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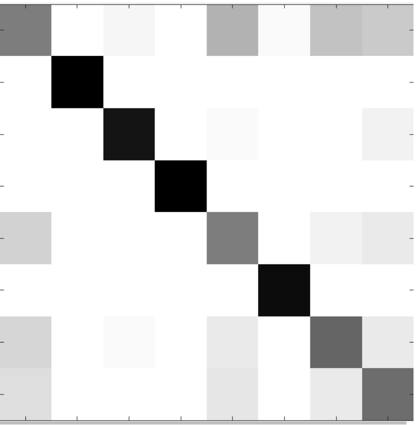

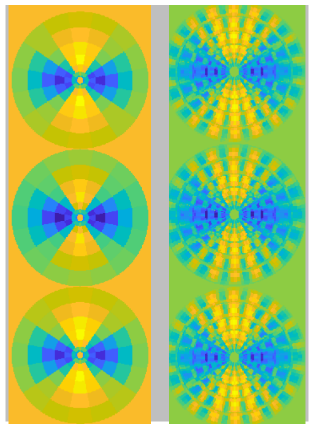

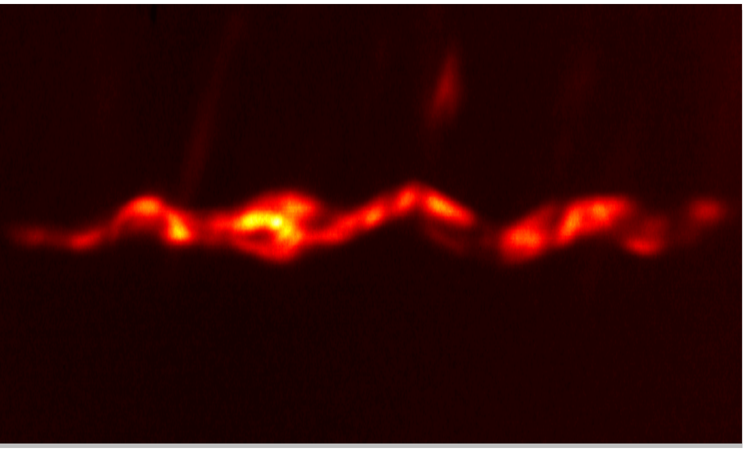

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Quantification of MagLIF stagnation morphology using the Mallat Scattering Transformation

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Stéphane Mallat (École Normale Supérieure)

presented at 59th Annual Meeting of the APS Division of Plasma Physics
25 October 2017 • Milwaukee, Wisconsin (paper PO8.00002)

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Outline of talk

- what is the Mallat Scattering Transformation and why does it perform well in classifying physical systems?
- verification of classification performance and regression of MST to stagnation morphology on a synthetic dataset
- application of the MST as a metric of MagLIF stagnation morphology to both Gorgon computer simulations and measured experimental data

2D Mallat Scattering Transformation (MST)

