

Continuous and discontinuous IMEX-FEM for the multi-fluid plasma model

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Multi-fluid plasma modeling

- Multi-fluid plasma models simulate each species independently and couple the species together through collisional and electromagnetic operators.
 - Primarily designed for systems with fast time scales where electron inertial effects and charge separation are resolved.
 - Useful for modeling interactions between ion/neutral species as the closures do not rely on complex equations of state.
- **Main drawback:** model is computationally expensive due to the necessity of resolving fast plasma scales associated with electron dynamics and Maxwell's equations.
- **Research objective:** Explore the behavior of strongly-coupled, multiple time-scale systems, specifically when stepping over fast time scales using mixed **implicit-explicit** (IMEX) time integration. Topics include:
 - How to split the multi-fluid plasma model for IMEX integration?
 - What happens to stability/accuracy when stepping over the faster time scales?

Multi-fluid plasma scales

- Scaling is strongly dependent on a species α 's mass, density, and temperature.
- Can be broken into **frequency scales**, **velocity scales**, and **diffusion scales**:

Plasma frequency

$$\omega_{p\alpha} = \sqrt{\frac{q_\alpha^2 n_\alpha}{m_\alpha \epsilon_0}}$$

Cyclotron frequency

$$\omega_{c\alpha} = \frac{q_\alpha B}{m_\alpha}$$

Collision frequency

$$\nu_{\alpha\beta} \sim \frac{n_\beta}{\sqrt{m_\alpha} T_\alpha^{\frac{3}{2}}} \frac{1 + \frac{m_\alpha}{m_\beta}}{\left(1 + \frac{m_\alpha T_\beta}{m_\beta T_\alpha}\right)^{\frac{3}{2}}}$$

Flow velocity

$$u_\alpha$$

Speed of sound

$$v_{s\alpha} = \sqrt{\frac{\gamma P_\alpha}{\rho_\alpha}}$$

Speed of light

$$c \gg u_\alpha, v_{s\alpha}$$

Momentum diffusivity

$$\nu_\alpha = \frac{\mu_\alpha}{\rho_\alpha}$$

Thermal diffusivity

$$\kappa_\alpha \sim \frac{k_\alpha}{\rho_\alpha}$$

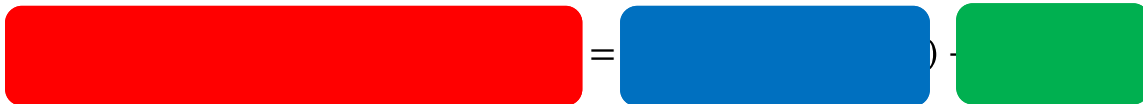
Multi-fluid plasma model

- Continuity equation:

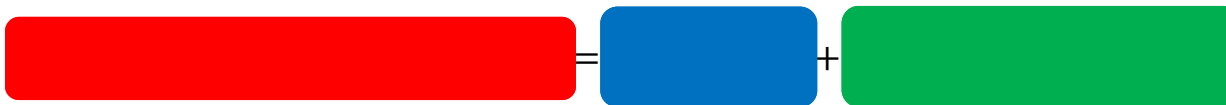


Each species α is represented by a separate density ρ , momentum $\rho \mathbf{u}$, and isotropic energy ϵ .

- Momentum equation:



- Energy equation:



- Ampere's Law:



Spatial operators are discretized using a finite element method.

- Faraday's Law:



Fluid
Electromagnetic
Inter-fluid

IMEX time integration

- IMEX gives a framework for splitting the model up into implicit and explicit terms:
 - Explicit for **slow**, non-stiff terms
 - Implicit** for **fast**, stiff terms

Implicit tableau

Explicit tableau

$$\begin{array}{c|c} c & A \\ \hline & b^t \end{array}$$

$$\begin{array}{c|c} \hat{c} & \hat{A} \\ \hline & \hat{b}^t \end{array}$$

$$\partial_t u = f(u, t) + g(u, t)$$

$$u^{(i)} = u^n + \Delta t \sum_{j=0}^{j<i} \hat{A}_{ij} f(u^{(j)}, t_n + \hat{c}_j \Delta t) + \Delta t \sum_{j=0}^{j \leq i} A_{ij} g(u^{(j)}, t_n + c_j \Delta t)$$

$$u^{n+1} = u^n + \Delta t \sum_{i=0}^{i<s} \hat{b}_i f(u^{(i)}, t_n + \hat{c}_i \Delta t) + \Delta t \sum_{i=0}^{i \leq s} b_i g(u^{(i)}, t_n + c_i \Delta t)$$

- Objective:** Combine the advantages of implicit and explicit solvers.
 - Take advantage of expensive implicit solver to overstep fast scales, and explicit solver to resolve slow scales.

