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Predicting Assembly Effective Mass from Two Component Effective Mass Models

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Nomenclature

m_i	effective mass of mode *i*
$\{\phi\}_R$	rigid body row vector
M	mass matrix
$\{\phi\}$	mode shape vector
k_i	effective stiffness of mode *i*
f_n	natural frequency
$f_{transition}$	transition frequency
EI	bending stiffness
KAG	shear stiffness
L	length of beam element

1 Abstract

Effective mass models are powerful tools that allow for a convenient means to calculate the energy associated with vibration response of a structure to a base input acceleration in a particular direction. This is useful for hardware qualification activities and margin assessment. Traditionally, these models are generated from purely analytical means such as a finite element model. However, experimental methods have recently been introduced as an intriguing alternative, particularly for applications where no finite element model is available. In this work, an effective mass modal model of a cable-connector assembly is desired, and neither component has a finite element model. Moreover, there can be multiple cable-connector combinations making analytical modeling as well as explicit testing of each combination impractical. This work develops the capability to combine an experimentally derived connector effective mass model with a simplified and easily extensible analytical cable model. The experimental connector effective mass model is generated through specialized modal testing. The simplified cable model is a Timoshenko beam finite element model whose properties are empirically derived from pinned-pinned cable modal data. The modeled length of the cable is appropriately adjusted for each configuration. Finally, the cable and connector component models can be combined to form the final assembly modal effective mass model for a given translational direction. This method lends itself to developing catalogues of connector and cable data, which can then be easily combined to form any number of assembly configurations without having to explicitly test/model them.

Keywords – Effective Mass, Cable Dynamics, Structural Dynamics, Margin Quantification, Modal Model

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2 Introduction and Motivation

An effective mass model is a modal model that simulates the response of a structure due to a base acceleration input in one direction, see Fig. 1. It can be used to calculate the actual energy in the structure during the base acceleration environment. This is a useful metric for comparing energy at failure relative to that in a qualification test (i.e. margin quantification). Standard methods for calculating an effective mass modal models require a finite element model [1]. Recently, however, methodologies have been developed to extract these models experimentally of a component on a fixture [2]. Typically, there are three effective mass modal models for a component, one for each translational direction (X, Y, and Z). Rotation directions are usually ignored since standard laboratory tests focus on a single translational direction.

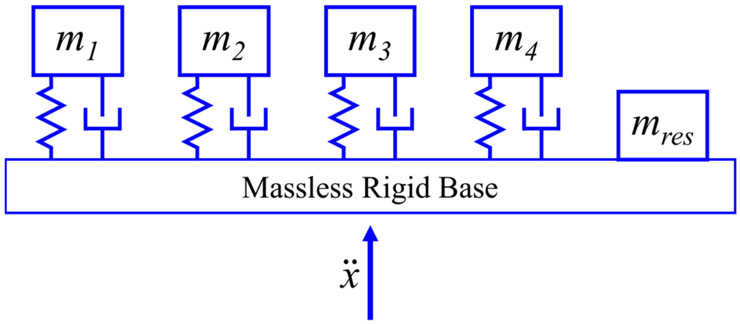

Fig. 1 Effective mass modal model

Developing effective mass models for cable-connector assemblies poses several challenges. The first is that finite element models (FEMs) are typically not developed for the assemblies nor their components. Additionally, there can be multiple cable-connector combinations that make analytical modeling as well as explicit testing of each combination impractical. The goal of this work is to develop a method that can calculate the effective mass modal model of a cable-connector assembly using limited dynamic information of the individual connector and cable components. This is accomplished by combining an experimentally derived connector effective mass modal model with a calibrated numerical model of the cable. The practical implementation would require modal testing each cable family at minimum and maximum lengths to create a catalog of empirically derived cable stiffness properties. Additionally, each connector type would have to be modal tested in order to develop a catalog of connector effective mass modal models. Thus as cable-connector designs change (either switching out the connector or changing the cable length), the important dynamic information (i.e. effective mass and corresponding natural frequencies) can be readily calculated using the process developed herein without the need to recreate a complex FEM nor re-test the combined physical hardware. This allows for the capability to quantify product margin during qualification activities with minimal effort.

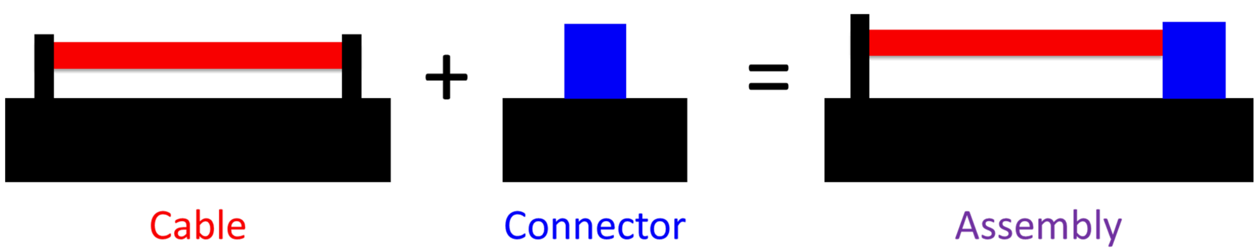

Fig. 2 Combining connector and cable components to create cable-connector assembly effective mass model

