequency proximity to the coupled acoustic mode. Simple changes, such as adding mass to the structure, could possibly be removed using a structural dynamic modification technique to result in the decoupled, un-altered structural response [7]. The effects of adding 1 and 2 kg of mass to the 12.5 kg shell structure is shown in Fig. 6. Though the modes are shifted, the coupling remains strong, perhaps indicating that for this coupled pair the shape similarity outweighs the frequency proximity effects [3].



**Fig. 6** Effects of added structural mass

### 3.7 Summary of Effects of System Changes

Here, a coupled system model was developed to explore how changes to the structure or acoustic volume would affect the structural response at a coupled mode. At this mode, the acoustic and structural mode shapes and frequencies are similar, causing the air inside the shell to change the response. It is desirable to understand if acoustoelastic coupling is occurring in a test structure and to remove the coupling effects to result in structure-only response measurements. Simulations in this work show that changing the damping in the air volume is most effective at changing the coupling without drastic changes to the structural mode FRF, as seen in Fig. 7.



**Fig. 7** Comparison of all system change approaches (a) and a comparison of the coupled system, shell-only, and the coupled system with damped air volume (b)

## 4 Conclusions

Simulation was used to design hardware to purposefully have coupled structural-acoustic modes. This was achieved by building several shell models using a parameterized meshing script and then comparing frequencies of the shell with acoustic mode frequencies for various ovaling mode orders. A shell geometry with good match to two acoustic modes was found and hardware built to use for experimental exploration of an acoustoelastic system.

Direct frequency response simulation was then used to explore the effects on the shell response of various decoupling methods including changing the air sound speed, adding mass the structure and increasing damping in the air. The simulation results match qualitatively with the experimental results for similar system changes employed in [4]. As such, simulation could be used to help design future tests of hollow structures or to diagnose coupling effects observed in tests.

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