Continuous Galerkin method

- Continuous finite element methods are derived from a weak form:

Weak form

$$\int_{\Omega} \phi \partial_t u \, dV + \int_{\Omega} \phi \nabla \cdot \mathbf{f} \, dV - \int_{\Omega} \phi s \, dV = 0$$

Apply divergence theorem with globally continuous test function ϕ_i

$$\int_{\Omega} \phi \partial_t u \, dV + \oint_{\partial\Omega} \phi_i \hat{\mathbf{n}} \cdot \mathbf{f} \, dS - \int_{\Omega} \mathbf{f} \cdot \nabla \phi_i \, dV - \int_{\Omega} \phi s \, dV = 0$$

Break global integral into elements $K \in \Omega$

$$\sum_K \left[\int_K \phi_i \partial_t u \, dV - \int_K \mathbf{f} \cdot \nabla \phi_i \, dV - \int_K \phi_i s \, dV \right] + \oint_{\partial\Omega} \phi_i \hat{\mathbf{n}} \cdot \mathbf{f} \, dS = 0$$

- Requires a global, nonlinear solve to find roots of residual.
 - Complex models like the multi-fluid plasma model require preconditioning schemes.

Compatible discretization for EM

- A physics compatible finite element discretization is used to enforce the divergence constraints for the electric and magnetic fields.
- Fluids are represented by an **HGrad** (node) basis $\rho \in V_{\nabla}$.
- The electric field is represented by an **HCurl** (edge) vector basis $\mathbf{E} \in V_{\nabla \times}$.
- The magnetic field is represented by an **HDiv** (face) vector basis $\mathbf{B} \in V_{\nabla \cdot}$.
- Compatibility is defined by the discrete preservation of the **De Rham Complex**:

$$\nabla \phi_{\nabla} \in V_{\nabla \times} \longrightarrow \nabla \times \phi_{\nabla \times} \in V_{\nabla \cdot} \longrightarrow \nabla \cdot \phi_{\nabla \cdot} \in V_{L_2}$$

- For Faraday's law, we choose a basis for the electric field such that its curl is fully represented by the basis used by the magnetic field.

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$

- Since the curl of the electric field is 'globally continuous' w.r.t. a divergence operator, the divergence of that curl is zero over the domain:

$$\nabla \cdot (\partial_t \mathbf{B} + \nabla \times \mathbf{E}) = \partial_t (\nabla \cdot \mathbf{B}) + \nabla \cdot \nabla \times \mathbf{E} = \partial_t (\nabla \cdot \mathbf{B}) + \sum_i E_i \cancel{\nabla \cdot \nabla} \times \phi_{\nabla \times}^i = \partial_t (\nabla \cdot \mathbf{B})$$

Result: The curl operator does not add divergence errors to the magnetic field

Satisfying Gauss' laws in plasmas

- **Goal:** Solve **plasma-coupled Maxwell's equations** and satisfy a **divergence constraint**:

$$\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} = -\frac{1}{\epsilon_0} \mathbf{j} \quad \partial_t \rho_c + \nabla \cdot \mathbf{j} = 0$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho_c$$

- In the **strong, non-discretized form**:

$$\nabla \cdot \left(\partial_t \mathbf{E} + \frac{1}{\epsilon_0} \mathbf{j} - c^2 \nabla \times \mathbf{B} \right) = \partial_t \nabla \cdot \mathbf{E} + \frac{1}{\epsilon_0} \nabla \cdot \mathbf{j} = \partial_t \left(\nabla \cdot \mathbf{E} - \frac{1}{\epsilon_0} \rho_c \right) = 0$$

- In the **weak form**: Choose a basis that supports the divergence constraint as HCurl does not support the divergence operation:

$$\begin{aligned} \int_{\Omega} \left(\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} + \frac{1}{\epsilon_0} \mathbf{j} \right) \cdot \nabla \phi_{\nabla} dV &= \int_{\Omega} \left(\partial_t \mathbf{E} \cdot \nabla \phi_{\nabla} + \frac{1}{\epsilon_0} \nabla \cdot \mathbf{j} \phi_{\nabla} \right) dV + c^2 \int_{\Omega} \mathbf{B} \cdot \nabla \times \nabla \phi_{\nabla} dV \\ &= \int_{\Omega} \partial_t \left(\mathbf{E} \cdot \nabla \phi_{\nabla} - \frac{1}{\epsilon_0} \rho_c \phi_{\nabla} \right) dV = 0 \end{aligned}$$

- Assumes that continuity equation is weakly satisfied:

$$\int_{\Omega} (\partial_t \rho_c - \nabla \cdot \mathbf{j}) \phi_{\nabla} dV = \int_{\Omega} (\partial_t \rho_c \phi_{\nabla} + \mathbf{j} \cdot \nabla \phi_{\nabla}) dV = 0 \rightarrow \int_{\Omega} \partial_t \rho_c \phi_{\nabla} dV = - \int_{\Omega} \mathbf{j} \cdot \nabla \phi_{\nabla} dV$$

IMEX splitting for CG

$$\partial_t \rho_\alpha + \mathbf{u}_\alpha \cdot \nabla \rho_\alpha = -\rho_\alpha \nabla \cdot \mathbf{u}_\alpha$$

$$u_\alpha < \frac{\Delta x}{\Delta t}$$

Each operator is associated with one or more plasma scales, which are grouped by color representing their approximate explicit stability limits.