The remainder of this report is outlined as follows. Section 3 describes the proposed method for calculating the effective mass modal model of a cable-connector assembly by combining a beam model representing the cable to an effective mass modal model of the connector. The methodology is verified with an analytical model. The proposed method is then employed on physical hardware in Section 4. Included in the section is a discussion of the connector effective mass modal model extracted from a modal test (4.1). Additionally, the cable stiffness properties are empirically derived from cable modal test data in Subsection 4.2. Subsection 4.3 uses the connector and cable information from 4.1 and 4.2 to calculate the cable-connector effective mass modal model and associated frequencies. These are then compared with the results of a truth test conducted on a physical cable-connector assembly. Included in these results are estimations of the uncertainties associated with the effective mass and natural frequencies for both the truth test data and the results of the proposed method. Conclusions are presented in Section 5.

3 Analytical Verification for Combining Effective Mass Models

The goal of this work is to have a method to calculate the natural frequencies and associated effective masses of the modes with high effective mass in a specific translational direction for a cable-connector assembly without the use of a complex finite element model and only knowing the cable type, cable length, and connector type. From experiments, it has been shown that the cable mode shapes can be accurately modeled as pinned-pinned beam mode shapes (see Subsection 4.2.3). Experiments can also be used to extract the connector effective mass. The analytical verification was to determine if we could couple the two separate results and achieve the analytical truth model response.

The remainder of this section discusses the analytical verification. Subsection 3.1 describes the truth model that was used to develop and validate the proposed method which is described in Subsection 3.2. The results of the verification are presented in 3.3.

3.1 Analytical Truth Model

An analytical truth model (see Fig. 3) was created to investigate the efficacy of the method used to compute cable-connector assembly effective masses and natural frequencies using cable and connector component dynamic information. It consisted of a cantilevered beam for the connector and a pinned beam for the cable. The two beams are coupled via a pinned connection.

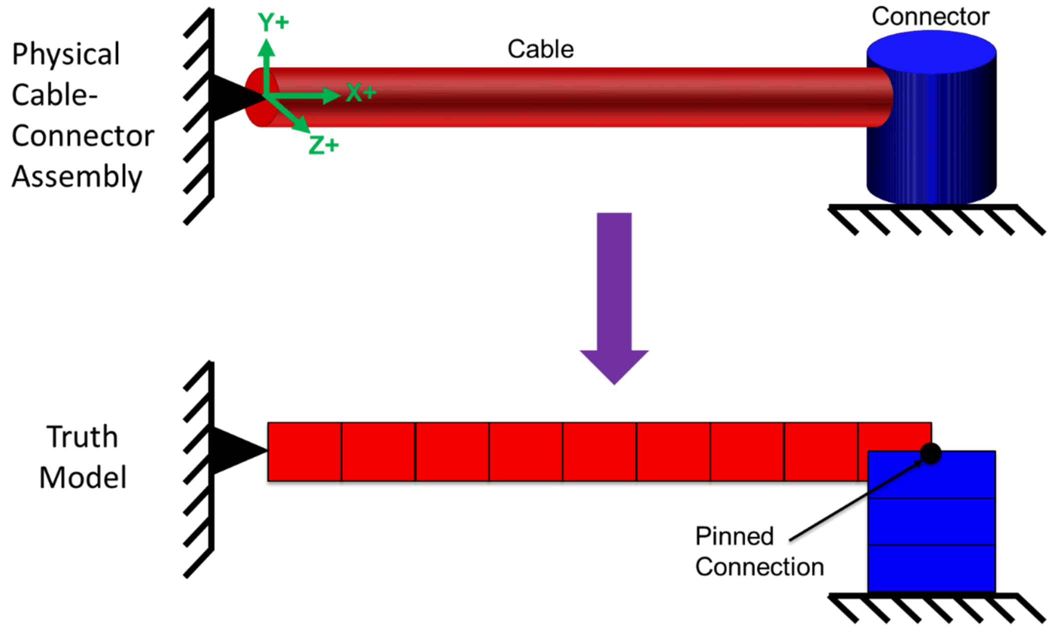

Fig. 3 Truth model for effective mass connection models

Natural frequencies and mode shapes were computed by solving the eigen value problem. The effective mass for each translational direction was calculated using equation (1) [1].

$$m_i = P_i^2 \tag{1}$$

where m_i is the effective mass of mode i and P_i is the modal participation factor for mode i. Equation (2) was used to compute P_i for each mode [1]:

$$P_i = \{\phi\}_R \mathbf{M} \{\phi\}_i \tag{2}$$

where $\{\phi\}_R$ is a rigid body row vector with a "1" at every entry corresponding to DOF oriented in the translation direction of interest and zero otherwise, $\mathbf{M}$ is the mass matrix, and $\{\phi\}_i$ is the shape of the mode for which the effective mas is calculated. Note that equations (1) and (2) are assuming unit modal mass. Typically, effective mass is presented as a percentage of the total mass of the object.

