SAND2017-11130C

MARGIN ASSESSMENT USING ENERGY QUANTITIES

88th Shock and Vibration Symposium

October 16-19, 2017

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Albuquerque, NM

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Topics

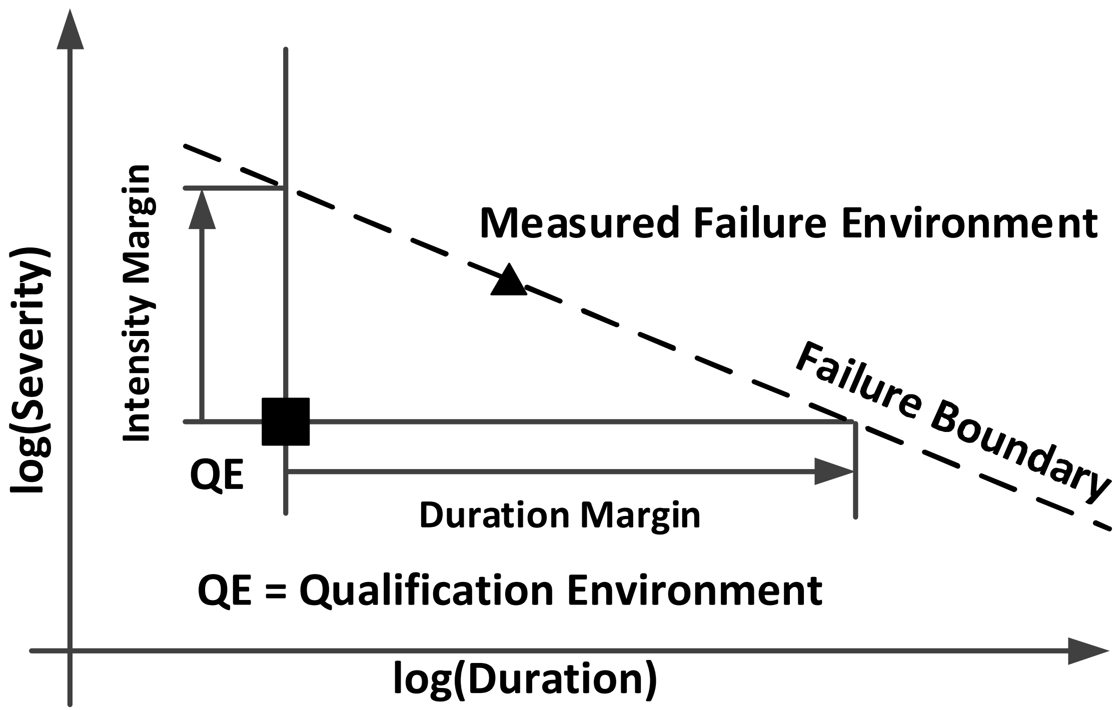
- Introduction to Margin Assessment
- Overview of Energy Quantities
  - Input Energy = Dissipated Energy
- Fatigue Damage and Energy
  - Cantilever Beam Tests
- Proposed Fatigue Damage Indicator
- Numerical Example
- Conclusions

Margin Assessment Introduction

- Systems are often tested to assess their structural integrity
  - Destructive and Evaluation Testing
- Margin assessments provide information about the robustness of a design above qualification environments
- If the qualification environments change in the future, the margin assessment data can be used to determine whether the design needs to be requalified
- If a production unit is exposed to an unintended vibration or shock, the margin assessment data can be used to determine if the unit has sufficient life to be fielded

Quantities of Interest for Margin Assessment

- Margins must be defined quantitatively
- The quantities of interest (QoI) must relate the severity of mechanical vibration to structural capacity
- QoI characteristics
  - Scalar quantity
  - Properly represent failure criteria
  - Capture localized failures
  - Consistent with QoIs used during design



Quantities of Interest for Margin Assessment

- Vibration (Fatigue)
  - Power Spectral Density (PSD)
  - Fatigue damage spectra
  - Sine spectra
  - Input power spectra
  - Miner's Rule
- Shock (Overstress)
  - Shock response spectra (SRS)
  - Pseudo velocity spectra
  - Absorbed energy spectra
- Spectra are not scalar quantities
- A scalar QoI can be obtained from energy spectra with minimal approximations

Characterize the effectiveness of energy-based methods for quantifying margins for vibration environments