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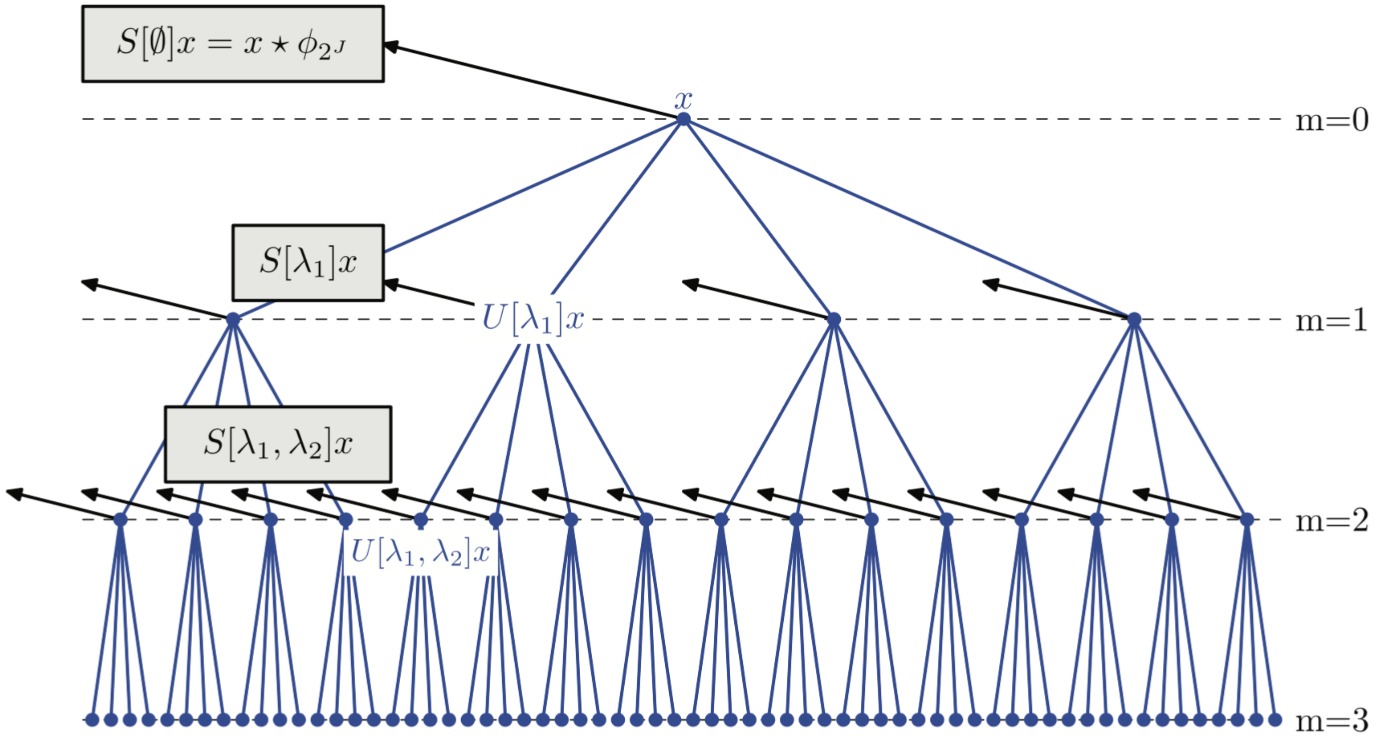
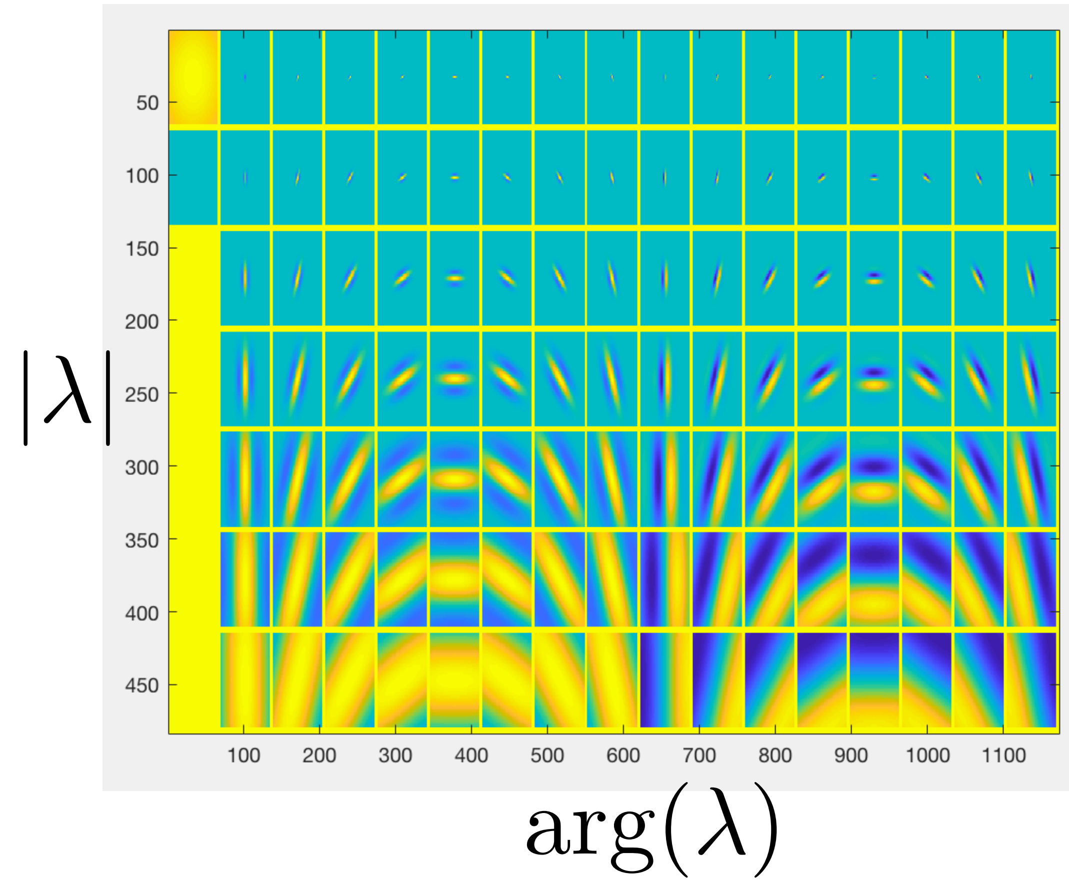
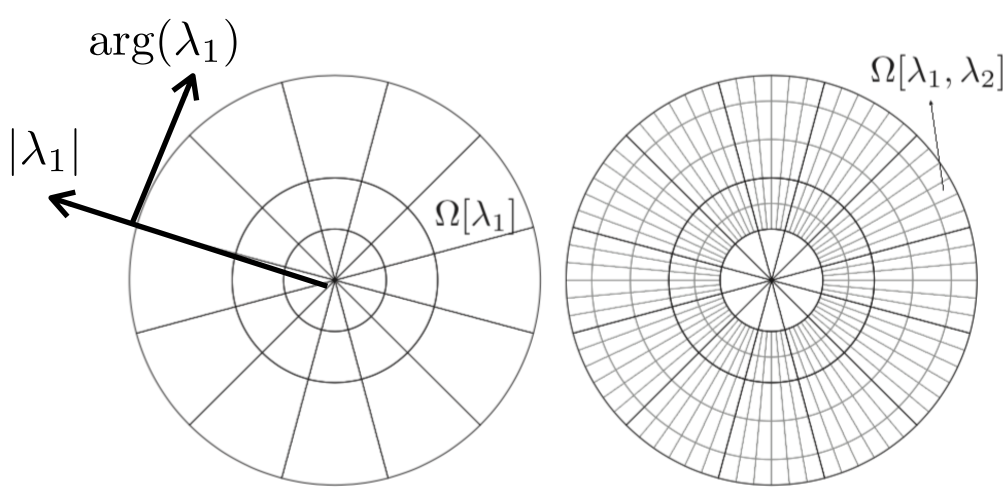
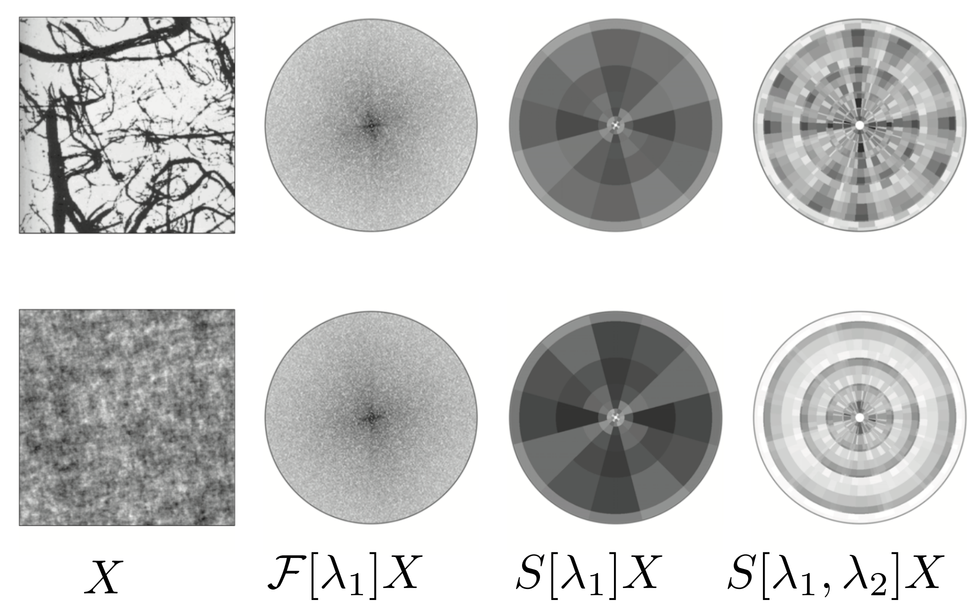






(a) (b)



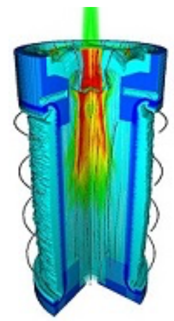
Bruno & Mallat, “Scattering transform for image classification: invariant Scattering Convolution Network”, IEEE Trans. on PAMI, vol. 35, no. 8, pp. 1872-1886, Aug. 2013.

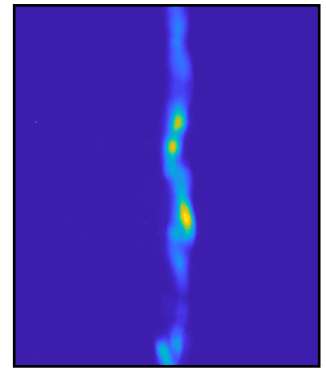
Relationship of MST to the physics: MST is the S-matrix!