The proposed method described in the following subsection was designed to match the natural frequencies and effective masses of the dominant modes of this truth model for transverse beam axes motion. Dominant modes are those that have large effective mass for the given direction (>5% of the total mass). The reason for this is that modes with the largest effective mass have the greatest potential to cause damage during qualification/operation environments.

As previously discussed, an effective mass model is produced for each translational direction. Effective mass and frequencies in the axial coordinate of the cable (X direction) were determined by another method. Thus the remainder of this paper focuses on the remaining transverse cable directions (Y and Z).

3.2 Creating Assembly Effective Mass Models from Component Dynamic Information

The proposed method assumes that an effective mass modal model of the connector and stiffness properties of the cable will be available. As discussed in the introduction, in practice these data will be contained in catalogs created from previously-conducted connector and cable testing. The basic concept of the proposed method is to create a simple numerical model of the cable beam from the desired length and stiffness properties then use the connector effective mass model to appropriately load the tip of the cable as it does in the cable-connector assembly. This is accomplished by sequentially connecting individual effective masses (m_{coi}) and stiffnesses (k_{coi}) of the connector to the tip element of the cable model, see Fig. 4. Mathematically, this is approximated by adding m_{coi} to the element in the mass matrix that corresponds to the cable tip DOF for the given translational direction and similarly adding k_{coi} in the stiffness matrix. The eigenvalue problem is then solved and the effective mass model is derived from the results and equation (1) to create the assembly submodel.

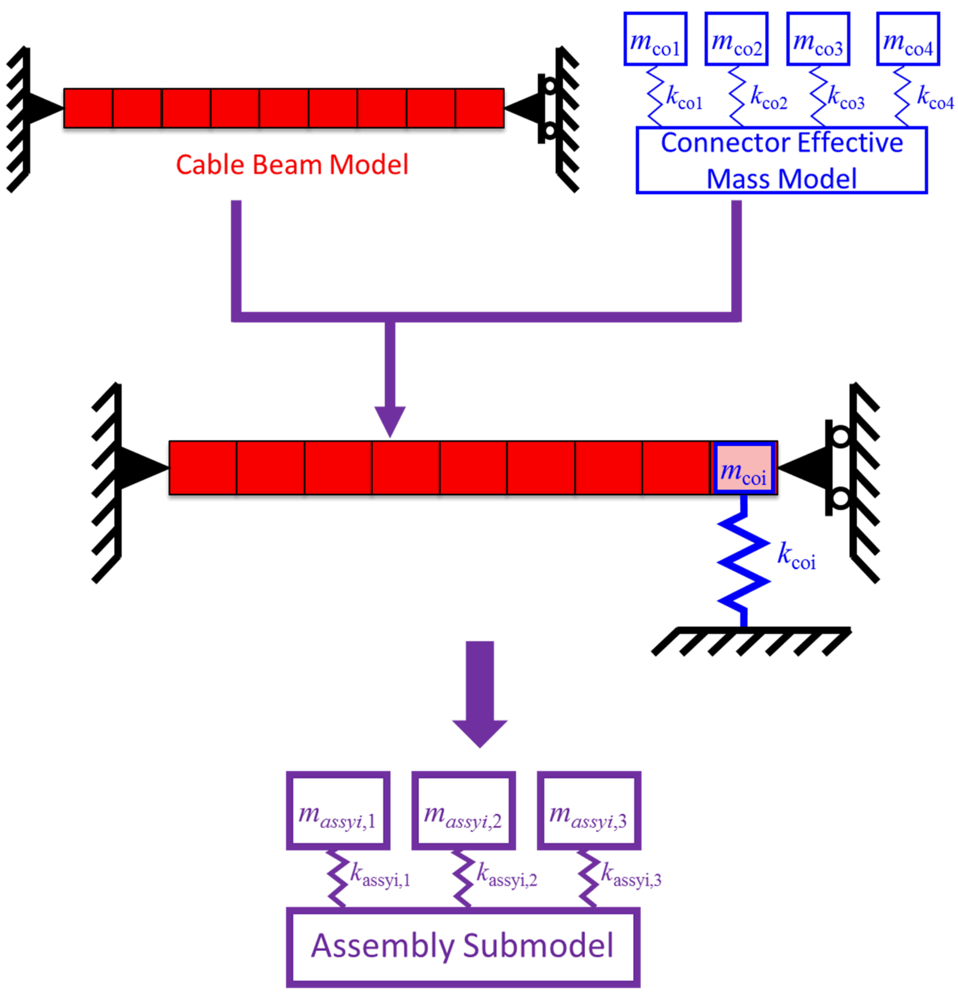

Fig. 4 Proposed method for calculating cable-connector assembly effective mass and natural frequencies from dynamic information from individual components

There could be multiple connector modes in the bandwidth of interest that are coupled to the cable. In this scenario, the first connector effective mass and stiffness are coupled to the cable and its effective mass model is calculated to create an assembly submodel. The second connector effective mass and stiffness are then used without the first (i.e. the connector effective mass added to the cable tip is just m_{co2} and not $m_{co1}+m_{co2}$) to create a second assembly submodel. This continues for every connector mode with large effective mass (see discussion below) in the bandwidth of interest. The final effective mass model of the assembly (called the final assembly model) is stitched together from each assembly submodel. The results from the first assembly submodel are included in the final assembly model up until a transition frequency. The results from the second assembly submodel are then used until a second transition frequency. For this work, each transition frequency was selected to be half-way between the subsequent connector natural frequencies. This process is depicted in Fig. 5 when two connector modes are used to compute the final assembly model.