Energy Spectra

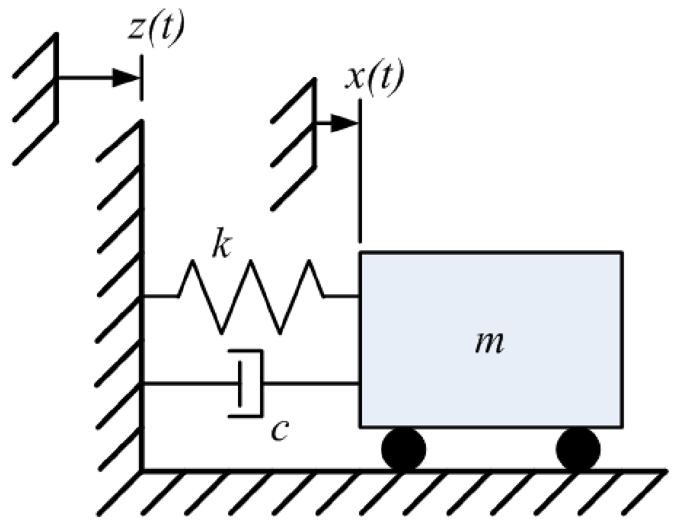
- SDOF oscillator equation of motion

$$m\ddot{x}(t) + c\big(\dot{x}(t) - \dot{z}(t)\big) + \ k(x(t) - z(t)) = 0$$

- Relative displacement equation of motion

$$w(t) = x(t) - z(t)$$

$$\ddot{w}(t) + 2\zeta\omega_n\dot{w}(t) + \ \omega_n^2 w(t) = -\ddot{z}(t)$$



- Output quantities

$$E_K{}^{R} = \frac{1}{2}\dot{w}^2(t)$$

$$E_D = 2\zeta\omega_n \int_0^{t_f} \dot{w}^2(t)dt$$

$$E_A = \frac{1}{2}\omega_n^2 w^2(t)$$

$$E_I = -\int_0^{t_f} \ddot{y}(t)\dot{w}(t)dt$$

Energy Response Spectra

- Energy Balance Equation

$$\frac{1}{2}\dot{w}^2(t)\Big|_0^{t_f} + 2\zeta\omega_n\int_0^{t_f}\dot{w}^2(t)dt + \frac{1}{2}\omega_n^2 w^2(t)\Big|_0^{t_f} = -\int_0^{t_f}\ddot{y}(t)\dot{w}(t)dt$$

Relative Kinetic Energy
E_K^R

Viscously Dissipated Energy
E_D

Absorbed or Potential Energy
E_A

Input Energy
E_I

- Total Input Energy = Total Dissipated Energy
- The integrals mean the input energy increases with multiple environments
  - Unlike the SRS

Input Power Spectra

- Vibration environments are defined in terms of base acceleration spectral density and exposure duration
- We compute specific input power spectra using Parseval's generalized theorem

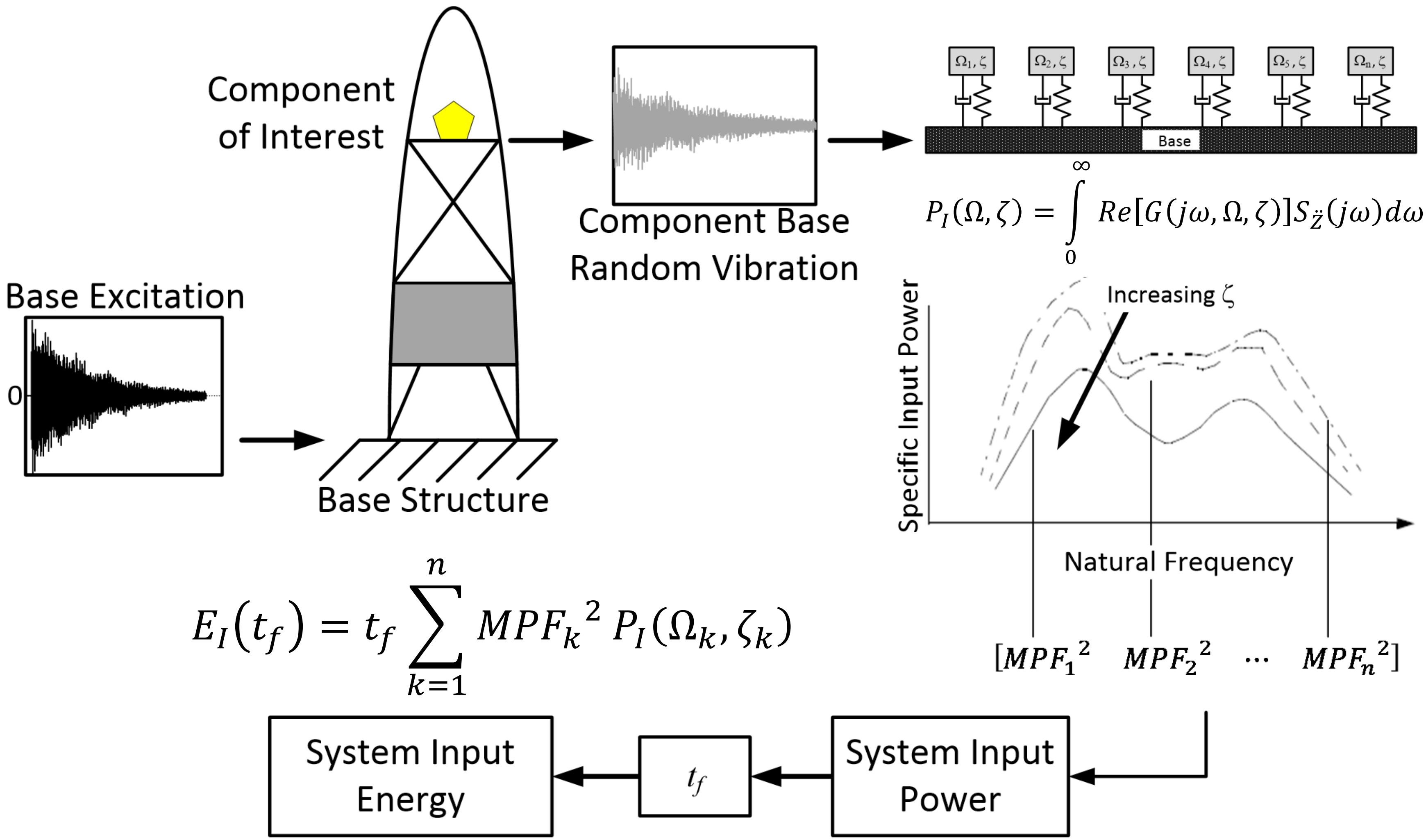
$$\hat{E}_I(\Omega,\zeta) = t_f P_I(\Omega,\zeta)$$