$$\partial_t \mathbf{u}_\alpha + \mathbf{u}_\alpha \cdot \nabla \mathbf{u}_\alpha = -\mathbf{u}_\alpha \nabla \cdot \mathbf{u}_\alpha - \frac{1}{\rho_\alpha} \nabla P_\alpha + \frac{1}{\rho_\alpha} \nabla \cdot \left(\mu_\alpha \left(\nabla \mathbf{u}_\alpha + \nabla \mathbf{u}_\alpha^T - \frac{2}{3} I \nabla \cdot \mathbf{u}_\alpha \right) \right)$$

$$u_\alpha < \frac{\Delta x}{\Delta t} \quad v_{s\alpha} < \frac{\Delta x}{\Delta t} \quad v_\alpha < \frac{\Delta x^2}{\Delta t}$$

$$+ \frac{q_\alpha}{m_\alpha} \mathbf{E} + \frac{q_\alpha}{m_\alpha} \mathbf{u}_\alpha \times \mathbf{B} - \sum_\beta \nu_{\alpha\beta} (\mathbf{u}_\alpha - \mathbf{u}_\beta)$$

$$\omega_{p\alpha} \Delta t < 1 \quad \omega_{c\alpha} \Delta t < 1 \quad \nu_{\alpha\beta} \Delta t < 1$$

$$\partial_t P_\alpha + \mathbf{u}_\alpha \cdot \nabla P_\alpha = -\gamma P_\alpha \nabla \cdot \mathbf{u}_\alpha + \nabla \cdot \left((\gamma - 1) k_\alpha \nabla T_\alpha \right) - \sum_\beta \frac{(\gamma - 1) \nu_{\alpha\beta} \rho_\alpha}{m_\alpha + m_\beta} (3(T_\alpha - T_\beta) - m_\beta (\mathbf{u}_\alpha - \mathbf{u}_\beta)^2)$$

$$u_\alpha < \frac{\Delta x}{\Delta t} \quad \kappa_\alpha < \frac{\Delta x^2}{\Delta t} \quad \nu_{\alpha\beta} \Delta t < 1$$

$$\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} = -\frac{1}{\epsilon_0} \sum_\alpha \frac{q_\alpha}{m_\alpha} \rho_\alpha \mathbf{u}_\alpha$$

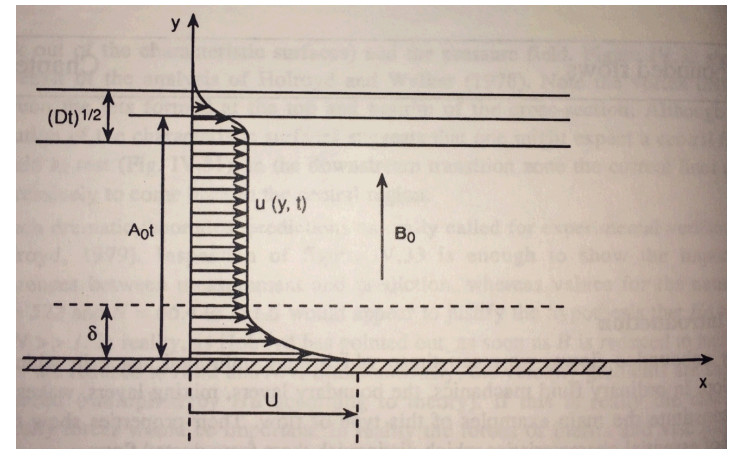
$$c < \frac{\Delta x}{\Delta t} \quad \omega_{p\alpha} \Delta t < 1$$

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$$

For IMEX-CG each operator can be moved between implicit and explicit evaluation depending on the explicit stability limits.

Resistive Alfvén wave problem

- Solution is derived from resistive/viscous MHD which **ignores Hall effects**:
 - Hall parameter $H = \frac{\omega_{ce}}{\nu_{ei}} = \frac{\eta B}{n_e e} \ll 1$.
 - Reducing Hall effects in magnetized multi-fluid model is tricky - requires large collision frequency.
- Problem used for verifying resistive, Lorentz force, and viscous operators:
 - Impulse shear due to a moving wall drives a **Hartmann layer**
 - Hartmann layer shear excites **Alfvén wave** traveling along magnetic field
 - Alfvén wave front diffuses due to momentum and magnetic diffusivity
 - Profile depends on the effective **Lundquist number** $S = \frac{L v_A}{\lambda}$