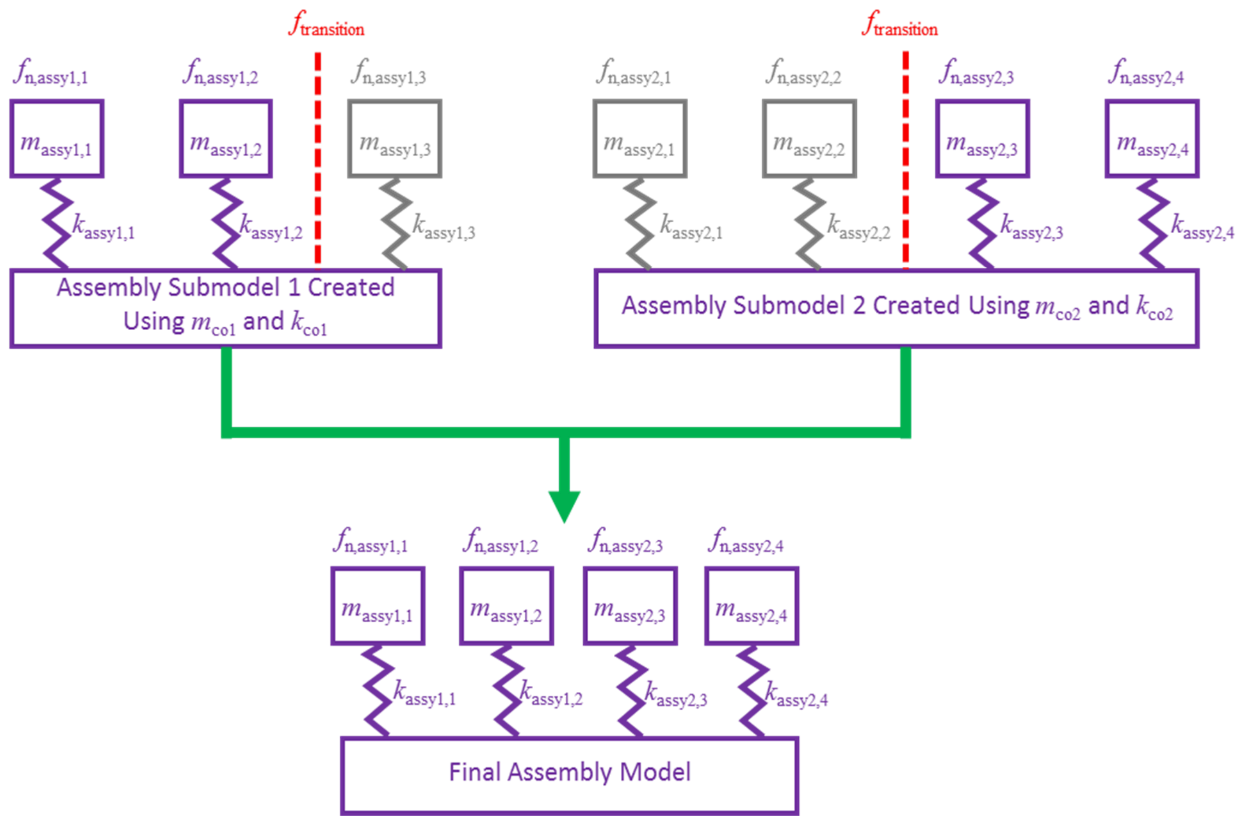

Fig. 5 Example of creating the final assembly model using different assembly submodels

It is important to note that only those modes of the connecter with large effective masses are used in this process. The reason for this is, if in a particular direction, a connector mode has a very small effective mass, it will correspondingly have a very low effective stiffness. When coupled to the cable as shown in Fig. 4, the tip of the cable would be virtually unconstrained and behave like a pinned-free beam which does not accurately represent the conditions of the true assembly. Therefore, only connector modes with large effective masses are included in this process.

In practice, an experimental effective mass modal model will be available for the connector. For the analytical verification work, however, the effective mass modal model of the connector was extracted from a cantilevered beam representing the connector in the truth model, see Fig. 6.