$$P_I(\Omega,\zeta) = \int_0^\infty Re[G(j\omega,\Omega,\zeta)] S_{\ddot{Z}}(j\omega) d\omega$$

$$P_I(\Omega,\zeta) \cong \frac{R_I}{2\zeta\Omega} S_{\ddot{Z}}(j\Omega)$$

$$G(j\omega,\Omega,\zeta) = \frac{-j\omega}{\Omega^2 - \omega^2 + j2\zeta\Omega\omega}$$

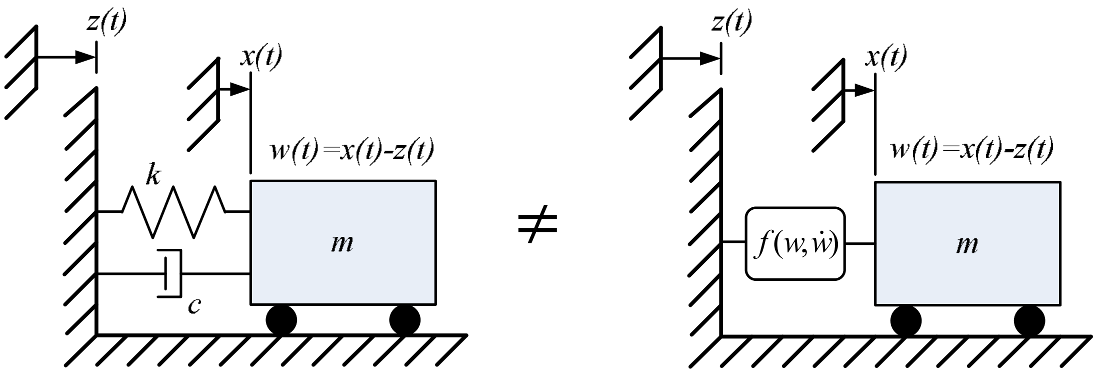
Input Energy



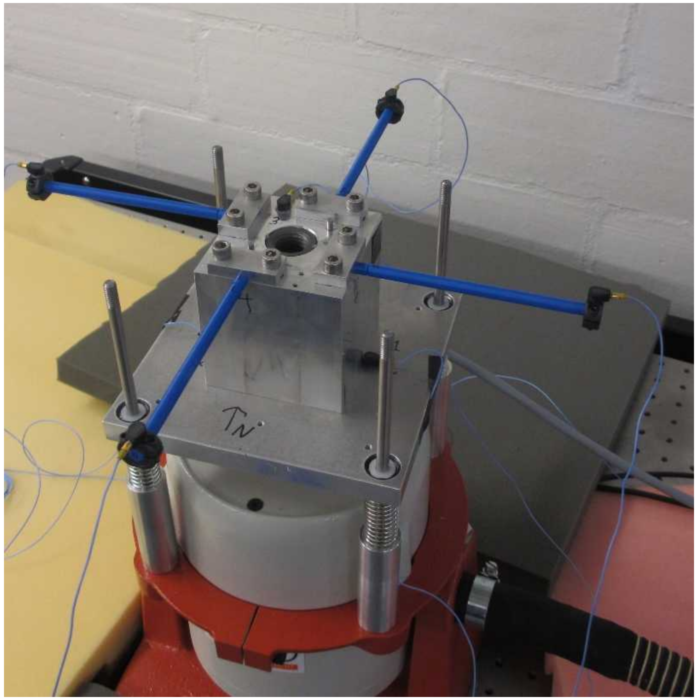
Fatigue Damage and Input Energy

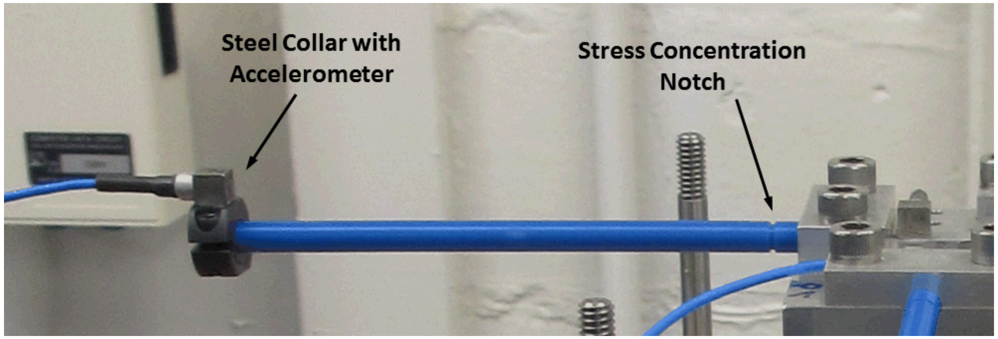
- Input energy has no knowledge of stress or cycle count
- Total Input Energy = Total Dissipated Energy
  - Unfortunately the energy is dissipated in a shock absorber and not by a damage inducing mechanism
- Tests have shown that viscous dissipated energy is not representative of fatigue damage mechanisms