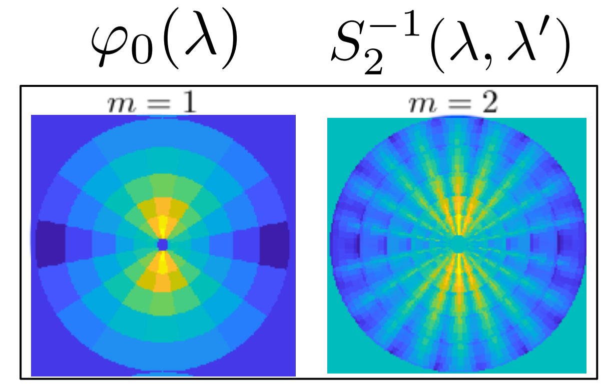
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla p$$

$$\mathbf{J} \times \mathbf{B} = \frac{(\mathbf{B} \cdot \nabla)\mathbf{B}}{\mu_0} - \nabla \left(\frac{\mathbf{B}^2}{2\mu_0} \right)$$







ODEs or PDEs with BC & IC → emergent behavior of self organized state → metric of self organization

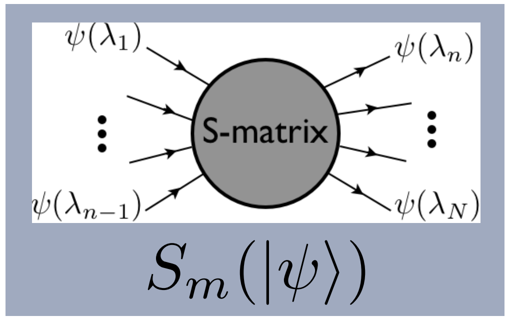
specified by state variables (coefficients of PDEs and BC)

system response

Mallat Scattering Transformation, texture of system response

Lagrangian L

state of system $|\psi\rangle$



generalized Green's function, physics as a function of scale running coupling constants (scale)

Lagrangian perspective $\mathcal{F}(L) = S_m(|\psi\rangle)$ canonical perspective

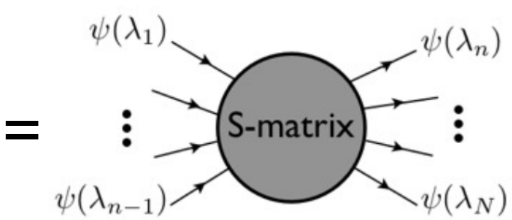
The MST is the S-matrix!

from the Lagrangian perspective define generating function:

$$Z[J] = N \int [d\psi(\lambda)]\, e^{(i/\hbar) S_0[\psi(\lambda)] + (i/\hbar)\int dx\, J(\lambda)\psi(\lambda)}$$

the connection to the canonical formulation is:

MST = $S_m(|\psi\rangle) = E(T_\lambda(\hat{\psi}(\lambda_1)\ldots\hat{\psi}(\lambda_N))\, F(f)) = \frac{1}{Z[J]}\frac{\delta}{\delta J(\lambda_1)}\cdots\frac{\delta}{\delta J(\lambda_N)} Z[J]\Big|_{J=0} = \mathcal{F}(L)$ =



define the effective action through a Legendre transformation

$$S[\varphi(\lambda)] = -\ln Z[J] + \int d\lambda\, J(\lambda)\, \varphi(\lambda)$$

expanding in S and φ it can be shown that:

$$E(\hat{\psi}(\lambda)F(f)) = \frac{1}{Z[J]}\frac{\delta Z[J]}{\delta J(\lambda)}\Big|_{J=0} = \varphi_0(\lambda)$$

= classical action averaged over fluctuations as a function of renormalization scale

$$E(\hat{\psi}(\lambda)\hat{\psi}(\lambda')F(f)) = \frac{1}{Z[J]}\frac{\delta^2 Z[J]}{\delta J(\lambda)\delta J(\lambda')}\Big|_{J=0} = -S_2^{-1}(\lambda,\lambda')$$

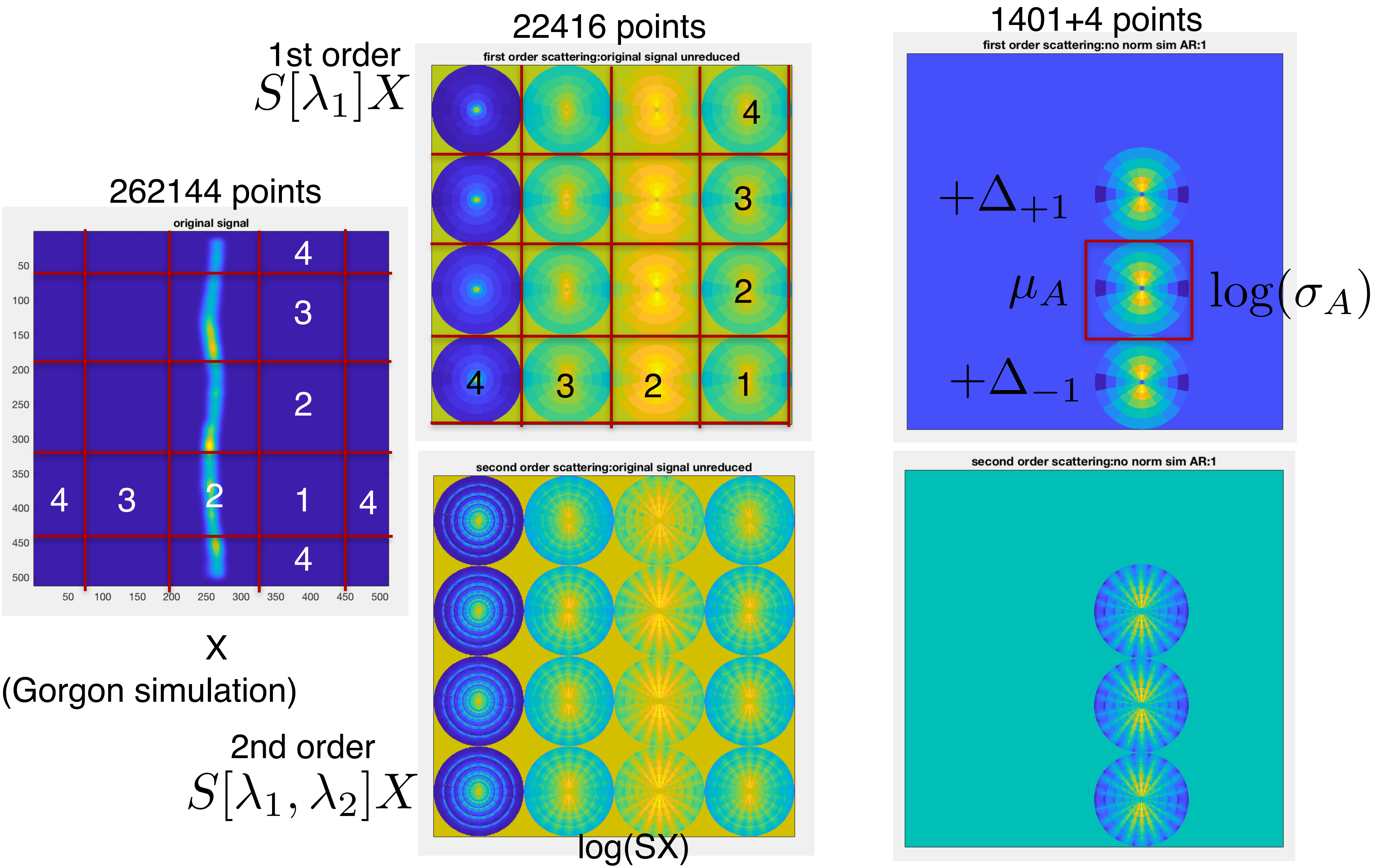
= transfer matrix (scale dependent renormalization mass) as a function of initial and final renormalization scale

effective physics as a function of scale
physics averaged at that scale
running coupling constants
renormalization