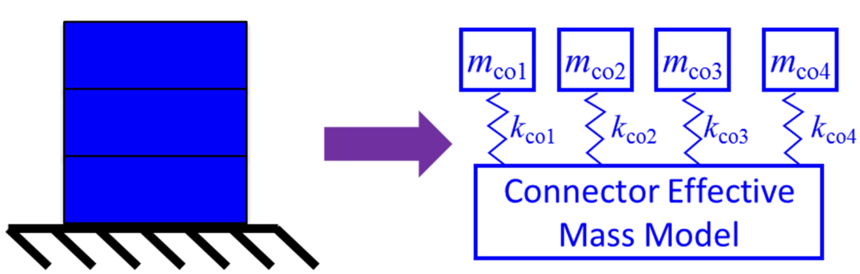

Fig. 6 Connector beam used to create the effective mass modal model for analytical verification

3.3 Analytical Verification Results

To demonstrate the capabilities and limitations of this process, stiffness and geometrical parameters were selected for the individual connector and cable beams shown in Fig. 3 and for the truth model, were coupled accordingly. The effective mass modal model was created for the connector using equation (1) in substitution for an experimental effective mass modal model which would be utilized in practice. The cable was then coupled to this model as described in Subsection 3.2 to create the final assembly model for the Y-direction and then for the Z-direction. For both directions, only two connector modes with significant effective mass were within the desired bandwidth of 7000 Hz. A comparison of the effective masses for each mode and corresponding natural frequency for the truth model and the final assembly models are shown in Fig. 7 and Fig. 8 for the Y- and Z-directions, respectively. The key quantity to notice is how close the vertices of each plot align, as they show both the frequency and effective mass of one of the modes. Additionally, the final assembly model is a reduced order model and as such has fewer DOFs than the truth model, resulting in fewer modes. Lastly, since only two connector modes were used to create the final assembly model, there is only one transition frequency for each direction.

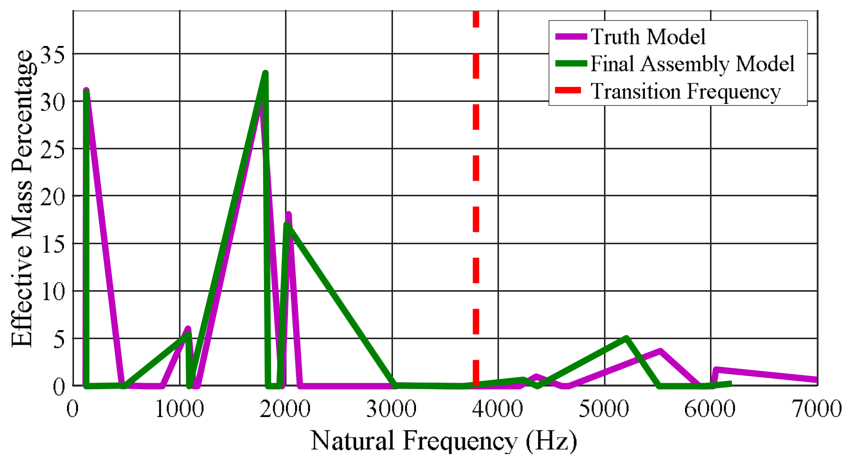

Fig. 7 Analytical verification results, truth model vs final assembly model comparison for Y-direction

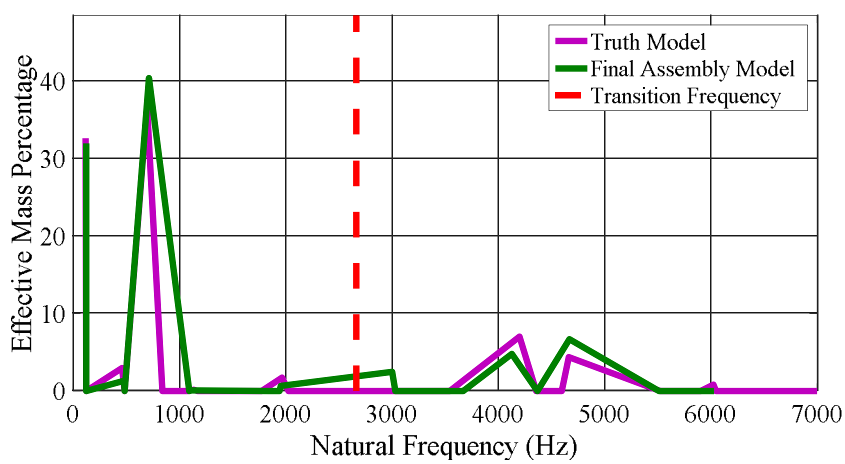

Fig. 8 Analytical verification results, truth model vs final assembly model comparison for Z-direction

In general, the final assembly model matches the effective mass and frequency of the truth model well for modes with large effective mass for both directions. It is also worth noting that the cable deformation in the mode shapes match well. For the modes in the vertical direction with greater than 4% effective mass, the final assembly model had a maximum frequency error of less than 6% (the predicted frequency was 5200 Hz when the truth was 5520 Hz) and the maximum effective mass error was less than 2%.

Likewise, the results for the Z-direction agree well for modes with effective mass larger than 4%. There appears to be larger effective mass errors for the modes near 4200 Hz and 4660 Hz. In the truth model, these modes correspond to the connector

2nd bending in phase and out of phase with the cable 6th bending. This coupling distributes the effective mass in the truth model between these two modes in a fashion that cannot replicated in the final assembly model since the connector is absorbed into the mass of the cable tip. However, it is worth noting that the sum of the effective masses of these two modes is nearly identical (~0.2% difference) for the truth and final assembly models. This indicates that the final assembly model is able to capture the general dynamic effect of this mode pair. These results were sufficiently accurate that this process was implemented on physical hardware.