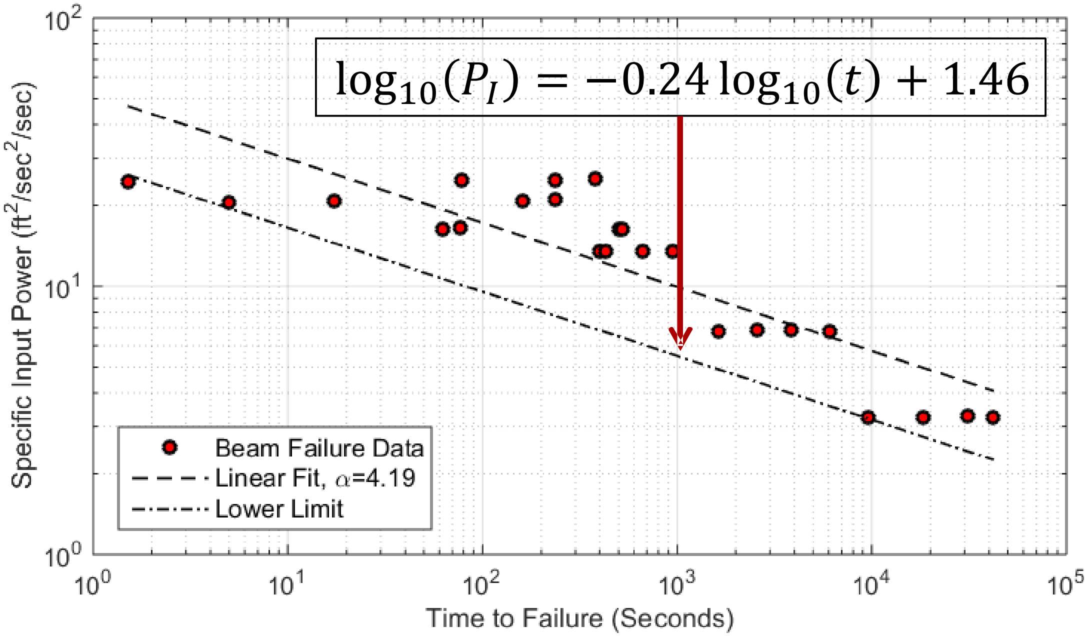
$E_I = t_f(P_I)$ $\neq$ Fatigue Damage



Fatigue Damage and Input Energy







- The line is a failure boundary
  - Like an S-N curve
- Lower limit line

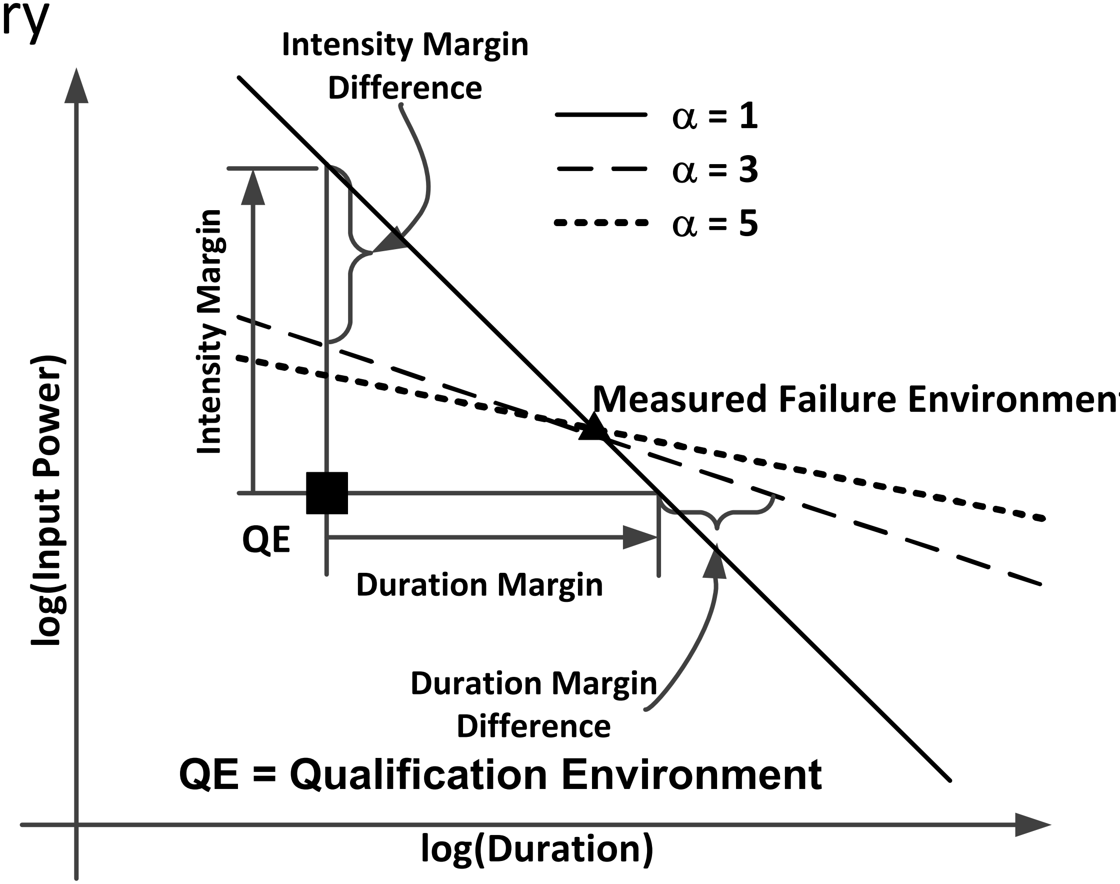
$$1.27x10^6 = tP_I^{\ 4.19}$$

Fatigue Damage, Energy, and Margin

- Define Fatigue Energy: $E_F = t_f (P_I)^{\alpha}$
- When $\alpha = 1 \quad E_F = E_I$
- On the failure boundary

  $E_F = Constant$