R. Moreau, Magnetohydrodynamics, 1990

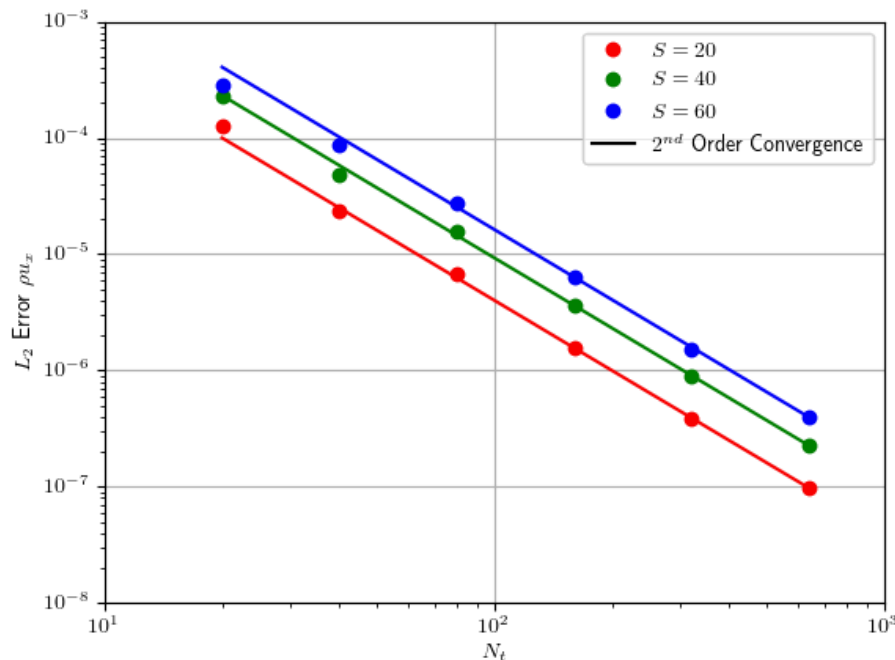
$$u_x = \frac{U}{4} \left(1 + \exp\left(\frac{v_A y}{\lambda}\right) \right) \operatorname{erfc}(\eta_+) + \frac{U}{4} \left(1 + \exp\left(-\frac{v_A y}{\lambda}\right) \right) \operatorname{erfc}(\eta_-)$$

$$B_x = \sqrt{\mu_0 \rho} \frac{U}{4} \left(1 - \exp\left(\frac{v_A y}{\lambda}\right) \right) \operatorname{erfc}(\eta_+) - \sqrt{\mu_0 \rho} \frac{U}{4} \left(1 - \exp\left(-\frac{v_A y}{\lambda}\right) \right) \operatorname{erfc}(\eta_-)$$

$$\eta_{\pm} = \frac{y \pm v_A t}{2\sqrt{\lambda t}}$$

Overstepping resistive plasmas

- Convergence tests show expected convergence even when massively overstepping non-MHD plasma scales.
- Realistic mass ratio $m_i = 1836 m_e$.



Plasma Scales for $S = 60$		
	Electrons	Ions
$\omega_p \Delta t$	$4 \cdot 10^7 - 1.3 \cdot 10^9$	$9.4 \cdot 10^5 - 3 \cdot 10^7$
$\omega_c \Delta t$	$1.7 \cdot 10^6 - 5.5 \cdot 10^7$	$9.4 \cdot 10^2 - 3 \cdot 10^4$
$v_{\alpha\beta} \Delta t$	$1.7 \cdot 10^{10} - 5.5 \cdot 10^{11}$	$9.4 \cdot 10^6 - 3 \cdot 10^8$
$v_s \frac{\Delta t}{\Delta x}$	$7 \cdot 10^{-3}$	$2 \cdot 10^{-4}$
$u \frac{\Delta t}{\Delta x}$	$2 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
$\frac{\mu}{\rho} \frac{\Delta t}{\Delta x^2}$	0.4 – 12	0.01 – 0.3
$c \frac{\Delta t}{\Delta x}$	167	

IMEX terms: **implicit/explicit**

Overstepping fast time scales is both stable and accurate. The inclusion of a resistive operator adds dissipation to the electron dynamics on top of the L-stable time integrator.

Ideal two-fluid vortex

- Extension of the ideal MHD vortex (*Balsara 2004*) where a current column (Z-pinch) is advected diagonally in a plane.
- Problem balances a pressure gradient with a combination of centrifugal and magnetic forces:

$$u_x^i = u_x^e = u_0 - u_1 \bar{y} \exp\left(\frac{1}{2}(1 - r^2)\right)$$

$$u_y^i = u_y^e = u_0 + u_1 \bar{x} \exp\left(\frac{1}{2}(1 - r^2)\right)$$

$$u_z^e = -\frac{1}{1 + M} \frac{B_0}{L \mu_0 e n_0} (2 - r^2) \exp\left(\frac{1}{2}(1 - r^2)\right)$$

$$u_z^i = \frac{M}{1 + M} \frac{B_0}{L \mu_0 e n_0} (2 - r^2) \exp\left(\frac{1}{2}(1 - r^2)\right)$$

$$P_e = \frac{1}{2} P_0 + \frac{1}{1 + M} P_B + \frac{M}{1 + M} P_u$$

$$P_i = \frac{1}{2} P_0 + \frac{M}{1 + M} P_B + \frac{1}{1 + M} P_u$$

$$P_u = -\frac{1}{2} \left(m_e + \frac{m_i}{Z_i} \right) n_0 u_1^2 \exp(1 - r^2)$$