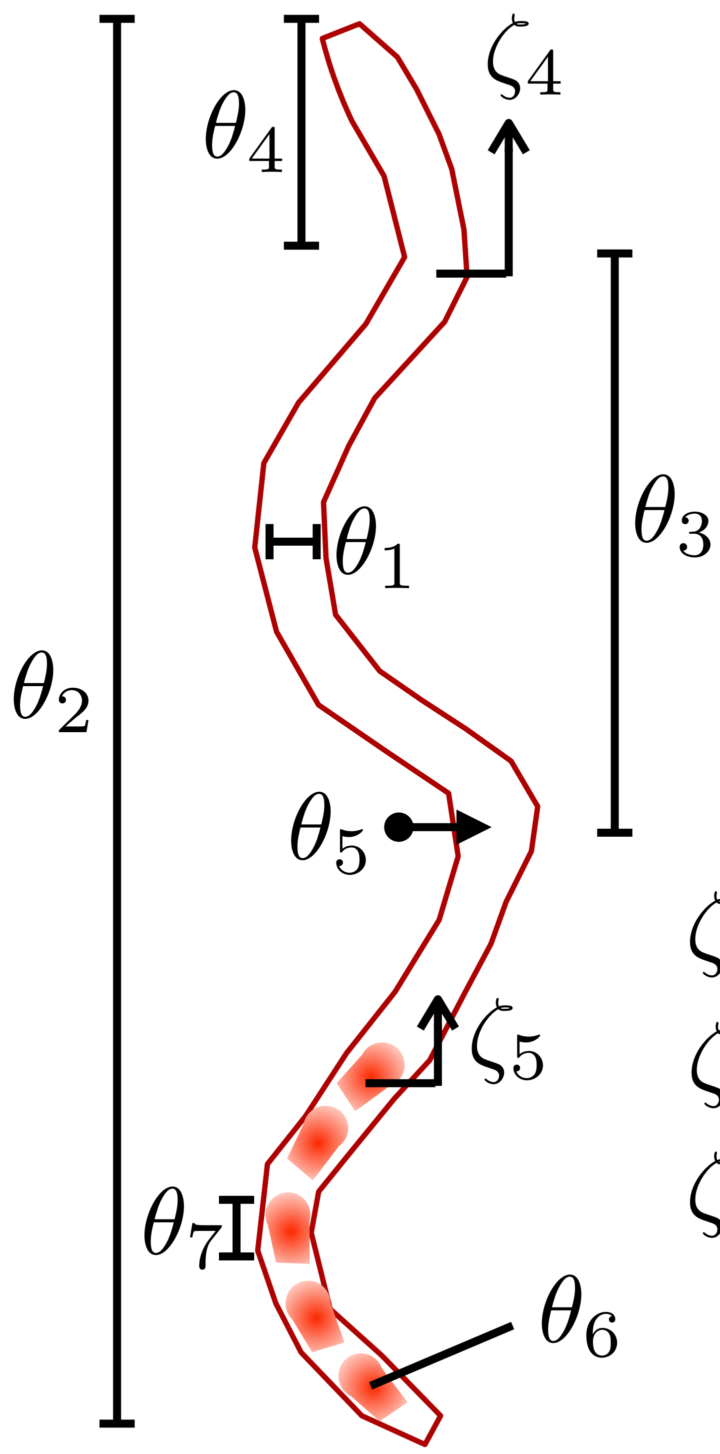
see Burby PoP **24**, 082104 (2017)
for MHD in Lagrangian perspective

Glinsky, arXiv:1106.4369

Parameterization for this problem



Synthetic model of stagnation images



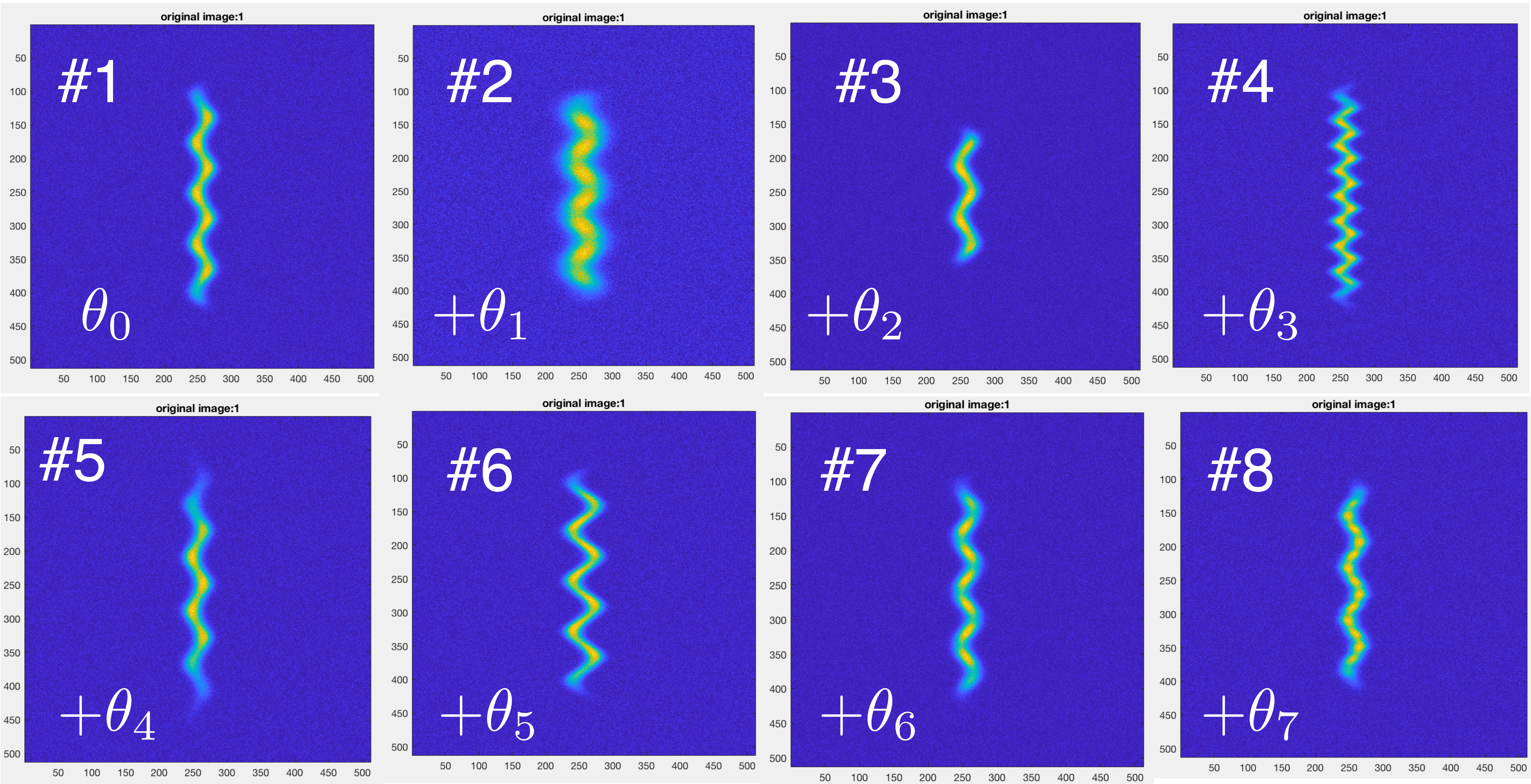
note: log10 used of all parameters except for stochastic phases