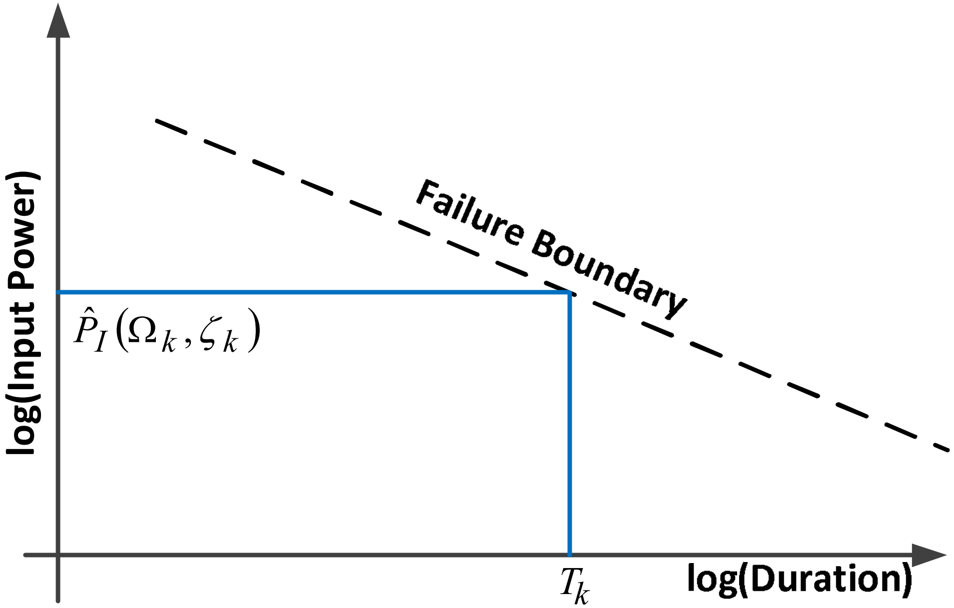
  $$C = (t_f)^{\frac{-1}{\alpha}} P_I$$



Fatigue Damage Indicator (FDI)

$$D_F = \sum_{k=1}^{n} \frac{t_f}{T_k} = \frac{t_f}{E_F} \sum_{k=1}^{n} \hat{P}_I(\Omega_k, \zeta_k)^{\alpha} \qquad \hat{P}_I(\Omega_k, \zeta_k) = MPF_k{}^2 P_I(\Omega_k, \zeta_k)$$

- Failure is predicted when $D_F = 1$
- This FDI is applicable to design
  - Need a failure boundary curve
    - Analogous to an SN curve



