

# Comparison of Time-Domain Objective Functions in Dynamic Fixture Optimization

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## ABSTRACT

Differences in impedance are usually observed when components are tested in fixtures at lower levels of assembly from those in which they are fielded. In this work, the Kansas City National Security Campus (KCNSC) test bed hardware geometry is used to explore the sensitivity of the form of the objective function on the adequate reproduction of relevant response characteristics at the next level of assembly. Inverse methods within Sandia National Laboratories' Sierra/SD code suite along with the Rapid Optimization Library (ROL) are used for identifying an unknown material (variable shear and bulk modulus) distributed across a predefined fixture volume. Comparisons of the results between time-domain based objective functions are presented. The development of the objective functions, solution sensitivity, and solution convergence will be discussed in the context of the practical considerations required for creating a realizable set of test hardware based on the variable-modulus optimized solutions.

**Keywords:** Material optimization, inverse method, dynamic optimization, fixture design, multi-DOF

## INTRODUCTION

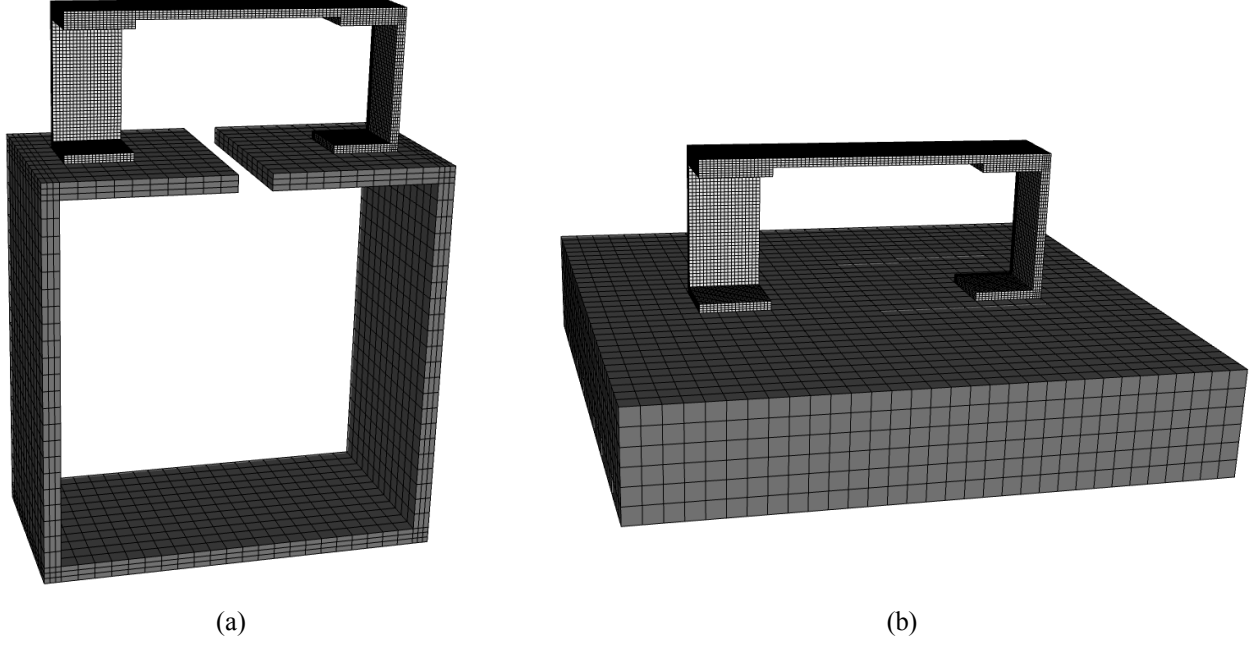
When performing ground-based component-level qualification testing it is common practice to use test fixtures that are effectively rigid across the frequency band of interest. It is also common practice to perform tests with single-axis inputs, with the understanding that the field inputs are much more complex. The implicit hope is that the process of deriving test inputs is adequately conservative to envelope all expected field responses. Although the manner in which component-level test environments are derived from field data typically supports the belief that they exceed field data, in general, there is no rigorous guarantee of that belief. Qualifying components at the next level of assembly could alleviate some of the issues associated with impedance mismatches observed when testing at lower levels of assembly, but still does not address issues related to multiple DOF inputs and boundary condition matching.

It is an axiomatic truth that if the field-level response characteristics of a component are preserved across all levels of assembly that any testing done will correctly inform any post-test assessments that need to be performed. For example, it is desired to gather correct evidence for qualifying components, calculating margin, assessing reliability, etc. Setting aside the philosophical discussion about whether reproducing field environments at the component-level is the actual goal of testing, if the intent is to reproduce actual field environments at a component, then it would be useful to characterize the effect of either optimizing applied input forces or optimizing test impedance subject to the appropriate test constraints. The latter optimization is explored here.

A purely numerical investigation is pursued to discover if there are mathematically-admissible solutions to a material inverse problem. Inverse methods within Sandia National Laboratories' Sierra/SD code suite [1, 2] along with the Rapid Optimization Library (ROL) [3] are used for identifying an unknown material (variable shear and bulk modulus) distributed across a predefined fixture volume. The KCNSC test bed hardware geometry [4] is used to explore how to construct time-domain objective functions that lead to material optimization solutions that preserve component response over multiple levels of assembly.