4 Proof of Concept on Physical Hardware

This section discusses the calculation of the effective mass modal model and corresponding natural frequencies for a physical cable-connector using the process developed in the previous section. The experimental effective mass modal model for the connector is first extracted from a modal test (Subsection 4.1). The stiffness parameters for the cable model are then empirically fit from modal tests of two different cable lengths in Subsection 4.2. The process developed in Subsection 3.2 is then employed to combine the connector effective mass modal model and the simplified cable FEM created from the cable stiffness and geometrical parameters. The results are compared to a truth test in Subsection 4.3.

4.1 Connector Effective Mass Experiment

This section discusses the extraction of the connector effective mass modal model from experimental data. The connector was installed into a stiff fixture and modal tested. Natural frequencies, damping values, and mode shapes were extracted. The method described in [2] was then used to extract the effective mass modal model of the connector, see Table 1 for the results. For brevity, the details of the technique are not given here, but [2] provides a detailed procedure on its implementation.

Table 1 – Experimentally Extracted Connector Effective Mass Model Results

Mode	Frequency (Hz)	Description	Normalized Effective Mass (%)		
			X	Y	Z
1	768	connector bending in Z	0	2.5	52.3*
2	785	connector bending in X	44.4*	1.4	0
3	1170	cable stub bending in Y	10.1	7	0
4	1264	cable stub bending in Z	0	0.1	0.4
5	1926	connector axial	0	80.2	0.1

*These values were reconfigured so that effective mass of connector bending is either purely in the X or Z direction

Note that the mode shapes showed that the bending modes 1 and 2 were not directly aligned with the coordinate axes, e.g. the connector did not bend purely in X, but also had some Z direction motion. This resulted in the effective mass for each of the first two modes being distributed between these two directions. The analytical verification from Section 3 had the connector modes aligned with the coordinate axes. Additionally, modes 1 and 2 have similar frequencies, so they could be oriented in any two orthogonal directions. Therefore, the bending axes were realigned to match the modeling axes, but with the same amount of effective mass is still retained in the appropriate direction.

4.2 Cable Model as a Timoshenko Beam

4.2.1 Creation of the Cable Model as a Timoshenko Beam

Previous work by Goodding, et. al. shows that cables can be represented by the bending (EI) and shear (KAG) stiffness terms of the Timoshenko beam without the rotary inertia term [3]. The EI and KAG values are used to create the stiffness matrix for the cable model. This cable is of the same length as that of the physical cable-connector assembly tested in the truth experiment in Subsection 4.3. The first step is to calculate the stiffness matrix of the Bernoulli-Euler beam, $\mathbf{K}_{cable,B}$, using EI and the length [4]. This is performed in one transverse direction using the translation and rotation at the first node and then the translation and rotation at the second node of the beam element. The shear beam stiffness matrix, $\mathbf{K}_{cable,S}$, is constructed from the elemental form given in (3) which was calculated from the wave equation [5].

$$\begin{bmatrix} \frac{KAG}{L} & 0 & -\frac{KAG}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{KAG}{L} & 0 & \frac{KAG}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

where L is the elemental length of the cable beam. Note that this elemental matrix will produce a $\mathbf{K}_{\mathbf{cable,S}}$ where all of the diagonal values will be 2KAG/L except the first and last. Additionally, the zero terms in (3) correspond to the rotational DOFs of the cable beam. The total stiffness matrix of the cable beam, K_{cable}, is then the sum of $\mathbf{K}_{\mathbf{cable,B}}$ and $\mathbf{K}_{\mathbf{cable,S}}$. Once $\mathbf{K}_{\mathbf{cable}}$ has been calculated (along with the cable mass matrix, $\mathbf{M}_{\mathbf{cable}}$), the cable is constrained with pinned boundary conditions for the stiffness parameter calibration to experimental data, see Subsection 4.2.2. The $\mathbf{M}_{\mathbf{cable}}$ and $\mathbf{K}_{\mathbf{cable}}$ comprise the cable model that is used in Subsection 4.3 along with the connector effective mass modal model from Subsection 4.1 to calculate the effective mass model and corresponding natural frequencies of a physical cable-connector assembly using the method outlined in Subsection 3.2.

There is the problem of shear locking over-stiffening the result using the formulation in equation (3) [6]. However, with this class of problem, the connector is sufficiently stiff that it minimizes the effect due to shear locking.

4.2.2 Modal Test of the Cable

The proposed method discussed in Subsection 3.2 requires the EI and KAG stiffness properties of the cable by itself in order to compute the simple FEM ($\mathbf{M}_{\mathbf{cable}}$ and $\mathbf{K}_{\mathbf{cable}}$). Since a cable is a complex, non-homogenous combination of wire strands and insulation material, the stiffness properties were empirically fit from modal testing of the cable clamped at either end, see Fig. 9. In order to achieve robust stiffness properties, two cable lengths were tested, and natural frequencies, damping values, and mode shapes were extracted for each.