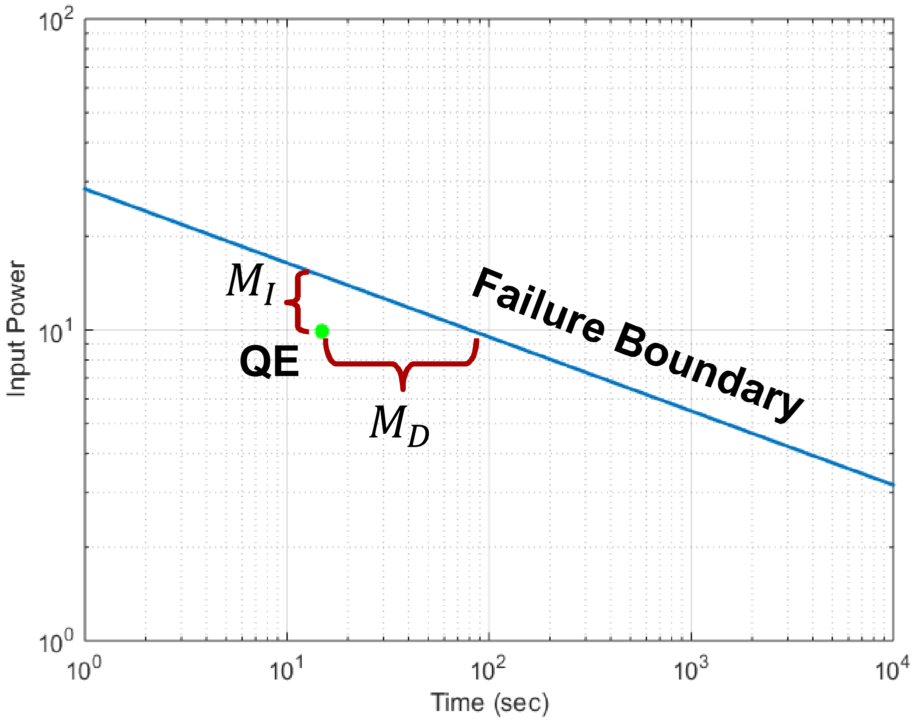
Fatigue Damage Indicator (FDI)

- This FDI is applicable to margin assessment
  - Duration Margin (Constant P_I)

$$M_D = \frac{(t_f)_B}{(t_f)_{QE}}$$

  - Intensity Margin (Constant t_f)

$$M_I = \frac{\left(\sum_{k=1}^{n} \hat{P}_I(\Omega_k, \zeta_k)^{-\alpha}\right)_B}{\left(\sum_{k=1}^{n} \hat{P}_I(\Omega_k, \zeta_k)^{-\alpha}\right)_{QE}}$$



FDI – PROs and CONs

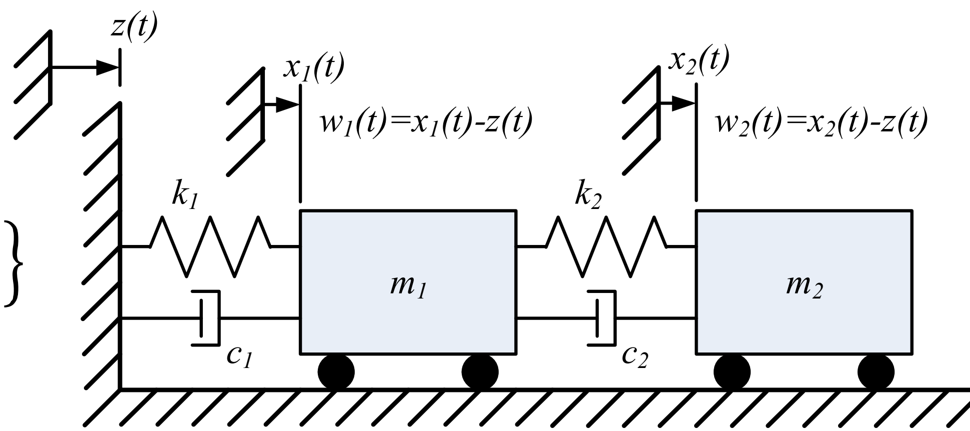
- PROs
  - Scalar quantity
  - Applicable for design
  - Applicable for comparing environments
  - Applicable to multiple environments
  - Applicable to environments with different spectral content
  - Applicable to multi-axial environments
  - Spiritually consistent with Miner's rule
- CONs
  - Non-standard
  - No experience base
  - Empirical