## PROBLEM DEVELOPMENT

The goal of this work is to investigate the theoretical potential of deducing a mechanical filter, via a heterogeneous test fixture, that can reproduce the system-level responses (kinematic and stress) at a component, when that component is isolated from the system assembly. The boundary condition challenge geometry provided in [4] serves as the system-level assembly definition. The component defined in that assembly is used with a standard-sized, continuous plate to represent the component-level definition. Finite element representations of both levels of assembly are shown in Figure 1.



**Fig. 1** Finite element model of the (a) boundary condition challenge system geometry and (b) boundary condition challenge component-level assembly on test fixture

All of the analysis is performed with Sandia National Laboratories' Sierra/SD code suite [1]. The first step of the analysis is to develop a set of "truth" data. This is achieved by exciting the system-level geometry (Figure 1(a)) with an arbitrary 3-DOF shock transient and predicting the acceleration and stress responses throughout the component. A representative 3-DOF acceleration time history response at the component is then derived from the system-level forward simulation and used as the input into the component-level simulation geometry (Figure 1(b)). In general, a forward simulation of the component-level geometry will not reproduce the responses observed in the system-level data, therefore, a solution will be sought by deducing a physically-admissible set of heterogeneous material properties across the elements of the discretized fixture in the component-level model. This is achieved by solving a time-domain inverse problem.

The time-domain inverse problem formulation is described in reference [2] and briefly summarized here. The solutions to the forward transient simulation are the measured acceleration histories (e.g.  $\mathbf{a}_m, \dots, \mathbf{a}_{mN}$ , and  $N$  is the number of time steps in a fully discrete formulation) at all nodes of interest. These acceleration time histories are used in an objective function,  $J$ , as

$$J(\{\mathbf{a}\}, \{\mathbf{p}\}) = \frac{\kappa}{2} (\{\mathbf{a}\} - \{\mathbf{a}_m\})^T [Q] (\{\mathbf{a}\} - \{\mathbf{a}_m\}) - \mathfrak{R}(\{\mathbf{p}\}) \quad (1)$$

where  $\mathbf{p}$  is the parameter set of bulk and shear moduli distributed heterogeneously throughout the fixture,  $[Q]$  is a general weight matrix (or a Boolean matrix to select a subset of measured degrees of freedom),  $\kappa$  is a scaling constant, and  $\mathfrak{R}$  is the regularization operator. In the above equation, the notation  $\{\mathbf{a}\} = \{\mathbf{a}_0, \mathbf{a}_1, \dots, \mathbf{a}_N\}$  is used to describe the acceleration history. The material identification problem is defined as an optimization problem by

$$\underset{\{\mathbf{p}\}}{\text{minimize}} \tilde{J}(\{\mathbf{p}\}) \text{ subject to } g_i(\{\mathbf{p}\}) \leq 0, i = 1 \dots n \quad (2)$$

and subject to the governing system of equations at time  $t_{k+1}$

$$[M(\{\mathbf{p}\})]\mathbf{a}_{k+1} + [C(\{\mathbf{p}\})]\mathbf{v}_{k+1} + [K(\{\mathbf{p}\})]\mathbf{u}_{k+1} = \mathbf{f}_{k+1} \quad (3)$$

with

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \Delta t \mathbf{v}_k + \frac{\Delta t^2}{2} [(1 - 2\beta)\mathbf{a}_k + 2\beta\mathbf{a}_{k+1}] \quad (4)$$

$$\mathbf{v}_{k+1} = \mathbf{v}_k + \Delta t [(1 - \gamma)\mathbf{a}_k + \gamma\mathbf{a}_{k+1}] \quad (5)$$

where  $\gamma$  and  $\beta$  are user-defined parameters in the Newmark-beta transition equations. The solution to Eqn. 2 is dependent on the number and spatial distribution of nodes that are retained in the objective function.

## DISCUSSION

For now, all of the practical matters of actually developing a test fixture that matches the theoretical distribution of material parameters have been ignored. Even if it were possible to do so using additive manufacturing or some other technique, a single fixture will not be adequate. The solution set,  $\mathbf{p}$ , is likely to have sensitive dependence on the input environments, so a unique fixture will be required for each unique input environment. The nature of the problem formulation and solution approach also guarantee non-uniqueness of the solutions. Since the solution converges only in the least squares minimization sense, there can be no expectation that the complicated field state can be reproduced closely enough to exercise the correct fatigue and damage mechanisms. The viability of this approach further relies on how faithfully the input environments can be produced within the constraints of current test capabilities. In the course of creating a more dynamically correct fixture have we sacrificed test repeatability and ease of test control?

The mathematical formulation given above was developed with respect to minimizing errors in local accelerations. Although acceleration is a common variable of interest for comparisons of environments and responses (along with other kinematic quantities, e.g. velocity and displacement), stress is a more natural variable when attempting to quantify fatigue, damage, consumed lifetime, or margin. The use of a stress-based objective function could be more appropriate for this type of optimization exercise. Such an objective function could also alleviate some of the concerns of matching the full-field kinematics. If certain regions are known, *a priori*, to be the primary locations of first failure, then satisfying the stress-state locally in those areas without regard to matching globally might be adequate (provided no induced stress states elsewhere shift where first failure occurs.)

## REFERENCES

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