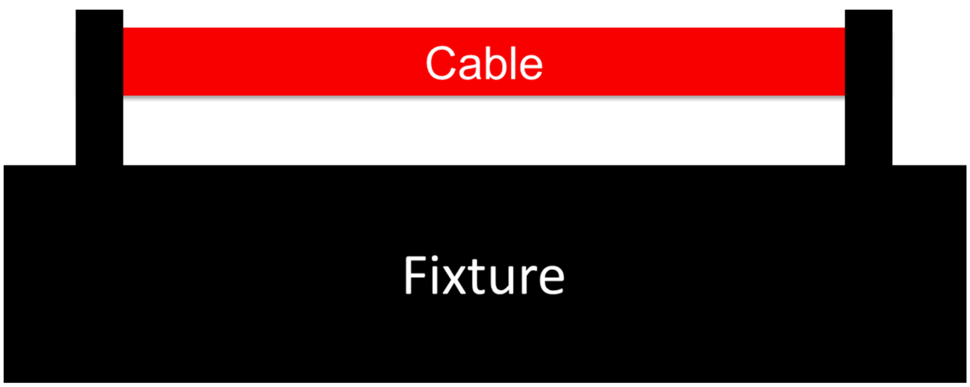

Fig. 9 Configuration for experiments used to empirically fit cable stiffness parameters

4.2.3 Fitting the Stiffness Parameters for the Timoshenko Beam

For the Timoshenko beam, two stiffness parameters need to be fit: EI (bending stiffness) and KAG (shear stiffness). This was accomplished by performing a sensitivity analysis according to the following:

$$\left\{f_{n,exp}\right\} = \left\{f_{n,model}\right\} + \left[\frac{\partial\left\{f_{n,model}\right\}}{\partial \mathrm{EI}} \quad \frac{\partial\left\{f_{n,model}\right\}}{\partial \mathrm{KAG}}\right]\left\{\begin{matrix}\Delta \mathrm{EI} \\ \Delta \mathrm{KAG}\end{matrix}\right\} \quad (4)$$

where $\{f_{n,exp}\}$ is a vector containing the experimental natural frequencies of the cable, and $\{f_{n,model}\}$ is a vector of the model natural frequencies calculated using an initial estimate of EI and KAG. Each $\{f_{n,model}\}$ were computed from an eigenvalue analysis of a Timoshenko beam defined by $\mathbf{M}_{\mathbf{cable}}$ and $\mathbf{K}_{\mathbf{cable}}$. Note that the models were created using pinned-pinned boundary conditions.

Table 2 displays the difference between the experimental natural frequencies of the cable and those of the Timoshenko beam generated from the converged EI and KAG values. It is important to note that several cable modes were affected by motion of clamps on the fixture which secure the two ends of the cable during the experiment. The influence on the frequencies could not be exactly quantified but are estimated to potentially shift frequencies up to 10%.

Table 2 – Results of Cable Stiffness Fit, Experiment vs Model

Length (in)	Mode	% Difference in f_n
8	1	9
	2	2
	3	-2
	4	5
6	1	-10
	2	0
	3	-11

The boundary conditions for the cable experiment were initially assumed to be clamped-clamped. However, no reasonable fit to the experimental frequencies could be achieved. The use of pinned boundary conditions was supported by the fact that the deflection of the ends of the cable in the experimental mode shapes visually appeared to replicate the motion of a pinned-pinned beam. Note that the KAG and EI values were one to two orders of magnitude below the values that one would obtain with the copper wire with the measured diameter. This reinforced that the experimental process of fitting these two parameters was required.

4.3 Comparison with Truth Test Results

The results from Subsection 4.2 were used to create a cable model of the physical hardware and this was coupled to the connector experimental effective mass modal model presented in Subsection 4.1 using the method proposed in Subsection 3.2 to create a final assembly model of the physical cable-connector assembly. During the course of this work, it was discovered that the cable lengths used in Subsection 4.2 to empirically fit the stiffness parameters did not bound the cable length used in the truth test. Thus the results presented below represent an extrapolation on the length effect, a poor engineering practice that lends itself to larger-than-expected errors. Schedule did not permit performing an additional cable test on the shorter length. Fig. 10 and Fig. 11 show the truth test results and the final assembly model calculations for the effective mass and natural frequencies for the cable-connector assembly. Note that for either direction only modes with the two largest effective masses are presented.