$\zeta_1 = \text{background noise}$

$\zeta_2 = \text{signal noise}$

$\zeta_3 = \text{amplitude of signal}$

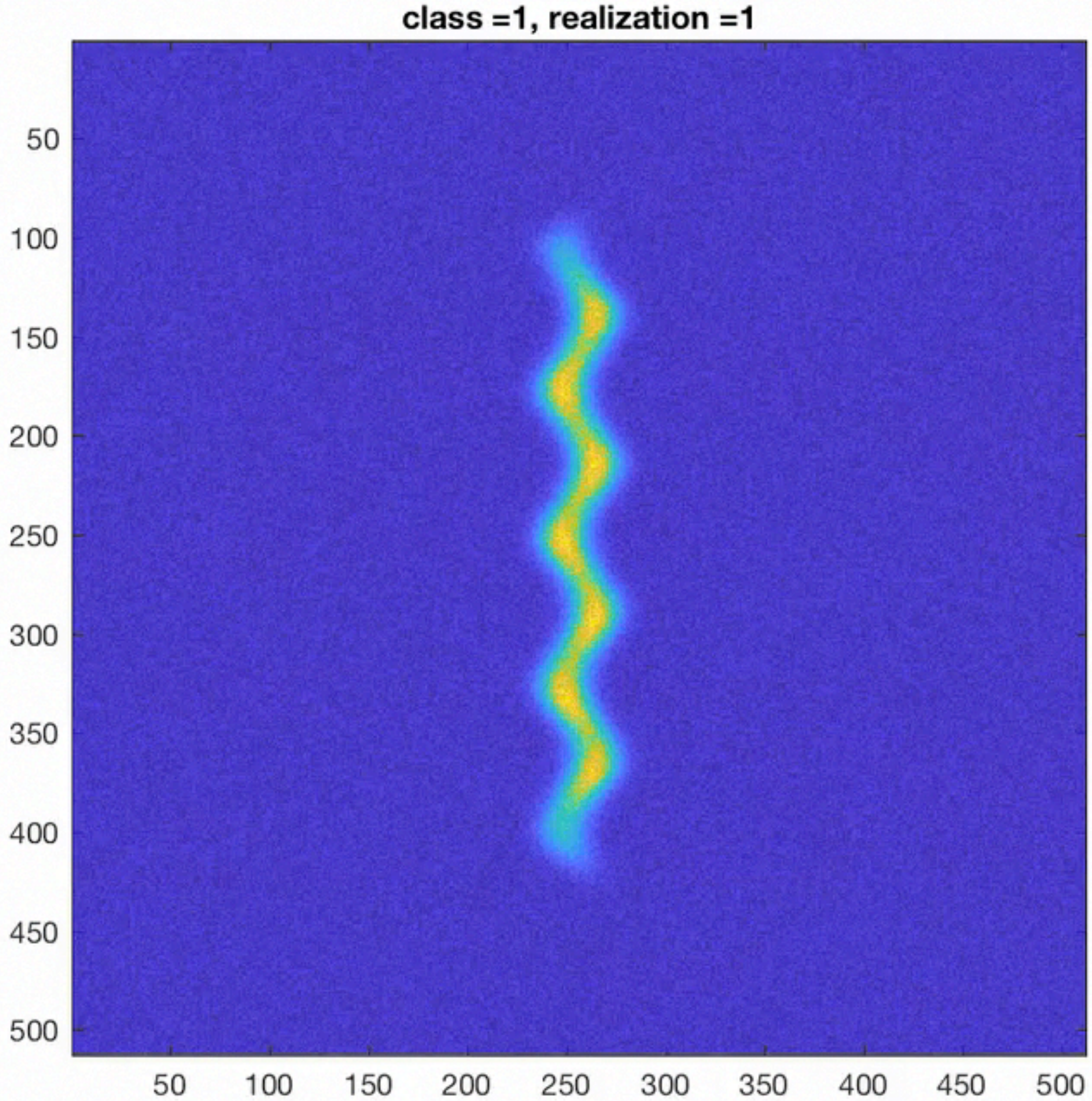
Eight classes in synthetic dataset

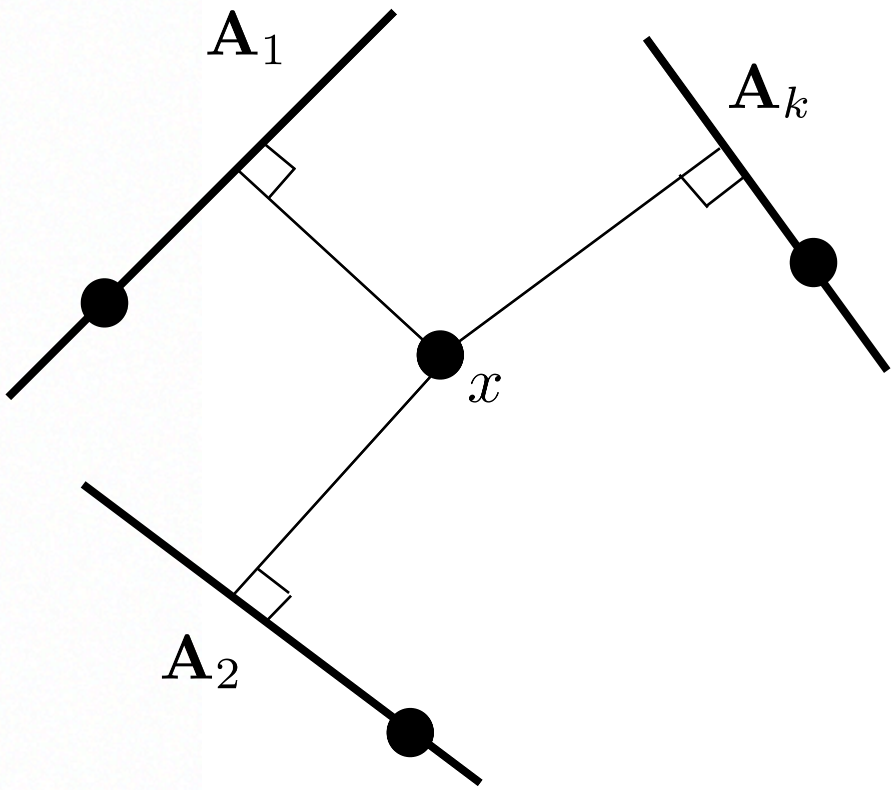


$$\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7)$$ model parameters (signal)