Example

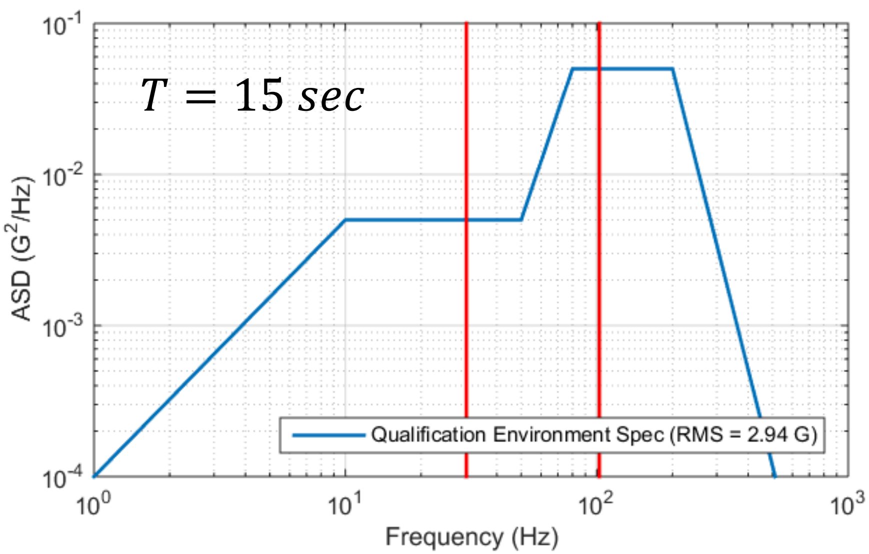
- 2-DOF linear system

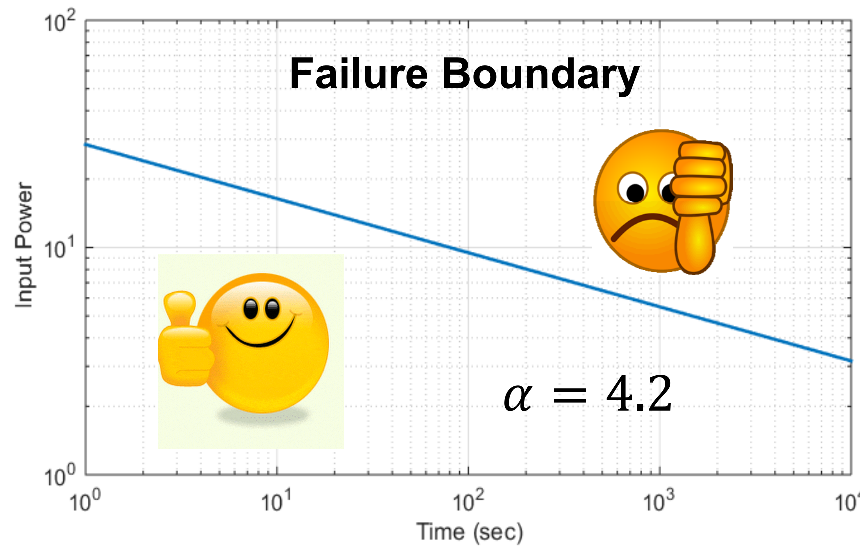
$$\Omega = \begin{Bmatrix} 30.3 \\ 102.4 \end{Bmatrix} Hz$$

$$MPF = \begin{Bmatrix} -1.58 \\ -0.71 \end{Bmatrix} \quad MEM = \begin{Bmatrix} 2.49 \\ 0.51 \end{Bmatrix}$$



- Qualification Environment





Example

- At the QE, we have adequate margins
  - Fatigue Damage Index

    $$D_F = 0.028$$

  - Duration Margin

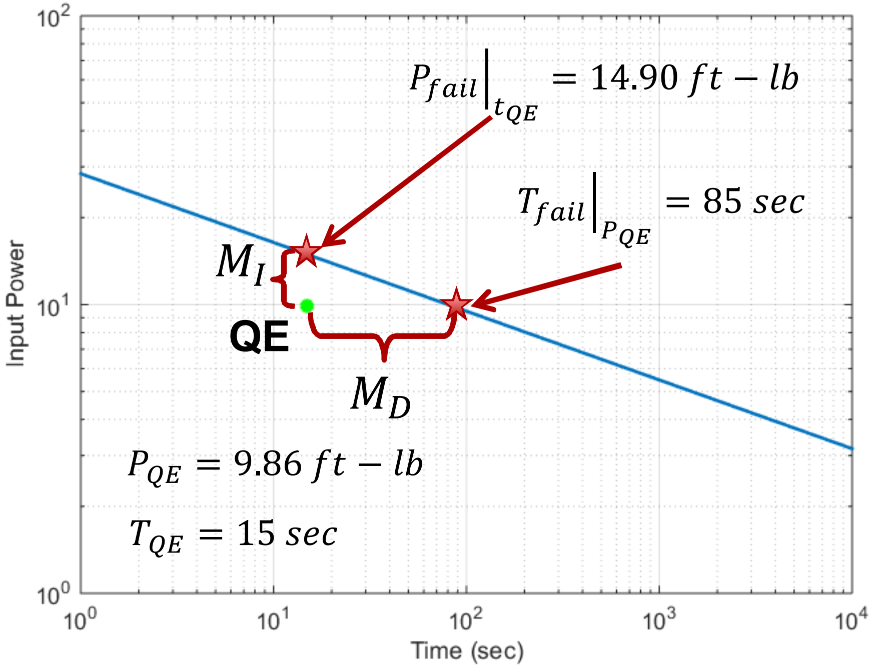
    $$M_D = \frac{T_{fail}\big|_{P_{QE}}}{t_{QE}}$$