$$P_m = \frac{B_0^2}{2\mu_0} (1 - r^2) \exp(1 - r^2)$$

$$\rho_e = m_e n_0$$

$$\rho_i = \frac{m_i}{Z_i} n_0$$

$$M = Z_i \frac{m_e}{m_i}$$

$$r^2 = \bar{x}^2 + \bar{y}^2$$

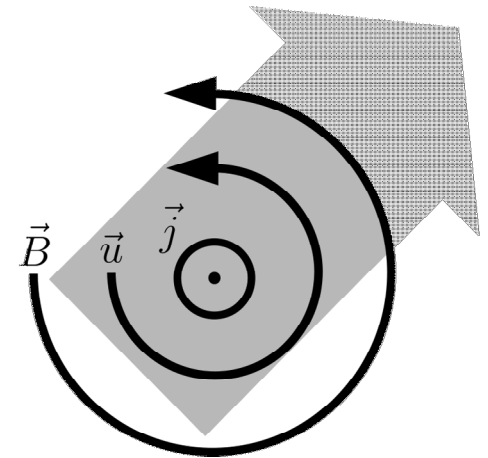
$$\bar{x} = \frac{x - u_0 t}{L}$$

$$\bar{y} = \frac{y - u_0 t}{L}$$

$$B_x = -B_0 \bar{y} \exp\left(\frac{1}{2}(1 - r^2)\right)$$

$$B_y = B_0 \bar{x} \exp\left(\frac{1}{2}(1 - r^2)\right)$$

$$E_z = -B_0 u_0 (\bar{x} + \bar{y}) \exp\left(\frac{1}{2}(1 - r^2)\right)$$

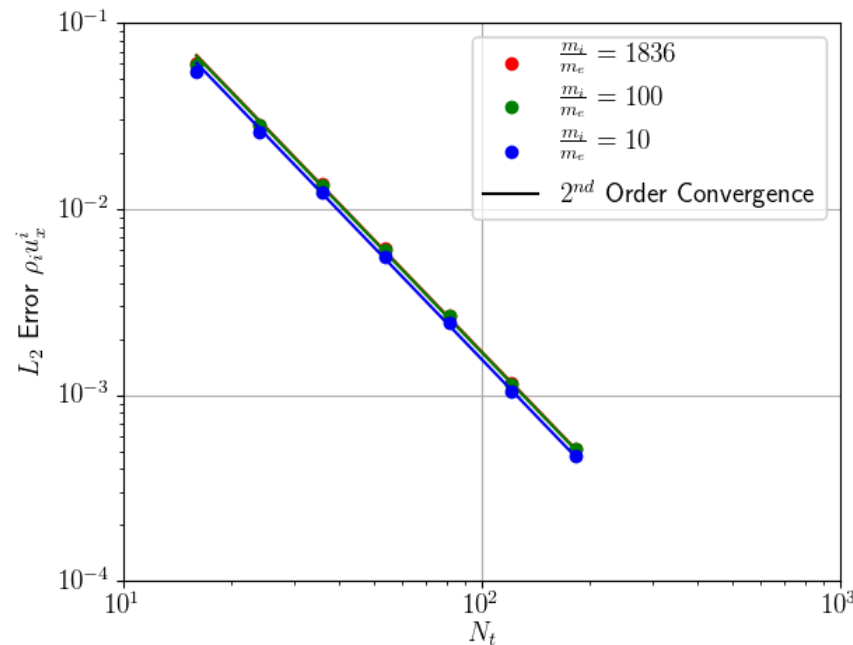


Overstepping dispersive plasmas

- Solution is independent of electron mass, however electrostatic/acoustic wave dispersion depends on mass ratio.
- Results show convergence for multiple mass ratios:

Plasma Scales for $m_i = 1836 m_e$		
	Electrons	Ions
$\omega_p \Delta t$	23 – 270	0.55 – 6.3
$\omega_c \Delta t$	1 – 12	$5.5 \cdot 10^{-4} - 6.3 \cdot 10^{-3}$
$v_s \frac{\Delta t}{\Delta x}$	0.25	$5.7 \cdot 10^{-3}$
$u \frac{\Delta t}{\Delta x}$	$6 \cdot 10^{-3}$	$6 \cdot 10^{-3}$
$c \frac{\Delta t}{\Delta x}$	6.3	

IMEX terms: **implicit/explicit**



Fast time scale plasma behaviors make it difficult to run this problem over MHD time scales – requires advanced preconditioning techniques – however, model is shown to converge properly, even when stepping over plasma and cyclotron frequencies.

Discontinuous Galerkin method

- Discontinuous Galerkin FEM does not assume a globally continuous test function:

Weak form

$$\int_{\Omega} \phi \partial_t u \, dV + \int_{\Omega} \phi \nabla \cdot \mathbf{f} \, dV - \int_{\Omega} \phi s \, dV = 0$$

Break into elements $K \in \Omega$ with discontinuous element test function ϕ_i^K

$$\sum_K \left[\int_K \phi_i^K \partial_t u \, dV + \int_K \phi_i^K \nabla \cdot \mathbf{f} \, dV - \int_K \phi_i^K s \, dV \right] = 0$$