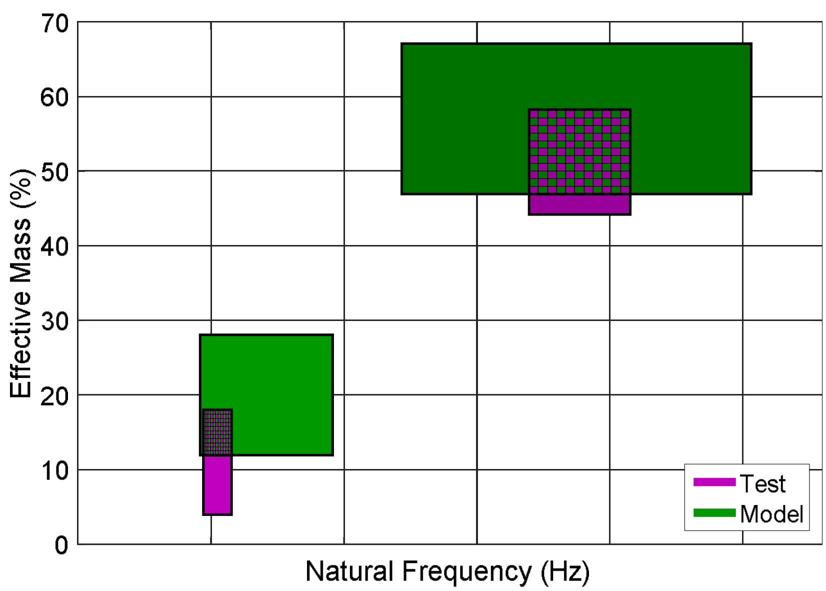

Fig. 10 Truth test vs final assembly model results, Y-direction

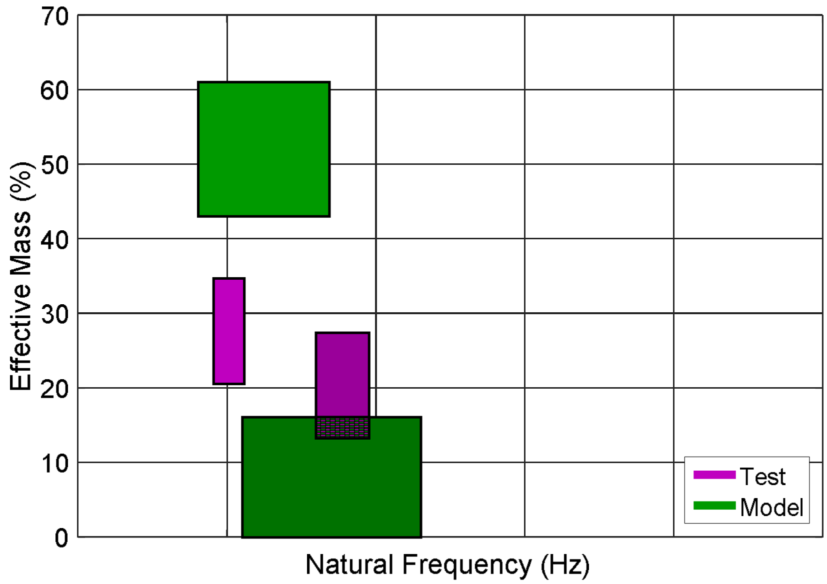

Fig. 11 Truth test vs final assembly model results, Z-direction

These results include estimated uncertainty bounds on both the effective mass and natural frequency for the truth test and final assembly model. They assume cable lengths interpolated within the tested bounds (an assumption that was not appropriately observed here). Uncertainties are valuable when computing the failure margin of a product. Due to cost and schedule limitations, an exhaustive uncertainty quantification effort could not be performed, so the error bounds in Fig. 10 and Fig. 11 were determined using engineering judgement and multiple experiments from previous work. For the truth test results, the frequency and effective mass uncertainties were chosen to be, respectively, ±10% of the extracted natural frequency and ±7% of the total mass of the cable-connector assembly. These numbers have been shown to be typical for effective mass experiments. The frequency and effective mass uncertainties for the final assembly model were selected to be a root-sum-square of the individual uncertainties from each source as listed in Table 3.

Table 3 – Sources of Uncertainty in Final Assembly Model

Parameter	Source		
	Connecter Experiment	**Max Error from Analytical Verification Results**	**Unit-to-Unit Variability**
Frequency	±10% of natural frequency	±6% of natural frequency	±30% of natural frequency
Effective Mass	±7% of total mass	±3% of total mass	±10% of calculated effective mass

Ideally, the final assembly model results in Fig. 10 and Fig. 11 would encompass the test results. The test frequencies are nearly within the model frequency bands, but this is not the case with the effective mass, particularly in the Z direction. The cable stiffness extrapolation, especially for shorter cables as explained above is impacting the model results, but at this stage we do not know if it is the only error. One phenomenon that was observed during the analytical verification work was that when modes of the individual connector and cable were close in frequency, they coupled in the cable-connector assembly which then distributed the effective mass between the two modes in a fashion that the final assembly model was not able to reproduce. However, the sum of the effective masses of these two modes was captured by the model. This is similar to the situation for the two modes in the Z direction.

5 Conclusion

A method for calculating the effective mass modal model of a cable-connector assembly using dynamic information of the components was developed. An analytical verification model showed that the method appeared to be sufficiently accurate to implement on physical hardware. Using a connector effective mass modal model extracted from experimental data and a simple cable FEM created from empirically fit stiffness parameters, the effective mass modal model of a simplified assembly was computed. The results were compared to a truth experiment with uncertainties applied to both the effective mass and natural frequencies for the model and experiment. While the model did not completely encompass the experimental results, its performance was encouraging especially when considering the influence of unanticipated extrapolation. Closely coupled cable and connector modes may also induce more error to the process.

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