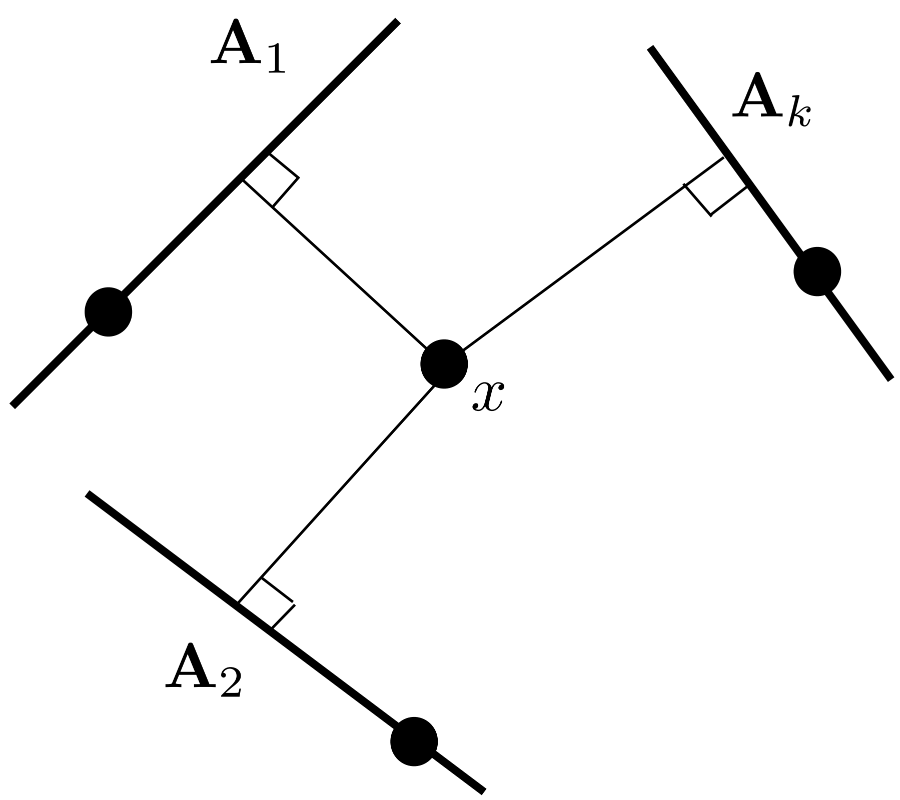
$$\zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5)$$ stochastic parameters (noise)

Ensemble of class realizations classified with an affine classifier





Affine classifier built using Mallat Scattering Transformation



$$x - P_{\mathbf{A}}(x) = x - \sum_{i=1}^{d} < x, A_i >$$

(projection perpendicular to an affine space)

$$\sigma_d^2 = N_C^{-1} \sum_{k=1}^{N_C} \frac{E(||SX_k - P_{\mathbf{A}_k}(SX_k)||^2}{E(||SX_k||^2)}$$

(precision)

$$(\sigma_d^2)_{kl} = \frac{E(||SX_k - P_{\mathbf{A}_l}(SX_k)||^2}{E(||SX_k - P_{\mathbf{A}_k}(SX_k)||^2}$$

(separation factors)

$$\sigma_d^2 = N_C^{-1} \sum_{k=1}^{N_C} \frac{E(\min_{l \neq k} ||SX_k - P_{\mathbf{A}_l}(SX_k)||^2}{E(||SX_k - P_{\mathbf{A}_k}(SX_k)||^2}$$

(separation factor)

Glossary

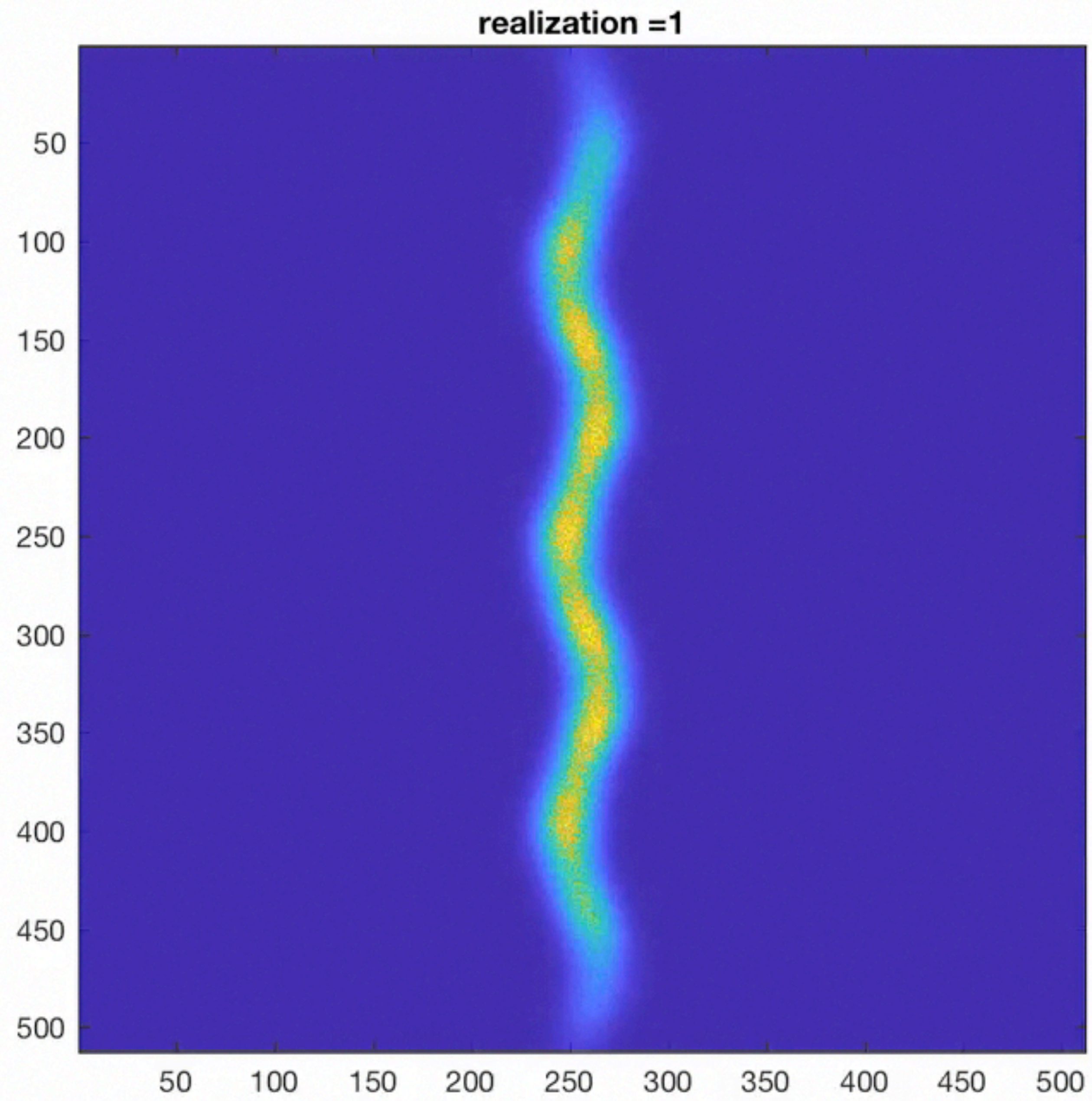
Precision – normalized average “distance” within the class, smaller the better, scales as variance

Separation – ratio of the “distance” between the classes to the distance within the class, bigger the better, scales as variance

Separation Matrix – matrix of separations between class #1 and class #2

Confusion Matrix – conditional probability of classified as class #1 given that it is of class #2