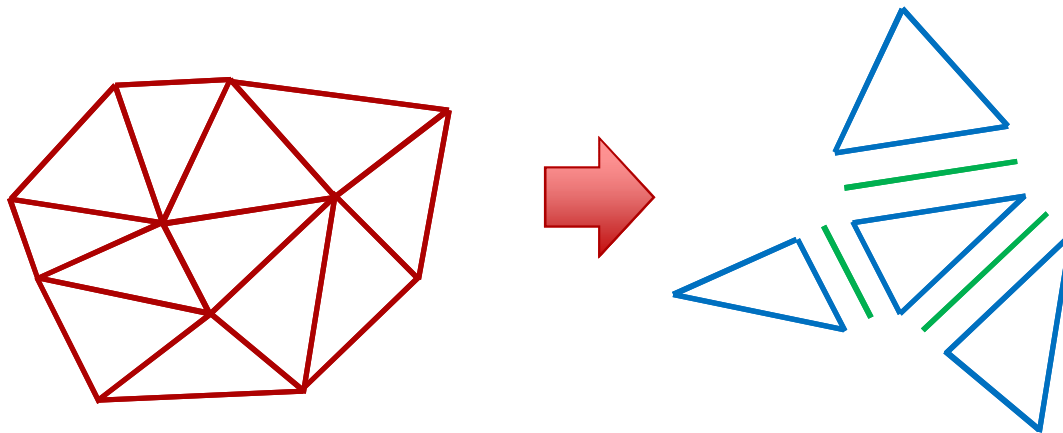
Apply divergence theorem to flux integral

$$\int_K \phi_i^K \partial_t u \, dV + \oint_{\partial K} \phi_i^K \hat{\mathbf{n}} \cdot \mathbf{f} \, dS - \int_K \mathbf{f} \cdot \nabla \phi_i^K \, dV - \int_K \phi_i^K s \, dV = 0$$

- Consistency:** Fluxes must be single valued on interfaces between elements.
 - Numerical Flux:** Solution to Riemann problem to generate consistent flux on interfaces.

Discretization has a local stencil

- DG breaks discretization into “local” interior and “non-local” interface operations.



- Interior operations: Internal fluxes, source terms
- Interface operations: Numerical fluxes
- Explicit DG can take advantage of this parallelization as the flux communication stencil is nearest neighbor -> Fast to compute, but limited by explicit CFL.
- Implicit DG requires large Jacobian stencil which couples all DOF between nearest neighbors -> Slow to compute.
- While the interface solve couples non-local cell information, this inter-cell coupling is limited to fluxes. **Source terms are cell-local.**

Localization speed-up for IMEX-DG

- For multi-fluid plasma model this allows implicit source time integration:
 - Allows overstepping of plasma, collision, reaction, and cyclotron frequencies.
 - Must resolve speed of light and plasma wave speeds.
 - Limits application to large scale space plasmas and atmospherics.
 - Implicit stage does not required dataset synchronization.
 - Advantageous for strong scaling.
 - Fast assembly, no preconditioner needed.
 - Fast dense solve using LAPACK.

Implicit block diagonal sparse solve

$$\begin{bmatrix} J_0 & & & \\ \text{---} & & & \\ & J_1 & & \\ \text{---} & & & \\ & & J_2 & \\ \text{---} & & & \\ & & & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ \text{---} \\ x_1 \\ \text{---} \\ x_2 \\ \text{---} \\ \vdots \end{bmatrix} = \begin{bmatrix} b_0 \\ \text{---} \\ b_1 \\ \text{---} \\ b_2 \\ \text{---} \\ \vdots \end{bmatrix}$$



Local dense solves

$$\begin{aligned} x_0 &= J_0^{-1} \cdot b_0 \\ x_1 &= J_1^{-1} \cdot b_1 \\ x_2 &= J_2^{-1} \cdot b_2 \\ &\vdots \end{aligned}$$

DG stabilization and IMEX

- Splitting the flux terms from the source terms allows us to treat stability separately for explicit/implicit terms.
- **Solution limiting:** Apply extrema bounds to solution to prevent oscillations.
 - **Pros:**
 - Works well for neutral fluid models.
 - Fast and easy to implement.
 - **Cons:**
 - Can cause issues with IMEX for strong implicit electrostatics when a divergence error cleaning scheme is applied.
 - Tends to have stability issues with high-order discretizations.
- **Artificial viscosity:** Apply a localized diffusion in oscillating regions.
 - **Pros:**
 - Stabilizes high-order discretizations.
 - Works well with cleaning schemes for Maxwell's equations.
 - **Cons:**
 - Expensive to calculate diffusion operators in DG.
 - Can impose a harsh time-step stability constraint depending on level of diffusion.
 - Difficult to identify where to apply diffusion.

Two-fluid electrostatic shock

- **Goal:** Test shock capturing techniques for DG in the presence of strong implicit source terms.
- The electrostatic shock is based on the Sod-shock problem for Euler fluids.
 - Electrons and ions are electrostatically coupled (no collisions).
 - Stiffness of electrostatic terms is governed by the plasma frequency.
 - Plasma frequency is controlled through permittivity ϵ_0 .
 - Converges to neutral/MHD shock in the limit of large plasma frequency and mass ratio.