    $$M_D = 5.67 = 15.1\ dB$$

  - Intensity Margin

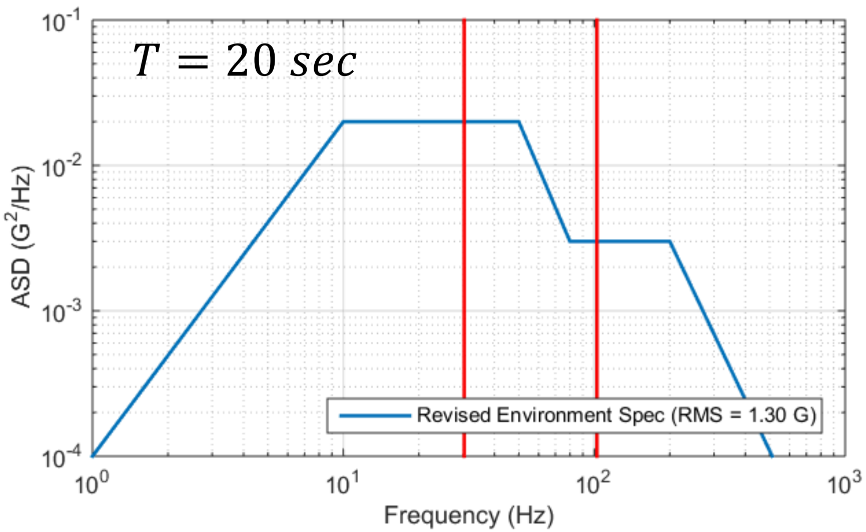
    $$M_I = \frac{P_{fail}\big|_{t_{QE}}}{P_{QE}}$$

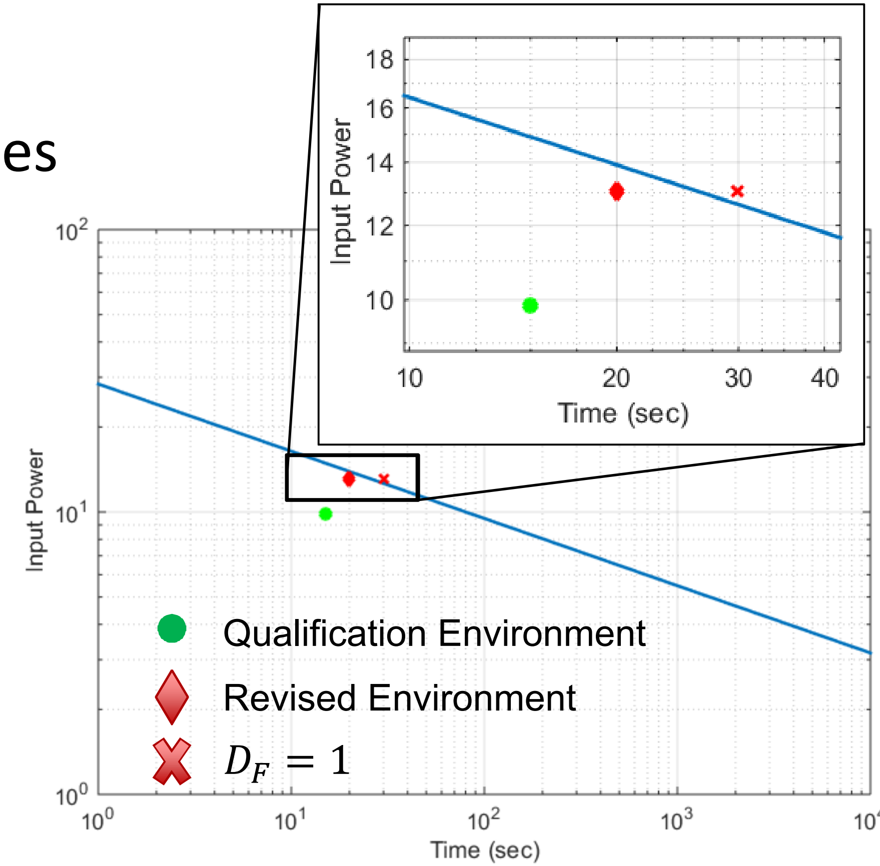
    $$M_I = 1.51 = 1.8\ dB$$



Example

- Assume the environment changes





- RMS is lower but the environment is more severe from a fatigue energy perspective
  - Combination of increased duration and spectral content

$$D_F = 0.67$$

$$D_F = 1\ @\ T = 29.8\ sec$$

$$M_D = 1.31 = 2.34\ dB$$

$$M_I = 1.14 = 0.57\ dB$$

Summary

- Attempted to characterize margin in terms of energy and power variables
  - Input energy and input power spectra can be easily computed
  - Cycle counting is not needed
- Input energy = Dissipated Energy but not Fatigue Damage
  - This is due to model form error
- Applied a correction factor and coined the term Fatigue Energy relating input power and exposure duration

$$E_F = t_f (P_I)^{\alpha}$$

- Suggested a Fatigue Damage Indicator to use to compare the severity of environments and compute margin
  - A work in progress….