Ensemble used to derive regressions for model parameters from MST

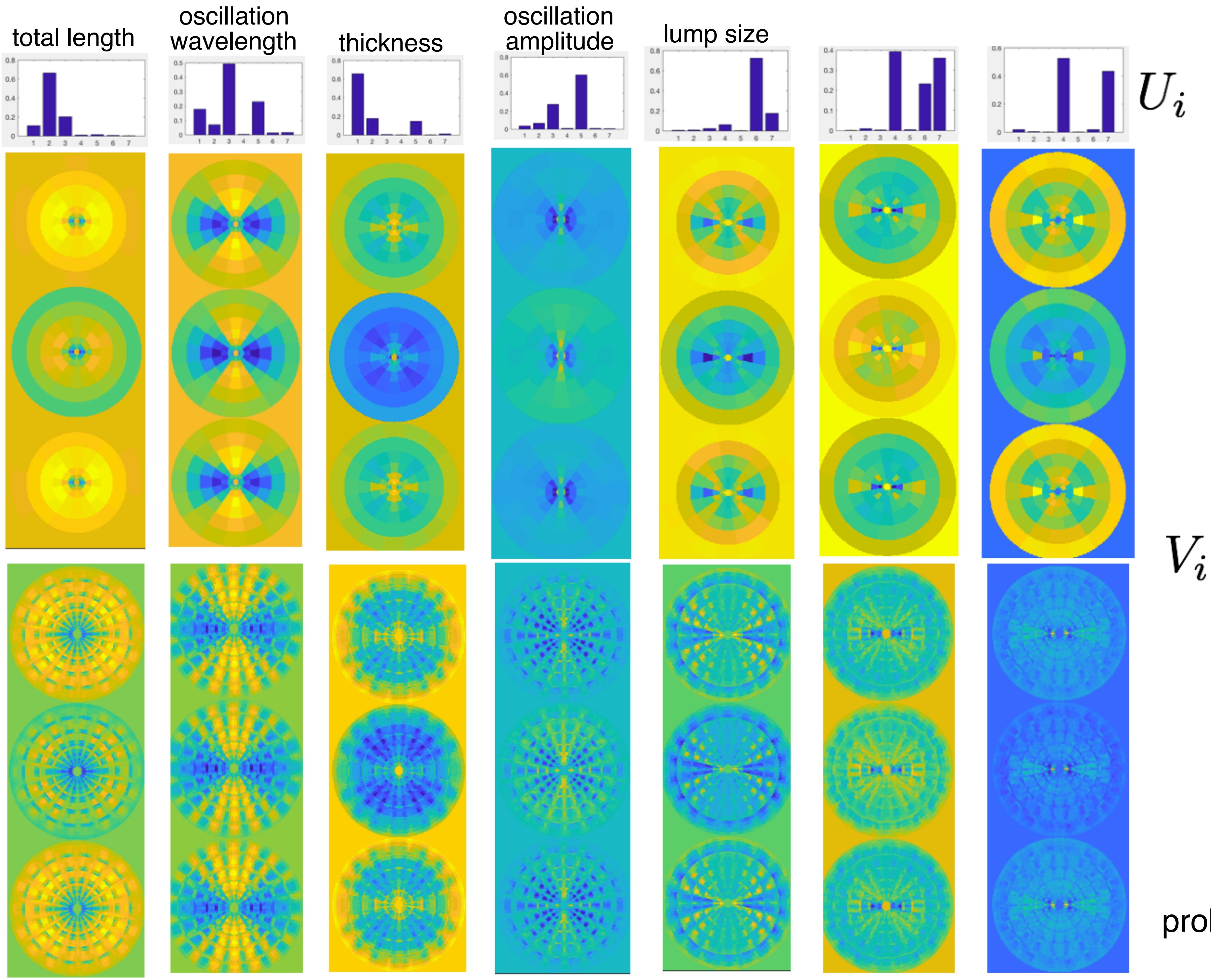


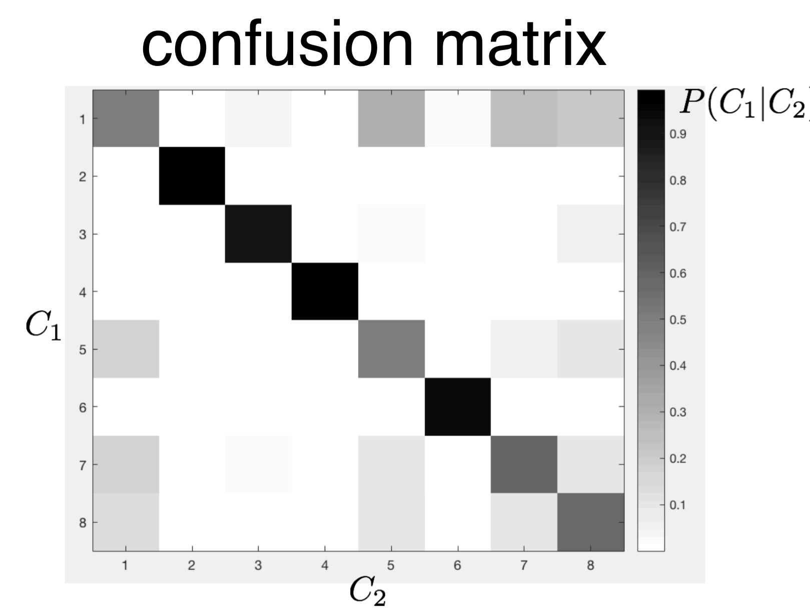
$$C(\bar{\theta}, SX) = \text{cross variance}$$