Plasma parameters

$$m_i = 1$$

$$m_e = 10^{-3}$$

$$q_i = 1$$

$$q_e = -1$$

$$\gamma = 5/3$$

Initial conditions

$$n_L = 1$$

$$n_R = 0.125$$

$$u_L^e = u_L^i = 0$$

$$u_R^e = u_R^i = 0$$

$$P_L^e = P_L^i = 0.5$$

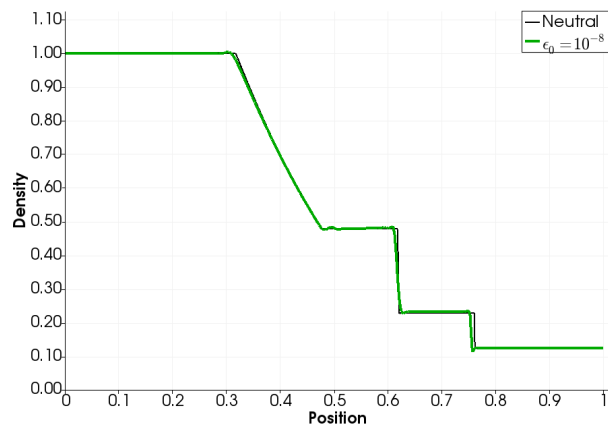
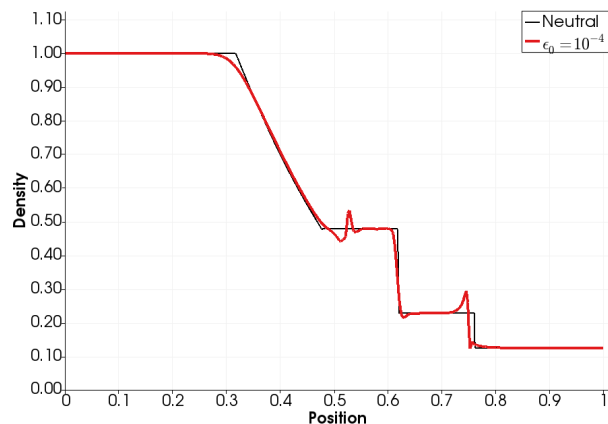
$$P_R^e = P_R^i = 0.05$$

$$E_L = B_L = 0$$

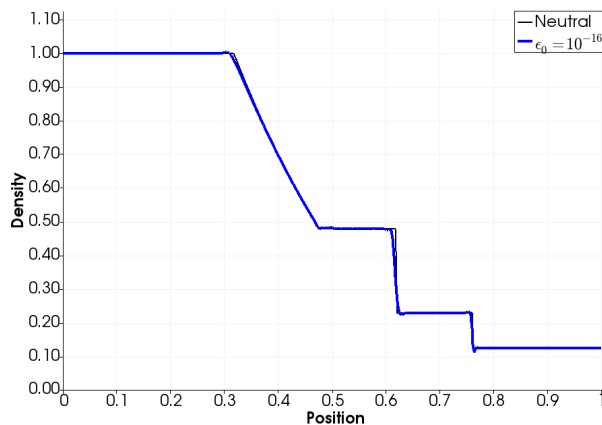
$$E_R = B_R = 0$$

Overstepping electrostatics

Multiple plasma frequencies are tested with the two-fluid plasma model at $t = 2$ with 250 cells with a 1st order basis against against a neutral shock solution.



As expected, the two-fluid solution shows agreement with the standard shock tube with increasing plasma frequency.

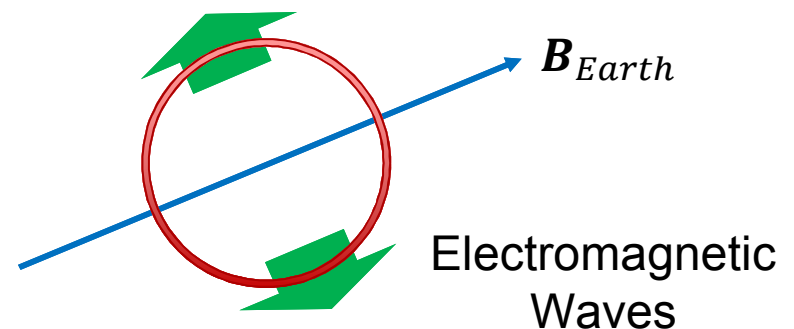
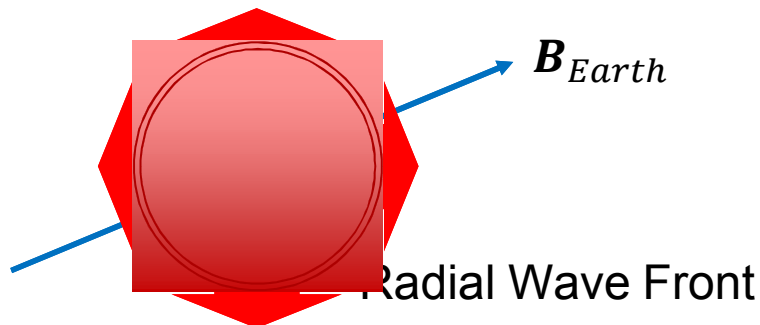


Artificial viscosity keeps the shock stable, while IMEX allows overstepping of plasma frequency.