$$C \overset{\text{SVD}}{=} U \Sigma V_T$$

note: z-normalize theta

Linear regression & classification of synthetic stagnation



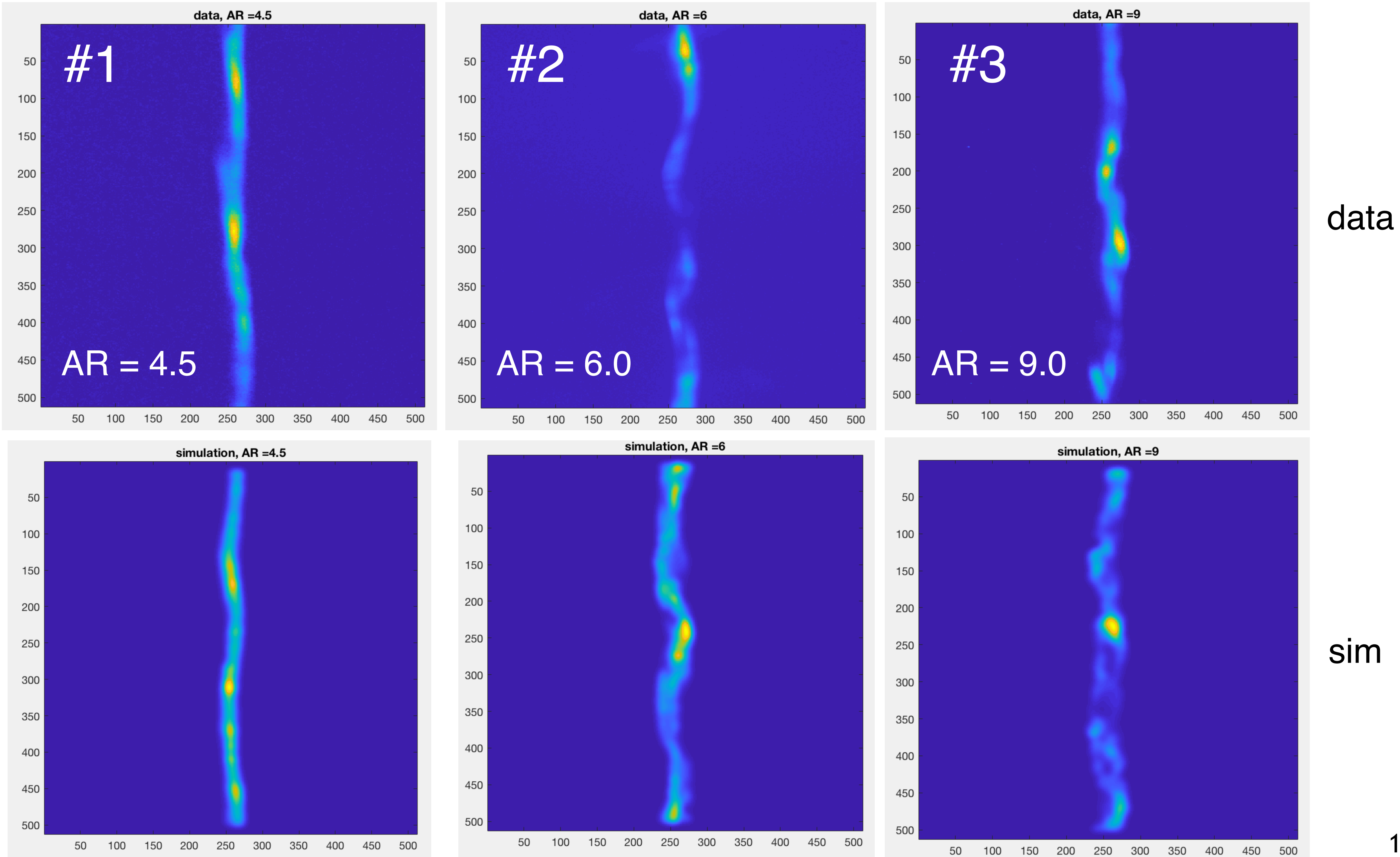


precision = 0.0016

separation = 24.9977

probability of incorrect classification = 0%

Comparison of Gorgon computer simulation to experimental data

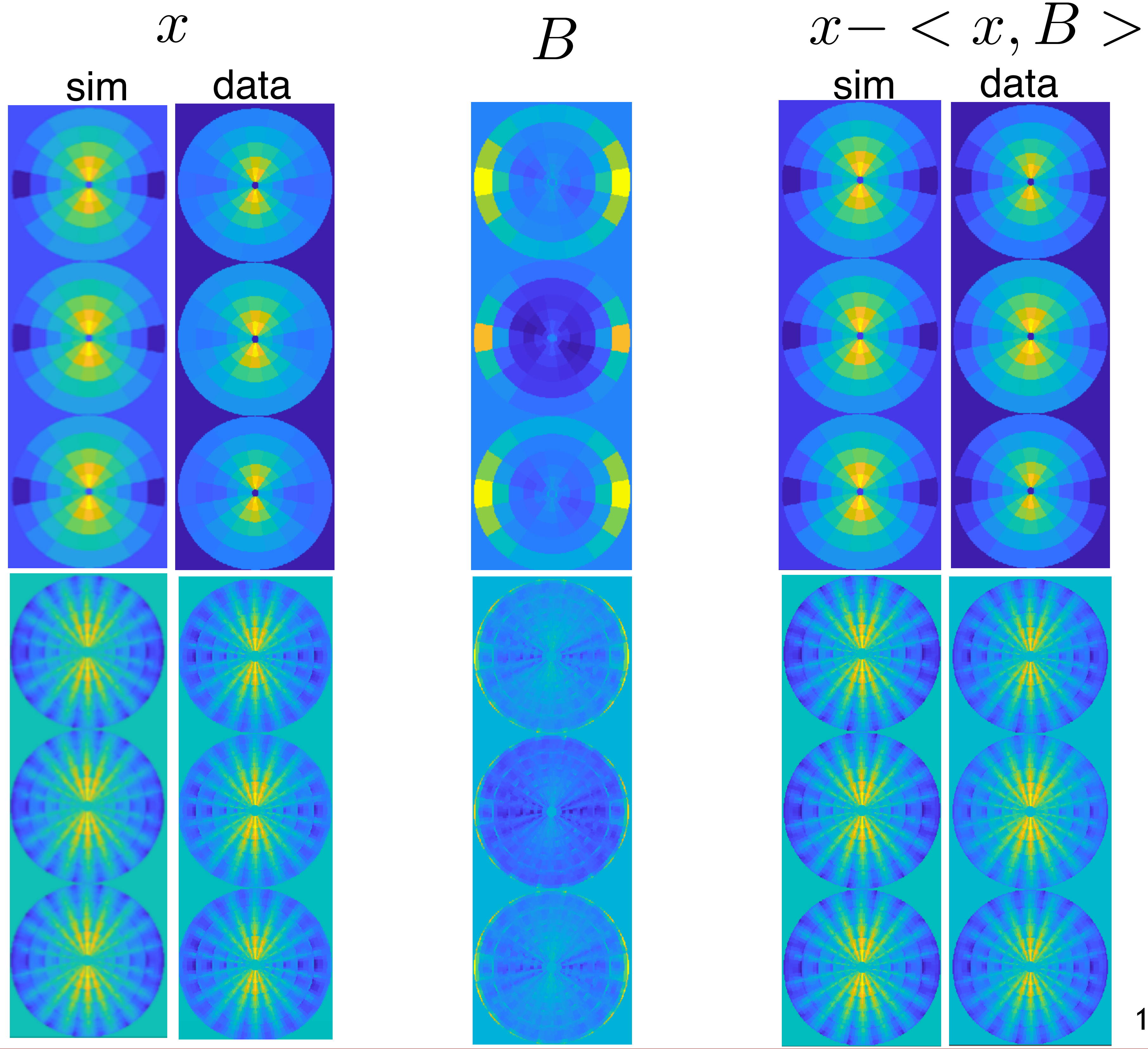


Significant distortion between simulation and experiment

solution: first principal component of simulation to data covariance projected out (effective background subtraction)

AR = 4.5

note: average interclass distance is about 10-20, while the average intraclass distance is about 2-5 (in the synthetic dataset)



d(sim,data) = 11.0

d(sim,data) = 4.2

Quantification of stagnation image morphology

- metric quantifies similarities between simulation and experiment
  - enables use of images in UQV
  - allows quantified statements to be made about morphology
- for example here we can state:
  - little difference between AR6 and AR9 data
  - AR6 simulation matches both AR6 and AR9 data
  - AR4 data significantly different and matches simulation well

```
separation_matrix =

    1.0000    3.7279    2.4980
    4.1202    1.0000    0.5997
    7.6733    1.7911    1.0000

confusion_matrix =

    0.5998    0.1914    0.1753
    0.2588    0.4696    0.4687
    0.1414    0.3389    0.3560
```