Plasma Scales			
		Electrons	Ions
$\epsilon_0 = 10^{-4}$	$\omega_p \Delta t$	0.095	0.003
$\epsilon_0 = 10^{-8}$	$\omega_p \Delta t$	9.5	0.3
$\epsilon_0 = 10^{-16}$	$\omega_p \Delta t$	95000	3000
	$v_s \frac{\Delta t}{\Delta x}$	0.22	0.007

Geomagnetic disturbance

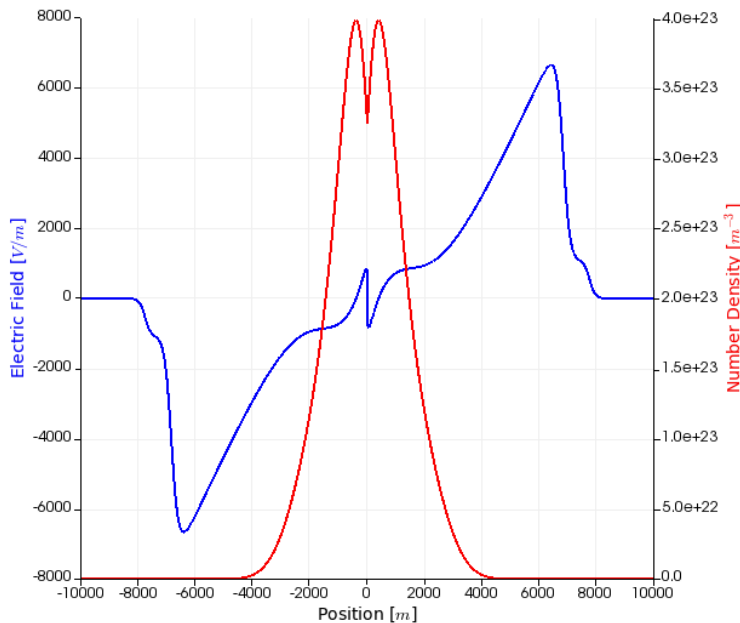
- **Goal:** Test the stability of the IMEX-DG multi-fluid model for multi-dimensional electromagnetics in the presence of steep density gradients.
- **Problem:**
 - Non-physical energy/density wave in atmosphere due to expanding source.
 - Initial density at nominal atmospheric density $n \approx 10^{11} [m^{-3}]$.
 - Source increases density by 12 orders of magnitude in cylindrical wave.
 - Current drive from source drives velocities to $u_e \approx 10^6 [m/s]$.
 - Thermal plasma response arises from $T \approx 100 [keV]$ source.
- Electrostatic interaction with background magnetic field generates electromagnetic waves propagating orthogonal to imposed field.



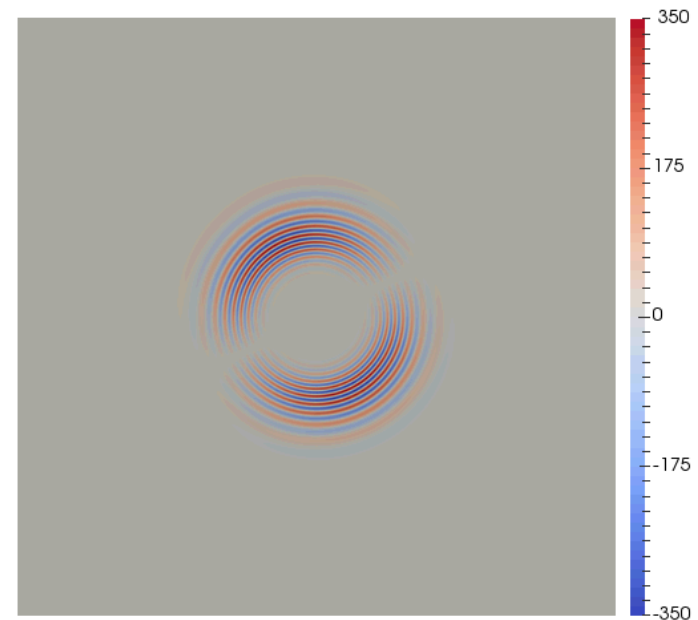
High-altitude response

- Test is run with a two-fluid model with realistic electron molecular-nitrogen plasma ($m_{N_2} = 51408 m_e$).

Electrostatic response at $t = 10 [\mu s]$



Induced current at $t = 0.5 [ns]$ $j_z [Am^{-2}]$



- Plasma frequency is initially resolved, but increases to $\omega_{pe} \Delta t \sim 10^6$.
- DG solution remains stable, and is able to resolve short time scale electromagnetics and long time scale electrostatics consistent with expectations.

Summary

- Discussed continuous and discontinuous FEM applications using IMEX time integration with a focus on **asymptotic preservation** of multi-fluid to MHD.
 - CG allows for stepping over explicit stability limits for flux and diffusion operators.
 - DG allows for stepping over source term stability limits with small impact on runtimes.
- Showed results for multi-fluid plasma model when resolving two-fluid behaviors while stepping over faster, less important physics.
 - CG: Showed convergence of slow dynamics to analytic solution for electrostatic and electromagnetic problems.
 - DG: Showed stability in the presence of stiff electrostatics for smooth and discontinuous problems.
- **Future work:**
 - Blending structure preserving Maxwell's equations with and DG fluid models to combine physics compatible discretizations for divergence constraints with fast nonlinear solves associated with stabilized DG.
 - Blending continuous and discontinuous discretizations for fluid models to step over electron acoustic and convective time scales.